Simulations Studies of Electron Lens Impact on Beam Parameters for Future Circular Colliders

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Introduction

This report summarizes the results of simulations about electron lenses for Future Circular Colliders, the impact on beam parameters is analyzed.

In general, an electron lens is characterized as a low energy electron beam which can interact with hadron beams. Electron guns are used in order to generate the electron beams. There are varied applications of e-lens such as compensation of beam-beam effects, provide Landau damping and collimation for halo particles. The impacts on the incident beam depend on the distribution of the electron lens.

In this report an incident proton beam was considered for all the analyses. The simulations started with a Gaussian distribution, the studies on this distribution include footprints generated for Landau damping and emittance increase per hour due to multiple displacements of the electron lens. The study of the emittance increase was extended to a cylindric hollow distribution, along with the effects of this e-lens on the incident beam distribution.

The cylindric hollow e-lens distribution does not contain any electric or magnetic fields inside the inner radius $R_{int}$ (used for the definition of this distribution in Eq. 1). However, it does have strongly nonlinear fields outside, and this effects can be observed from the kick amplitude in Fig. 1 b). Applying this e-lens distribution to the incident beam can increase the collimation efficiency.

Electron lens distributions

The cylindric hollow distribution used is defined as:

$$\rho(x, y) = \begin{cases} \frac{1}{\pi(R_{ext}^2 - R_{int}^2)} & \text{if } R_{int} < r < R_{ext} \\ 0 & \text{otherwise} \end{cases} \quad \text{with } r^2 = (x - x_0)^2 + (y - y_0)^2, \quad (1)$$

where $R_{ext}$ and $R_{int}$ delimits the electrons distribution. For all simulations, these values were in terms of the $\sigma$ size of the proton beam, which can be calculated as:

$$\sigma = \sqrt{\beta(s) \frac{\epsilon_n}{\gamma \beta}}, \quad (2)$$

where $\epsilon_n$ corresponds to the normalized emittance, $\beta(s)$ is the betatron function at the location of the e-lens, $\gamma$ and $\beta$ are the relativistic factors.

The Gaussian distribution is

$$\rho(r) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right) \quad \text{with } r^2 = (x - x_0)^2 + (y - y_0)^2. \quad (3)$$

These e-lens distributions are defined in Poisson Solver and they are used to compute the kicks given by the e-lens according to the number of electrons ($N_e$) for a desired current. The value of $N_e$ can be calculated as

$$N_e = \frac{IL}{e\beta_e c}, \quad (4)$$

where $I$ corresponds to the e-lens current, $\beta_e$ is the relativistic factor for electrons, $L$ the electron gun length, $e$ the electron charge and $c$ the speed of light.
**Electron lens kicks**

The kicks produced on the incident beam are highly dependent on the electron lens distribution and its current. These kicks can be computed numerically based on the initial distribution. The potential is calculated and the electric field is obtained.

All the kicks used in the simulations of this report were computed in the Poisson Solver, which applies the steps previously explained starting from a given discretized 2-D grid and the e-lens distribution.

The cylindric hollow distribution is the main candidate as a possible collimator. There is an analytical approximation for the angle kicks amplitude produced by a perfect centered cylindric hollow distribution. The mathematical approximation is defined as [1]

\[
\Theta = \Theta_{\text{max}} \begin{cases} 
0, & r < R_{\text{int}} \\
\frac{r-R_{\text{int}}}{R_{\text{max}}-R_{\text{min}}}, & R_{\text{int}} < r < R_{\text{ext}} \\
R_{\text{ext}}, & r > R_{\text{ext}}
\end{cases}
\]

with \( \Theta_{\text{max}} \) [urad] = 0.2\( \frac{L[m]I[A]}{(B\rho)} \left(\frac{1 + \beta_e}{\beta_e}\right) \), (5)

where \( \beta_e \) is the relativistic beta of the e-lens and \( B\rho \) is the magnetic rigidity, which in the FCC corresponds to 11 Tkm and 166.8 Tkm for 3.3 TeV and 50 TeV, respectively [7]. The kick amplitude can also be expressed in terms of distance instead of radians with \( \Delta x = \beta \Theta \), where \( \beta \) is the betatron at the IR. However, this relatively simple approximation was not used for the simulations, instead the kicks were obtained from the electron distribution as previously explained.

Figure 1 represents the cylindric hollow distribution and the produced kicks in terms of radial distance. As can be seen, there are no kicks produced below \( R_{\text{int}} \), so the effect of the e-lens in this zone is null. On the other hand, all particles located above \( 4\sigma \) get a radial kick, so they are spread to farther positions and the cumulative distribution decreases. However, this simple analysis only takes into consideration the shape of the kicks amplitude produced on the incident proton beam, but there could be effects on beam parameters, such as emittance which is explored later in this report.

![Fig. 1: a) Distribution amplitude of a cylindric hollow with \( R_{\text{int}} = 4\sigma \) and \( R_{\text{ext}} = 7\sigma \), b) Kick amplitude produced by the same distribution.](image1)

A Gaussian electron lens produces a different kick amplitude distribution, as can be seen in Fig. 2 b). This e-lens affects the particles of the incident proton beam at different positions. The slope of the kick amplitude distribution is an important parameter, because it is related with the maximum tune shift of the 0-amplitude particle. The Gaussian distribution is used as a suppressor of collective beam stabilities via a Landau damping mechanism, thanks to the detuning with amplitude that is
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generated in the incident proton beam.

Fig. 2: a) Distribution amplitude of a Gaussian e-lens distribution, b) kick amplitude produced by the Gaussian distribution.

In reality there is no perfect aligned electron lens. Some studies in this report were also made on misaligned Gaussian and cylindric hollow distributions, and the results are presented in the following sections. These displacements can have negative impacts on the incident proton beam parameters, such as the emittance increase.

Model for simulations

The tools used for the simulations were Poisson solver and COherent Multi Bunch Interaction (COMBI). In order to have different comparisons, the proton beam was simulated for cases with 3.3 TeV and 50 TeV. The table 1 shows the principal parameters used for the simulation implemented in COMBI.

<table>
<thead>
<tr>
<th>Collider</th>
<th>Circumference (m)</th>
<th>Energy (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC</td>
<td>97749.14</td>
<td>3.3</td>
</tr>
<tr>
<td>FCC</td>
<td>97749.14</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1

Distinct electron lenses were used for the studies. In table 2 the main parameters are shown, where the principal difference was the current of the e-lens.

<table>
<thead>
<tr>
<th>Energy (KeV)</th>
<th>Current (A)</th>
<th>Length (m)</th>
<th>Number of electrons (s⁻¹)</th>
<th>$R_{\text{int}}(\sigma)$</th>
<th>$R_{\text{ext}}(\sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.5</td>
<td>2.1</td>
<td>$1.18 \times 10^{11}$</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>2.1</td>
<td>$7.08 \times 10^{11}$</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>2.1</td>
<td>$1.18 \times 10^{12}$</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>2.1</td>
<td>$1.65 \times 10^{12}$</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 2

The maximum number of turns simulated for all cases was 20000. For the FFC, which has a revolution frequency of $f_{\text{FCC}} = 3066.957$ Hz, the simulations represent around 6.5 seconds of its operation.
Poisson Solver

The Poisson Solver was the principal tool used to obtain the kicks generated by the e-lens distribution. It uses integrated green functions to solve the Poisson equation, obtaining the electric potential. From this point, the electric field is calculated as the negative gradient of the electric potential.

\[
E = -\nabla V.
\] (6)

The kicks can be calculated once the electric field is known. The definition of the electron lens distribution is the first step taken into the Poisson Solver. Multiple distributions are available such as a gaussian, cylindric hollow, gaussian with a hole in the middle, parabolic, a lorentzian and a double cylindric hollow distribution.

COMBI: COherent Multi Bunch Interaction

The COherent Multi Bunch Interaction is a tool that helps to simulate the evolution of ion beams with a large range of adjustable and optional parameters, which are called actions. Some of the actions available are the application of a transverse field kick (e-lens), phase advance, synchrotron motion, noise sources. The use of an electron lens in COMBI should be called as an action code, which requires the \( \beta_e \) (relativistic beta), \( \beta_{\text{e-lens}} \) (\( \beta \) function at the location of the e-lens), an optional displacement that can be placed in both planes accordingly to a chosen frequency.

During the implementation of the electron lens in the simulations, some parts of the code about the randomness was found to be inconsistent. The application of the random displacement in the simulations was erroneous. After some verifications this bug was corrected, which improved the simulation code and its application.
Simulation results

Tune spread

An electron lens can help to suppress collective instabilities by Landau damping. The results of this application is visualized in a so-called tune footprint diagram. The tune shifts can be related to every particles at different amplitudes, and then to a general tune spread. It is possible to extract information from the COMBI results in order to plot a footprint. The diagram was obtained by tracking all particles from 0σ to 6σ and then applying a Fast Fourier Transform in order to get the oscillation frequencies of the particles.

The maximum tune shift given by a gaussian electron beam distribution can be calculated analytically for the 0-amplitude amplitude particle as:

$$\Delta Q_{x,y}\big|_{p_{-elens}} = \frac{N_e r_0 \beta_{elens}}{4\pi \gamma \sigma_e^2} (1 + \beta_e),$$  

(7)

where $N_e$ is the number of electrons, $r_0$ is the classical radius of the incident particle, $\beta_{elens}$ is the betatron function at the location of the electron lens, $\beta_e$ corresponds to the relativistic beta of the e-lens, and $\sigma_e$ is the beam size.

Different simulations for the FCC were run varying the currents of the gaussian e-lens distribution. The resultant footprints are shown in Fig. 3. The tunes were set at $Q_x = 0.31$ and $Q_y = 0.32$, and the footprint shows a significant tune spread.

For the 0-amplitude particle, which is located in the highest slope of the kick amplitude, the maximum tune spread in the x plane are $\Delta Q_x = 0.005505$, $\Delta Q_x = 0.007340$, $\Delta Q_x = 0.009175$ for 0.3 A, 0.4 A and 0.5 A, respectively. The simulations presented are very close to the analytical maximum tune spread expected from Eq. (7).

![Footprints for gaussian distributions and multiple currents.](image)

Fig. 3: Footprints for gaussian distributions and multiple currents.

The effect of increasing the current of the e-lens in the tune spread is evident, as the current gets higher the maximum tune spread also increases as shown in Fig. 3
Displacement noise

A misalignment of the electron lens will introduce undesirable noise in the incident proton beam, which leads to an emittance increase with consequent reduction of the luminosity reach of the collider. Some simulations with random displacements in the x-plane and/or y-plane were carried out, and the results on the emittance increase per hour are analyzed. The simulations presented correspond to the FCC with the injection energy of 3.3 TeV. The number of proton particles was $10^5$, and the random e-lens displacements were varied from 50 nm ($6 \times 10^{-4} \sigma_p$) to 1000 nm ($1.2 \times 10^{-2} \sigma_p$). These displacements were applied on each turn to the electron lens with a current of 1 A. In this first part, the results for a gaussian electron lens distribution are presented.

A displacement of 200 nm ($2.4 \times 10^{-3} \sigma_p$) introduces an emittance increase around 5%. However, for displacements of 400 nm ($4.8 \times 10^{-3}$), the emittance increase per hour in the y-plane is already around 20%, and at 1000 nm is around 80%. The emittance growth in the y-plane almost doubles the one in the x-plane.

The emittance increase leads to negative impacts on the collider performance (luminosity reach compromised). Fig. 5 shows the results for reverted tunes ($Q_x = 0.32$ and $Q_y = 0.31$). The same pattern can be observed as a function of the e-lens displacement.
The emittance increase in the x-plane almost doubles the growth of the other plane. Therefore, as the tunes were swapped, the emittance increase of the two planes is reversed.

**Displacement noise for a cylindric hollow electron lens**

The same study was carried out for a cylindric hollow distribution of $R_{\text{int}} = 4\sigma$ and $R_{\text{ext}} = 7\sigma$. There was no clear pattern for the emittance increase per hour from the simulations. However, as it can be seen in Fig. 6, the emittance increase for all the displacements applied was always relatively low. The maximum emittance increase per hour was around 4%.

The negative increase and the lack of a pattern can be produced by numerical noise. Different parameters may be implemented for future studies, such as increasing even more the displacements and the current for the e-lens.

![Fig. 6: Amplitude emittance growth for random displacements with $Q_x = 0.32$ and $Q_y = 0.31$ and a cylindric hollow distribution.](image-url)
Cylindric hollow collimation

This second part of the studies is focused on the effects of the cylindric hollow e-lens on the proton distribution. The currents used vary from 0.5 A to 7 A. The simulations were run in COMBI with a maximum of 20,000 turns for proton beams of energy of 3.3 TeV and 50 TeV. The e-lens for these simulations was perfectly aligned without any displacement.

Due to computational limits, all simulations based on an initial gaussian distribution for the incident particles did not have a clear effect on the tails. In order to better visualize the impact above $4\sigma$ particles, an initial uniform distribution defined from $-10\sigma$ to $10\sigma$ was used. Then, in order to make the comparison as a more realistic case, the initial and final tracked distributions were weighted with a gaussian distribution. An example of the initial distribution is shown in Fig 13 together with the weighted distribution using a gaussian function.

![Initial flat distribution](image1.png)

![Initial flat distribution weighted with gaussian function](image2.png)

Fig. 7: a) Initial flat distribution, b) initial flat distribution weighted with gaussian function.

Most of the simulations in this part were implemented for proton beams with 3.3 TeV, unless otherwise stated.

Tracked distributions

In this section some plots are shown from the original tracked distributions (without the gaussian weight function), so the effects of the e-lens can be seen directly on the distribution. The population of the tails depends on the current applied as seen in Fig. 8, the particle distribution above $10\sigma$ of 7 A is the highest for all turns. In 5000 turns, the electron lens with 0.5 A starts to populate almost at $15\sigma$ (the initial distribution is defined from $-10\sigma$ to $10\sigma$), whereas there are protons already located almost at $40\sigma$ for the e-lens of 7 A.

The particles do not get accumulated in one specific range, but they are kicked out farther away at each turn. This was expected as the amplitude kick beyond the electron lens still exists and it is proportional to $1/r$. In this way also the effects of an electron lens with high current on the proton distribution may be easier to visualize. In the last turn of the simulation, the impacts in the core for the 5 A and the 7 A e-lens are evident.
Fig. 8: Distributions of the \( x \)-plane for multiple currents and after a) 5000 turns, b) 10000 turns, c) 15000 turns and d) 20000.

Fig. 9 shows the effect of an electron lens with 3 A on the proton distribution, so the patterns seen in one dimension are extended into the beam profile.

Fig. 9: Effect of electron lens of 3 A on proton distribution after 20000 turns.
Tracked distributions with weight function

The comparisons made in this section takes the tracked distributions and weight them as explained earlier. The initial distribution can be labeled as $\Phi_0$ and the tracked ones as $\Phi_{x,y}$. In order to see the effects of the e-lens on the tracked proton distribution, the comparisons are made using the ratio $\Phi_{x,y}/\Phi_0$.

There are important effects seen in the tracked distributions, Fig.10 shows the behavior of the x-plane distribution under the influence of the electron lens with different currents and for multiple turns, and it is compared as defined before, with the initial distribution as a ratio.

At the first 5000 turns all currents present a similar effect on the distribution, the edges start to decrease. For 10000 turns the differences start to be visible, the edges for the case of 7 A are lower than the rest of the currents, the kicks amplitude are stronger so particles get kicked out more easily. The same behavior is still present for 15000 turns. In the case of 20000 turns the effect of the electron lens is more evident. For 0.5 A, 3 A and even 5 A, the distribution inside the e-lens remains almost constant and the drop of particles begins at $4\sigma$. However, in the 7 A the core of the distribution is being affected by the e-lens and there is a general decrease, even from 10000 turns this effect is visible.

Fig. 10: Distribution profile in the x-plane for 3.3 TeV incident particles with 0.5 A, 3 A, 5 A and 7 A e-lens currents. a) Shows the distribution after 5,000 turns, b) for 10,000 turns, c) 15,000 turns and d) 20,000 turns.
Fig. 11: Linear fits of x-plane for particle distribution of 0.5 A and 3 A with 3.3 TeV. The slope shows the diffusion rate per sigma.

A linear fit is applied to the edges of the distributions, and the slope gives an approximation of the diffusion rate per sigma. In Fig 11 the evolution of the particle distribution in the x-plane is plotted for the cases 0.5 A and 3 A from $-8\sigma$ to $-4\sigma$, and a linear fit is also shown. In all cases the slope from the 3 A case is higher, which contributes to a stronger diffusion rate per sigma. However, there is not a clear pattern of the evolution of these slopes for one case. The slopes in both cases increase until 15000 turns, but at 20000 turns they become smaller. As the turns increase, the 0.5 A e-lens case almost reaches the behaviour of the 3 A, so the difference of the diffusion rate of these two cases start to be less significant.

The effects on the vertical plane due to multiple electron lenses are shown Fig. 12, the currents of the e-lens applied were the same as in the x-plane. The results obtained show the same patterns, the effects of the e-lens start to be visible from 5000 turns and is more clear in 20000 turns, the 7 A case is also affecting the core of the distribution in the y-axis.
The same simulations were performed for a proton beam with an energy of 50 TeV. Fig. 14 and 13 show the results for the x-plane and y-plane, respectively. The evolution of beam distribution keeps the same shape as the 3.3 TeV, which was expected. However, the kicks computed from the Poisson solver for each case have different amplitude. The expected result was to have a lower diffusion rate for the 50 TeV case, as its kicks amplitude are much smaller, so there is not a clear reason for this behaviour. But this changes did not have an impact on the model used.

Fig. 13: a) Kicks on 3.3 TeV beam, b) kicks on 50 TeV beam.
Fig. 14: Distribution profile in the x-plane for 50 TeV incident particles with 0.5 A, 3 A, 5 A and 7A e-lens currents. a) Shows the distribution in 5,000 turns, b) for 10,000 turns, c) 15,000 turns and d) 20,000 turns.

Fig. 15: Distribution profile in the y-plane for 50 TeV incident particles with 0.5 A, 3 A, 5 A and 7A e-lens currents. a) Shows the distribution in 5,000 turns, b) for 10,000 turns, c) 15,000 turns and d) 20,000 turns.
A comparison for different turns within the same currents is shown in Fig. 16. The diffusion rate for both cases stays almost constant with the increase of the turns, and this can be seen from the slopes of the distributions beyond $4\sigma$, so for cases of large currents (3 A and 7 A) the rate at which the particles are kicked out is independent of the number of turns. For the case of 3 A the core of the distribution is unaffected. On the other hand, the evolution of the proton distribution when the 7 A electron lens is applied shows a different pattern. As the turns are increased there is a general drop of the distribution, so the electron lens is also affecting almost every particle, including the ones in the core.

The behaviour of the distributions when an electron lens with relatively high current is applied could be related to an emittance increase. The percentage of emittance increase are plotted for 0.5 A, 3 A and 7 A in Fig. 17, showing the values for the entire simulation. There is different behaviour for the three cases, with a relatively low current of 0.5 A the emittance increase is completely linear. After 20000 turns the increase is about 0.25%, which can be considered inside the working range.

For 3 A, the emittance increase is almost linear in both planes, but the rate is much higher than the 0.5 A case. At the end of the simulation, the emittance increase is already 250%, and this is not optimum for the collider performance. In the case of a cylindric hollow of 7 A, the emittance increase starts to behave linearly but around 11000 turns there is a fast exponential increase, so it starts to grow quickly.
The oscillation amplitudes for the three cases are plotted in Fig. 18. Around 10000 turns, when the emittance starts to grow in an exponential way, the oscillation amplitudes also increase abruptly. The impact of the electron lens of 7 A on the beam is really strong and the beam actually becomes unstable. On the other hand, for the cases of 0.5 A and 3 A the beams are still stable, but the emittance also increases for the 3 A case.

As can be visible in Fig. 19 where the frequency spectrum of the beam is plotted as a function of the number of turns, the dark red spot around 10000 turns shows a clear sign of instabilities with a consequent fast exponential growth of the emittance. Therefore, the core of the protons beam is affected accordingly to the behaviour observed in Fig 10, 11, 13, 14, 15 for a current of 7 A.
Particles kicked outside $6\sigma$

In a collider, the first physical collimator walls are located around $6\sigma$, so the principle of the cylindric hollow e-lens is to improve the collimation efficiency by increasing the diffusion rate to spread to particles out.

In this remain part, a comparison of the particles that are located outside $6\sigma$ for the tracked and initial distributions is performed. A new quantity is introduce, let the number of proton particles for a tracked distribution be $\phi_c$, where the subscript $c$ is labeled depending on the current used for the electron lens. The number of particles located outside $6\sigma$ for the initial distribution can be labeled in a similar way as $\phi_0$. The comparison of these quantities is carried out as a relative difference in the form

$$\%\text{Relative difference} = \left( \frac{\phi_c - \phi_0}{\phi_0} \right) \times 100\%.$$  \hspace{1cm} (8)

The results are plotted in Fig. 20, where the extreme case of 7 A presents the highest increase rate of relative error per turn, so this can suggests that the number of particles outside $6\sigma$ increases on each turn. On the other hand, electron lens of 0.5 A has almost no real effect on the number of particles outside $6\sigma$. The relative difference for 5000 turns is the highest value for this case, but some statistical noise can be involved. From 10000 to 20000 turns this values is almost constant. The cases for 3 A and 5 A exhibit an increase of the relative difference, and the 5 A growth represents a higher rate.
Conclusions

The first part of the report showed simulations with a Gaussian electron lens. The interaction with the e-lens has a significant impact on the tune spread of the proton beam. The maximum tune shift increases along with the current, and also decreases if the current is reduced. Simulation results agree with the expected values of the maximum tune shift.

A study about the noise introduced by a misalignment of the proton beam and the e-lens was also presented, and it was shown that it produces an increase of the emittance. A random displacement of 200 nm ($2.4 \times 10^{-3} \sigma_p$) as maximum on a Gaussian e-lens of 1 A keeps the emittance increase per hour below 5%. An asymmetric behaviour has been observed between the horizontal and vertical planes: the emittance increase in the y-plane almost doubles the x-plane. The difference of the tunes between H-V planes produces the asymmetry.

The simulations of the displacement noise for the cylindric hollow e-lens resulted to be affected by numerical noise. This is due to the small displacements applied, and only particles above $4\sigma$ undergo the electron lens force. As the current was relatively low, these particles do not contribute to any emittance increase as observed in the simulations.

The last part of the study was focused on the impact of a cylindric hollow distribution as a collimation system, the e-lens was defined with $R_{int} = 4\sigma$ and $R_{ext} = 7\sigma$. The results from these simulations showed that indeed, the tails of the distributions are populated in every turn due to the interaction of the incident proton beam and the e-lens. In all cases, the proton particles did not concentrate in a specific range, but they were kicked out farther away from the distribution on each turn, as the kicks are still present beyond $7\sigma$. Distinct effects were observed as the current was varied. The e-lens with a relative low current showed impacts on the proton distribution that were expected, as the turns increased the particles beyond $4\sigma$ were kicked out to more distant positions and the core was not affected. No signs of emittance increase were observed. However, when the current was as high as 7 A, the core of the distribution began to be affected as the turns increased.

The electron lenses with a relative high current (7 A and 3 A) have consequences on the emittance that need to be considered carefully. For the case of the e-lens with 7 A, the percentage emittance increase begins to behave almost linear but after 10000 turns it has an exponential growth. The stability of the beam starts to be compromised, and the effects of the cylindric hollow distribution extends even to the core of the beam distribution. For these reasons, an electron lens with 7 A and 3 A can be considered as extreme cases.

The presented studies can be extended by including a longitudinal tilt of the e-lens in order to analyze the impacts on the beam parameters produced by both the Gaussian distribution and the cylindric hollow distribution. Convergency studies could also improve the understanding of the noise introduced by the e-lens.
References


