RADIATIVE CORRECTIONS
TO SUPERSYMMETRIC HIGGS BOSON MASSES

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Abstract
We have computed radiative corrections to Higgs bosons masses in the Minimal Super­symmetric Standard Model. The one-loop corrected physical masses are properly obtained by finding the propagator poles, instead of approximating them with the second derivatives of the effective potential. The theoretical and numerical relation between the two approaches is discussed.

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It was recently discovered [1-4] that in the Minimal Supersymmetric extension of the Standard Model (MSSM) the Higgs boson masses are affected by large radiative corrections, which significantly change the tree level predictions. This fact has obviously important phenomenological implications, which have been discussed by other speakers at this Meeting [5]. Here we will rather concentrate on the following point: computations of the corrected masses were mainly [1, 2, 4, 6] performed using an approximate method, based on the effective potential. The question rises whether this is really a good approximation. We have recently computed [7, 8] the corrected Higgs masses following a different method, which essentially consists in identifying the corrected masses with the (real part of the) poles of the corrected propagators\(^1\), instead of approximating them with the second derivatives of the effective potential. At the same time we studied the theoretical relation between the two approaches, and we could also check the numerical accuracy of the effective potential method.

We briefly recall the situation at tree level. The MSSM [13] contains two Higgs doublets \(H_1, H_2\) (in total 8 real d.o.f.). The minimization of the tree potential

\[
V_0 = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2(H_1H_2 + h.c.) + \frac{g_1^2 + g_2^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g_2^2}{2}|H_1^\dagger H_2|^2
\]

leads to nonvanishing v.e.v.'s \(v_1, v_2\) for the neutral components of the Higgs fields, so that gauge symmetry is spontaneously broken. The (covariant) kinetic terms \(|D_\mu H_1|^2 + |D_\mu H_2|^2\) generate mass terms for the \(W\) and \(Z\) gauge bosons, with

\[
m_W^2 = \frac{1}{2} g^2 (v_1^2 + v_2^2) , \quad m_Z^2 = \frac{1}{2} (g^2 + g'^2) (v_1^2 + v_2^2) .
\]

To obtain the Higgs boson spectrum, one can insert in the tree potential the 'shifted' expressions of the Higgs fields

\[
H_1 = \left( v_1 + \frac{1}{\sqrt{2}} (S_i + iP_i) \right) , \quad H_2 = \left( v_2 + \frac{1}{\sqrt{2}} (S_i + iP_i) \right) .
\]

Then one obtains mass terms for the (neutral CP-even) \(S\), fields, for the (neutral CP-odd) \(P\), fields and for the (charged) \(H^\pm\) fields:

\[
\frac{1}{2} S_i \left( M_S^2 \right)_{ij} S_j , \quad \frac{1}{2} P_i \left( M_P^2 \right)_{ij} P_j , \quad H_i^\dagger \left( M_H^2 \right)_{ij} H_j^\pm .
\]

The mass eigenstates and eigenvalues in each of the three sectors are easily obtained by diagonalizing the three mass matrices \(M_S^2, M_P^2, M_H^2\), also taking into account the minimization conditions of the potential. In the neutral CP-odd sector one finds a massless (unphysical) Goldstone boson \(G\) and a physical field \(A\), with \(m_A^2 = m_1^2 + m_2^2\). The charged sector, too, contains a massless (unphysical) Goldstone boson \(G^\pm\) and a physical field \(H^\pm\), whereas the neutral CP-even sector contains two physical fields \(H\) and \(h\), with masses

\[
m_{H^\pm}^2 = m_W^2 + m_A^2 , \quad m_{H^\pm}^2 = \frac{1}{2} \left[ m_Z^2 + m_A^2 \pm \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2m_A^2 \cos^2 2\beta} \right] .
\]

In summary, the Higgs boson masses \(m_{H^\pm}, m_h, m_H\) can be expressed as functions of the other Higgs boson mass \(m_A\), of the gauge boson masses \((m_W^2, m_Z^2)\) and of the \(\beta\) parameter, defined by \(\tan \beta = v_2/v_1\). This can be done also at one loop level\(^2\).

\(^1\)For related work, see also refs. [3, 9-12].

\(^2\)Of course, further parameters coming from one-loop diagrams will appear.
The tree-level predictions for the Higgs boson masses are significantly modified by one-loop radiative corrections. For example, the one-loop computations show that the tree level inequalities \( m_h < m_Z, m_h < m_A, m_{H^\pm} > m_W \) can be violated. The dominant contributions to one-loop corrections come from top and stop loops, due to the large top Yukawa coupling \( h_t \). In particular, the largest corrections are proportional to the fourth power of \( m_t \).

We will now sketch a comparison between two different methods of computing the radiatively corrected masses (for a more complete discussion and for details, see [7, 8]). In the 'effective potential approach' the one-loop corrected masses are identified with the eigenvalues of the matrix of the second derivatives of the one-loop effective potential, at the minimum. [Actually, this is done for scalars (Higgs bosons): in the case of gauge bosons, one takes the tree-level expressions (2) for the masses, using however the one-loop corrected v.e.v.'s.] In this way one obtains only 'approximate' physical masses (which we will denote with a 'prime'). Instead, in the 'diagrammatic approach' the one-loop corrected masses are obtained, more properly, as poles of one-loop corrected propagators: these are the 'true' physical masses. Before describing schematically the difference between the two methods, I anticipate that in either approach one tries to obtain the \( H^\pm, H, h \) 'physical' (=one-loop corrected) masses in terms of the \( A, W, Z \) 'physical' masses, thus generalizing the tree level procedure. I also recall for completeness the expression of the one-loop effective potential, in the Landau gauge and \( \overline{\text{DR}} \) renormalization scheme:

\[
V^{\text{EFF}}(\phi) = V_0(\phi) + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4(\phi) \left( \log \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right).
\]  

The one-loop contribution to \( V^{\text{EFF}} \) contains the field-dependent generalized mass matrix \( \mathcal{M}^2(\phi) \), which in a sense summarizes the interactions of all the particles of the model with the Higgs fields (denoted collectively with \( \phi \)). The above \( V^{\text{EFF}} \) does not depend (at one-loop order) on the renormalization scale \( Q \), thanks to the implicit \( Q \)-dependence of fields and parameters. However, the derivatives of \( V^{\text{EFF}} \) are \( Q \)-dependent, just because the fields are.

We first discuss the charged Higgs case [9, 6, 8]. Very schematically, the computation in each of the two approaches can be divided into three steps (although in the actual computation steps 2 and 3 are performed simultaneously).

**Effective Potential Approach (E.P.A.)**

1. Minimization of \( V^{\text{EFF}} \): \[ \left[ \frac{\partial V^{\text{EFF}}}{\partial \phi_i} \right]_{\text{min}} = 0. \] This defines the one loop corrected \( v_1, v_2 \).

2. Computation of the matrix \[ \left[ \frac{\partial^2 V^{\text{EFF}}}{\partial \phi_i \partial \phi_j} \right]_{\text{min}}, \] interpreted as 'one-loop mass matrix' in the neutral CP-odd sector, and extraction of its nonzero eigenvalue, \((m_A^2)'\) [the zero eigenvalue corresponds to the Goldstone boson \( G \)].

3. Computation of the 'one-loop mass matrix' in the charged sector, \[ \left[ \frac{\partial^2 V^{\text{EFF}}}{\partial H^{\pm}_i \partial H^{\mp}_j} \right]_{\text{min}}, \] and extraction of its nonzero eigenvalue, \((m_{H^\pm}^2)'\) [the zero eigenvalue corresponds to the Goldstone boson \( G^\pm \)]. It turns out that the quantity \((m_{H^\pm}^2)'\) can be expressed in terms of \((m_A^2)'\) and \((m_W^2)'\), i.e. one obtains a relation among the (approximate) one-loop corrected masses of the form

\[
(m_{H^\pm}^2)' = (m_W^2)' + (m_A^2)' + \Delta',
\]  

\[(7)\]
where the correction $\Delta'$ is a function of the derivatives of the field dependent mass matrix $M^2(\phi)$. The explicit computation of $\Delta'$, including quark and squark one-loop contributions, was performed in ref.[6].

**Diagrammatic Approach (D.A.)**

1. ‘No-tadpole condition’: the tadpole counterterms are adjusted so to cancel tadpole diagrams. This insures that the shifted fields have zero one-point functions, i.e. that $\nu_1, \nu_2$ are the one-loop corrected v.e.v's. This step is then equivalent to the corresponding first step in the E.P.A.

2. Computation of the matrix $(G^P(p^2))_{ij}$, the one-loop corrected propagator in the neutral CP-odd sector, and extraction of its nonzero pole, $m^2_A$ [the zero pole corresponds to the Goldstone boson $G$].

3. Computation of the matrix $(G^\pm(p^2))_{ij}$, the one-loop corrected propagator in the charged sector, and extraction of its nonzero pole $m^2_{H^\pm}$ [the zero pole corresponds to the Goldstone boson $G^\pm$]. It turns out that the quantity $m^2_{H^\pm}$ can be expressed in terms of $m^2_A$ and $m^2_W$, where $m^2_W$ is the pole of the one-loop corrected $W$-propagator. Thus one obtains a relation among the one-loop corrected masses of the form

$$m^2_{H^\pm} = m^2_W + m^2_A + \Delta,$$  \hspace{1cm} (8)

where the correction $\Delta$ is the following combination of on-shell self-energies:

$$\Delta = \hat{\Pi}_W(m^2_W) + \hat{\Pi}_A(m^2_A) - \hat{\Pi}_{H^\pm}(m^2_{H^\pm})$$  \hspace{1cm} (9)

The explicit computation of $\Delta$, including quark and squark one-loop contributions, was performed in ref.[7]. Contributions from other sectors had been evaluated in ref.[9].

What is the relation between the two approaches? Working in the same renormalization scheme and exploiting the fact that the second derivatives of the effective potential at the minimum correspond to the inverse propagators at zero momentum, the following relations among ‘approximate’ and ‘true’ one-loop corrected masses can be obtained [7]:

$$(m^2_A) = m^2_A + \Delta \hat{\Pi}_A(m^2_A), \quad (m^2_{H^\pm}) = m^2_{H^\pm} + \Delta \hat{\Pi}_{H^\pm}(m^2_{H^\pm}), \quad (m^2_W) = m^2_W + \hat{\Pi}_W(m^2_W),$$  \hspace{1cm} (10)

where we define, for a generic self-energy $\hat{\Pi}(p^2)$, $\Delta \hat{\Pi}(p^2) \equiv \hat{\Pi}(p^2) - \hat{\Pi}(0)$. It is also possible to rewrite the E.P.A. correction $\Delta'$ in the D.A. language:

$$\Delta' = \hat{\Pi}_A(0) - \hat{\Pi}_{H^\pm}(0).$$  \hspace{1cm} (11)

Eq. (10), or the comparison of (9) and (11), show clearly that the approximation implicit in the E.P.A. corresponds essentially to neglecting the gauge boson self-energies and to evaluating the scalar self-energies at zero momentum instead than on-shell. We also add that only the masses obtained as propagator poles are scale and gauge independent.

The case of the neutral CP-even Higgs bosons is a bit more complicated, since mixing occurs between physical states. However, the first two steps (determination of the one-loop corrected v.e.v.'s and $A$-mass) are the same as before, both in the E.P.A. and in the

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3It is understood that self-energies are $\overline{DR}$-renormalized, and that only their real part is taken.
D.A.: we now sketch the final step. In the E.P.A., the one-loop corrected masses \( (m_H^2)' \) and \( (m_H^2)' \) are obtained as eigenvalues of the effective potential second derivative matrix in the neutral CP-even sector, which turns out to have the form:

\[
\left[ \frac{\partial^2 V^{EFF}}{\partial S_i \partial S_j} \right]_{mn} = M^2_S((m_A^2)' , (m_Z^2)' ) + \Delta' .
\]

(12)

It is the sum of a matrix with the same form as the tree level mass matrix \( M^2_S \), but with the (approximate) one-loop corrected A- and Z-masses replacing the tree level ones, plus a correction matrix, \( \Delta' \), computable in terms of the derivatives of the field dependent mass matrix \( M^2(\phi) \). The explicit computation of \( \Delta' \), including quark and squark one-loop contributions, was performed in the second of ref.\[2\].

In the D.A., after the first two steps mentioned earlier, the relevant quantity to be computed is the one-loop corrected propagator in the neutral CP-even sector. The inverse propagator turns out to have the form:

\[
\left( G^{-1}_S(p^2) \right)_{ij} = \delta_{ij} p^2 - \left( M^2_S(m_A^2 , m_Z^2 ) + \Delta' + \Delta(p^2) \right)_{ij} .
\]

(13)

The tree-level mass matrix now contains the one-loop corrected A- and Z-masses, obtained as propagator poles. The remaining terms contain Higgs and gauge boson self-energies, and we have separated the piece \( \Delta' \) which corresponds to the E.P.A. correction. Now, the one-loop corrected masses \( m_{h_1} \) and \( m_{h_2} \) are the poles of \( G_S(p^2) \). Then they are the two values of \( p^2 \) which make \( \det \left( G^{-1}_S(p^2) \right) \) vanish, i.e. the two solutions of

\[
\det \left[ p^2 - \left( M^2_S( m_A^2 , m_Z^2 ) + \Delta' + \Delta(p^2) \right) \right] = 0 .
\]

(14)

One can try to solve this equation perturbatively, and it seems safer to include in the unperturbed part not only \( M^2_S \), but also \( \Delta' \); indeed the E.P.A. computations have shown that the one-loop corrections contained in \( \Delta' \) can be large (even comparable with \( M^2_S \)). Proceeding this way, the first order solutions of eq.(14) can be written [8]:

\[
m_{h_1}^2 = \bar{m}^2_{h_1} + \cos^2(\beta + \alpha)\hat{\Pi}_Z(m_Z^2) + \sin^2(\beta - \alpha)\Delta \hat{\Pi}_A(m_A^2) - \Delta \hat{\Pi}_H(m_{h_1}^2) ,
\]

(15)

\[
m_{h_2}^2 = \bar{m}^2_{h_2} + \sin^2(\beta + \alpha)\hat{\Pi}_Z(m_Z^2) + \cos^2(\beta - \alpha)\Delta \hat{\Pi}_A(m_A^2) - \Delta \hat{\Pi}_H(m_{h_2}^2) ,
\]

(16)

where \( \bar{m}_{h_1}^2, \bar{m}_{h_2}^2 \) and \( \alpha \) are the eigenvalues and the mixing angle of \( M^2_S(m_A^2 , m_Z^2 ) + \Delta' \) (i.e. essentially the E.P.A. quantities). Again we see explicitly that the D.A. improves the E.P.A. computation in taking into account the gauge boson self-energies and the external momentum dependence of Higgs boson self-energies.

Explicit analytical formulæ for the radiatively corrected Higgs masses in the D.A., including top, bottom, stop and sbottom one-loop contributions, can be found in refs.\[7, 8\]. Here we just report some graphical results. As regards the squark sector, in each of the following examples we have assigned a common value \( m_{soft} \) to the 'soft' squark mass terms and a common value \( A \) to the parameters \( A_t, A_b \) appearing in the 'soft' trilinear terms. The \( \mu \) parameter is the coefficient of the \( H_1 H_2 \) term in the superpotential. Fig. 1 shows an example of the radiatively corrected \( m_H's \) as a function of \( m_t \), for four different \( \tan \beta \) values and for fixed values of \( m_A, m_{soft}, A, \mu \). The D.A. results are compared with the E.P.A. and the tree level results. One can notice that the corrections are generally small, also taking into account the fact that extreme values of \( \tan \beta \) are theoretically disfavoured.
Moreover, the E.P.A results are a good approximation to the full D.A. results. Fig. 2 shows an example of the radiatively corrected \( m_H \) and \( m_h \) as functions of \( m_A \), for fixed values of \( m_t, \tan \beta, m_{soft}, A, \mu \). Again, the D.A. results are compared with the E.P.A. results (both for 'D-terms' neglected and included) and with the tree-level prediction. Here one-loop corrections are larger: one can notice, e.g., that the tree level upper bound for \( m_h \) (\( m_h < m_Z \)) can be violated by more than 25 GeV. The E.P.A. results turn out to be approximatively correct, with errors of a few GeV. Finally, we show in Fig. 3 an example of qualitative disagreement between the two methods. The ratio of \( m_H \) to its tree level value is plotted for a higher range of \( m_A \) values than before, with the shown choice of the other parameters. The external momentum of Higgs boson self-energies (which is \( m_A \) or \( m_H \)) is here comparable with the masses of the particles circulating in the loops, and the 'cusps' in the D.A. results are due to the non-smooth behaviour of self-energies when the external momentum crosses a two-particle threshold (e.g., the first peak corresponds to the two-light-stop threshold). The E.P.A. results obviously don't feel such effects since the external momentum is always zero in that approach. However, the quantitative consequences of such discrepancies are rather limited, because the above phenomenon occurs in a region where the corrections to \( m_H \) themselves are not large (a few percent or less).

To summarize, we have schematically discussed the computation of the radiatively corrected Higgs boson masses in the D.A., also outlining a theoretical and numerical comparison with the E.P.A. calculation. In particular, the D.A. calculations confirm the fact that corrections to \( m_{H \pm} \) are small, typically not more than a few GeV, whereas larger corrections affect \( m_H \) and \( m_h \). The typical error of the E.P.A. can be quantified in a few GeV. In particular, if threshold effects are present, the E.P.A. certainly misses them. We have not mentioned radiative corrections to vertices involving Higgs bosons: some E.P.A. calculations [6, 14] have shown that they can be large, again due to a \( m_t \) dependence. Again one can wonder whether the E.P.A. misses some important effects: D.A. computations are in progress on this subject.

References


Fig. 1 - Results for the one-loop $m_{H^\pm}$ versus $m_t$ in the D.A. (solid) and E.P.A. (dashed), compared to the tree level prediction (dash-dotted), for $m_A = 40$ GeV, $m_{soft} = 500$ GeV, $A = \mu = 0$ and for different values of $\tan \beta$. 

[13] For reviews and references see, e.g.:
H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75;
Fig. 2 - Compared results for $m_h$ and $m_H$ as function of $m_A$, for the shown choices of the parameters $\tan \beta$, $m_t$, $m_{soft}$, $A$ and $\mu$. The E.P.A. results are drawn both in the case of D-terms neglected and included.

Fig. 3 - Example of threshold effects in the diagrammatic result for $m_H$. The 'cusps' correspond to the $t_1$-$t_2$, the $t$-$t$ and the $t_1$-$t_2$ thresholds.