Analyzing the Transverse Profile of Self-Modulated Proton Bunches at AWAKE

Mathis Gerdes
Supervisors: Marlene Turner, Edda Gschwendtner

September 2018

Abstract

The AWAKE experiment is designed to study electron acceleration in plasma wakefields driven by self-modulated proton bunches. This project focuses on analyzing the transverse, time-integrated profile of the proton bunches after self-modulation in rubidium plasma. The size and shape of these profiles can be used to verify the presence of self-modulation for each event and to study the influence of experimental parameters on the modulation process.

1 Introduction

Plasma based acceleration technology promises to reach acceleration gradients multiple orders of magnitude larger than those of conventional RF accelerators [1], which is especially relevant for light particles benefiting from linear acceleration. The AWAKE experiment studies plasma wake field technology based on a 400 GeV/c drive proton bunch, accelerating a witness electron bunch [2]. Effective wakefield excitation requires a proton bunch length on the order of the plasma wavelength (≈ 1 mm at AWAKE), while the available proton bunch length is $\sigma_z = 12 \text{ cm}$ [2]. The experiment therefore relies on the development of transverse seeded self-modulation (SSM) in which the wakefield focuses and de-focuses protons into a train of microbunches. By using a co-propagating and co-axial laser pulse ionizing a neutral vapor in the plasma cell, the SSM is seeded [3]. For this project it is relevant to note that this leads to one part of the proton bunch being unaffected by plasma interactions followed by a second, self-modulated part.

The presented analysis uses data from the 9th and 10th of September 2017, as it provides ranges of several experimental parameters with an invariant measurement setup. Due to an update of the measurement setup relative to the June data, the data acquisition and processing of the September data is more reliable.

This report outlines the methodology and initial results of an analysis [4] of the transverse bunch profile after self modulation in 10 m of rubidium plasma (see setup in figure [1]), but does not attempt to explain the physical causes. An average elliptic asymmetry of the halos around the bunch core with a preferred vertical direction, also for events without any self-modulation, is found and discussed in section 3.1. Section 3.2 finds an overall increase of the mean bunch profile diameter and dependencies on different experimental parameters. The timing (longitudinal offset between laser pulse and proton bunch) is found to have a significant impact on the mean diameter in section 3.3. Finally,
2 Measuring the Shape of the Bunch Profile

Using the measurement setup schematically outlined in figure 1 with two scintillating screens (1 mm of Chromox) at positions 2 m and 10 m behind the plasma, the transverse, longitudinally integrated bunch profile is captured. Since the intensity of the bunch profile covers several orders of magnitude, each imaging system (IS; IS1 and IS2) is equipped with two cameras tuned to measure the core of the bunch and the halo of the bunch, respectively.

The pixel density of the cameras and the relative shift and rotation can be determined experimentally and are therefore known. The transverse center of the bunch can be determined as the peak of the core image and translated to the halo image.

As the goal of this project is to analyze the bunch profile, and specifically the asymmetry in connection with experimental parameters, the halo images have to be reduced to numerical measures. The focus here is on extracting the radial extent of the proton bunch profile in the form of a set of radii with associated angles (in a polar coordinate system with origin at the bunch center). Other measures, such as the average deviation from a theoretical distribution, are conceivable but not further discussed here.

Two methods for extracting the extent are considered in sections 2.3 and 2.2, after a discussion of practical challenges and issues.

2.1 Challenges

An issue with the extent of the bunch profile is the ambiguity of defining the edge of a distribution without a sharp edge (such as a normal distribution; theoretically the edge is well-defined by the outermost proton, however the background noise and limited sensitivity of the measurement devices make detection of this edge, at present, impractical). A first approach may be to set an intensity level (above noise), above which values are considered part of the bunch. This leads to something like the contour method discussed in section 2.2. Another approach, discussed in section 2.3, is based on the radial gradient of the measured intensity.
An issue with general methods independent of a known theoretical shape is that multiple edge points may be found, potentially contradicting each other regarding the maximal extent. This can be caused either by plateaus within the bunch profile (the radius may locally increase radially) or dust particles (both of these issues can be observed in the present data).

Markings on the screen used to determine the pixel density and orientation may, for large radii, cause problems as they block the bunch intensity from being observed. Since the camera’s fields of view are kept constant during a run of the experiment, individual events may be discarded based on an overlap with the markings. The methods outlined in the following were extended to circumvent the problem to a certain degree by using linear spline interpolation and smoothing, however the reliability of this method was not further tested or analyzed. All present results deal with halos that do not touch the markings.

### 2.2 Contour Based Method

The shape of the bunch profile may be found as the longest connected contour, using established methods (here `skimage.measure` form scikit-image\(^5\) is used). However, to make the extent of multiple shapes comparable, one maximal radius per angle is desired. This may be achieved by transforming the found contour points to polar coordinates (see appendix \(A\)), and using linear spline interpolation (as done here) or binning to handle overlapping edge points. To find a contour, one has to set an intensity level. By determining the noise level as the average pixel intensity of pixels far from the center, the contour level can be given in relative terms. Figure 2 shows the result of the contour method for two different levels.

[Figure 2: Halo (left) and core (middle) image with bunch center shown with dashed orange lines and area of markings colored white. The results of the contour based method for two different levels are shown on the right in orange and red. All camera images are shown in logarithmic scale.]({https://scikit-image.org/})
2.3 Gradient Based Method

By defining the extent as the position where the radial gradient stops being significantly negative (in some local environment; significance must be chosen based on the background noise level in combination with other smoothing parameters), the ambiguity in choosing the level for the contour method is removed. The gradient method used here first transforms the image into polar coordinates (A.1 in the appendix) and then computes the gradient of the logarithm of the data along the radial direction. This data is turned into either 1 if the gradient is negative or 0 if not. Applying a Gaussian filter to this gives a measure of the negativity of the gradient in the local environment for each point. If parameters for smoothing and filtering are chosen adequately, the edge can be measured as the transition between the negative and positive gradient areas (given some threshold of negativity). Figure 3 shows the result of the gradient based method on an especially symmetric event. It is worth noting that the range of radii varies significantly, maybe more than a qualitative inspection of the raw image leads to believe. Origin of this may be a faulty setting of the bunch center, due to an incorrect translation from the core to the halo image. This shift leads to all extracted radii (of multiple events) being biased in the same way, which can be detected by averaging all radii. Fitting an ellipse with arbitrary center (see sections A.2 and A.3 in the appendix) to this mean radii of a large enough set of events may allow finding the actual center and shifting the radii in hindsight. One should, however, be careful with this method because any actual, physically relevant bias would also be removed. Figure 4 shows the result of this procedure for radii extracted from IS2 images.

Figure 3: Results of the gradient based method with selected radial slices of the bunch profile and corresponding found radii (left), the mean and extremal radii compared with the average radial profile (averaged over $\phi$) and a laser-off event (no SSM), and the radii on top of the original image on the right.

For the following results the gradient based methods with settings as for figure 3 are used.

3 Results

3.1 Elliptic Asymmetry

Based on data taken on the 10th of September 2017, the average shape (mean over all radii for the set of events of that date, for each angle) was found to be elliptical with an eccentricity of $0.4456 \pm 0.0011$ and $0.3942 \pm 0.0014$ for IS1 and IS2, respectively (see
Figure 4: Processing of the overall average radii per angle for events from the 10th of September 2017 at IS2, with the raw radii being shifted to be centered around the origin, and fitted with an ellipse. A circle with the mean radius is shown on the right in comparison to the average radii.

Figure 5: Stacked histograms (left and middle) of the angles of the long axis of fitted ellipses for events from the 10th of September 2017. The right plot shows the relation between the eccentricity and the angle.

To obtain this result, first a circle with arbitrary center was fitted to the overall average radii which is then used to shift the average radii to be centered around the origin (see also section 2.3). The eccentricity is computed by fitting an ellipse to this shifted data.

The statistically significant value of the eccentricity implies there is a systematic asymmetry in the bunch profile shape with a preferred direction. Using the same data set and applying the same method as for the overall average radii to each event, a histogram of the angle of the long axis of the fitted ellipses as in figure 5 is obtained. This confirms the previous finding of a preferred asymmetry direction. Furthermore, it is noteworthy that the laser off events (no self modulation) show a clear preference for the angle of the long axis, and the direction of laser-on and laser-off events align. The relation between the eccentricity and the preferred direction does not show a clear correlation between the two measures, but shows that the asymmetry for laser-off events seems greater for IS1 than IS2.

Figure 6 shows the eccentricity versus several of the experimental parameters. None seem to fully explain the asymmetry, but several parameters, such as the timing and the vapor density may have an impact.
Figure 6: Eccentricity of fitted ellipses for events from the 10th of September 2017 versus several available experimental parameters.

3.2 Proton Bunch Population Scan

The events of the 10th of September 2017 range over a number of proton populations. Figure 7 shows the mean diameter for each of the events plotted against the proton count, displaying in color various correlations with other experimental parameters. Overall, the mean radius increases with the number of protons in the bunch, but saturates for large values. Both the timing (distance between the ionizing laser pulse and the proton bunch center in time; visible as the coloring in the upper right plot) and the vapor density (coloring in the lower left plot) can be seen to affect the mean diameter size. The upper left plot of figure 7 displays in color the laser pulse energy, which can be used to distinguish between laser-off events (colored red) and laser-on events (colored green; self modulation present). The laser-off events show a linear increase of the mean diameter with the number of protons in the bunch (without any self modulation, the mean diameter of the transverse profiles increases with the proton count). The bottom right plot shows the vapor density gradient in color.

3.3 Laser Pulse - Bunch Center Alignment Scan

The set of events explored in the previous section contains a range of different values for the distance between the laser pulse and the proton bunch center (timing), as can be inferred by the coloring in figure 7. Figure 8 shows the correlation between the mean bunch profile diameter and the timing, on the left. The second plot between the vapor gradient and the mean diameter indicates the gradient does not significantly affect the bunch profile size, within the available range of values.

3.4 Laser Energy Scan

The events from the 9th of September 2017 provide a range of different laser energy values. Figure 9 shows the mean diameter for these events in dependence of the laser energy as well as the vapor density. An increase in the diameter, as well as in the spread
Figure 7: Mean diameter of events from the 10th of September 2017 measured at IS2 for a range of proton counts, in connection with various experimental parameters. The laser-off events are only visible in the upper-left plot.

Figure 8: Mean diameter of events from the 10th of September 2017 in dependence of the timing (left), and the vapor gradient (right). Laser-off events are excluded.
of the diameters with laser energy is observable. The positions of the proton bunch and the laser pulse at the entrance and exit of the vapor source, as well as the respective sizes were measured and extracted previously\(^6\). By labeling events where the proton bunch is not fully contained within the radius of the laser pulse as misalign (the radius of the proton bunch and laser pulse were taken to be 0.2 mm and 1 mm, respectively), as done in figure 9, the hypothesis of a misalignment causing an increase in the spread of diameters can be tested. Figure 9 shows the mean diameter in relation to the vapor density gradient and the timing, using color as third axis. The result does not support the conclusion that the misalignment is causing the increase in the spread of diameters with laser energy.

By defining the range of diameters (maximal diameter minus minimal diameter) divided by the mean diameter as a measure for asymmetry, the spread can be further analyzed. Figure 10 shows this relative diameter range with respect to the laser energy together with the mean vapor density, the density gradient and the laser alignment. None of the inspected parameters seems to explain the diameter spread relative to the laser energy. While the vapor density (left plot) may have an impact, too few data points are shown to make a significant claim.

\(^6\)by Anna-Maria Bachmann (anna-maria.bachmann@cern.ch)
4 Summary

Two methods for extracting the radial extent of the transverse bunch profile captured at AWAKE were analyzed and used to find correlations in the increase of size and asymmetry of the shape relative to experimental parameters. An overall asymmetric (elliptic) shape of the bunch profile was found, both with and without self modulation, as well as a preferred direction of the asymmetry. This serves as motivation to further analyze the impact of an initially asymmetric bunch on the SSM, and to find the potential experimental source of the asymmetry.

The mean diameter is shown to change with the proton bunch population, the laser to proton bunch alignment, the vapor density, and the laser pulse energy. These claims must be taken relative to the present parameter ranges, outside of which some saturation may be expected. The spread of diameters was found to increase with greater laser pulse energy. An inspection of the asymmetry as well as correlations with various parameters did, however, not conclusively explain this effect.


A Geometry

A.1 Polar Transformation

The image is, by the nature of the cameras, given on a two dimensional grid with Cartesian coordinates referred to as $x$ (horizontal) and $y$ (vertical). Generally the transformation between polar coordinates $\phi$, $r$ and Cartesian coordinates is

$$
\begin{align*}
  x &= r \sin(\phi), \\
  y &= r \cos(\phi).
\end{align*}
$$

(1)

Given a numpy array $I$ representing the image, with the first axis being the y-axis, the following Python pseudo-code computes a transformed image with $r$ and $\phi$ as axes (in that order):

```python
1 def polar_to_cart(center, r, phi):
2     return center.x + r * cos(phi), center.y + r * sin(phi)

3 def image_in_polar(I, center, max_radius, angle_count=360):
4     r = np.arange(1, max_radius)
5     phi = np.linspace(0, 2 * pi, angle_count)
6     mesh_grid = np.meshgrid(r, phi)
7
8     # evaluate image at these coordinates
9     x, y = polar_to_cart(center, r, phi)

10     flat_image = I[y.astype(int).flatten(), x.astype(int).flatten()]
11     return flat_image.reshape(x.shape)
```

Algorithm 1: Transforming an image given in Cartesian coordinates into polar coordinates

A.2 Radius of an Ellipse

The radius of an ellipse with extreme diameters of length $a$ and $b$ is described by

$$
  r(\phi) = \frac{ab}{\sqrt{a^2 \sin^2(\phi) + b^2 \cos^2(\phi)}},
$$

(2)

where $\delta$ sets the rotation of the ellipse such that for $\delta = 0$, $a$ is just the diameter along the x-axis. For $b \geq a$, $c$ gives the angle of the long axis. The eccentricity of an ellipse is given by

$$
  \epsilon = \sqrt{1 - \left(\frac{\min(a, b)}{\max(a, b)}\right)^2}.
$$

(3)
A.3 Shifting Radii

If the transverse center of the bunch was not identified correctly (shifted by some distance), one might want to get the radii as they would have been observed with the correct center. Furthermore, the ability of shifting radii can be used to get the radii of an ellipse (including a circle), centered arbitrarily (as long as the observation point is within the ellipse shape).

Given some radius $r$ and angle $\phi$ uniquely specifying a certain point, the radius $r'$ and angle $\phi'$ that would have been observed from an origin at $(x_0, y_0)$ is given by

$$r' = \sqrt{x_0^2 + y_0^2 + r^2 - 2r(x_0 \cos \phi + y_0 \sin \phi)}$$

and

$$\cos \phi' = \frac{r}{r'} \cos(\phi) - \frac{x_0}{r'}.$$  

It is important to note that under this transformation not only the radii but also the angles change. If one would like to have the radii at the same angles as previously, interpolation methods may be used or the angles may be taken approximately equal if the shift is small.

References

