We propose a method to probe the coupling of the Higgs to strange quarks by tagging strange jets at future electron-positron colliders. For this purpose we describe a jet-flavor observable, $J_F$, that is correlated with the flavor of the light quark in the hard part of event. The main idea behind $J_F$ is that strange-quark jets tend to contain a hard collinear kaon. We then use a simplified version of this variable to set up a strangeness tagger aimed at studying $h \to s\bar{s}$. Employing previously studied methods for suppressing the background from non-Higgs events and from Higgs decays to vector bosons or taus, we determine the sensitivity of our method to the strange Yukawa coupling, and find it to be of the order of the standard-model expectation.

I. INTRODUCTION

The hierarchical structure of the masses and mixing angles of the fundamental, point-like matter fields is one of the unsolved mysteries of the standard model (SM) of elementary particles. This flavor puzzle extends beyond the SM, since new-physics models are severely constrained by experimental observations, which favor the flavor structure of the SM without modification. Within the SM, the flavor structure is trivially encoded by the structure of the Yukawa interactions: the Yukawa coupling of the fermion $f$ to the physical Higgs is given by the ratio of the fermion mass to the Higgs vacuum expectation value, 

$$y_f = \frac{m_f}{v}.$$  

(1)

Beyond the SM, various alternatives motivated by the flavor puzzle predict significant deviations from the structure of Eq. (1) (see, e.g., Refs. [1–6] for relevant discussions). Before the Higgs boson was discovered at the LHC, the question of whether Eq. (1) was satisfied by nature was academic. Only recently, as we have entered the era of precision Higgs measurements, one can begin to tackle this question experimentally, opening a new direction in flavor physics.

Experimental tests of Eq. (1) are performed in terms of the signal strength $\mu_P$, defined as the ratio of the measured production cross section or branching ratio of the process denoted by $P$ to the SM expectation. The latest results from the ATLAS [7–9] collaboration are

$$\mu_{th} = 1.39_{-0.26}^{+0.28}, \quad \mu_{bb} = 1.01_{-0.20}^{+0.20}, \quad \mu_{\tau\tau} = 1.09_{-0.30}^{+0.35},$$  

(2)

where $P = th$ refers to the process $pp \to th$ and $P = bb$ denotes the decay $h \to bb$, etc. Similarly, the CMS [10] collaboration has measured

$$\mu_{th} = 1.18_{-0.27}^{+0.30}, \quad \mu_{bb} = 1.12_{-0.29}^{+0.29}, \quad \mu_{\tau\tau} = 1.02_{-0.24}^{+0.26}.$$  

(3)

Within the large uncertainties, these results are in good agreement with the expectation of Eq. (1) for the third-generation fermions, for which the Yukawa couplings are large and thus easier to measure. Precision will improve as the experiments collect more data.

However, it is important to note that the flavor puzzle is related to the mass hierarchy among all three generations of up-type quarks ($u, c, t$), down-type quarks ($d, s, b$) and charged fermions ($e, \mu, \tau$). Therefore, direct measurements of the smaller Higgs couplings to the first two generations of the different sectors is also necessary. So far, only upper bounds on the corresponding signal strengths have been obtained [11–14],

$$\mu_{\mu\mu} \lesssim 2.8, \quad \mu_{ee} \lesssim 3.7 \times 10^5, \quad \mu_{\mu\mu} \lesssim 110, \quad \mu_{\tau\tau} \lesssim 7.2 \times 10^8.$$  

(4)

The tightest limit on $\mu_{ee}$ is obtained by identifying charmed hadrons in $h \to c\bar{c}$ decays, while that on $\mu_{\tau\tau}$ is extracted from the search for the “exclusive” decay $h \to \phi\gamma$.

There is a clear path towards measuring the Yukawa couplings of the muon [11], charm quark [15], and possibly even the electron [16]. For the light quarks, the situation is much less clear. It is possible that their Yukawa couplings could be probed exclusively [3] or indirectly [17–20]. However, these methods are characterized by small rates and/or are subject to large QCD backgrounds that are difficult to control (see, e.g., Refs. [21, 22]).

In this paper we propose using a strangeness tagger, inspired by $Z \to s\bar{s}$ measurements at the DELPHI [23] and SLD [24] experiments, for probing the Higgs coupling to the strange quark. In Section II we present the basic idea and describe a new variable for determining the light flavor of a jet or other collection of particles. In Section III we implement a simplified algorithm of this variable and extend it by a method to reject background from heavy-flavor jets with the aim to distinguish $h \to s\bar{s}$ decays from other Higgs decays into two quarks or two gluons. We refer to such decays as $h \to j\bar{j}$. In practice, the only relevant $h \to j\bar{j}$ background decays are the ones with large branching fractions, namely, $h \to bb$, $h \to c\bar{c}$ and $h \to gg$.

In Section IV we consider the non-negligible background from events without a Higgs, as well as from Higgs decays into final states other than $jj$. Both classes of background are in the following referred to as non-$h \to jj$. Since the branching ratio for $h \to s\bar{s}$ is small, the measurement requires very clean event samples with a
large fraction of $h \rightarrow jj$ events. This can be achieved only at lepton colliders producing many Higgs bosons. Therefore, we limit the discussion to such facilities, having in mind the International Linear Collider (ILC) [25], the Future Circular Collider in electron mode (FCC-ee) [26] and the Circular Electron Positron Collider (CEPC) [27]. The final results of our study are presented in Section V, and conclusions are given in Section VI. More details on the $s$-tagger will be provided in a companion paper [28].

II. LIGHT FLAVOR TAGGING

Flavor tagging relies on the idea that the flavor of a parton that belongs to the hard part of the event is correlated with properties of the collection of final-state hadrons originating from that parton. This collection may be the hadrons in a jet or any other part of the event, e.g., a hemisphere defined by a particular axis. We use the term jet as a generic umbrella term for both, with jet flavor referring to the flavor of the primary parton of the collection. The process of determining jet flavor is referred to as tagging.

While QCD respects flavor, it does impact the particle composition of a jet, thus affecting jet flavor tagging. For example, flavor-blind gluon splitting can result in a quark appearing inside the jet while its partner antiquark does not, resulting in the creation of flavor that is differential in phase space [29]. In the case of heavy quarks, this can be mitigated perturbatively by modifying the clustering algorithm used to construct the jet. This is done either by undoing the flavor creation by gluon splitting [29] or by allowing the jet to form a higher representation (beyond fundamental) of the SM global flavor group [30].

In this paper, we attempt to tackle the same issue for the light-flavor quarks. The price to pay is that we must give up some perturbative control and hence calculability. Specifically, the jet-flavor variable that we define below is fundamental) of the SM global flavor group [30].

The dynamics that sets the flavor in our case can be very roughly characterized by two distinctive scales. The first is a hard, perturbative scale given by the Higgs mass, and the second is the soft hadronization scale $\Lambda_{QCD}$. In calculations and simulations of jet properties, the evolution from the hard to the soft scale, referred to as showering, is described via perturbation theory. At the end of the evolution, fragmentation functions (FFs) are used to determine in a statistical way the flavor composition of the event. As in the case of parton distribution functions, the FFs cannot be calculated from first principles and must be extracted from data (see, e.g., Ref. [31] for a recent review). Since plans for future lepton colliders include high-luminosity runs on the $Z$-pole, improved measurements of the FFs at the large energy scale will be available for studies of $h \rightarrow jj$ at these facilities. Similarly, at higher collision energies, where $WW$ production is kinematically accessible, the FFs can be measured in the decay of the $W$ bosons, albeit with lower statistics.

While the processes of showering and hadronization degrades the flavor-tagging capability, some of the correlation between the primary parton and the final-state hadrons remains. To exploit this, we define a new jet-flavor variable,

$$J_F = \frac{\sum_H \tilde{p}_H \cdot \hat{s} R_H}{\sum_H \tilde{p}_H \cdot \hat{s}}. \quad (5)$$

Here, the sum is over all hadrons $H$ inside the jet, $\tilde{p}_H$ is the momentum vector of the hadron, $R_H$ is its quantum number or numbers in the flavor representation of interest, and $\hat{s}$ is the normalized jet axis. In our case of a Higgs boson produced approximately at rest and undergoing a $h \rightarrow jj$ decay, $|\sum \tilde{p}_H| \approx m_h/2$, so the denominator is nonzero, and $J_F$ is well defined.

As our focus here is on strangeness tagging, in our computations below we use only the SU(3) flavor composition for evaluating $R_H$. For further simplicity, we consider only pions and kaons, which constitute the majority of final-state hadrons. Since $m_{u,d} \ll m_s \sim \Lambda_{QCD}$, the SU(3) flavor symmetry is broken in a rather strong way by the strange mass. Therefore, it is enough to consider only strangeness, assigning $R_{K^\pm} = \pm 1$ and $R_H = 0$ for $H = \pi^\pm, \pi^0$. The presence of neutral kaons weakens the ability to perform strangeness tagging for two reasons. First, it is impossible to determine whether a neutral kaon has strangeness 1 or $-1$. Second, only the $K_S$ decays in a clearly distinguishable manner, while the $K_L$ gives rise to calorimeter energy deposits that are not unique. Therefore, a reasonable, minimal-bias approach is to assign $R_{K_S} = 1$ or $-1$ to each $K_S$ in the jet, choosing the set of values that minimizes the value of $|J_s|$.

Let us consider several cases to investigate the usefulness of $J_s$ for studying $h \rightarrow ss$.

- In the simple and rare case in which the $s$ ($\bar{s}$) fragments to just a $K^-$ ($K^+$) that carries all of the momentum, the $K^\pm$ would have $J_s = \pm 1$.
- When soft gluon emission gives rise to production of soft hadrons, their small weights $\tilde{p}_H \cdot \hat{s}$ result in relatively little impact on $J_s$.
- The case of collinear gluons emitted by the strange quark gives rise to two effect. The first is reduction of the momentum of the otherwise hard kaon that originates from the strange quark, which decreases the absolute value of $J_s$. The second involves the case in which a collinear gluon splits into an $ss$ pair. If the two resulting kaons have similar values of $\tilde{p}_H \cdot \hat{s}$ and opposite values of $R_K$, the impact on $J_s$ will be small. However, a larger shift in $J_s$ will occur if one kaon is much harder than the other, if only one of the two kaons is identified, or if only one kaon is associated to the jet by the jet reconstruction algorithm. Since these effects are strangeness-blind, the average effect is again reduction of the absolute value of $J_s$. Thus, $J_s$ is not safe against collinear emission. Consequently, it cannot be calculated from first principles, and its distribution for different jet flavors needs to be extracted from experiment.
S. TAGGER FOR $h \rightarrow s\bar{s}$

Next, we explore the problem of identifying strange jets in $h \rightarrow s\bar{s}$ events. For this purpose we use simulated samples, as described in Section III A. In Section III B we discuss the event reconstruction and the flavor-tagging algorithm used for this study, referred to as the s-tagger. The s-tagger uses a simplified version of $J_s$, and further takes advantage of the long lifetimes of charm and bottom hadrons to suppress $h \rightarrow c\bar{c}$ and $h \rightarrow b\bar{b}$ background. While the background considered here arises only from other $h \rightarrow jj$ decays, the effect of the non-$h \rightarrow jj$ background is discussed in Section IV.

A. Simulated event samples

Electron-positron colliders proposed for detailed study of the Higgs boson [25, 26] are designed to operate around a center-of-mass energy $\sqrt{s} = 250$ GeV, where the cross section for associated Higgs production, $e^+e^- \rightarrow Zh$, peaks at $\sigma \approx 210$ fb for unpolarized beams [38, 39]. We therefore adopt this scenario and use it for generating signal samples. Another Higgs production mode at this energy is $W^+W^-$-fusion, $e^+e^- \rightarrow hh\nu\bar{\nu}c$. However, this process contributes only a few percent to the total Higgs production cross section and is thus not included in our simulation.

We use PYTHIA version 8.219 to generate, decay and shower $e^+e^- \rightarrow Zh$ events. As a cross check, we also generate events with Herwig version 7.1.4, and find good agreement in the relevant kinematic distributions [28].

For the purpose of studying background from hadronic $W$ decays in Sec. IV, we also generate a sample of $e^+e^- \rightarrow W^+W^-$ events using PYTHIA version 8.219.

B. Candidate reconstruction and selection

Our study is based on truth-level information, without full detector simulation. In order for the estimate to be more realistic, we make several assumptions about the reconstruction performance, as described below.

We assume that in a $e^+e^- \rightarrow Zh$ event, the final-state hadrons originating from the Higgs decay are correctly associated to it. Therefore, all particles from the $Z$ decay are ignored at the truth level. This approximation is especially justified since we consider the invisible decay mode of the $Z$ boson later on.

As we saw in the previous section, kaons are the crucial particles for distinguishing strange-quark jets from other light jets. For the purpose of evaluating our $s$-tagger, we make the following assumptions about kaon reconstruction.

Neutral kaons. Among the neutral kaons and their decays, only $K_S^0$ mesons decaying into two charged pions can be efficiently reconstructed. This decay has a branching fraction of about 69% and is easily identified from the two pion tracks that form a displaced vertex. We require that the $K_S^0$ decay takes place at a (3-dimensional) distance of between 5 mm and 1 m from the $e^+e^-$ interaction point (IP) of the event. The lower cutoff removes prompt background, and the upper one ensures that the decay occurs sufficiently deep in the detector for the pion tracks to be well reconstructed. Optimizing the cut values based on an actual detector is expected to yield relatively small changes in the efficiency. For $K_S$ mesons satisfying these requirements, we assume a total reconstruction efficiency of 85%.
Charged kaons. We assume that a charged pion or kaon is reconstructed with an average efficiency of 95%. We further take into account the possibility that the detector has a particle identification (PID) system, to discriminate pions from kaons. For simplicity, we suppose that the PID observable is normally distributed with unit width, and that the two distributions are separated by $2\sqrt{2}$. In order to achieve this separation in the relevant momentum range, the energy deposited by a particle per unit length, $dE/dx$, needs to be measured with a resolution of 5-6%. This is consistent with the anticipated $dE/dx$ resolution of the (ILD) [40], one of the proposed designs for a detector at the ILC. Within this simplified approach, choosing a PID efficiency for kaons automatically determines the mis-ID rate for pions. E.g., for a high PID efficiency for kaons, $\epsilon_{K^\pm} = 95\%$, the mis-ID rate for pions is $\epsilon_{\pi^\pm} \approx 12\%$. In the following we use “kaon” to refer both to actual kaons and to pions that are misidentified as kaons.

The $s$-tagger employed for this study uses the reconstructed $K^\pm$ and $K_S$ mesons in a simplified application of $J_s$. We divide the event into two hemispheres in the Higgs rest frame, defined by the direction of the momenta $p_{K^\pm}$ and $p_K^0 = -p_{K^\pm}$ of the two Higgs daughter partons. In hemisphere $i = 1, 2$, the direction $\hat{s}$ is set to $p_{K^\pm}$, and the value of $p_{K^\pm} = \hat{p}_K \cdot \hat{s}$ is calculated for each kaon in its respective hemisphere. We have verified that $p_{K^\pm}$ is generally well aligned with the sphericity axis. In each hemisphere we select one kaon, so that the sum of the $p_{K^\pm}$ values of the two kaons is maximal while excluding kaon combinations for which the total charge is $\pm 2$. The use of only the hardest kaon stems from the expectation that this kaon dominates the calculation of $J_s$. This approach was also employed in the DELPHI [23] and SLD [24] studies of $Z \rightarrow s\bar{s}$ decays. The event is rejected if no kaon pair fulfilling the charge requirement is found. The value of $p_{K^\pm}$ in each hemisphere is then used in the $s$-tagger as a measure of jet strangeness.

The events that pass these cuts are categorized according to the charges of the two kaons as charged-charged (CC), charged-neutral (CN), or neutral-neutral (NN). Given that for neutral kaons we use only $K_S^0 \rightarrow \pi^+\pi^-$, the relative production abundances of the three event categories are CC : CN : NN $\approx 9 : 6 : 1$. Hence it is clear that the CC channel is most promising, whereas the other channels will be less sensitive, particularly given the smaller reconstruction efficiency of the $K_S$ and the fact that the sign of its strangeness cannot be determined. Therefore, for simplicity, in what follows we use only the CC channel.

Higgs decays to bottom and charm quarks are a significant background in the study of $h \rightarrow s\bar{s}$, warranting special consideration. In addition to using $p_{K^\pm}$, we take advantage of the fact that bottom and charm hadrons have sufficiently large lifetimes that their decays occur visibly away from the IP. Consequently, a charged-kaon track produced in a heavy-flavor decay has a non-vanishing transverse impact parameter $d_0$ with respect to the IP. By contrast, in a strange jet the kaon selected by the above criteria is usually produced promptly. Taking advantage of this, we further suppress the heavy-quark background by requiring a small $d_0$ value for charged kaons. This requirement cannot be applied for a $K_S^0$, since its detector signature occurs far from the IP, resulting in poor $d_0$ resolution.

Since we do not fully simulate the detector response, we account for the $d_0$ measurement resolution by smearing the true value of $d_0$ for a given track with a Gaussian distribution. The Gaussian width is taken to be $\sigma_{d_0} = \sqrt{\Delta^2_{IP} + \Delta^2_{d_0}}$, where $\Delta_{IP}$ is the uncertainty in the location of the IP, and $\Delta_{d_0}$ is the uncertainty in the location of the track. We parameterize $\Delta_{d_0}$ as a function of the kaon track momentum $p$ and polar angle $\theta$ in the laboratory frame [41],

$$\Delta_{d_0} = \sqrt{(5 \text{ \mu m})^2 + \left(\frac{10 \text{ GeV}}{p \sin^{3/2} \theta} \text{ \mu m}\right)^2}. \quad (6)$$

At ILC, the beamspot size is of order tens of nanometers [42], so that $\Delta_{IP}$ is negligible. By contrast, at FCC-ee the beamspot size in the x-direction will be 38 $\mu$m [43], necessitating determination of the IP position in every event. This can be done using the fragmentation tracks produced in the Higgs decay (excluding the kaons for which $d_0$ is to be determined). Eq. (6) suggests that the average value of $\Delta_{d_0}$ for a 1 GeV fragmentation track is of order 20 to 30 $\mu$m. Assuming that about 15 to 25 Higgs-daughter fragmentation tracks are used in the IP-position determination, this implies $\Delta_{IP} \sim 10 \mu$m. When the $Z$ decays visibly, its daughter tracks can also be used to improve the measurement of the IP position. For the sake of simplicity, we take $\Delta_{IP} = 5 \mu$m for all events. Fig. 2 shows the obtained normalized distributions after requiring $p_{K^\pm} > 10$ GeV for both of the kaons. As discussed below, this momentum cut is found to be approximately optimal for a luminosity of 5 ab$^{-1}$.

Heavy-flavor background can be further reduced by vetoing a kaon if it is part of a jet that contains additional...
large-$d_0$ tracks or a multitrack vertex that is displaced from the IP in a manner consistent with charm or bottom decay. Algorithms relying on such signatures are regularly used to select heavy-flavor jets (see, e.g., Ref. [44]). Implementing these algorithms as a heavy-flavor veto involves experimental aspects that are beyond the scope of our analysis. In particular, we note that the hard-kaon requirement tends to select bottom and charm hadrons that undergo few-body decays, necessitating careful study of veto performance as a function of $p_T$ rather than relying on studies reported in the literature for generic decays. In addition, a veto on leptons within the jet, as done, for example, in charm-jet tagging [45, 46], could be used to somewhat improve the rejection of heavy-flavor jets. Further improvements to background rejection can be achieved by using additional observables, as jet-shape variables and variants of particle multiplicity. Since we do not implement these additional algorithms, our results on the ability of the s-tagger to reject heavy-flavor jets can be taken as conservative.

IV. NON-\(h \to jj\) BACKGROUND

After defining the parameters of the s-tagger for \(h \to s\bar{s}\) decays, we now prepare to apply it to a realistic lepton-collider scenario. Since \(e^+e^- \to Zh\) is the dominant Higgs production mode of interest, the decay modes of the \(Z\) boson determine the selection strategy. The best results are obtained for \(Z\) decays into neutrino pairs \([47, 48]\). This channel has a relatively large branching fraction of 20\% and a clean, missing-energy signature that provides good rejection of non-Higgs background and Higgs decays into non-\(jj\) final states. Therefore, for the following estimates we consider only the \(Z \to \nu\bar{\nu}\) decay mode.

We rely on previous studies of \(h \to jj\) for rejection of non-\(h \to jj\) background. A detailed study for ILC, which assumed a Higgs mass of \(m_h = 120\) GeV, appears in Ref. [47]. That analysis employed a cut-and-count technique, in which the signal and background yields were counted after applying selection criteria. By using modern machine-learning techniques and accounting for the larger mass separation between the Higgs and the electroweak gauge bosons, improved background rejection can be achieved. Ref. [48] contains an estimate of the performance of such an analysis, using a boosted decision tree (BDT) at the CEPC collider. Similar results are given in the CEPC conceptual design report [49]. A similar study has been performed for CLIC [50].

The rejection of non-\(h \to jj\) events was studied in all three references, their aim was to measure the relatively large branching ratios of \(h \to gg, bb, cc\), whereas here we study the much rarer process \(h \to s\bar{s}\). Therefore, one expects that optimizing the methods employed in these studies for \(h \to s\bar{s}\) will lead to somewhat better background rejection than what we assume here.

These studies used a modular approach to first separate \(h \to jj\) from all non-\(h \to jj\) events, and then apply a flavor tag on the selected signal-rich sample. We refer to these two sets of cuts as preselection and flavor cuts, respectively, and refer to their combination as final event selection. Adopting the same modular approach, we assume that the preselection cuts are orthogonal to the flavor cuts. The most effective preselection cuts, in the sense that they achieved the best rejection of non-\(h \to jj\) background, were obtained in Ref. [48]. This reference also provides the most detailed information on the composition of the events after the preselection cuts. Starting from this event composition, we use the s-tagger selection efficiency for each event type to determine the composition of the sample after the final event selection.

Table I lists the dominant non-\(h \to jj\) background event types and their percentages after the preselection cuts in Ref. [48]. The main background is \(e^+e^- \to W^+W^-\), where one \(W\) decays into \(\nu\bar{\nu}\), thus generating missing energy and mass, and the other \(W\) decays hadronically, which we assume to fake the Higgs.

A. s-tagger efficiencies for non-\(h \to jj\) background

In what follows we explain how we obtain the s-tagger efficiency for the various non-\(h \to jj\) event types.

For the \(e^+e^- \to W^+W^-\) background with one \(W\) decaying hadronically, we obtain the s-tagger efficiency from the simulated \(e^+e^- \to W^+W^-\) sample, with one \(W\) kept stable and the other \(W\) decaying hadronically. The efficiency for the remaining backgrounds that do not have an actual Higgs boson is taken from the \(e^+e^- \to Zh\) simulation, accounting for the flavor composition the background as given in Table II. In the case of background from Higgs decays to non-\(jj\) final states, the dominant decays have the following branching fractions:

\[
B(h \to WW^*)B(W \to qq^*) \approx 14\% \quad B(h \to ZZ^*)B(Z \to qq^*) \approx 1.8\% \quad \text{(7)}
\]

\[
B(h \to \tau\tau)B(\tau \to \text{hadrons})^2 \approx 2.6\% .
\]

Observing that \(h \to WW^*\) decays constitute the majority of this type of background, we take their contribution to be 100\%, for simplicity. Since the selected hard kaon is more likely to originate from the decay of the on-shell \(W\), the performance of the s-tagger obtained for the \(e^+e^- \to WW^*\) background is also assumed for the \(h \to WW^*\) background. In principle, this procedure may be biased due to the fact that the momentum distribution of the partons in \(e^+e^- \to Zh, h \to WW^*\) differs from the one in \(e^+e^- \to WW\). To check this, we generate \(e^+e^- \to WW\) events at \(\sqrt{s} = 200\) GeV and find the s-tagger performance to be very similar to that of \(e^+e^- \to WW\) at \(\sqrt{s} = 250\) GeV. This validates the use of the same s-tagger efficiency for \(e^+e^- \to WW\) and \(h \to WW^*\).
TABLE I. The dominant background processes in the analysis of $e^+e^- \rightarrow Zh$ with $h \rightarrow jj$ and $Z \rightarrow \nu\bar{\nu}$, as obtained in Ref. [48]. Each column shows the primary processes produced in the $e^+e^-$ collision, the final state following the primary-particle decays, and the fraction of such events in the non-$h \rightarrow jj$ background. Final states that are not shown contribute less than 1% each.

<table>
<thead>
<tr>
<th>Final state</th>
<th>$WW$</th>
<th>$Z(Z+\gamma^*)$</th>
<th>$Zh+\nu\bar{\nu}$</th>
<th>$Z(Z+\gamma^*)$</th>
<th>$Zh$</th>
<th>$Z\nu$</th>
<th>$WW$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction [%]</td>
<td>$47.1$</td>
<td>$18.0$</td>
<td>$13.7$</td>
<td>$12.2$</td>
<td>$2.7$</td>
<td>$2.5$</td>
<td>$2.0$</td>
</tr>
</tbody>
</table>

TABLE II. Relative composition of the hadronic part of the non-$h \rightarrow jj$ event that is assumed to fake the $h \rightarrow jj$ candidate. The values are based on the entries of Table I. $W$ refers to the case where a $W$ boson is falsely identified as Higgs, in both $e^+e^- \rightarrow WW$ and $h \rightarrow non-jj$ events. The other compositions stem mainly from $Z/\gamma^* \rightarrow q\bar{q}$.

<table>
<thead>
<tr>
<th>Higgs candidate</th>
<th>$W$</th>
<th>$bb$</th>
<th>$uu$</th>
<th>$dd$</th>
<th>$cc$</th>
<th>$ss$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction [%]</td>
<td>$65.3$</td>
<td>$9.8$</td>
<td>$6.1$</td>
<td>$6.0$</td>
<td>$6.4$</td>
<td>$6.0$</td>
</tr>
</tbody>
</table>

V. RESULTS

The results of our $s$-tagger study are summarized in Fig. 3 as a function of the number of non-$h \rightarrow jj$ events ($N_{non-jj}$) vs. the number of $h \rightarrow jj$ events ($N_{jj}$) obtained after the preselection but before the flavor cuts. To be precise,

$$N_{jj} = L \sigma_h B(h \rightarrow jj) \epsilon_{jj},$$

where $L$ is the integrated luminosity, $\sigma_h$ is the production cross section for $e^+e^- \rightarrow \nu\bar{\nu}h$ via both $Zh$ and $WW$-fusion, $B(h \rightarrow jj)$ is the total branching fraction for $h \rightarrow jj$ processes, and $\epsilon_{jj}$ is the efficiency for such an event to pass the preselection criteria. Similarly, the number of non-$h \rightarrow jj$ events after preselection is

$$N_{non-jj} = L \sum_{i \in non-jj} \sigma_i \epsilon_i,$$

where the sum is over all the non-$h \rightarrow jj$ processes, $\sigma_i$ is the cross section for the process, including relevant branching fractions, and $\epsilon_i$ is the efficiency for events of the process to pass the preselection criteria. Fig. 3 shows diagonal lines of constant $N_{jj}/N_{non-jj}$, with the values of this ratio being those obtained using the cut-and-count [47] and BDT [48] techniques. On these lines, we show the points corresponding to integrated luminosities of $0.25$, $5$, and $50$ ab$^{-1}$.

We scan the points in the plot plane and each value of $N_{jj}$, we determine the number of $h \rightarrow jj$ background events for the dominant decays $h \rightarrow b\bar{b}, c\bar{c}$, and $gg$ given the SM branching fractions [51]. We also determine the number of $h \rightarrow ss$ signal events, taking $B(h \rightarrow ss) = B(h \rightarrow cc) (m_s/m_c)^2 \approx 2.3 \times 10^{-4}$, where we evaluate the charm and strange quark masses at the Higgs mass scale using RunDec version 3.0 [52].

Given this composition of the $h \rightarrow jj$ events and that of the non-$h \rightarrow jj$ events shown in Table II, we evaluate the number of signal and background events as a function of the cuts on $d_0$, $\epsilon_{K_S}$, and $p_T$. We then select the cuts that yield the strongest upper limit on the signal strength $\mu_{ss}$ at 95% confidence level, and present the upper limits as contours in Fig. 3. These limits do not include systematic uncertainties. However, given the small number of signal events, the analysis is statistics limited.

For small values of the ratio $N_{jj}/N_{non-jj}$—around and above the cut-and-count line—very loose cuts that effectively switch off the $s$-tagger yield the strongest, yet weak limits on $\mu_{ss}$. For the larger values of $N_{jj}/N_{non-jj}$ obtained by the BDT analysis [48], using the $s$-tagger becomes beneficial in helping improve the limit on the signal strength. In both points indicated by the crosses on the BDT line, the best cuts are around $d_0 \gtrsim 18 \mu m$, $p_T \gtrsim 10$ GeV, and $\epsilon_{K_S} \approx 96\%$. With this working point, an upper limit on the signal strength of $\mu_{ss} < \mathcal{O}(15)$ and $\mathcal{O}(5)$ is obtained for integrated luminosities of $5$ and $50$ ab$^{-1}$, respectively. The limit is weakened to $\mu_{ss} < \mathcal{O}(75)$ for an integrated luminosity of $250$ fb$^{-1}$.

VI. CONCLUSION

In this paper we assessed the possibility to constrain the strange Yukawa coupling at future lepton colliders.
For this purpose we first introduced the notion of light-jet flavor by defining a new observable $J_F$, based on the idea that the hard hadrons in a jet are correlated with the flavor of the initial light parton. We showed that the distribution of $J_F$ does capture this correlation and hence allows to separate different light-jet flavors. The $J_F$ observable is robust against soft radiation but not against collinear emission. We then constructed a strangeness-tagging algorithm based on a simplified version of $J_F$ combined with very simple heavy-flavor hadron rejection. This $s$-tagger relies on $dE/dx$-based kaon identification that can be obtained with a particle tracker, the kaon momentum, and the impact parameter of the kaon track. We find that a 95% confidence-level upper limit on $\mu_{ss}$ of order 75, 15, or 5 is within reach at a future collider with an integrated luminosity of 0.25, 5, or 50 ab$^{-1}$, respectively. By comparison, the estimated limit from the HL-LHC with 3 ab$^{-1}$, obtained by rescaling the current limit in Eq. (4) by the ratio of luminosities, is $\mu_{ss}^{HL-LHC} \lesssim 9.4 \times 10^9$. Thus, even with a simple tagging algorithm, a very large improvement can be obtained at a lepton collider, and a high-luminosity lepton collider is sensitive to the strange Yukawa coupling at the level of several times the SM prediction.

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