Lepton Universality Violation at LHCb
LHCbでのレプトン普遍性違反

“Les quarks beauté bousculent l’universalité leptonique”
CERN Courier March 2018

Steve Playfer
on behalf of the LHCb collaboration

Workshop on Hints of New Physics in Heavy Flavours
Nagoya University, November 15th 2018
Outline

• Introduction to Lepton Flavour Universality
• The LHCb detector
• Electroweak penguin decays
• The ratios $R_K$ and $R_{K^*}$
• Anomalies in Branching Fractions and angular distributions
• The semileptonic ratios $R_{D^*}$ and $R_{J/\psi}$
• Outlook for the future
Lepton Flavour Universality

EW bosons in SM:

\[ Z^0, \gamma \]

\[ W^- \]

\[ \ell^- \]

\[ \ell^+ \]

\[ \ell \in \{ e, \mu, \tau \} \]

Coulplings of Z, W and \( \gamma \) to \( \ell \) do not depend on lepton flavour.

Differences in decay rates are driven by the different masses:

\[ m_e = 0.511 \text{MeV}, \ m_\mu = 105 \text{MeV}, \ m_\tau = 1777 \text{MeV} \]

Semileptonic b decays to e and \( \mu \) almost identical.

Leptonic \( B \rightarrow \ell \nu, \ B \rightarrow \ell\ell \) helicity-suppressed by \( m_\ell^2 \)

Charged Higgs in NP:

\[ H^- \]

\[ \tau^- \]

\[ \nu_\tau \]

Heavy \( Z' \) boson in NP:

\[ Z' \]

\[ \ell^- \]

\[ \ell^+ \]

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Nagoya University, November 15\textsuperscript{th} 2018
The LHCb Detector

Forward detector (2<\eta<5)

The LHCb detector is not lepton flavour universal!

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Triggering at LHCb

Hardware (Level 0)

• L0M: Muons identified with $p_T > 1.5$ to $1.8$ GeV/c
• L0E: Electrons identified with $E_T > 2.5$ to $3.0$ GeV
• L0H: Any $\pi/K$ from the signal decay with $E_T > 3.5$ GeV
• L0I: Other high $p_T$ tracks independent of the signal decay

Software

• 2,3 or 4-track vertices displaced from the primary vertex and consistent with the signal decay mode
Bremsstrahlung recovery < 100% with $E_T(\gamma) > 75\text{MeV}$

**Electron Reconstruction at LHCb**

- **Part. Reco.**
- **Bremsstrahlung**

**JHEP 08(2017) 055**

**JHEP 04 (2015) 064**

**Low mass tail**

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**Electroweak Penguin Decays**

Flavour changing neutral current transitions require loops/boxes in the SM.

Can replace $W$, $Z$, $t$ with charged Higgs, $Z'$, SUSY partners, leptoquarks or other NP.

Could in principle have tree level FCNC couplings, but these are strongly constrained by other measurements.
Effective Theory

$$\mathcal{H}_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_e}{4\pi} \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

Integrate out scales above $\mu \sim m_b$

SM calculations of inclusive rates give (10% accuracy):

$C_7 \sim -0.3$ from the photon

$C_9 \sim +4$ from EW vector

$C_{10} \sim -4$ from EW axial-vector

($'$) indicate RH contributions (suppressed by $m_s/m_b$ in SM)
Map of $K(\ast)\ell\ell$ Contributions

Photon pole enhancement (no pole for $B\to K\ell\ell$ decays)

Form-factors from LCSR calculations

$C_7^{(i)}$, $C_9^{(i)}$

interference

Spectrum dominated by narrow charmonium resonances.

Typically removed in analyses

Long distance contributions from $c\bar{c}$ above open charm threshold

Form-factors from Lattice QCD

$4\left[m(\ell)\right]^2$

$d\Gamma/dq^2$

$q^2$ dilepton mass squared

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Measure Double Ratios

Reduces dependence on simulation for selection and reconstruction efficiency.

\[
R_K = \left( \frac{N_{K^+\mu^+\mu^-}}{N_{K^+e^+e^-}} \right) \left( \frac{N_{J/\psi(e^+e^-)K^+}}{N_{J/\psi(\mu^+\mu^-)K^+}} \right) \left( \frac{\epsilon_{K^+e^+e^-}}{\epsilon_{K^+\mu^+\mu^-}} \right) \left( \frac{\epsilon_{J/\psi(\mu^+\mu^-)K^+}}{\epsilon_{J/\psi(e^+e^-)K^+}} \right)
\]

L0 electron   L0 hadron   L0 signal independent   L0 muon

\begin{align*}
&\text{J/ψ(ee)} \\
&\text{Candidates} / (40 \text{ MeV}/c^2) \\
&\text{LHCb} \\
&62324 \pm 318 \\
&9337 \pm 124 \\
&16796 \pm 165 \\
&1226(41) \\
&3/\text{fb at 7-8TeV} \\
&1<q^2<6\text{GeV}^2 \\
&\text{K\ μ\ μ\ events} \\
&\text{PRL 113, 151601 (2014)}
\end{align*}

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Result for $R_K$

LHCb PRL 113, 151601 (2014): $R_K = 0.745^{+0.09}_{-0.07}$ (stat) $\pm 0.036$ (syst)

3/fb at 7-8TeV Window $1 < q^2 < 6$ GeV$^2$ 2.6$\sigma$ away from SM $R_K=1(10^{-2})$

Bordone, Isidori & Pattori
EPJC 76, 440 (2016)

Errors are almost entirely from Kee samples. Dominant systematics from fit shapes and bremsstrahlung correction.

For comparison:

BaBar PRD86, 032012 (2012): $R_K = 0.74^{+0.31}_{-0.25}$ (stat) $\pm 0.07$ (syst)

Window $0.1 < q^2 < 8$ GeV$^2$

Belle PRL103, 171801 (2009): $R_K = 1.03 \pm 0.19$ (stat) $\pm 0.06$ (syst)

All $q^2$ apart from $J/\psi$ and $\psi'$ regions

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Nagoya University, November 15th 2018
285 $K^* \mu \mu$ events

353 $K^* \mu \mu$ events

274k $J/\psi(\mu \mu)$ events

89 $K^*ee$ events

111 $K^*ee$ events

58k $J/\psi(ee)$ events
Results for $R_{K^*}$

LHCb JHEP 08, 055 (2017) 3/fb at 7-8TeV

$R_{K^*} = 0.66^{+0.11}_{-0.07}$ (stat) $\pm 0.03$ (syst) for $0.045 < q^2 < 1.1$ GeV$^2$ 2.2$\sigma$ away from SM prediction of $R_{K^*}=0.926(4)$ Altmannshofer et al EPJC 77, 377 (2017)

$R_{K^*} = 0.69^{+0.11}_{-0.07}$ (stat) $\pm 0.05$ (syst) for $1.1 < q^2 < 6$ GeV$^2$ 2.5$\sigma$ away from SM prediction of $R_{K^*}=1(10^{-2})$ Bordone, Isidori & Pattori EPJC 76, 440 (2016)

For comparison:

BaBar PRD86, 032012 (2012): $R_{K^*} = 1.06^{+0.48}_{-0.33}$ (stat) $\pm 0.08$ (syst) Window $0.1 < q^2 < 8$ GeV$^2$

Belle PRL103, 171801 (2009): $R_{K^*} = 0.83 \pm 0.17$ (stat) $\pm 0.05$ (syst) All $q^2$ apart from $J/\psi$ and $\psi'$ regions

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**K(*)\(\mu\mu\) Branching Fractions**

3/fb at 7-8TeV

- **JHEP 11, 017 (2016)**
  - Corrected for 10% S-wave \(K\pi\) using angular/mass fit

- **JHEP 06, 133 (2014)**
  - Are \(K(*)\mu\mu\) BFs below SM?
  - \(K(*)\)ee BFs agree with SM!

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More Branching Fractions

3/8 fb at 7-8 TeV

These BFs also below SM?

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Angular analysis of $K^{*}\mu\mu$

3/fb at 7-8TeV

$P_5' = S_5 / \sqrt{F_L(1-F_L)}$

Is an angular coefficient that is designed to be insensitive to form factors

DHMV = Descotes-Genon et al
JHEP 12, 125 (2014)

LHCb says disagreement with SM is at the level of 3.4$\sigma$

Supported by Belle and maybe ATLAS.
Not confirmed by CMS.
$K^*\mu\mu$ $F_L$ and $A_{FB}$

3/fb at 7-8TeV

Longitudinal $K^*$ polarisation

Lepton forward-backward asymmetry

ABSZ = Altmannshofer & Straub
EPJC 75, 882 (2015)

JHEP 02,014 (2016)

Zero-crossing point shifted up in $q^2$
Angular analysis of $K^{*}ee$

$F_L = 0.16 \pm 0.06 \pm 0.03$

$A^{(2)}_T = -0.23 \pm 0.23 \pm 0.05$

$A^{\text{Im}}_T = +0.14 \pm 0.22 \pm 0.05$

$A^{\text{Re}}_T = +0.10 \pm 0.18 \pm 0.05$

“For SM values of C7 the ratio C7'/C7 is compatible with zero”

3/fb at 7-8 TeV

JHEP 04, 064 (2015)

Low $q^2$ region is described by photon pole.

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Global Fits for Wilson coefficients

Consistent with $\Delta C_9 \sim -1$ due to NP. Could also be a small shift in $\Delta C_{10}$. 

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Designed for 40MHz readout with a full trigger in software. Instantaneous luminosity $2 \times 10^{33}$/cm$^2$/s (increase x5)

LHCb Upgrade 2019-2020

New Si Upstream Tracker
New SciFi Downstream Tracker
New ECAL/HCAL electronics
MaPMT for RICH Photosensors
New RICH 1 Optics
New Muon electronics
New Pixel VELO

LHCb Upgrade TDR
CERN-LHCC-2012-007

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Outlook for $R_K$ and $R_{K^*}$


- All results based on $3/fb$ at 7-8TeV (Run 1 2010-2012)
  - $1 < q^2 < 6$ GeV$^2$
  - $\sigma(R_K) \quad \sigma(R_{K^*})$
    - 0.09 0.11 (stat)
    - 0.036 0.050 (syst)

- We have another $6/fb$ at 13TeV (Run 2 2015-2018)
  - x4 in B statistics due to increased production X-section
    - 0.043 0.052

- LHCb upgrade during shutdown (2019-2020)
  - 40MHz readout and trigger entirely in software
  - Better calorimeter granularity and timing for electrons
    - After upgrade can reduce syst errors

- Integrated luminosity $50/fb$ in Runs 3 & 4
  - Higher instantaneous luminosity $2 \times 10^{33}$/cm$^2$/s
    - 0.017 0.020

- Possible major upgrade in ~2030
  - Much higher luminosity $2 \times 10^{34}$, with target of $300/fb$
    - 0.007 0.008
    - similar to $\sigma$(SM)

More on upgrades in talk by Eugeni Grauges Pous
Semileptonic D(*)$\ell\nu$ decays

Review by Ciazenek et al
Nature 546, 227 (2017)

SM predictions of $R(D^{(*)}) = D^{(*)}\tau\nu/D^{(*)}\mu\nu$

$R(D) = 0.299(6), R(D^*) = 0.252(3)$

BaBar (HT)
HT Hadronic tags
Belle (ST)
ST Semileptonic tags
LHCb

$R(D)$
0.375 ± 0.069
0.440 ± 0.072

$R(D^*)$
0.293 ± 0.041
0.336 ± 0.040
0.332 ± 0.030
0.302 ± 0.032

World averages (2017): $R(D) = 0.397(49), R(D^*) = 0.316(19)$ are $2\sigma/3\sigma$ above SM

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R(D*) with $\tau \to \mu \nu \nu$

Measure ratio of:

$B(D^*\tau \nu) = 1.7\%$

$B(\tau \to \mu \nu \nu) = 17.4\%$

To:

$B(D^*\mu \nu) = 4.9\%$

- Same visible final state particles
- 3 $\nu$ in signal mode, 1 $\nu$ in normalisation mode
- Separated by fit to missing mass $m_{\text{miss}}^2$, muon energy $E_\mu^*$, leptonic $q^2$
- Backgrounds from $D^{**}$, $B \to D(s)D^*$, combinatorics, muon mis-ID
- Mostly dealt with by control samples, e.g. wrong-sign combinations, additional charged track at B decay vertex ...

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Result for $R(D^*)$ with $\tau \rightarrow \mu \nu \nu$

- $B \rightarrow D^* \mu \nu$ dominates at low $q^2$
- $B \rightarrow D^* \tau \nu$ is visible at high $q^2$, high $m_{\text{miss}}^2$, low $E_\mu^*$
- Backgrounds from $D^{**} \mu \nu$, $B \rightarrow D_{(s)} D^*$
  combinatorics, muon mis-ID

$R(D^*) = 0.336 \pm 0.027 \text{ (stat)} \pm 0.030 \text{ (syst)}$

Systematic limited by size of MC sample!

1.9$\sigma$ above SM

LHCb PRL 115, 112991 (2015)
R(D*) with $\tau \to 3\pi(\pi^0)\nu$

Measure ratio of:

$B \to D^* \tau \nu$

$B(\tau \to 3\pi \nu) = 9.3\%$

$B(\tau \to 3\pi \pi^0 \nu) = 4.6\%$

To:

$B \to D^*3\pi(\pi^0)$

$B(D^*3\pi) = 0.72\%$

$B(D^*3\pi \pi^0) = 1.76\%$

- Same visible final state particles (we don’t require the $\pi^0$)
- $2\nu$ in signal mode, $0\nu$ in normalisation mode
- Signal extracted by fit to $q^2$, $\tau$ lifetime, and a BDT (to suppress $D_sD^*$)

- Backgrounds from $D^{**}$, $B \to D_{(s)}D^*$, $B \to D^*3\pi X$, combinatorics
- Mostly dealt with by control samples
Result for $R(D^*)$ with $\tau \to 3\pi \nu$

- $B \to D^* \tau \nu$ is visible at high $q^2$, high BDT, and with non-zero $t_\tau$
- Backgrounds from $D^{**}$, $B \to DD^*$, $B \to D_sD^*$, combinatorics

$R(D^*) = 0.291 \pm 0.019$ (stat) $\pm 0.026$ (syst) $\pm 0.054$ (norm)

Consistent with SM and $R(D^*)$ from $\tau \to \mu \nu \nu$

LHCb PRL 120, 171802 & PRD 97,07213 (2018)
R(J/ψ) with $\tau \rightarrow \mu \nu \nu$

Measure ratio of:

$B_c \rightarrow J/\psi \tau \nu$

SM prediction

$R(J/\psi) = 0.25-0.28$

form factors not measured

- Same visible final state particles
- 3ν in signal mode, 1ν in normalisation mode
- Separated by fit to $m_{\text{miss}}^2$, $E_\mu^*$, $q^2$ and using $\tau(B_c)=0.5\text{ps}$
- Backgrounds from other charmonium, combinatorics, muon mis-ID
- Mostly dealt with by control samples

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Result for $R(J/\psi)$ with $\tau \rightarrow \mu \nu \nu$

- $B_c \rightarrow J/\psi \mu \nu$ is visible at zero $m_{\text{miss}}^2$, and with small $\tau(B_c)$
- $B_c \rightarrow J/\psi \tau \nu$ is visible at high $q^2$, high $m_{\text{miss}}^2$, and with small $\tau(B_c)$
- Main backgrounds from mis-ID, combinatorics

$$R(J/\psi) = 0.71 \pm 0.17 \text{ (stat)} \pm 0.18 \text{ (syst)}$$

Systematic dominated by form factors

$2\sigma$ above SM

LHCb PRL 120, 121801 (2018)
Summary of $R(D), R(D^*)$ and $R(J/\psi)$

Three experiments
Nine measurements all above SM
Different techniques and final states

R(D), R(D*), R(J/\psi) are 2/3/2σ above SM
Combined significance 4σ
SM uncertainties are small

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Outlook for $R(D^*)$ and $R(J/\psi)$

*Physics case for LHCb Upgrade II*  
*arXiv:1808.08865 (2018)*

- All results based on $3/\text{fb}$ at 7-8TeV (Run 1 2010-2012)
  - $\sigma(R_{D^*}) = 0.027$  
  - $\sigma(R_{J/\psi}) = 0.17$ (stat)

- We have another $6/\text{fb}$ at 13TeV (Run 2 2015-2018)
  - $4\times$ in B statistics due to increased production X-section
  - $\sigma(R_{D^*}) = 0.030$  
  - $\sigma(R_{J/\psi}) = 0.18$ (syst)

- LHCb upgrade during shutdown (2019-2020)
  - 40MHz readout and trigger entirely in software
  - Better vertexing for reducing backgrounds to $\tau$, D and B
  - After upgrade can reduce syst errors

- Integrated luminosity $50/\text{fb}$ in Runs 3 & 4
  - Higher instantaneous luminosity $2\times10^{33}/\text{cm}^2/\text{s}$
  - $\sigma(R_{D^*}) = 0.007$  
  - $\sigma(R_{J/\psi}) = 0.07$

- Possible major upgrade in $\sim$2030
  - Much higher luminosity $2\times10^{34}$, with target of $300/\text{fb}$
  - $\sigma(R_{D^*}) = 0.002$  
  - $\sigma(R_{J/\psi}) = 0.02$

*More on upgrades in talk by Eugeni Grauges Pous*

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Nagoya University, November 15th 2018
More analyses to come ...

• \( b \rightarrow s \ell \ell \): \( R(B_s \rightarrow \phi), \ R(\Lambda_b \rightarrow \Lambda), \ R(K^{**}) \),

• Full angular analysis of \( K^*\)ee

• \( b \rightarrow d \ell \ell \): \( R(\pi), \ R(\rho), \ R(B_s \rightarrow K^*) \)

• \( b \rightarrow c \tau \nu \): \( R(D), \ R(B_s \rightarrow D_s^{(*)}), \ R(\Lambda_b \rightarrow \Lambda_c) \)

• Angular analysis of \( \tau \rightarrow 3\pi\nu \) to determine spin structure of NP in \( R(D^*) \)

• \( b \rightarrow u \tau \nu \): \( \Lambda_b \rightarrow p\tau\nu, \ \bar{B} \rightarrow \bar{p}\rho\tau\nu \)
Summary and Conclusions

• There are a number of $2-3\sigma$ anomalies that have appeared in $b \rightarrow s \ell\ell$ and $b \rightarrow c \ell\nu$ since 2012

• $R(K)$ and $R(K^*)$ both suggest a 30% deficit in muons compared to electrons in $b \rightarrow s \ell\ell$ ($1<q^2<6\text{GeV}^2$)

• $R(D)$, $R(D^*)$ and $R(J/\psi)$ all suggest an enhancement in $\tau$ compared to $\mu$ in $b \rightarrow c \ell\nu$

• LHCb can push these lepton universality tests to the % level or better in the next 10-20 years
BACKUP
The three methods used to determine the angles in each fit is used to determine the zero-crossing points of fit. Section 7.2 discusses the determination of the same set of observables using a principal describes the determination of the observables in bins of simulation.

**K*μμ** as a function of q^2 after integrating over decay angles

K*ee as a function of q^2 bin and L0 trigger after integrating over decay angles

Normalised to J/ψ = 1

<table>
<thead>
<tr>
<th></th>
<th>ε_{ℓ+ℓ−}/ε_{J/ψ(ℓ+ℓ−)}</th>
<th>low-q^2</th>
<th>central-q^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ^+μ^−</td>
<td></td>
<td>0.679 ± 0.009</td>
<td>0.584 ± 0.006</td>
</tr>
<tr>
<td>e^+e^− (L0E)</td>
<td></td>
<td>0.539 ± 0.013</td>
<td>0.522 ± 0.010</td>
</tr>
<tr>
<td>e^+e^− (L0H)</td>
<td></td>
<td>2.252 ± 0.098</td>
<td>1.627 ± 0.066</td>
</tr>
<tr>
<td>e^+e^− (L0I)</td>
<td></td>
<td>0.789 ± 0.029</td>
<td>0.595 ± 0.020</td>
</tr>
</tbody>
</table>
# K*\(\ell\ell\) Systematics

## Table 4: Systematic uncertainties on the \(\Delta R_{K^*0}/R_{K^*0}\) ratio for the three trigger categories separately (in percent). The total uncertainty is the sum in quadrature of all the contributions.

<table>
<thead>
<tr>
<th>Trigger category</th>
<th>(\Delta R_{K^*0}/R_{K^*0}) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low-(q^2)</td>
</tr>
<tr>
<td>Corrections to simulation</td>
<td>(\text{L0E} \quad \text{L0H} \quad \text{L0I})</td>
</tr>
<tr>
<td>Trigger</td>
<td>2.5 4.8 3.9</td>
</tr>
<tr>
<td>PID</td>
<td>0.1 1.2 0.1</td>
</tr>
<tr>
<td>Kinematic selection</td>
<td>2.1 2.1 2.1</td>
</tr>
<tr>
<td>Residual background</td>
<td>— — —</td>
</tr>
<tr>
<td>Mass fits</td>
<td>1.4 2.1 2.5</td>
</tr>
<tr>
<td>Bin migration</td>
<td>1.0 1.0 1.0</td>
</tr>
<tr>
<td>(r_{J/\psi}) ratio</td>
<td>1.6 1.4 1.7</td>
</tr>
<tr>
<td>Total</td>
<td>4.0 6.1 5.5</td>
</tr>
</tbody>
</table>
Checks of $K^*(\ell\ell)$ Results

- $R_{K^*}(J/\psi) = 1.043 \pm 0.006$ (stat) $\pm 0.045$ (syst)
- $BF(J/\psi K) = 1.01 \times 10^{-3}$ and $BF(J/\psi K^*) = 1.27 \times 10^{-3}$
- $BF(K^*\mu\mu) = 0.342 \pm 0.006$ (stat) $\pm 0.045$ (syst) $\times 10^{-7}$ \quad 1.1 < q^2 < 6 \text{GeV}^2$
- $BF(K^*\gamma) = 4.2 \times 10^{-5}$ from photon contribution to low $q^2$ region
- Take double ratios with respect to $\psi'$
- Compare kinematic distributions and other selection variables (sPlot method)

All checks are ok to better than 10%
The data are not estimated. The distributions are normalised to unity. The hatched areas correspond to the statistical uncertainties.

Figure 8: Fraction of candidates [%].

$K^*_{\ell\ell}$ sPlots

$q^2$

$M(K\pi)$

$\theta_\ell$
Angular Analysis

- Multibody final-states:
  - Angular distribution provides many observables that are sensitive to BSM physics.
  - Constraints are orthogonal to branching fraction measurements, both in their impact in global fits and in terms of experimental uncertainties.

For example, a decay described by three angles and $q^2$:

- $K^+ \pi^- K^*_{0 \pi}$
- $\mu^+ \mu^- B^0 \pi^-$

Lepton decay angle $\theta_\ell$ and $K^*$ decay angle $\theta_K$ defined in B rest frame.

- Acoplanarity angle $\phi$ defined in B rest frame.
- Reverses sign for $\bar{B}$.
### K*\(\ell\ell\) Angular Coefficients

<table>
<thead>
<tr>
<th>(i)</th>
<th>(I_i)</th>
<th>(f_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s</td>
<td>(\frac{3}{4} \left[</td>
<td>A_{L</td>
</tr>
<tr>
<td>1c</td>
<td>(</td>
<td>A_0</td>
</tr>
<tr>
<td>2s</td>
<td>(\frac{1}{4} \left[</td>
<td>A_{L</td>
</tr>
<tr>
<td>2c</td>
<td>(-</td>
<td>A_0</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{2} \left[</td>
<td>A_{L</td>
</tr>
<tr>
<td>4</td>
<td>(\sqrt{\frac{2}{3}} \text{Re}(A_{L</td>
<td>}^0 A_{L\perp}^* + A_{R</td>
</tr>
<tr>
<td>5</td>
<td>(\sqrt{\frac{2}{3}} \text{Re}(A_{L</td>
<td>}^0 A_{L\perp}^* - A_{R</td>
</tr>
<tr>
<td>6s</td>
<td>(2\text{Re}(A_{L</td>
<td>}^0 A_{L\perp}^* - A_{R</td>
</tr>
<tr>
<td>7</td>
<td>(\sqrt{\frac{2}{3}} \text{Im}(A_{L</td>
<td>}^0 A_{L\perp}^* - A_{R</td>
</tr>
<tr>
<td>8</td>
<td>(\sqrt{\frac{1}{3}} \text{Im}(A_{L</td>
<td>}^0 A_{L\perp}^* + A_{R</td>
</tr>
<tr>
<td>9</td>
<td>(\text{Im}(A_{L\perp}^* A_{L\perp}^* + A_{R\perp}^* A_{R\perp}^*))</td>
<td>(\sin^2 \theta_K \sin^2 \theta_l \sin 2\phi)</td>
</tr>
<tr>
<td>10</td>
<td>(\frac{1}{3} \left[</td>
<td>A_S</td>
</tr>
<tr>
<td>11</td>
<td>(\sqrt{\frac{4}{3}} \text{Re}(A_S^0 A_{L\perp}^* + A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\cos \theta_K)</td>
</tr>
<tr>
<td>12</td>
<td>(-\frac{1}{3} \left[</td>
<td>A_S</td>
</tr>
<tr>
<td>13</td>
<td>(-\sqrt{\frac{4}{3}} \text{Re}(A_S^0 A_{L\perp}^* + A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\cos \theta_K \cos 2\theta_l)</td>
</tr>
<tr>
<td>14</td>
<td>(\sqrt{\frac{2}{3}} \text{Re}(A_S^0 A_{L\perp}^* + A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\sin \theta_K \sin 2\theta_l \cos \phi)</td>
</tr>
<tr>
<td>15</td>
<td>(\sqrt{\frac{2}{3}} \text{Re}(A_S^0 A_{L\perp}^* - A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\sin \theta_K \sin \theta_l \cos \phi)</td>
</tr>
<tr>
<td>16</td>
<td>(\sqrt{\frac{2}{3}} \text{Im}(A_S^0 A_{L\perp}^* - A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\sin \theta_K \sin \theta_l \sin \phi)</td>
</tr>
<tr>
<td>17</td>
<td>(\sqrt{\frac{2}{3}} \text{Im}(A_S^0 A_{L\perp}^* + A_{S\perp}^0 A_{R\perp}^*))</td>
<td>(\sin \theta_K \sin 2\theta_l \sin \phi)</td>
</tr>
</tbody>
</table>

\[
S_i = \left( I_i + \bar{I}_i \right) / \left( \frac{d\Gamma}{dq^2} + \frac{\bar{d}\Gamma}{dq^2} \right) \\
A_i = \left( I_i - \bar{I}_i \right) / \left( \frac{d\Gamma}{dq^2} + \frac{\bar{d}\Gamma}{dq^2} \right)
\]

\[
F_L = S_1 c \quad A_{FB} = \frac{3}{4} S_6 s
\]

\[
P_1 = \frac{2 S_3}{1 - F_L} = A_T^{(2)}
\]

\[
P_2 = \frac{2 A_{FB}}{3 (1 - F_L)}
\]

\[
P_3 = \frac{-S_9}{1 - F_L}
\]

\[
P'_{4,5,8} = \frac{S_{4,5,8}}{\sqrt{F_L (1 - F_L)}}
\]

\[
P'_6 = \frac{S_7}{\sqrt{F_L (1 - F_L)}}
\]
D*τν Control Samples

- D*−h+ for muon mis-identification using D⁰(Kπ), Λ(ρπ) to calibrate particle identification
- D*−µ− for combinatorial background
- Additional charged track at B vertex for D** and partially reconstructed backgrounds in hadronic final states
- Additional neutral energy in ECAL about D* or τ direction
- D*−Dˢ⁺(KKπ), D*−D⁺(Kππ), D*−D⁰(Kπ) for double charm
- D*−Dˢ⁺(3π), D*−D⁺(3π) for dominant backgrounds in τ → 3πν