Storage Ring to Search for Electric Dipole Moments of Charged Particles

Feasibility Study

F. Abusaif,1 A. Aggarwal,2 A. Aksentev,19 B. Alberdi-Esuain,3 A. Atanasov,6 L. Barion,4 S. Basile,4 M. Berz,5 M. Beyß,3 C. Böhme,1 J. Böker,1 J. Borburgh,6 C. Carli,6 I. Ciepał,7 G. Ciullo,4 M. Contalbrigo,4 J.-M. De Conto,5 S. Dymov,14 O. Felden,1 M. Gagosidze,10 M. Gaisser,3 R. Gebel,1 N. Giese,1 K. Grigoryev,1 D. Grzonka,1 M. Haj Tahar,6 T. Hahnraiths,1 D. Heberling,11 V. Hejny,1 J. Hetzel,1 D. Hölscher,3 O. Javaakhishvili,10 L. Jorat,6 A. Kacharava,4 V. Kamerdzhev,1 S. Karanth,2 C. Käseberg,3 I. Keshelashvili,1 I. Koop,12 A. Kulikov,9 K. Laihem,3 M. Lamont,6 A. Lehrach,1 P. Lenisa,4 N. Lomidze,13 B. Lorentz,1 G. Macharashvili,13 A. Magiera,2 K. Makino,5 S. Martin,1 D. Mchedlishvili,13 U.-G Meißner,1,6,14,25 Z. Metreveli,10 J. Michaud,8 F. Müller,3 A. Nass,3 G. Natour,15 N. Nikolaev,16 A. Nogga,1,6 A. Pesce,4 V. Poncza,1 D. Prasuhn,1 J. Pretz,1 F. Rathmann,1 J. Ritman,1 M. Rosenthal,6 A. Saleev,1 M. Schott,18 T. Sefzick,1 Y. Senichev,19 D. Shergelashvili,13 V. Shmakova,1 S. Siddique,3 A. Silenko,20 M. Simon,1 J. Slim,3 H. Soltner,15 A. Stahl,3 R. Stassen,1 E. Stephenson,21 H. Straatmann,15 H. Ströher,1 M. Tabidze,13 G. Tagliente,22 R. Talman,23 Yu. Uzikov,9 Yu. Valdau,1 E. Valetov,5 T. Wagner,1 C. Weidemann,1 A. Wirzba,1,6 A. Wrońska,2 P. Wüstner,15 P. Zupranski,24 M. Żurek,26

Abstract: The proposed method exploits charged particles confined as a storage ring beam (proton, deuteron, possibly helium-3) to search for an intrinsic electric dipole moment (EDM) aligned along the particle spin axis. Statistical sensitivities could approach $10^{-29}$ e.cm. The challenge will be to reduce systematic errors to similar levels. The ring will be adjusted to preserve the spin polarisation, initially parallel to the particle velocity, for times in excess of 15 minutes. Large radial electric fields, acting through the EDM, will rotate the polarisation. The slow rise in the vertical polarisation component, detected through scattering from a target, signals the EDM.

The project strategy is outlined. It foresees a step-wise plan, starting with ongoing COSY activities that demonstrate technical feasibility. Achievements to date include reduced polarisation measurement errors, long horizontal-plane polarisation lifetimes, and control of the polarisation direction through feedback from the scattering measurements. The project continues with a proof-of-capability measurement (precursor experiment; first direct deuteron EDM measurement), an intermediate prototype ring (proof-of-principle; demonstrator for key technologies), and finally the high precision electric-field storage ring.

CERN-PBC-REPORT-2019-002
CPEDM Collaboration
December 2019
Acknowledgements

The authors would like to acknowledge important discussions with, and significant contributions by, Yannis K. Semertzidis and his colleagues from the Center for Axion and Precision Physics (CAPP) of the Korean Advanced Institute of Science and Technology (KAIST, South Korea) to this study; appendices B and F were authored by them.

This report is supported by an ERC Advanced Grant of the European Commission (srEDM, #694340, PI Hans Ströher (Forschungszentrum Jülich, Germany)). We also acknowledge the support for Transnational-Access-to-COSY within HORIZON2020 (STRONG-2020, #824093) by the European Union.

This research was supported in part by the DFG and the NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” (NSFC Grant No. 11621131001, DFG Grant No. TRR110) and by the National Science Foundation under Grant No. NSF PHY-1125915.

The work of N. Nikolaev was partly supported by the Russian MOS program (No. 0033-2019-0005). The Georgian collaborators acknowledge support by the Shota Rustaveli National Science Foundation of the Republic of Georgia, SRNSFG grant No. #217854, “A first-ever measurement of the Electric Dipole Moment (EDM) of the deuteron at COSY”.

Contents

Acronyms and abbreviations viii

Executive Summary 1

1 Introduction 13
  1.1 Project Scope ........................................ 13
  1.2 Key Accomplishments .................................. 14
  1.3 European and global context ............................ 14
  1.4 Contents of the report by chapter ....................... 15
  1.5 Special Appendices ..................................... 16
  1.6 Appendices Describing New Ideas ...................... 17
  1.7 Final Comments ......................................... 17

2 Physics Case for CPEDM 19
  2.1 Introduction ........................................... 19
     2.1.1 Current experimental bounds ......................... 19
     2.1.2 Scientific potential of a proton EDM measurement . 20
  2.2 Dimensional analysis .................................... 21
     2.2.1 Naive EDM estimate based on known physics .......... 21
     2.2.2 BSM scale estimate ................................ 22
     2.2.3 Estimate of the strong CP-violating QCD \bar{\theta} parameter . 23
  2.3 EDM analysis based on non-perturbative methods ......... 23
     2.3.1 Determination of the \bar{\theta} induced nucleon EDM .... 23
     2.3.2 Estimates of the nucleon EDM terms in the BSM scenario . 24
     2.3.3 Estimates of the nuclear EDM matrix elements for light nuclei 25
  2.4 Option for oscillating EDM searches at storage rings .... 26

3 Historical Background 31
  3.1 Beginnings at Brookhaven National Laboratory (BNL, USA) . 31
  3.2 Continuation at the Forschungszentrum Jülich (FZJ, Germany) . 31
  3.3 Ongoing activity: the precursor experiment at COSY ........ 33
  3.4 Charged-particle EDM Initiative and experience of the collaboration . 33
  3.5 Further developments .................................... 34
  3.6 Summary ................................................ 34

4 Experimental Method 35
  4.1 Introduction ........................................... 35
  4.2 Spin evolution in electric and magnetic fields .......... 35
  4.3 The storage ring EDM search ............................ 36
     4.3.1 Frozen spin concept ................................ 37
     4.3.2 Dual beam operation ............................... 38
8.1 Introduction – BNL design ................................................. 77
8.2 Preparedness for the full-scale ring .................................. 80
8.3 New ideas ................................................................. 82

9 Electric Fields .............................................................. 83
9.1 Assumptions and boundary conditions ............................. 83
9.2 Electrode material ....................................................... 83
9.3 Ring elements ........................................................... 84
9.3.1 Main dipoles ......................................................... 84
9.3.2 Quadrupole .......................................................... 87
9.3.3 Injection equipment ................................................. 88
9.4 Required R&D ........................................................... 90
9.5 Summary ................................................................. 90

10 Sensitivity and Systematics ............................................. 93
10.1 Statistical Sensitivity .................................................... 93
10.2 Systematic Effects ...................................................... 93
10.2.1 Recap of the proposal .............................................. 94
10.2.2 Sources for systematic Errors and general Comments ......... 96
10.2.3 Radial magnetic Field leading to a systematic Error proportional to the Perturbation ...... 97
10.2.4 Second order Effects .............................................. 99
10.2.5 Summary ........................................................... 103

11 Polarimetry ................................................................. 106
11.1 Introduction to Polarimetry .......................................... 106
11.2 Polarimeter Spin Formalism ......................................... 106
11.3 Beam Preparation ...................................................... 107
11.4 Main Ring Polarimeter Design Goals .............................. 110
11.5 Implementation of the Polarimeter ................................. 111
11.6 Choice of analysing reaction ........................................ 113
11.7 Target operation in a storage ring .................................. 115
11.8 Development of calorimeter detectors for an EDM polarimeter ........................................ 116
11.9 Use of the polarimeter to maintain frozen spin .................. 118
11.10 Correction of rate and geometry errors in the polarimeter ....... 120
11.11 Polarimeter rotations, energy loss, and deuteron tensor polarisation effects ..................... 122
11.12 Time-reversed experiment ......................................... 123

12 Spin Tracking ............................................................... 126
12.1 Introduction ............................................................. 126
12.2 Simulation Programs ................................................... 126
12.3 Status and Plans ........................................................ 127
12.4 Spin tracking Simulations for Deuteron and Proton EDM Measurements ......................... 127
12.4.1 Precursor Experiment for Deuterons at COSY ................. 127
### 12.4.2 Proton EDM Storage Ring

#### 13 Roadmap and Timeline

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.1 CPEDM Strategy</td>
<td>136</td>
</tr>
<tr>
<td>13.2 Timeline</td>
<td>137</td>
</tr>
</tbody>
</table>

### Appendices

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Results and achievements at Forschungszentrum Jülich</td>
<td>139</td>
</tr>
<tr>
<td>A.1 Results and achievements at COSY</td>
<td>139</td>
</tr>
<tr>
<td>A.1.1 High precision spin tune measurements</td>
<td>139</td>
</tr>
<tr>
<td>A.1.2 Long horizontal polarisation lifetime</td>
<td>140</td>
</tr>
<tr>
<td>A.1.3 Feedback and control of polarisation</td>
<td>140</td>
</tr>
<tr>
<td>A.1.4 Invariant spin axis measurements</td>
<td>141</td>
</tr>
<tr>
<td>A.1.5 Radio-Frequency Wien filter for spin manipulation</td>
<td>141</td>
</tr>
<tr>
<td>A.1.6 Measurements of deuteron carbon and proton carbon analysing powers</td>
<td>143</td>
</tr>
<tr>
<td>A.1.7 Orbit control</td>
<td>143</td>
</tr>
<tr>
<td>A.1.8 Beam Based Alignment</td>
<td>143</td>
</tr>
<tr>
<td>A.1.9 Beam Position Monitor</td>
<td>144</td>
</tr>
<tr>
<td>A.1.10 Electrostatic and combined Deflector development</td>
<td>144</td>
</tr>
<tr>
<td>A.1.11 &quot;Spin-Offs&quot;</td>
<td>147</td>
</tr>
<tr>
<td>A.2 Results and achievements from the Jülich/Bonn theory group</td>
<td>148</td>
</tr>
<tr>
<td>B Mitigation of background magnetic fields</td>
<td>153</td>
</tr>
<tr>
<td>B.1 Static magnetic field configurations</td>
<td>153</td>
</tr>
<tr>
<td>B.1.1 Static radial magnetic field</td>
<td>153</td>
</tr>
<tr>
<td>B.1.2 Static longitudinal magnetic field</td>
<td>156</td>
</tr>
<tr>
<td>B.1.3 Static vertical magnetic field</td>
<td>157</td>
</tr>
<tr>
<td>B.2 Effect of alternating magnetic fields and the geometric phases</td>
<td>157</td>
</tr>
<tr>
<td>B.3 Magnetic shielding</td>
<td>159</td>
</tr>
<tr>
<td>B.3.1 Residual field</td>
<td>160</td>
</tr>
<tr>
<td>B.3.2 Time stability of the residual field</td>
<td>160</td>
</tr>
<tr>
<td>B.4 Summary</td>
<td>161</td>
</tr>
<tr>
<td>C Statistical Sensitivity</td>
<td>164</td>
</tr>
<tr>
<td>C.1 Statistical error on EDM</td>
<td>164</td>
</tr>
<tr>
<td>C.2 Precursor Experiment</td>
<td>167</td>
</tr>
<tr>
<td>C.3 Summary</td>
<td>167</td>
</tr>
<tr>
<td>D Gravity and General Relativity as a ‘Standard Candle’</td>
<td>169</td>
</tr>
<tr>
<td>E Additional Science Option: Axion Search</td>
<td>172</td>
</tr>
<tr>
<td>E.1 Concept of Search for Axion-like Particles</td>
<td>172</td>
</tr>
<tr>
<td>E.2 Technical Considerations for an Axion Search</td>
<td>173</td>
</tr>
</tbody>
</table>
H.5 Spin wheel at the beam energy of frozen spin ............................................. 202
H.6 Other possibilities for EDM measurements with countercirculating beams ........ 203
H.6.1 An option with two RF Wien filters in the prototype EDM ring .................. 203
H.6.2 An option with static Wien filters in the prototype EDM ring .................... 203
H.7 Summary and outlook ............................................................................. 204

I New ideas: Deuteron EDM Frequency Domain Determination ........................................ 205
I.1 Motivation ............................................................................................... 205
I.2 Universal SR EDM measurement problems ............................................. 206
I.2.1 Spin motion perturbation ..................................................................... 206
I.2.2 Expected machine imperfection SW roll rate ....................................... 207
I.2.3 Spin decoherence ................................................................................ 208
I.2.4 Machine imperfections ........................................................................ 208
I.3 Main methodology features ..................................................................... 209
I.4 EDM estimator statistic .......................................................................... 209
I.5 Effective Lorentz factor ......................................................................... 210
I.6 Guide field flipping ................................................................................ 211
I.7 Statistical precision ................................................................................. 212

J New ideas: Distinguishing the effects of EDM and magnet misalignment by Fourier analysis .......................................................... 214

K New ideas: External Polarimetry ................................................................. 218
K.1 Pellet-extracted beam sampling ............................................................... 218
K.1.1 Pellet-extracted beam sampling; qualitative ........................................ 218
K.1.2 Experimental confirmation of wire and pellet beam extraction at COSY ...... 221
K.1.3 Re-interpretation and revision of COSY moving wire beam experiments ..., 223
K.1.4 Quantitative formulation of pellet beam sampling ............................... 224
K.1.5 Derivations of required formulas ......................................................... 225
### Acronyms and abbreviations

<table>
<thead>
<tr>
<th>Letter</th>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td></td>
<td>ALP</td>
<td>Axion-like Particle</td>
</tr>
<tr>
<td></td>
<td>ANKE</td>
<td>Name of detector at COSY</td>
</tr>
<tr>
<td>B</td>
<td>BMT</td>
<td>Bargmann-Michel-Telegdi (equation)</td>
</tr>
<tr>
<td></td>
<td>BNL</td>
<td>Brookhaven National Laboratory</td>
</tr>
<tr>
<td></td>
<td>BPM</td>
<td>Beam Position Monitor</td>
</tr>
<tr>
<td></td>
<td>BSM</td>
<td>Beyond Standard Model (of elementary particle physics)</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>Charge (symmetry)</td>
</tr>
<tr>
<td></td>
<td>CAPP</td>
<td>Center for Axion and Precision Physics (Research) (Daejeon, South Korea)</td>
</tr>
<tr>
<td></td>
<td>CCW</td>
<td>Counter Clock-Wise</td>
</tr>
<tr>
<td></td>
<td>CDR</td>
<td>Conceptual Design Report</td>
</tr>
<tr>
<td>CeNTREX</td>
<td>Name of experiment to search for proton EDM in Tl-nuclei</td>
<td></td>
</tr>
<tr>
<td>CERN</td>
<td></td>
<td>Conseil Européan pour la Recherche Nucléaire</td>
</tr>
<tr>
<td>CESR</td>
<td></td>
<td>Cornell Electron-Positron Storage Ring</td>
</tr>
<tr>
<td>ChPT</td>
<td></td>
<td>Chiral Perturbation Theory</td>
</tr>
<tr>
<td></td>
<td>CKM</td>
<td>Cabibbo-Kobayashi-Maskawa (matrix)</td>
</tr>
<tr>
<td></td>
<td>COSY</td>
<td>Cooler Synchrotron (storage ring) (Forschungszentrum Jülich, Germany)</td>
</tr>
<tr>
<td></td>
<td>CP</td>
<td>Charge-parity (invariance)</td>
</tr>
<tr>
<td></td>
<td>CPEDM</td>
<td>Charged Particle Electric Dipole Moment (collaboration)</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>Charge-parity-time reversal (symmetry)</td>
</tr>
<tr>
<td></td>
<td>CSR</td>
<td>Cryogenic Storage Ring (Max-Planck Institute, Heidelberg, Germany)</td>
</tr>
<tr>
<td></td>
<td>CW</td>
<td>Clock-Wise</td>
</tr>
<tr>
<td>D</td>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td></td>
<td>DESY</td>
<td>Deutsches Elektronen Synchrotron (Hamburg, Germany)</td>
</tr>
<tr>
<td></td>
<td>DORIS</td>
<td>Name of a detector at DESY</td>
</tr>
<tr>
<td>E</td>
<td>EDM</td>
<td>Electric Dipole Moment</td>
</tr>
<tr>
<td></td>
<td>ELENA</td>
<td>Extra Low Energy Antiproton (ring) (CERN)</td>
</tr>
<tr>
<td>F</td>
<td>FNAL</td>
<td>Fermi National Accelerator Laboratory (Chicago, USA)</td>
</tr>
<tr>
<td></td>
<td>FRM-II</td>
<td>Forschungsreaktor München (Heinz Maier-Leibnitz, München, Germany)</td>
</tr>
<tr>
<td>G</td>
<td>G</td>
<td>Magnetic anomaly</td>
</tr>
<tr>
<td></td>
<td>GR</td>
<td>General Relativity</td>
</tr>
<tr>
<td>H</td>
<td>HGF</td>
<td>Helmholtz-Gemeinschaft Deutscher Forschungszentren</td>
</tr>
<tr>
<td>I</td>
<td>IBS</td>
<td>Institute for Basic Science (South Korea)</td>
</tr>
<tr>
<td></td>
<td>IKP</td>
<td>Institut für Kernphysik (Institute for Nuclear Physics of FZJ) (Jülich, Germany)</td>
</tr>
<tr>
<td></td>
<td>ILL</td>
<td>Institut Laue-Langevin (Grenoble, France)</td>
</tr>
<tr>
<td></td>
<td>ISOLDE</td>
<td>Isotope Separator On Line Device (CERN)</td>
</tr>
<tr>
<td>J</td>
<td>JEDI</td>
<td>Jülich Electric Dipole moment Investigations (collaboration)</td>
</tr>
<tr>
<td></td>
<td>J-PARC</td>
<td>Japan Proton Accelerator Research Complex (Tokai, Japan)</td>
</tr>
<tr>
<td></td>
<td>JULIC</td>
<td>Jülich Light Ion Cyclotron (FZJ, Germany)</td>
</tr>
<tr>
<td>K</td>
<td>KAIST</td>
<td>Korea Advanced Institute of Science and Technology (South Korea)</td>
</tr>
<tr>
<td></td>
<td>KM</td>
<td>Kobayashi-Maskawa (mixing matrix)</td>
</tr>
<tr>
<td></td>
<td>KVI</td>
<td>KVI - Center for Advanced Radiation Technology (Groningen, The Netherlands)</td>
</tr>
<tr>
<td>L</td>
<td>LANL</td>
<td>Los Alamos National Laboratory (Los Alamos, USA)</td>
</tr>
<tr>
<td></td>
<td>LC</td>
<td>Inductance-Capacitor</td>
</tr>
<tr>
<td>M</td>
<td>MDM</td>
<td>Magnetic Dipole Moment</td>
</tr>
</tbody>
</table>
Executive Summary

Science context and objectives
Symmetry considerations and symmetry-breaking patterns have played an important role in the development of physics in the last 100 years. Experimental tests of discrete symmetries (e.g., parity $P$, charge-conjugation $C$, their product $CP$, time-reversal invariance $T$, the product $CPT$, baryon- and/or lepton number) have been essential for the development of the Standard Model (SM) of particle physics.

Subatomic particles with nonzero spin (regardless whether of elementary or composite nature) can only support a nonzero permanent electric dipole moment (EDM) if both time-reversal ($T$) and parity ($P$) symmetries are violated explicitly while the charge symmetry ($C$) can be maintained (see e.g. [2]). Assuming the conservation of the combined $CPT$ symmetry, $T$-violation also implies $CP$-violation. The $CP$-violation generated by the Kobayashi-Maskawa (KM) mechanism of weak interactions contributes a very small EDM that is several orders of magnitude below current experimental limits. However, many models beyond the Standard Model predict EDM values near the current experimental limits. Finding a non-zero EDM value of any subatomic particle would be a signal that there exists a new source of $CP$ violation, either induced by the strong $CP$ violation via the $\theta_{QCD}$ angle or by genuine physics beyond the SM (BSM). In fact, the best upper limit on $\theta_{QCD}$ follows from the experimental bound on the EDM of the neutron. $CP$ violation beyond the SM is also essential for explaining the mystery of the observed baryon-antibaryon asymmetry of our universe, one of the outstanding problems in contemporary elementary particle physics and cosmology. A measurement of a single EDM will not be sufficient to establish the sources of any new $CP$-violation. Complementary observations of EDMs in multiple systems will thus prove essential. Up to now measurements have focused on neutral systems (neutron, atoms, molecules). We propose to use a storage ring to measure the EDM of charged particles.

The storage ring method would provide a direct measurement of the EDM of a charged particle comparable to or better than present investigations on ultra-cold neutrons. The neutron investigations measure the precession frequency jumps in traps containing magnetic and electric fields as the sign of the electric field is changed. These experiments are now approaching sensitivities of $10^{-26}$ e·cm [3] and promise improvements of another order of magnitude within the next decade. Because proton beams trap significantly more particles, statistical sensitivities may reach the order of $10^{-29}$ e·cm [4] with a new, all-electric, high-precision storage ring. Indirect determinations for the proton produce model-dependent EDM limits near $2 \times 10^{-25}$ e·cm using $^{199}$Hg [5]. Thus storage rings could take the lead as the most sensitive method for the discovery of an EDM.

It should be noted that the rotating spin-polarised beam used in the EDM search is also sensitive to the presence of an oscillating EDM resulting from axions or axion-like fields, which correspond to the dark-matter candidates of a pseudo-scalar nature. These may be detected through a time series analysis of EDM search data or by scanning the beam’s spin-rotation frequency in search of a resonance with an axion-like mass in the range from $\mu$eV down to $10^{-24}$ eV [6, 7].

Methodology
The electric dipole must be aligned with the particle spin since it provides the only axis in its rest frame. The EDM signal is based on the rotation of the electric dipole in the presence of an external electric field that is perpendicular to the particle spin. The particles are formed into a spin-polarised beam. Measurements are made on the beam as it circulates in the ring, confined by the ring electromagnetic fields that always generate an electric field in the particle frame pointing to the centre of the ring.

For a particle propagating in generic magnetic $\vec{B}$ and electric $\vec{E}$ fields the spin motion is described...
by the Thomas-Bargmann-Michel-Telegdi (Thomas-BMT) equation and its extension for the EDM [8]:

$$\frac{d\vec{S}}{dt} = \left(\vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}}\right) \times \vec{S},$$  \hspace{1cm} (1)

$$\vec{\Omega}_{\text{MDM}} = \frac{-q}{m} \left[ \left( G + \frac{1}{\gamma} \right) \vec{B} - \frac{\gamma G}{\gamma + 1} \vec{\beta} \cdot \vec{B} - \left( G + \frac{1}{\gamma + 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right],$$

$$\vec{\Omega}_{\text{EDM}} = -\frac{\eta q^2}{2mc} \left[ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{\beta} \left( \vec{\beta} \cdot \vec{E} \right) + c\vec{\beta} \times \vec{B} \right].$$

The angular velocities, \(\vec{\Omega}_{\text{MDM}}\) and \(\vec{\Omega}_{\text{EDM}}\), act through the magnetic dipole moment (MDM) and electric dipole moment (EDM) respectively. \(\vec{S}\) in this equation denotes the spin vector in the particle rest frame, \(t\) the time in the laboratory system, \(\vec{\beta}\) and \(\gamma\) the relativistic Lorentz factors. The magnetic anomaly \(G\) and the electric dipole factor \(\eta\) are dimensionless and introduced via the magnetic dipole moment \(\vec{\mu}\) and electric dipole moment \(\vec{d}\), which are both pointing in the same direction and are proportional to the particle’s spin \(\vec{S}\):

$$\vec{\mu} = g \frac{q h}{2m} \vec{S} = (1 + G) \frac{q h}{m} \vec{S}, \hspace{1cm} \vec{d} = \eta \frac{q h}{2mc} \vec{S},$$  \hspace{1cm} (2)

where \(q\) and \(m\) are the charge and the mass of the particle, respectively.

The effect of the torque for a positively charged particle is illustrated in Fig. 1. In this example the magnetic and electric fields are purely vertical and purely radial, respectively. A particle is confined on an ideal planar, closed orbit in the ring. Its velocity \(\vec{v} = c\vec{\beta}\) is along the orbit. The spin axis is given by the purple arrow that rotates in a plane perpendicular to \(\vec{E}\). If the initial condition begins with the spin parallel to the velocity, then the rotation caused by the EDM will make the vertical component of the beam polarisation change. This rotation receives a contribution from both the external field \(\vec{E}\) and the motional electric field \(c\vec{\beta} \times \vec{B}\), and becomes the signal observed by a polarimeter located in the ring. This device allows beam particles to scatter from nuclei in a fixed bulk material target (black). The difference in the scattering rate between the left and right directions (into the blue detectors) is sensitive to the vertical polarisation component of the beam. Continuous monitoring will show a change in the relative left-right rate difference during the time of the beam storage if a measurable EDM is present.

Fig. 1: Diagram showing a particle travelling around the storage ring confined by purely vertical magnetic and purely radial electric fields. The polarisation, initially along the velocity, precesses slowly upward in response to the radial electric field acting on the EDM. The vertical component of this polarisation is observed through scattering in the polarimeter.

The angular velocities \(\vec{\Omega}\) in Eq. (1) describe the rotation of the spin vector of the particle as it travels around the ring. Because the magnetic moments of all particles carry an anomalous part, the polarisation will in general rotate in the plane of the storage ring relative to the beam path. This rotation

\[More details on the application of the Thomas-BMT equation for circular accelerators and the inclusion of gravity effects are discussed in Chap. 4.\]
must be suppressed by matching $\vec{\Omega}_{\text{MDM}}$ to the cyclotron frequency

$$\vec{\Omega}_{\text{cycl}} = -\frac{q}{\gamma m} \left( \vec{B}_\perp - \frac{\vec{\beta} \times \vec{E}_r}{\beta^2 c} \right),$$  \hspace{1cm} (3)$$

i.e. $\vec{\Omega}_{\text{MDM}} = \vec{\Omega}_{\text{cycl}}$, a condition called “frozen spin”. Under this condition, the vertical polarisation can build up. In a magnetic ring, this condition requires that (since $\vec{\beta} \cdot \vec{B} = 0$) a radial electric field is added to the ring bending elements with

$$E_r = \frac{GB\beta \gamma^2}{1 - G\beta^2 \gamma^2}.$$  \hspace{1cm} (4)$$

For particles such as the proton where $G > 0$, it is also possible to build an all-electric ring ($\vec{B} = 0$) provided that one can choose $\gamma = \sqrt{1 + 1/G}$. For the proton, this gives $p = 0.7007$ GeV/c. The kinetic energy of $T = 232.8$ MeV fortuitously comes at a point where the spin sensitivity of the polarimeter is near its maximum (e.g., carbon target), creating an advantageous experimental situation.

The statistical error for one single machine cycle is given by [4]

$$\sigma_{\text{stat}} \approx \frac{2\hbar}{\sqrt{Nf\tau PAE}}.$$  \hspace{1cm} (5)$$

Assuming the parameters given in Table 2, the statistical error for one year of running (i.e., 10000 cycles of 1000 s length) is

$$\sigma_{\text{stat}}(1\text{ year}) = 2.4 \times 10^{-29} \text{ e} \cdot \text{ cm}.$$  \hspace{1cm} (6)$$

The challenge is to suppress the systematic error to the same level.

Table 2: Parameters relevant for the statistical error in the proton experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam intensity</td>
<td>$N = 4 \cdot 10^{10}$ per fill</td>
</tr>
<tr>
<td>polarisation</td>
<td>$P = 0.8$</td>
</tr>
<tr>
<td>spin coherence time</td>
<td>$\tau = 1000$ s</td>
</tr>
<tr>
<td>electric fields</td>
<td>$E = 8$ MV/m</td>
</tr>
<tr>
<td>polarimeter analyzing power</td>
<td>$A = 0.6$</td>
</tr>
<tr>
<td>polarimeter efficiency</td>
<td>$f = 0.005$</td>
</tr>
</tbody>
</table>

Many of the systematic errors in the EDM search may be eliminated by looking at the difference between two experiments run with clockwise (CW) and counter-clockwise (CCW) beams in the ring. One beam represents the time-reverse of the other, and the difference will show only time-odd effects such as the EDM. For the proton, the choice of an all-electric ring allows the two beams to be present in the ring at the same, an advantage when suppressing systematic effects. Figure 2 illustrates two features of the all-electric proton experiment, the counter-rotating beams and the alternating direction of the polarisation (along or against the velocity) in separate beam bunches, which is important for geometric error cancellation in the polarimeter.

In general any phenomena other than an EDM generating a vertical component of the spin limits the sensitivity (i.e. the smallest detectable EDM) of the proposed experiment. Such systematic effects may be caused by unwanted electric fields due to imperfections of the focusing structure (such as the misalignment of components) or by magnetic fields penetrating the magnetic shielding or produced inside the shield by the beam itself, the RF cavity, or gravity. A combination of several such phenomena, or a combination of an average horizontal spin and one of these phenomena, may as well lead to such systematic effects.
Fig. 2: Electric storage ring with simultaneously clockwise and counter clockwise circulating beams (dark and light blue arrows), each with two helicity states (green and red arrows for each beam). The gray circles represent electric field plates.

In many cases, as for example effects due to gravity, the resulting rotations of the spin into the vertical plane do not mimic an EDM because the observations for the two counter-rotating are not compatible with a time-odd effect. In this case, the contributions from the two counter-rotating beams tend to cancel, provided the forward and reverse polarimeters can be calibrated with sufficient precision. In some cases, for example, magnetic fields from the RF cavity, the resulting spin rotations into the vertical plane can be large.

The most important mechanism dominating systematic effects is an average static radial magnetic field that mimics an EDM signal. For a 500 m circumference frozen-spin EDM ring, an average magnetic field of about $10^{-17}$ T generates the same vertical spin precession as the EDM of $10^{-29}$ e·cm the final experiment aims at being able to identify. In order to mitigate systematic effects, the proposed ring will be installed in state-of-the-art magnetic shielding that reduces residual fields to the nT level [4]. The vertical position difference between the two counter-rotating beams that is caused by the remaining radial field will be measured with special pick-ups that must be installed at very regular locations around the circumference to measure the varying radial magnetic field component created by the bunched beam separation. A complete, thorough study of systematic errors in the EDM experiment is very delicate and not yet available. Studies of systematic effects have been carried out and are underway by several teams in the CPEDM collaboration to further improve the understanding of basic phenomena to be taken into account and to estimate the achievable sensitivity. The preliminary conclusion is that the intended sensitivity is very challenging. Meeting this challenge requires that we proceed in a series of stages (see Fig. 3) where each one depends on the knowledge gained from the preceding stage’s experience.

Readiness and Expected Challenges

The JEDI (Jülich Electric Dipole moment Investigations) Collaboration has worked with COSY (COoler SYNchrotron at the Forschungszentrum Jülich in Germany) for the last decade to demonstrate the feasibility of critical EDM technologies for the storage ring. Historically, these studies were begun with deuterons, and the switch has not been made yet to protons in order to preserve and build on the deuteron experience. These studies are briefly itemised below.

- The beam may be slowly brought to thick ($\sim 2$ cm) target blocks for scattering particles most
efficiently into the polarimeter detectors. Favouring elastic scattering events yields the best polarimeter performance [9].

- After suitable calibration, a comparison of the left-right asymmetries for oppositely polarised beam bunches may be used to lower the polarisation systematic error below one part per million [9].
- Time marking polarimeter events [10] leads to an unfolding of the precession of the in-plane polarisation and the measurement of the spin tune \( \nu_S = G\gamma \), polarisation revolutions per turn) to a part in \( 10^{10} \) in a single cycle of 100 s length [11]. The polarimeter signals permit feedback stabilisation of the phase [12] of the in-plane precession to better than one part per billion \( (10^9) \) over the time of the machine store. This is necessary to maintain frozen spin.
- By using bunched beam, electron cooling, and trimming of the ring fields to sextupole order, the polarisation decoherence with time may be reduced, yielding a lifetime in excess of 1000 s [13].

Observation of the spin tune variations allows for the measurement of the direction of the invariant polarisation axis with a precision of about 1 mrad [14].

With deuterons in the COSY ring at 970 MeV/c with non-frozen spin, the polarisation precesses in the horizontal plane at 121 kHz relative to the velocity. The EDM is associated with a tilt of the invariant spin axis away from the vertical direction. This tilt of the spin rotation axis generates an oscillation of the vertical spin component. This effect is too small for a measurement with a reasonable sensitivity. However, using an RF Wien filter in the ring with fields oscillating synchronously with the spin precession in the horizontal plane, a vertical spin component builds up over the whole duration of the fill. This has become the basis for the precursor experiment (see Fig. 3, stage 1). Initial running reveals that EDM-like signals arise from systematic perturbations of the deuteron spin as it goes around COSY. These resemble the effects of small rotational misalignments of the Wien filter about the beam line and longitudinal polarisation changes induced by a solenoid located across the ring from the Wien filter. Using these two effects for reference, measurements can lead to the location of the invariant spin axis.

In parallel, studies and preparations for the proton EDM measurement in a fully-electric frozen-spin ring are on-going. Some of the key technologies are currently under development for the final ring. These include:

- electrostatic deflector design that requires testing full scale prototypes in a magnetic field with beam to levels of at least 8 MV/m;
- beam position monitors are needed to operate at a precision of 10 nm for a measurement time of 1000 s;
- the ring must be shielded to provide isolation from systematic radial magnetic fields to the nT level [4].

Spin tracking calculations are needed to verify the level of precision needed in the ring construction and the handling of systematic errors. For a detailed study during beam storage and buildup of the EDM signal, one needs to track a large sample of particles for billions of turns. The COSY-Infinity [15] and Bmad [16] simulation programs are utilised for this purpose. Given the complexity of the tasks, particle and spin tracking programs have been benchmarked and simulation results compared to beam and spin experiments at COSY to ensure the required accuracy of the results.

Finally, a strategy will be needed to verify any signal produced by the experiment after the CW-CCW subtraction through a series of critical tests and independent analyses.

When constructed, the proton EDM experiment will be the largest electrostatic ring ever built. It will have unique features, such as counter-rotating beams and strenuous alignment and stability requirements. It may also require stochastic cooling and weak magnetic focusing consistent with dual beam operation. Intense discussions within the CPEDM collaboration have concluded that the final ring cannot be designed and built in one step; instead, a smaller-scale prototype ring (see Fig. 3, stage 2) must be constructed to confirm and refine the following critical features.
– The ring stores high beam intensities for a sufficiently long time.
– Beam injection must allow for multiple polarisation states (longitudinal fore and aft, sideways for polarisation coherence monitoring) in both CW and CCW beams.
– The ring must circulate CW and CCW beams simultaneously, both horizontally polarised.
– The ring must support frozen spin.
– Magnetic shielding must operate to reduce the ambient magnetic fields (esp. radial) to suitable levels while allowing full operation of the ring high voltage, vacuum, monitoring, and control.
– Polarimeter measurements must be made for both CW and CCW beams using the same target. A second polarimeter is needed with independent beam extraction onto its target.
– Beam cooling (electron-cooling before injection, or stochastic) is required to reduce the beam phase space.

CPEDM Strategy
As emphasised above this challenging project needs to proceed in stages that are also outlined in Fig. 3.

1. COSY will continue to be used as long as possible for the continuation of critical R&D associated with the final experiment design. An important requirement is to test as many of the results as possible with protons where the larger anomalous magnetic moment leads to more rapid spin manipulation speeds.
2. The precursor experiment will be completed and analysed. Some data will be taken with an improved version of the Wien filter with better electric and magnetic field matching.
3. The next stage is to design, fund, and build a prototype ring (discussed in detail below) to address critical questions concerning the features of the EDM ring design. At 30 MeV, the ring with only an electric field can store counter-rotating beams, but they are not frozen spin. At 45 MeV with an additional magnetic field, the frozen spin condition can be met. But the magnetic fields also prevent the CW and CCW beams from being stored at the same time. Even so, an EDM experiment may be done with these two beams used on alternating fills.
4. Following step 3, the focus will be to create the final ring design, then fund and construct it.
5. Once the ring is ready, the longer term activity will be to commission and operate the final ring, improving it with new versions as the systematic errors and other experimental issues are understood and improved.

| 1 | Precursor Experiment | 2 | Prototype Ring | 3 | All-electric Ring |
|---|---|---|---|---|
| **dEDM proof-of-capability**<br>(orbit and polarization control; first dEDM measurement) | **pEDM proof-of-principle**<br>(key technologies, first direct pEDM measurement) | **pEDM precision experiment**<br>(sensitivity goal: $10^{-29} \text{ e cm}$) |
| - Magnetic storage ring  
- Polarized deuterons  
- d-Carbon polarimetry  
- Radiofrequency (RF) Wien-filter | - High-current all-electric ring  
- Simultaneous CW/CCW op.  
- Frozen spin control (with combined E/B-field ring)  
- Phase-space beam cooling | - Frozen spin all-electric (at $p = 0.7 \text{ GeV/c}$)  
- Simultaneous CW/CCW op.  
- B-shielding, high E-fields  
- Design: cryogenic, hybrid,… |
| Ongoing at COSY (Jülich) 2014 → 2021 | Ongoing within CPEDM 2017 → 2020 (CDR) → 2022 (TDR) Start construction > 2022 | After construction and operation of prototype > 2027 |

**Fig. 3:** Summary of the important features of the proposed stages in the storage ring EDM strategy.

Future scientific goals may include conversion of the ring to crossed electric and magnetic field...
operation so that other species besides the proton could be examined for the presence of an EDM. Analysis of the data may be made for signs of axions using a frequency decomposition and investigation of counter-rotating beams with different species used in novel EDM comparisons.

The prototype ring and the CPEDM stages 2 and 3 are host-independent. If the prototype is built at COSY, it would take advantage of the existing facility for the production of polarised proton (and deuteron) beams, beam bunching, and spin manipulation. COSY itself could be used for producing electron-cooled beams. It may also be built at another site (e.g., CERN) provided that a comparable beam preparation infrastructure is made available. In either case, the lattice design will mimic that of the high-precision ring in order to test as many features as possible on a smaller scale.

Details of the Prototype EDM ring

The prototype ring (PTR) will be small (circumference of 100 m) and operate in two modes (see stage 3 in Fig. 3 and Table 3). The ring will be as inexpensive as possible, consistent with being capable of achieving its goals. The first mode would operate with all-electric bending (at 30 MeV), a demonstration that such a concept works and may be used to demonstrate feasibility of the ring with simultaneous counter-rotation beams. The second would extend the operating range to 45 MeV with the addition of magnetic bending (air core). With this combination, frozen spins could be demonstrated for a proton beam, other spin manipulation tools developed, and a reduced-precision proton EDM value measured. Alternating fills in counter-rotating directions would allow cancellation of the average radial magnetic field \( \langle B_r \rangle \) that is the leading cause of systematic error (though with a large systematic error associated with the needed magnetic field reversal).

This section describes a starting-point lattice in terms of geometry, type and strength of the elements. The ring is square with 8 m long straight sections. The basic beam parameters are given in Table 3.

Table 3: Basic beam parameters for the Prototype ring

<table>
<thead>
<tr>
<th></th>
<th>( E ) only</th>
<th>( E, B )</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>kinetic energy</td>
<td>30</td>
<td>45</td>
<td>MeV</td>
</tr>
<tr>
<td>( \beta = v/c )</td>
<td>0.247</td>
<td>0.299</td>
<td></td>
</tr>
<tr>
<td>( \gamma ) (kinetic)</td>
<td>1.032</td>
<td>1.048</td>
<td></td>
</tr>
<tr>
<td>momentum</td>
<td>239</td>
<td>294</td>
<td>MeV/c</td>
</tr>
<tr>
<td>magnetic rigidity ( B \rho )</td>
<td>0.981</td>
<td></td>
<td>T·m</td>
</tr>
<tr>
<td>Electric field only</td>
<td>6.67</td>
<td></td>
<td>MV/m</td>
</tr>
<tr>
<td>Electric field ( E ) (frozen spin)</td>
<td>7.00</td>
<td></td>
<td>MV/m</td>
</tr>
<tr>
<td>Magnetic field ( B ) (frozen spin)</td>
<td>0.0327</td>
<td></td>
<td>T</td>
</tr>
</tbody>
</table>

Prototype ring requirements and goals

The foremost goal of the prototype ring is to demonstrate the ability to store enough protons (\( \sim 10^{10} \)) to be able to perform proton EDM measurements in an electric storage ring, recognising that some superimposed magnetic bending is likely to be necessary to meet this goal.

Since ultimate EDM precision will require simultaneously counter-circulating beams a prototype ring has to demonstrate the ability to store and control simultaneously two such beams.

Cost-saving measures in the prototype, such as room temperature operation, minimal magnetic shielding, and avoidance of excessively tight manufacturing and field-shape matching tolerances, are expected to limit the precision of any prototype ring EDM measurement. Nonetheless, data for reliable cost estimation and extrapolation of the systematic error evaluation to the full scale ring has to be obtained.
Prototype ring design

The lattice has fourfold symmetry, as shown in Fig. 4. The basic parameters for the prototype ring are given in Tables 4 and 5. The bending, for example for 45 MeV protons, is done by eight 45° electric/magnetic bending elements. The acceptance of the ring is to be 10 mm·mrad for 10^10 particles. The lattice is designed to allow a variable tune between 1.0 and 2.0 in the radial plane and between 1.6 and 0.1 respectively in the vertical plane.

<table>
<thead>
<tr>
<th>Table 4: Geometry</th>
<th>Table 5: Bend elements, 45 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td># B-E deflectors</td>
<td>Electric</td>
</tr>
<tr>
<td># arc D quads</td>
<td>gap between plates</td>
</tr>
<tr>
<td># arc F quads</td>
<td>plate length</td>
</tr>
<tr>
<td>quad length</td>
<td>electric field</td>
</tr>
<tr>
<td>straight length</td>
<td>total bending length</td>
</tr>
<tr>
<td>bending radius</td>
<td>total straight length</td>
</tr>
<tr>
<td>electric plate length</td>
<td>bend angle per unit</td>
</tr>
<tr>
<td>arc length (45°)</td>
<td>magnetic field</td>
</tr>
<tr>
<td>circumference total</td>
<td>current density</td>
</tr>
<tr>
<td>emittance ε_x = ε_y</td>
<td>1.0 π mm-mrad</td>
</tr>
<tr>
<td>acceptance a_x = a_y</td>
<td>10.0 π mm-mrad</td>
</tr>
<tr>
<td></td>
<td>gap between plates</td>
</tr>
<tr>
<td></td>
<td>plate length</td>
</tr>
<tr>
<td></td>
<td>electric field</td>
</tr>
<tr>
<td></td>
<td>total bending length</td>
</tr>
<tr>
<td></td>
<td>total straight length</td>
</tr>
<tr>
<td></td>
<td>bend angle per unit</td>
</tr>
<tr>
<td></td>
<td>magnetic field</td>
</tr>
<tr>
<td></td>
<td>current density</td>
</tr>
<tr>
<td></td>
<td>windings/element</td>
</tr>
</tbody>
</table>

The injector: Injection into the prototype ring will closely resemble injection into a nominal all-electric ring. In particular there will be an even number of bunches in each beam, with alternating sign polarisations, whether in single beam or counter-circulating beam operation. The injector for the prototype ring could be the electron-cooled beam from COSY or make use of equipment at CERN. The beams will be protons in the 30 to 45 MeV range, in a cooled phase space of 1 πmm-mrad, with the beams bunched into 2, 4, 6 or 8 bunches to be fed into counter-circulating beams in the prototype ring.

Injection into the prototype ring will be done using switching magnets distributing the beams into clockwise (CW) or counter clockwise (CCW) direction as sketched in Fig. 4. All beam bunches are transferred with vertical polarisation, either up or down.

Electric bends: The electrostatic deflectors consist of two cylindrically-shaped parallel metal plates with equal potential and opposite sign. With the zero voltage contour of electric potential defined to be the centre line of the deflector, the ideal orbit of the design particle stays on the centre line. The electrical potential vanishes on the centre line of the bends, as well as in drift sections well outside the bends. So the electric potential vanishes everywhere on the ideal particle orbit. With the electric potential seen by the ideal particle continuous at the entrance and exit of the deflector, its total momentum is constant everywhere (even through the RF cavity).

The designed ring lattice requires electric gradients in the range from 5 to 10 MV/m. This is more than the standard values for most accelerator deflectors separated by a few centimetres. Assuming 60 mm distance between the plates, to achieve such high electric fields we have to use high voltage power supplies. At present, two 200 kV power converters have been ordered for testing deflector prototypes. The field emission, field breakdown, dark current, electrode surface and conditioning will be studied using two flat electrostatic deflector plates, mounted on the movable support with the possibility of changing the separation from 20 to 120 mm. The residual ripple of power converters is expected to be in the order of ±10^{-5} at a maximum of 200 kV. This will lead to particle displacement on the order of millimetres. A smaller ripple or stability control of the system will be a dedicated task for investigations planned at the test ring facility.

Magnetic bends: The experiments require periodic reversals of the magnetic bending field to use sym-
The basic layout of the prototype ring, consisting of 8 dual, superimposed electric and magnetic bends; 3 families of quadrupoles (Focusing, Defocusing, and Straight-section); and four 8-m long straight sections. The total circumference is about 100 m. Injection lines for injecting counter-circulating beam are represented just as stubs. Costs given in the Addendum are restricted to just the prototype ring, which is truly site-independent. The possibly greater infrastructure costs associated with producing appropriately polarised beams are neither given nor site-independent.

Symmetry to suppress systematic deviations. The reversal of the magnetic field should be done with best possible reproducibility. This is why the magnetic field production will iron-free (see Fig. 5).

Other components: All quadrupoles will be electrostatic. Their design will follow the principles of the Heidelberg CSR ring [17]. Both DC and AC Wien filters and solenoids will be required for spin control. The RF cavity design is under study.

The requirement for the vacuum is mainly given by the minimum beam lifetime requirement of about 1000 s. The emittance growth in the ring caused by multiple scattering from the residual gas is 0.005 mm-mrad/s. At $10^{-12}$ Torr vacuum, the emittance at the beginning, assumed to be 1 mm-mrad, will have increased to 5 mm-mrad within 1000 s, assuming a nitrogen ($N_2$) partial pressure. This is about the cooling rate expected for stochastic cooling. (One notes in passing that stochastic cooling becomes impractical for very low tunes.) For such an ultra-high vacuum only cryogenic or NEG pumping systems can be used. Bake-out must be foreseen for either cryogenic or NEG systems.

The choice of NEG requires a beam pipe with a diameter of 300 mm over the full circumference of 100 m. This can easily be plated with the NEG material. We will then have an active area of $\approx 120 \text{ m}^2$ for the whole ring. The roughing speed will be about 5000 liter/s per meter of length of vacuum pipe.

There are beam position monitors located around the ring. A BPM is placed at the entrance and the exit of each bending unit. One BPM will be placed additionally in close connection to the quadrupoles in the straight sections. A new type of BPM, of Rogowski coil design [19], has been developed at the IKP of the FZ-Jülich. These pick-ups are presently in a development stage. The position resolution is measured to be 10 $\mu$m over an area with a diameter of about 90 mm. These BPMs require only a short beam insertion length of 60 mm and an offset-bias free response to counter-circulating beams.

All-electric storage ring

This document describes the vision of CPEDM culminating in the design, construction, and operation of a dedicated, high-precision storage ring for protons. Operating at the all-electric, frozen-spin momentum...
of 0.7 GeV/c, the signals from counter-rotating beams aim to measure the proton electric dipole moment with a sensitivity of $10^{-29}$ e·cm. The major challenge is the handling of all systematic errors to obtain an overall sensitivity of a similar size. The main source of systematic uncertainties will be due to any unknown or unidentified radial magnetic field acting through the much larger magnetic dipole moment and leading to a false EDM signal. The level at which this can be mitigated remains to be determined.

Invaluable results and experiences are expected from the intermediate step, the construction of a smaller, prototype ring. The attempts to examine the control of counter-rotating beams and study directly the conditions for frozen spin will have a huge impact on the detailed outline of the high-precision ring design.

The concept of an all-electric storage ring with extremely well-fabricated and aligned elements running two longitudinally polarised proton beams in opposite directions in the absence of significant magnetic fields serves as the current starting point. There are new ideas under development that offer the prospect of further mitigation of the systematic issues:

- A hybrid electric/magnetic ring [20] with magnetic focusing (in addition to electric deflector contributions) will change the electromagnetic environment in significant ways. Even in the presence of uncontrolled radial magnetic fields, this geometry offers at least one point at which the magnetic field vanishes. Beam-based alignment techniques will tend to find these points and place the beam there. This substantially relaxes the requirement that radial magnetic fields be made to nearly vanish. The magnetic focusing, however, does not produce counter-rotating beams with the same phase space profile. So periodic reversal of the magnetic focusing would be required to provide a set of signals that must be averaged to obtain an EDM value.

- It is possible to find pairs of unlike polarised beams for which the same superimposed electric and magnetic bending yields a frozen spin condition for both (e.g. protons and $^3$He) [21]. Since the two beams would not have the same revolution frequency, to circulate simultaneously they would run with appropriately different RF harmonic numbers. Though not yielding either EDM value directly, the resulting EDM difference will be independent of the (otherwise dominant) radial magnetic field systematic error. Any EDM signal differences would be interpreted as the presence of an EDM on at least one of the two beams.
Work on these concepts can proceed using the prototype ring with the possibility of yielding new physics results.

Prototype ring costs

Preliminary prototype ring cost estimates are given in Table 6. Many items are currently receiving R&D funding. The bend element high voltage supplies are presently under development. Neither building nor injection line costs are included. The accuracy of this cost estimation is preliminary. The magnetic bend equipment for the frozen spin experiments in a 2\textsuperscript{nd} stage will require additional costs for the magnets and a Wien Filter of about 7000 k€.

<table>
<thead>
<tr>
<th>component</th>
<th>cost [k€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>bends</td>
<td>9200</td>
</tr>
<tr>
<td>electric-quads</td>
<td>1700</td>
</tr>
<tr>
<td>vacuum</td>
<td>1800</td>
</tr>
<tr>
<td>pick-ups</td>
<td>400</td>
</tr>
<tr>
<td>control</td>
<td>2000</td>
</tr>
<tr>
<td>polarimeter</td>
<td>1200</td>
</tr>
<tr>
<td>RF equipment</td>
<td>300</td>
</tr>
<tr>
<td>sum machine</td>
<td>16600</td>
</tr>
</tbody>
</table>

Table 6: Summary of preliminary cost estimates for the prototype ring first stage.

Roadmap and Timeline

Fig. 6: GANTT chart of major activities and events for CPEDM.

A staged approach to the CPEDM project (outlined in Fig. 3 and expanded in detail in Fig. 6) is currently ongoing with work on the precursor experiment and feasibility studies. This is partially funded by an ERC Advanced Grant that runs until September 2021 (event 3). Meanwhile, this longer “Yellow Report” having been published will be followed by an Helmholtz-Gemeinschaft Deutscher Forschungszentren (HGF) evaluation (event 1) and preparation for the start of the new HGF funding period (event 2).
Since 2017, preparation has been underway on the design for a prototype electric and mixed field storage ring to verify CPEDM concepts that will appear in a CDR/TDR available for funding consideration by 2021. With approval, construction and commissioning of the prototype ring will begin. In parallel, experimental work at COSY would refocus on feasibility studies for proton beams. By the beginning of subsequent funding period, the first prototype results should show the best techniques for the all-electric full-energy ring. These will be the subject of another CDR/TDR study. If approved, efforts will switch to the construction and running of this new ring.

The storage ring EDM feasibility studies made so far show encouraging results. Handling systematic errors is the main challenge. The path to addressing this lies through the construction and operation of a small-scale prototype ring from which will come the design for the high-precision ring with the best sensitivity to new physics.

References
[16] For Bmad, see https://www.classe.cornell.edu/bmad.
Chapter 1

Introduction

1.1 Project Scope

An experiment is described to detect a permanent electric dipole moment (EDM) of the proton with a sensitivity of $10^{-29} \text{ e-cm}$ by using counter-rotating polarised proton beams at the “magic” momentum of 0.7007 GeV/c in an all-electric precision storage ring.

The science case for such a project is based on the fact that measurements of EDMs of fundamental particles provide “a unique, extraordinarily sensitive way to probe for a physical phenomenon of profound significance, [the] violation of microscopic time-reversal invariance” (F. Wilczek). Assuming CPT-symmetry conservation, T-violation implies violation of the combined CP-symmetry, one of the ingredients required for explaining the matter-antimatter asymmetry of our Universe. EDM searches are sensitive to new physics beyond the Standard Model of elementary particle physics at a scale of the order of 1000 TeV. Moreover, the storage ring technology will also allow a search for oscillating EDMs, which may be connected with axions or axion-like particles. The physics motivation is thus evident and well supported by the community.

The storage ring concept has been well developed over the years, including a detailed examination of the experimental method, required technologies and involved systematics. R&D has progressed in parallel on essential storage ring components such as electrostatic deflectors, beam instrumentation, magnetic shielding and polarimetry.

A good understanding of the key systematic errors has been achieved, and their potential constraints on the ultimate sensitivity of the storage ring approach have been quantified. The leading systematic uncertainty is due to a residual radial magnetic field interacting with the magnetic moment to mimic the EDM signal. A radial magnetic field will lead to a vertical separation of the counter-rotating beams. Measurement of this separation will provide a handle to mitigate this systematic.

The ultimate goal is to design, build, and operate an all-electric storage ring for protons at their magic momentum (0.7 GeV/c) with clockwise (CW) and counter-clockwise (CCW) longitudinally polarised beams to achieve a sensitivity of the order of $10^{-29} \text{ e-cm}$. To this end a number of ring lattice options have been developed. These options make reasonable assumptions about the achievable electric field, deflector size, instrumentation requirements etc., and have led to the adoption of a baseline ring design of some 500 m in circumference.

To fully confirm the validity of the approach, a small all-electric prototype ring is proposed. This would allow:

(i) to deploy and test key hardware components of the all-electric ring;
(ii) to verify that an intense proton beam can be stored for at least 1000 s;
(iii) to deploy and use beam instrumentation, like the polarimeter;
(iv) to demonstrate the ability to master key systematics via the use of counter-clockwise beams.

The prototype is seen as a key step in demonstrating the credibility of the full ring proposal. A baseline proposal for this prototype has been developed and foresees two phases (see chapter 7 “Prototype Ring”):

- Phase 1: All-electric ring for 30 MeV proton beams (CW and CCW)
- Phase 2: Combined E- and B-fields for 45 MeV proton beams (frozen spin) to allow a first pEDM measurement
It is expected that the prototype ring will provide invaluable information for the outline and design of the final ring.

1.2 Key Accomplishments

Significant insight into the storage ring EDM project and accompanying technological advances have been achieved in the past few years. These include early findings of the srEDM collaboration at BNL (USA) and contributions from KAIST (South Korea). Recently a new level of measurement-technology achievements has been reached by the JEDI collaboration working at the Cooler Synchrotron COSY at the Forschungszentrum Jülich (Germany). These achievements are enumerated below.

1. A deuteron beam polarisation lifetime near 1000 s in the horizontal plane of the magnetic storage ring COSY has been observed [1]. This long “spin coherence time” (SCT) was obtained through a combination of beam bunching, electron cooling, sextupole field corrections, and the suppression of collective effects through beam current limits. This record lifetime is required for a storage ring search for an intrinsic electric dipole moment on the deuteron and paves the way for similarly large SCT for protons.

2. A new method to determine the spin tune was established and tested [2]. In an ideal planar magnetic storage ring, the “spin tune” — defined as the number of spin precessions per turn — is given by 

\[ \nu_S = \gamma G \]  

(\gamma \) is the Lorentz factor, \( G \) the gyromagnetic anomaly). At 0.97 GeV/c, the deuteron spins coherently precess at a frequency of about 121 kHz in COSY. The spin tune was deduced from the up-down asymmetry of deuteron-carbon scattering. In a time interval of 2.6 s, the spin tune was determined with a precision of the order \( 10^{-8} \), and to \( 10^{-10} \) for a continuous 100 s accelerator cycle. This renders the new method a precision tool for accelerator physics; observing and controlling the spin motion of particles to high precision is again mandatory for the measurement of electric dipole moments of charged particles in a storage ring.

3. The successful use of feedback from a spin polarisation measurement to the revolution frequency of a 0.97 GeV/c bunched and polarised deuteron beam in COSY has been realised in order to control both the precession rate (\( \approx 121 \) kHz) and the phase of the horizontal polarisation component [3]. Real time synchronisation with a radio frequency (RF) solenoid made possible the rotation of the polarisation out of the horizontal plane, yielding a demonstration of the feedback method to manipulate the polarisation. In particular, the rotation rate shows a sinusoidal function of the horizontal polarisation phase (relative to the RF solenoid), controlled to within a one standard deviation range of \( \sigma = 0.21 \) rad. The minimum possible adjustment was 3.7 mHz out of a revolution frequency of 751 kHz, which changes the precession rate by 26 mrad/s. Such capability meets the requirement for the use of storage rings to look for an intrinsic electric dipole moment of charged particles.

4. Procedures have been developed and tested that allow for systematic errors in the measurement of the vertical polarisation component (that carries the EDM signal) to be corrected to a level below one part in \( 10^5 \) [4]. This requires a prior calibration of the polarimeter for rate and geometric error effects and the use of two opposite polarisation states in the measurement. The extra polarisation state allows for an independent estimate of the size of the systematic error. Such corrections may be made in real time. This meets the sensitivity requirement to measure the small vertical component polarisation changes expected in the EDM search.

Further details will be given in later chapters of this report.

1.3 European and global context

Permanent electric dipole moments are sought in various elementary and complex systems; the most recent experimental limits are given in the “Physics Motivation” Chapter 2 below. A rather complete list of the international EDM efforts can be found in [5].
Neutron EDM searches are conducted/under development/proposed at nuclear fission reactor facilities (ILL Grenoble, FRM-2 Munich, PNPI Gatchina) and spallation neutron sources (PSI Villigen, ESS Lund) in Europe as well as in the US and Canada (SNS Oak Ridge, LANL Los Alamos, TRIUMF, Vancouver). Molecular and atomic EDMs are sought by a numerous groups worldwide including projects at radioactive beam facilities such as ISOLDE (CERN). The muon EDM is measured as a by-product of $(g - 2)_\mu$ experiments at FNAL (Batavia, USA) and J-PARC (Tokai, Japan).

A new experiment, called CeNTREX, has recently been launched at Yale University (USA) to search for a deformation in the shape (nuclear Schiff moment) of the atomic nucleus $^{205}$Tl inside a thallium fluoride (TlF) molecule [6]. The experiment will be complementary to $^{199}$Hg and primarily sensitive to the proton EDM and $\theta_{QCD}$. It is expected that it will improve the indirect proton EDM limit of $2 \times 10^{-25}$ e·cm by more than one order of magnitude in the coming years.

Storage ring EDM searches for the proton and other light nuclei (deuteron, $^3$He) have been discussed for a few years (see “Background” Chapter 4). It is our strong belief that eventually a result for the pure proton system will be required to complement the free neutron EDM. This is currently pursued by the JEDI and CPEDM collaborations and constitutes the motivation for the present document.

1.4 Contents of the report by chapter

- The Executive Summary was prepared and submitted in December 2018 for the European Strategy for Particle Physics (ESPP) update to consider in their review of the storage ring EDM search along with other experimental programs in nuclear and high energy physics research. The Summary described the concept of the experiment, the strategic path forward that includes a prototype ring for further feasibility testing, and an outline of plans for the final EDM ring.

[2] The Physics Motivation begins with a summary of the status of other major searches for an EDM on various systems and discusses both measured and derived upper bounds. These are compared with what one would expect for an intrinsic EDM on the basis of a naïve dimensional analysis. But an EDM appears to be suppressed in nature, as illustrated by the small size of the $CP$-violating $\theta_{QCD}$ parameter. The size of this parameter may be estimated from the current limit on the EDM in the neutron. A brief mention is made of the possibility to search for axion-like particles through a search for an oscillating EDM with a frequency related to the axion mass.

[3] The Background chapter gives a summary of the history of the storage ring EDM search from the original ideas developed at the Brookhaven National Laboratory in the USA. The story is followed as it moved into feasibility testing at the COSY storage ring located at the Forschungszentrum Jülich in Germany. This led to the first direct measurement of an EDM upper limit for the deuteron using a Wien filter located on the storage ring. The section also explores the experience of the collaboration members and work being done on supporting technologies for the final EDM ring.

[4] The Experimental Method chapter describes the storage ring EDM search beginning with the most basic concept of the experiment and developing all of the essential ideas needed for this experiment. Various experimental possibilities are presented and considered. Essential formalism is shown for both the ring and the polarisation measurement. There is also a discussion of systematic effects and the challenge of managing all aspects of the experiment. This section is intended for the novice to EDM searches.

[5] The Strategy describes briefly the idea to build a prototype ring featuring both electric and mixed field designs so that more of the critical technologies for the EDM search may be demonstrated. This will lead to a design for the final ring.

[6] The Precursor Experiment at COSY adds an RF Wien filter to the existing COSY storage ring so that the cancellation of the EDM signal due to the in-plane rotation of the polarisation is broken. This allows in principle for a sensitivity to the deuteron EDM, which is the beam currently in use for EDM feasibility studies. A series of first measurements were completed in the fall of 2018 and the preliminary results are presented here.
The **Prototype Ring** chapter presents the detailed design considerations for the small ring to be built as a site for the continuation of feasibility studies for the final EDM experiment. The prototype will operate in two modes: (i) an all-electric setup that allows for the simultaneous storage of both clockwise and counter-clockwise travelling beams, and (ii) a combined electric and magnetic field ring that creates the conditions for frozen spin operation. In both cases, systematic errors may be studied in an environment that uses the same electric field structures that are proposed for the final EDM ring.

The **All-electric Proton EDM Ring** is described more fully in this chapter, based on the design for the lattice in the prototype ring. Ring specifications are provided. A table is included that shows the status and preparedness of various aspects of the projects.

The **Electric Fields** chapter reviews the status of various accelerator systems that will be needed for the EDM ring. Electric fields pose a particular challenge since the best experiment is associated with large field strength. This then generates requirements on voltage holding capability and the ability to suppress dark currents. Focusing and beam injection elements will also be needed, and the prototype ring offers a chance to develop various designs.

The **Sensitivity and Systematics** chapter describes in detail the considerations that lead to an expected statistical sensitivity reach of $10^{-29} \text{e cm}$ for charged particle EDMs in a storage ring. The main part of this section is devoted to an assessment of the size and nature of systematic effects that can mimic an EDM signal and means, such as simultaneous clockwise and counter-clockwise measurements, that may be used to cancel these errors.

The **Polarimetry** chapter begins with the items needed during the beam preparation phase of the experiment to verify the polarisation of the beam. For the main EDM ring, the polarimeter target (most likely carbon) and the detectors needed for making online polarisation measurements and cancelling the systematic errors associated with this process. The requirements of the polarimeter are explained and details given for the choice of detector acceptance in order to maximise the figure of merit of the device. Examples are provided for both the prototype and final EDM ring designs. This section also details the work so far accomplished in fashioning the calorimeter detectors and other event tracking hardware that will be needed. Details are provided on the use of the polarimeter as a device for maintaining the frozen spin operating condition in the EDM ring.

**Spin Tracking** consists of those calculations needed to describe the history of the polarised beam as it circulates in the EDM ring. It is also a testing laboratory where we can explore various sources of systematic error (e.g., magnet misalignment) and ways to mitigate it. This calls for reliably calibrated programs using well-understood techniques for treating electric and magnetic field effects.

The last formal chapter of the report covers a **Roadmap and Timeline**.

1.5 **Special Appendices**

[A] The **Results and Achievements at Forschungszentrum Jülich** covers (i) polarimetry, (ii) high precision tune measurements, (iii) long horizontal polarisation lifetime, (iv) feedback control of polarisation, (v) invariant spin axis measurements, (vi) RF Wien filter construction, (vii) reference data bases for deuteron and proton-induced reactions on carbon, (viii) progress in orbit measurement and control (ix) electrostatic deflector development, (x) EDM and axion theory, and (xi) spin tracking simulations.

[B] The **Mitigation of Background Magnetic Fields** chapter describes the influence of magnetic fields on the EDM experiment and why they need to be small. Stray magnetic fields need to be managed with shielding and perhaps some active elements. And the effects of any residual field need to be well understood. For injection and perhaps for spin manipulation, time-varying magnetic fields may be needed, so their effects on the experiment need to be explored.

[C] The appendix on **Statistical Sensitivity** gathers the derivations for the contribution of event col-
lection statistics to the final EDM result. Connections are shown to critical ring and experimental requirements.

[D] The appendix on **Gravity and General Relativity as a ‘Standard Candle’** contains the inclusion of gravity as an explicit item in the Thomas-BMT equation. From this the level of the signal from gravity acting on the beam may be estimated, opening the possibility of using it as a marker of sensitivity.

[E] The **Axion Search** appendix contains preliminary plans to use the rotating polarisation of the COSY beam to search for an oscillating EDM that is a possible signature of an axion-like particle in nature. The first experiment to develop this techniques was scheduled to run in April 2019.

### 1.6 Appendices Describing New Ideas

[F] The **Hybrid Scheme** addresses the problems of minimising the residual horizontal magnetic field in the all-electric storage ring by imposing a magnetic focusing system. This system along with beam-based alignment techniques draws the beam toward the point in each quadrupole where the field vanishes. This reduces the requirements on the elimination of the residual background field by orders of magnitude. This does break the symmetry between the CW and CCW rotating beams. Symmetry may be restored by operating with both focusing field polarities and averaging the results. Independent confirmation of this scheme is underway.

[G] The appendix on a **Doubly Magic EDM Measurement Method** describes the possibility for having CW and CCW beams being different polarised particle species (e.g. protons and $^3$He) circulating at different energies under “frozen spin” conditions for both beams. This would enable precise comparisons of EDM properties and magnetic moments across the two species.

[H] The **Spin Tune Mapping for EDM Searches** appendix explores a more generalised method of making EDM searches by replacing the requirement of “frozen spin” with corrections applied by a Wien filter mounted in the storage ring. This method may be generalised to allow comparison of multiple particle species.

[I] The **Frequency Domain** appendix introduces the notion of utilising “spin wheel” rotation of the polarisation about the horizontal transverse axis to obtain sensitivity to the magnetic and electric dipole contributions together. By measuring the frequency of the resulting rotation, a precise subtraction to obtain the EDM contribution becomes possible.

[J] The **EDM from Fourier Analysis** appendix explores the idea of separating EDM effects from systematic due to machine errors through the use of a Fourier analysis of the experimental signals.

[K] The **External Polarimetry** appendix addresses the problem that a block target located at the edge of the beam does not necessarily sample the polarisation across the full beam. This allows the effects of a polarisation distribution across the beam to become a systematic error in the results. The scheme presented in this appendix uses pellets dropped through the beam to extract a fraction of the beam into a channel branching from the main beam line where it strikes a large and thick polarimeter target that spans the entire beam profile. The efficiency for this scheme is expected to be comparable to the block target scheme used at COSY.

### 1.7 Final Comments

We would like to emphasise that this write-up is a status report of what has been achieved and what is known at the time of the editorial deadline (December 2019) – work is ongoing at COSY, CERN and other places towards the realisation of the storage ring EDM project.

### References

[6] See https://demillegroup.yale.edu/research (Demille Group, search for the electric dipole moment of the electron)
Chapter 2

Physics Case for CPEDM

2.1 Introduction

Both continuous and discrete symmetries combined with possible breaking patterns have been decisive for the development of physics in the last 100 years. This was exemplarily demonstrated by the construction of the Standard Model (SM) of particle physics. Measurements of sizes or limits with which discrete symmetries (as, e.g., parity \( P \), charge-conjugation \( C \), their product \( CP \), time-reflection invariance \( T \), the product \( CPT \), baryon- and/or lepton number) are respectively broken or conserved have been essential for this task in the second part of the last century. These tests currently play, and will continue to play, an essential role for constructing and identifying physics beyond the SM (BSM).

As it is the case for all stationary states of finite and parity–non-degenerate quantum systems, the ground-state of any of the known non-selfconjugate subatomic particles with nonzero spin\(^1\) (regardless of elementary or composite nature) can only support a nonzero permanent electric dipole moment (EDM), if both time-reflection (\( T \)) and parity (\( P \)) symmetries are violated explicitly, while the charge symmetry (\( C \)) can be maintained. Assuming the conservation of the combined \( CPT \) symmetry, \( T \) violation also implies \( CP \) violation.

The \( CP \) violation generated by the Kobayashi-Maskawa (KM) mechanism of weak interactions induces a very small EDM that is several orders of magnitude below current experimental limits. However, many models beyond the standard model predict EDM values near these limits. Hence, there is a window in which the search for nonzero EDMs corresponds to a search for \( CP \) violation beyond the weak interaction one. In fact, finding a non-zero EDM value for any subatomic particle (above the KM limit of the SM which experimentally is out of reach for the foreseeable future) will be a signal that there exists a new source of \( CP \) violation, either induced by the strong \( CP \) violation via the QCD $\theta$ angle\(^2\) or by genuine physics beyond the SM. The latter is essential for explaining – within the framework of the Big Bang and inflation – the mystery of the observed baryon-antibaryon asymmetry of our universe, one of the outstanding problems in contemporary elementary particle physics and cosmology.

2.1.1 Current experimental bounds

Over the years, the quest in improving the bounds of the permanent EDM of the neutron, \( d_n \), pioneered more than 60 years ago by the work of Purcell, Ramsey, and Smith [2], has served to rule out or, at least, to severely constrain many models of \( CP \) violation, demonstrating the power of sensitive null results. The current bound of the neutron EDM resulting from these efforts is

\[
|d_n| < 3.0 \times 10^{-26} \text{ e \cdot cm} \quad (90\% \text{ C.L.}) \ [3,4]
\]

which corresponds to \( |d_n| < 3.6 \times 10^{-26} \text{ e \cdot cm} \) at a 95% confidence upper limit [4]. As reported below, the prediction of the CKM matrix is at least four orders of magnitude smaller: \( |d_{n}^{\text{SM}}| \lesssim 10^{-30} \text{ e \cdot cm} \), see Chap. 2.2.1 for more details.

There are complimentary constraints from atomic and molecular physics experiments. Especially, the EDM bounds on paramagnetic atoms, e.g.,

\[
|d_{\text{Cs}}| < 1.4 \times 10^{-23} \text{ e \cdot cm} \quad (95\% \text{ C.L.}) \ [5],
\]

\(^1\)E.g. the $\rho^0$ and $\omega$ vector mesons are particles with nonzero spin. But as they are selfconjugate, i.e. their own antiparticles, they cannot possess an electric dipole moment, while the $\rho^\pm$ or the $K^*$ as non-selfconjugate vector mesons have this possibility.

\(^2\)Actually the best upper limit on this parameter of Quantum Chromodynamics follows from the experimental bound on the EDM of the neutron.
and the constraints from dipolar molecules and molecular ions indirectly lead to the following upper limits on the electron EDM:\footnote{Note that the EDMs of paramagnetic atoms and the $P$- and $T$-violating observables in polar molecules are dominated by system-dependent linear combinations of the electron EDM and the nuclear-spin–independent electron-nucleon interaction, which couples to the scalar current components of the pertinent nuclei. An extraction of an electron EDM value $d_e$ cannot independently be performed from the extraction of this semi-leptonic four-fermion interaction $C_7$, while the quoted $|d_n|$ bounds assume that the measured paramagnetic systems are saturated by the electron EDM alone. For more details on this issue, on further EDM bounds, and also on the analogous extractions of the $|d_n|$ and $|d_p|$ bounds of valence and core nucleons for diamagnetic atoms see, e.g., the reviews \cite{7, 8} and quoted references therein.}

\begin{align*}
|d_{205\text{Tl}}| &< 1.1 \times 10^{-24} \, e \cdot \text{cm} \ (95\% \ C.L.) \ [6, 7], \\
|d_{\text{e}}^{\text{YbF}}| &< 1.1 \times 10^{-27} \, e \cdot \text{cm} \ (90\% \ C.L.) \ [9], \\
|d_{\text{e}}^{\text{ThO}}| &< 1.1 \times 10^{-29} \, e \cdot \text{cm} \ (90\% \ C.L.) \ [10–12], \\
|d_{\text{e}}^{\text{HFF}^+}| &< 1.3 \times 10^{-28} \, e \cdot \text{cm} \ (90\% \ C.L.) \ [13].
\end{align*}

These bounds should be put into perspective since they are quite large compared to the prediction of the CKM mechanism in the SM: $|d_{\text{e}}^{\text{SM}}| \sim 10^{-14} \, e \cdot \text{cm}$, see, e.g., \cite{14}.

In contrast to the paramagnetic cases which are sensitive to the their electron clouds, in diamagnetic atoms the EDM-defining spin is carried by the pertinent nucleus. Corresponding upper limits on the EDMs of diamagnetic atoms are, e.g.,

\begin{align*}
|d_{\text{129Xe}}| &< 6.6 \times 10^{-27} \, e \cdot \text{cm} \ (95\% \ C.L.) \ [15], \\
|d_{\text{225Ra}}| &< 1.4 \times 10^{-23} \, e \cdot \text{cm} \ (95\% \ C.L.) \ [16], \\
|d_{\text{199Hg}}| &< 7.4 \times 10^{-30} \, e \cdot \text{cm} \ (95\% \ C.L.) \ [1].
\end{align*}

Because of Schiff screening, the indirect bounds on the neutron and proton EDM obtained by applying nuclear physics methods \cite{17} are much weaker than their parent atom bounds. From the currently best case, $^{199}\text{Hg}$, the following indirect bounds on the neutron and proton EDM could be derived \cite{1}:

\begin{align*}
|d_{n}^{199\text{Hg}}| &< 1.6 \times 10^{-26} \, e \cdot \text{cm} \ (95\% \ C.L.), \\
|d_{p}^{199\text{Hg}}| &< 2.0 \times 10^{-25} \, e \cdot \text{cm} \ (95\% \ C.L.).
\end{align*}

The indirect bound on $|d_p|$ is by a factor of 13 or 6 weaker than the indirect or direct $|d_n|$ counterparts, respectively, and therefore not really competitive.

The current status of the already excluded EDM regions derived from the experimental upper limits of the various particles mentioned above are summarised in Figure 2.1.

\subsection*{2.1.2 Scientific potential of a proton EDM measurement}

In this proposal, we discuss an experimental opportunity, provided by the storage ring technology, to push a direct measurement of the proton EDM to $10^{-28} e \cdot \text{cm}$ sensitivity, corresponding to an improvement by nearly 5 orders of magnitude. Such dramatic improvement is made possible by new ideas and techniques described in this document. Several new neutron EDM experiments involving ultra-cold neutrons (UCN) have already been started worldwide with the aim to eventually approach $|d_n| \sim 10^{-28} e \cdot \text{cm}$ sensitivity. Compared to that, the storage ring studies target a $|d_p|$ sensitivity more than an order of magnitude beyond $|d_n|$ expectations which are primarily limited by the achievable number of trapped UCNs. Such an improved sensitivity might be crucial in reaching the forefront of the underlying mechanisms behind baryogenesis and BSM-induced $CP$ violation. In view of the entirely unknown isospin properties of the latter, even at the lower $10^{-27 \sim 28} e \cdot \text{cm}$ sensitivity the proton EDM studies are complementary
2.1 Current status of excluded regions of electric dipole moments. Shown are direct and/or derived EDM bounds of the particles and a selection of atoms discussed in Chap. 2.1.1, and, additionally, the upper limit on the EDM of the muon, $|d_\mu| < 1.8 \times 10^{-19} \text{ e} \cdot \text{cm} (95\% \text{ C.L.})$, by the muon $(g - 2)$ collaboration [18].

Fig. 2.1: Current status of excluded regions of electric dipole moments. Shown are direct and/or derived EDM bounds of the particles and a selection of atoms discussed in Chap. 2.1.1, and, additionally, the upper limit on the EDM of the muon, $|d_\mu| < 1.8 \times 10^{-19} \text{ e} \cdot \text{cm} (95\% \text{ C.L.})$, by the muon $(g - 2)$ collaboration [18].

to the neutron ones and will be essential in discriminating between – or at least constraining – various mechanisms for baryogenesis or competing models of $CP$ violation, e.g., variants of supersymmetric (SUSY) models, multi-Higgs models, left-right symmetric ones, or the strong $CP$ violation from the QCD $\theta$ term. Note that a priori the results for $d_n$ and $d_p$ are independent and could have significantly different values. Only when interpreted within the context of a specific theoretical framework do their values become related and a comparison is meaningful. Even if $d_n$ is found to differ from zero, the measurement of $d_p$ (and perhaps additional measured EDMs of light nuclei, e.g., deuteron or helium, which might also be studied in a storage ring experiments) will prove crucial in unfolding the new source of $CP$ violation.

2.2 Dimensional analysis

2.2.1 Naive EDM estimate based on known physics

Because of its inherent $P$ and $CP$ violation, the upper limit on the permanent EDM ($d_N$) of the nucleon (i.e. neutron or proton) can be estimated [19] from the product of the $P$- and $CP$-conserving nuclear magnetic moment (approximated by the nuclear magneton $\mu_N = \frac{e}{2m_N} \sim 10^{-14} \text{ e} \cdot \text{cm}$) times a suppression scale counting the $P$ violation ($\sim G_F f_\pi^2 \sim 10^{-7}$ in terms of the Fermi constant $G_F$ and the axial decay constant of the pion $f_\pi$) times the additional $CP$-violating scale ($\sim 10^{-3}$ derived from the absolute ratio of the $CP$-forbidden and $CP$-allowed amplitudes $A(K_L \to \pi\pi)$ and $A(K_S \to \pi\pi)$, respectively). Thus the absolute value of the nucleon EDM cannot be much larger than the natural scale

$$|d_N| \lesssim \mu_N \times 10^{-7} \times 10^{-3} \sim 10^{-24} \text{ e} \cdot \text{cm} \quad (2.1)$$
without getting into conflict with known physics constraints – on top of the experimental neutron EDM bound which nowadays is even more restrictive, see above.

In the absence of the QCD $\bar{\theta}$ term, the SM only possesses a nonzero $CP$-violating phase if the CKM-matrix involves at least three generations, such that in this case the above estimate inherently implies a flavour change. The EDM, however, is flavour-neutral. Therefore, the upper bound for the nucleon EDM in the SM with zero $\bar{\theta}$ term necessarily involves a further $G_F f_p^2 \approx 10^{-7}$ suppression factor to undo the flavour change:

$$|d_{N}^{\text{SM}}| \lesssim 10^{-7} \times 10^{-24} e \cdot \text{cm} \sim 10^{-31} e \cdot \text{cm},$$

(2.2)

This simple estimate agrees in magnitude with the three-loop calculations of Refs. [20, 21] and also with the two-loop calculations of Refs. [22, 23] that include a strong penguin diagram and the long-distance effect of a pion loop. It is even consistent with modern loop-less calculations involving charm-quark propagators [24, 25]. From the bounds (2.1) and (2.2) one can infer that an EDM of the nucleon measured in the window

$$10^{-24} e \cdot \text{cm} > |d_{N}| \gtrsim 10^{-30} e \cdot \text{cm}$$

(2.3)

will be a clear sign for new physics beyond the KM mechanism of the SM: either strong $CP$ violation by a sufficiently large QCD $\bar{\theta}$ term or $CP$ violation by BSM physics, as, e.g., supersymmetric models, multi-Higgs models or left-right symmetric models.

2.2.2 BSM scale estimate

A rough estimate of the scale of BSM physics probed by EDM experiments can be derived from an expression of a subatomic EDM $d_i$ that is solely based on dimensional considerations and that holds for dimension-six extensions of the SM, since the SM symmetries and the pertinent chirality constraints preclude any contribution from dimension-five operators:

$$d_i \approx \frac{1}{16\pi^2} \frac{m_i}{\Lambda_{\text{BSM}}^2} e_i \sin \phi.$$  

(2.4)

Here $e_i$ and $m_i$ are the charge and mass, respectively, of the relevant quark or lepton, $\sin \phi$ results from the $CP$-violating BSM phases, and $\Lambda_{\text{BSM}}$ is the mass scale of the underlying BSM physics. In general, the coupling of BSM physics to subatomic particles induces at least one quantum loop and therefore a $g^2/(16\pi^2) \sim 10^{-2}$ suppression factor (assuming $g \sim 1$) is included in addition.

For current quark masses of the order $m_q \sim 5 \text{ MeV}$, we might therefore expect

$$|d_{N}| \sim 10^{-24} \left(\frac{1 \text{ TeV}}{\Lambda_{\text{BSM}}}\right)^2 |\sin \phi| e \cdot \text{cm}.$$  

(2.5)

If $\Lambda_{\text{BSM}} \gtrsim 1 \text{ TeV}$ and $\sin \phi \sim 1$, this result is compatible with the upper limit (2.1) derived from the naive estimate, i.e. it is compatible with all the known physics, except the constraints from direct or indirect EDM measurements. The projected sensitivity for $|d_{p}| \sim 10^{-29} e \cdot \text{cm}$ would in turn allow one to test the $CP$-violating phase $\phi$ of a theory of mass scale $M \sim 1 \text{ TeV}$ down to values of $\phi \gtrsim 10^{-5}$, while for natural values of the $CP$-violating phases, $\phi \sim 1$, a mass scale range up to $M \sim 300 \text{ TeV}$ can be probed.\(^5\)

\(^4\)Strictly speaking, the quark masses are scale- and scheme-dependent.

\(^5\)These numbers refer to one-loop processes as, e.g., supersymmetric extensions. They are suppressed by about two orders of magnitude for two-loop (so-called Barr-Zee [26]) processes as, e.g., in multi-Higgs scenarios, while they are enhanced by the same factor for loop-free particle exchanges as, e.g., for leptoquarks.
2.2.3 Estimate of the strong $CP$-violating QCD $\bar{\theta}$ parameter

Even if a natural-sized $\bar{\theta}$ parameter (which is given by the sum of the original $\theta$ that couples to the product of the gluon and dual gluon field strength tensors and the phase of the determinant of the quark mass matrix) is removed by the Peccei-Quinn mechanism [27], it can not be excluded that a fine-tuned $\bar{\theta}$, compatible with the $|d_n|$ bound [3, 4], will reemerge from Planck-scale physics upon UV completion.

The scale of the nucleon EDM induced by the $\bar{\theta}$ parameter can be estimated, in a similar way to the expression given in (2.1), by [28–30]

$$|d_N^\theta| \sim |\bar{\theta}| \cdot \frac{m_q^*}{\Lambda_{\text{QCD}}} \cdot \frac{e}{2m_N} \sim |\bar{\theta}| \cdot 10^{-16} e \cdot \text{cm},$$

(2.6)

where $m_q^* = m_u m_d m_s / (m_u m_d + m_u m_s + m_s m_d) \sim m_u m_d / (m_u + m_d)$ is the reduced quark mass. The additional suppression factor given by the ratio of the reduced quark mass to the QCD scale parameter $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$ takes into account that the $\bar{\theta}$ induced EDM would have to vanish if any quark mass were vanishing, since in that (chiral) limit the complete $\bar{\theta}$ term could be rotated away by an axial $U(1)$ transformation acting on the quark with zero mass [30]. Applying the above estimate (2.6) and utilising the empirical bound on the neutron EDM [3, 4], one finds the following upper limit for $\bar{\theta}$:

$$|\bar{\theta}| \lesssim 10^{-10}.$$

Taking into account the limit (2.2) of the Kobayashi-Maskawa induced nucleon EDM, the accessible window for determining $\bar{\theta}$ by nucleon EDM measurements is therefore

$$10^{-10} \gtrsim |d_n| \gtrsim 10^{-14},$$

while the projected sensitivity for $|d_p| \sim 10^{-29} e \cdot \text{cm}$ would allow a measurement of the value of, or the bound on, the parameter $\bar{\theta}$ down to the order $10^{-13}$.

2.3 EDM analysis based on non-perturbative methods

EDM measurements are of low-energy in nature and therefore all predictions of EDM values of sub-atomic particles, especially nucleons belong to the realm of non-perturbative QCD.

2.3.1 Determination of the $\bar{\theta}$ induced nucleon EDM

The QCD $\bar{\theta}$-term is manifestly a flavour-neutral, isoscalar source of $CP$ violation. It is instructive that the underlying non-perturbative physics nonetheless entails $d_p \neq d_n$.

The best way to predict the ratios $d_p/\bar{\theta}$ or $d_n/\bar{\theta}$ in the $\bar{\theta}$ term scenario would be the application of lattice QCD methods. Unfortunately, all current high-precision lattice calculations dedicated to these predictions have been based on the computation of the $T$- and $P$-violating $F_3$ form factors of the neutron or proton and have not taken into account that, in a finite volume, the Dirac states of the nucleon acquire an axial rotation in the mass term, such that there is sizeable admixture of the large $F_2$ (Pauli) in the small $F_3$ (EDM) form factor, as first pointed out in Ref. [31]. When the reported numbers of $d_n/\bar{\theta}$ or $d_p/\bar{\theta}$ had been reanalysed, the corrected values turned out to be compatible with zero within one standard deviation [31, 32].

It is expected that lattice estimates with better accuracy will become achievable in the next 1-2 years, though.\(^6\)

\(^6\)Note that no general consensus has been reached about the need to perform this correction to the form factors. The authors of Ref. [33] assert that in their method, which differs from other attempts by a purely imaginary value of the vacuum angle, the expansion is about a topologically non-trivial vacuum and that therefore the mixing-angle dependence has been included.

\(^7\)A step in that direction has been made in Ref. [34]. Using the gradient-flow method with proper axial rotation and by extrapolating from dynamical quark masses corresponding to admittedly large pion masses of (700, 570, 410) MeV this paper predicts the following results: $d_e/\bar{\theta} = -1.86(0.59) \cdot 10^{-16} e \cdot \text{cm}$ and $d_p/\bar{\theta} = 1.5(1.2) \cdot 10^{-14} e \cdot \text{cm}$ which in turn imply $|\bar{\theta}| < 1.61(51) \cdot 10^{-10}$ as an upper bound on the QCD theta angle from the experimental bound on the EDM of the neutron.
In the meantime, chiral perturbation theory (ChPT) can be applied to estimate the contribution of the pion one-loop terms to the $\bar{\theta}$-induced neutron and proton EDMs [35] – note that the leading term which involves a $CP$-violating but isospin-conserving pion-nucleon vertex was already estimated nearly forty years ago [29], while the loop diagram with the isospin-breaking counterpart is subleading. Both diagrams are divergent and have logarithmic scale dependence, which in principle can be cured by the addition of two independent $CP$-violating photon-nucleon contact terms [36, 37]. The signs and sizes of the latter, however, cannot be determined in ChPT and need external input, either from experiment or from lattice QCD which currently, as mentioned above, produces inconclusive results. The leading pion loop term predicts a contribution (at the mass scale of the nucleon) of $^{8}$

$$\Delta d_p/\bar{\theta} = - \Delta d_n/\bar{\theta} = (1.8 \pm 0.3) \cdot 10^{-16} e \cdot \text{cm}$$  (2.7)

that is of isovector nature – see [35] with input parameters from [38]. Note, however, that here the subleading isoscalar loop-term is neglected and that the sizes and signs of the two missing contact terms are not known, such that ChPT itself cannot predict the ratio of the proton to the neutron EDM.

For a real test or falsification of the $\bar{\theta}$ hypothesis as the leading (i.e. dimension-four) $CP$-violating mechanism in case $d_n$ and $d_p$ are measured, one needs the anticipated results of lattice QCD. However, even without lattice QCD calculations, additional measurement of the deuteron or helion or both EDMS would enable independent tests, since ChPT and chiral effective field theory methods can be used to get an estimate of the genuine nuclear contributions of these light nuclei (including triton) [38], i.e.

$$d_{2H} = -0.94d_p - 0.94d_n/\bar{\theta} = (0.89 \pm 0.30) \cdot 10^{-16} e \cdot \text{cm},$$  (2.8)

$$d_{\text{He}} = -0.90d_n + 0.03d_p/\bar{\theta} = -(1.01 \pm 0.42) \cdot 10^{-16} e \cdot \text{cm},$$  (2.9)

$$d_{3H} = -0.92d_p + 0.03d_n/\bar{\theta} = (2.37 \pm 0.42) \cdot 10^{-16} e \cdot \text{cm}.$$  (2.10)

These numbers are comparable with the predictions for the single-nucleon EDM case—cf. Eq. (2.7) and footnote 7. Therefore, they can equally well be used to test or constrain the value of the $\bar{\theta}$ term to $\sim 10^{-13}$ level, assuming that the above listed EDMS can be measured to $10^{-29} e \cdot \text{cm}$ sensitivity.

### 2.3.2 Estimates of the nucleon EDM terms in the BSM scenario

Again lattice QCD is the first choice for an estimate of the EDM contributions of the dimension-six $CP$-violating operators, which can be grouped [39–41] into quark operator terms ($CP$-violating photon–quark vertex terms), the quark-chromo operator terms ($CP$-violating gluon–quark vertex term), the isoscalar Weinberg three-gluon term [42], isospin-conserving $CP$-violating four-terms, and isospin-breaking four quark terms which can be traced back to left-right symmetric models. While there do not exist lattice QCD calculations for any of the four-quark operators, exploratory studies have just started in the Weinberg three-gluon case. In the quark-chromo scenario there already exist promising signals for the connected contributions, but results with (quark-)disconnected diagrams, non-perturbative mixing and renormalisation are still missing – see [32, 43] and reference therein for further details. In the quark EDM case, however, lattice QCD has delivered since the pertinent weight factors of the $u$-, $d$-, and $s$-quark EDMS follow via a chiral rotation from the corresponding flavour-diagonal quark tensor charges, $g_{\Gamma}^{u,d,s}$, i.e. $^9$

$$d_n = d_u^{q}g_{\Gamma}^{u} + d_d^{q}g_{\Gamma}^{d} + d_s^{q}g_{\Gamma}^{s},$$  (2.11)

$$d_p = d_u^{q}g_{\Gamma}^{u} + d_d^{q}g_{\Gamma}^{d} + d_s^{q}g_{\Gamma}^{s},$$  (2.12)

where the predictions of the tensor charges improved considerably from 2015 to 2018:

$$g_{\Gamma}^{u} = 0.774(66), \quad g_{\Gamma}^{d} = -0.233(28), \quad g_{\Gamma}^{s} = -0.008(9)$$  [45]; (2.13)

$^8$Here and in the following, the signs of the EDMS always refer to the convention $e > 0$.

$^9$Note that the flavour assignments of the tensor charges refer to the proton case, while for the neutron the role of $u$ and $d$ have to be interchanged. The cited values refer to the $\overline{\text{MS}}$ scheme at 2 GeV [44].
\[ g^u_T = 0.782(21), \quad g^d_T = -0.219(17), \quad g^n_T = -0.00319(72) \quad [46]; \quad (2.14) \]
\[ g^u_T = 0.784(30), \quad g^d_T = -0.204(15), \quad g^n_T = -0.0027(16) \quad [47]. \quad (2.15) \]

While the ratio \( g^u_T / g^d_T \approx -4 \) is compatible with the estimate from the naive non-relativistic quark model and the one from QCD sum rules [39], the absolute values of \( g^u_T \) and \( g^d_T \) are smaller, approximately reduced by a factor 3/5 relative to the naive quark model estimate and the central values in the QCD sum rule case (see also below).

The above predictions of the tensor quark charges allow for stringent tests of the split SUSY scenario with gaugino mass unification [48–50], since in this case there is a strong correlation between the electron and neutron (or proton) EDMs [51], the latter governed by the quark EDM operators, while all other \( CP \)-violating operators are highly suppressed. In particular, the results (2.15) and the indirect experimental bound \( |d_e| < 1.1 \cdot 10^{-29} e \cdot cm \quad [12] \) imply \( |d_n| < 4.1 \cdot 10^{-29} e \cdot cm \) as upper bound in the split-SUSY scenario [47]. This limit is still in the range of sensitivity of a dedicated proton EDM storage ring experiment.

With the exception of the quark EDM case mentioned above, currently there do not exist any predictions of lattice QCD or ChPT for any of the other \( CP \)-violating BSM operators. In the latter cases, only QCD sum rule estimates of the quark and quark-chromo contributions to the nucleon EDMs are available [39].

\[
d_n \approx (1 \pm 0.5) \times \left\{ 1.4 \left( d^u_d - 0.25d^u_u \right) + 3.2 \left( e_ud^u_d - 0.25e_ud^u_u \right) \right\} \pm (0.02 \text{ GeV}) e d^W, \quad (2.16) \\
d_p \approx (1 \pm 0.5) \times \left\{ 1.4 \left( d^u_d - 0.25d^u_u \right) + 3.2 \left( e_ud^u_d - 0.25e_ud^u_u \right) \right\} \pm (0.02 \text{ GeV}) e d^W, \quad (2.17) 
\]

where \( d^u_{u,d} \) and \( d^u_{u,d} \) denote the u-flavour and d-flavour quark and quark-chromo EDMs, respectively with \( e_u,d \) the corresponding quark charges, while \( d^W \) (of dimension mass\(^{-2}\)) stands for the prefactor of the Weinberg term. Taking these large uncertainties into account, currently we have no reliable prediction of the ratio of the proton to neutron EDM for any of the BSM extensions (SUSY, multi-Higgs models, left-right symmetric models), with the notable exception of the above discussed split-SUSY case (assuming that quark EDM ratios follow the quark mass (times quark charge) ratios).

### 2.3.3 Estimates of the nuclear EDM matrix elements for light nuclei

If, however, storage ring experiments are planned to measure the deuteron and/or helion EDMs, these results would determine the genuine nuclear EDM contributions. The relevant, i.e., leading, \( CP \)-violating nuclear matrix elements are governed by tree-level operators and are predicted in the framework of chiral effective field theory (chEFT) with reasonable uncertainties [38, 52, 53]:

\[
d_{^2\text{H}} - 0.94(1)(d_p + d_n) = \{ 0.18(2)g_1 - 0.75(14)\Delta_{3\pi} \} \text{ e } \cdot \text{fm}, \quad (2.18) \\
d_{^3\text{He}} - 0.90(1)d_n + 0.03(1)d_p = \{ 0.11(1)g_0 + 0.14(2)g_1 + 0.61(14)\Delta_{3\pi} \\
\quad - 0.04(2)C_1/\text{fm}^3 + 0.09(2)C_2/\text{fm}^3 \} \text{ e } \cdot \text{fm}, \quad (2.19) \\
d_{^4\text{He}} - 0.92(1)d_p + 0.03(1)d_n = \{ -0.11(1)g_0 + 0.14(2)g_1 - 0.60(14)\Delta_{3\pi} \\
\quad + 0.04(2)C_1/\text{fm}^3 - 0.09(2)C_2/\text{fm}^3 \} \text{ e } \cdot \text{fm}. \quad (2.20) 
\]

Here \( g_0 \) and \( g_1 \) are the dimensionless low-energy constants of the isospin-conserving and isospin-breaking \( CP \)-violating pion-nucleon vertices, respectively, while \( \Delta_{3\pi} \cdot m_N \) is the prefactor of the \( CP \)-violating three-pion term and \( C_1 \) and \( C_2 \) are the coefficients of the two leading \( CP \)-violating four-nucleon terms. The values of the three hadronic low-energy constants \( g_0, g_1 \) and \( \Delta_{3\pi} \) can be predicted from the coefficients of the \( CP \)-violating terms of the underlying theory at the quark-gluon level, e.g., from \( \theta \) in the case of QCD [38, 54] or from the prefactors of the quark-chromo [55] or the left-right-model-induced four-quark terms – see [56] and references therein. While the \( \theta \) mechanism assigns a dominant role to \( g_0 \), the quark-chromo mechanism predicts \( g_0 \) and \( g_1 \) of about equal magnitude, whereas
$g_1$ dominates in the left-right scenario. There do not exist analogous predictions for the hadronic coefficients $C_1$ and $C_2$. The order of their contributions can so far only be estimated by naive dimensional analysis and thus has to be included in the theoretical uncertainties. Note that the role and magnitude of the $CP$-violating four-nucleon and three-pion terms have not been investigated for $A > 3$ nuclei – see [7, 8] for more information on EDM calculations for heavy nuclei.

2.4 Option for oscillating EDM searches at storage rings

The storage ring technology also allows to search for time-varying (oscillating) components of the EDM in addition to the static (permanent) one [57, 58] and therefore to test the hypothesis that the dark matter content in our galaxy is (at least partially) saturated by a classical oscillating field 10 of axions or axion-like particles (ALPs), even if the axion/ALP mass $m_a$ were in the range from $10^{-7}$ eV to $10^{-22}$ eV [59,60].11 This mass range is very challenging for any other technique to reach, since, e.g., the resonance cavities of the microwave (haloscope) method 12 would have to be unwieldy large in size [62]. There are, though, some astrophysical constraints from the bounds of supernova energy losses, Big Bang nucleosynthesis, and the spatial extent of dwarf galaxies [63]. For instance, the latter give an upper bound on the de Broglie wavelength and therefore the lower bound of $10^{-22}$ eV on the mass of a non-relativistic bosonic particle trapped in the halo of a dwarf galaxy.

All interactions of the axions/ALPs are either suppressed by the very large axion/ALP decay constant $f_a$ or are just of gravitational nature. Thus, in the so-called pre-inflationary Peccei-Quinn symmetry breaking scenario [61], the initial displacement (misalignment) of axion/ALP field $a$ from the minimum of its potential energy density, given by $m_a^2 c^2 / 2$, 13 leads to a coherent oscillation of the classical axion/ALPs field at a Compton frequency $\omega_a = m_a c^2 / h$. The idea is to equate the energy density in these oscillations with the mass-energy associated with dark matter [59, 60]. The axions/ALPs trapped in the halo of our galaxy and to be observed in terrestrial experiments acquire in addition a velocity $v$ of the size of the virial velocity of our solar system relative to the centre of our galaxy $\sim 10^{-3} c$. Thus their frequency is second-order Doppler-shifted, $\omega' \approx \omega_a \left(1 + v^2 / 2c^2 \right)$. This implies a finite coherence time of order $\tau_a \approx h / m_a c^2$, thus a $Q$-value of the size $(c/v)^2 \sim 10^6$, and a coherence length of order $h/(m_a v)$. For any terrestrial experiment smaller than this coherence length, which is at least 0.5 km for $m_a c^2 \lesssim 0.1 \mu eV$, the oscillating axion field corresponds to [57, 58]

$$a(t) = a_0 \cos(\omega'(t-t_0) + \phi_0) \approx a_0 \cos(\frac{1}{2} m_a c^2 (t-t_0) + \phi_0),$$

(2.21)

where the undetermined local phase $\phi_0$, which is approximately constant as long as the measurement period $|t - t_0|$ is smaller than the coherence time $\tau_a$, is correlated with the choice of the start point $t_0$ of the measurement cycle.14 The amplitude $a_0$ of this classical field oscillating at the frequency $\omega' \approx \omega_a$ can be estimated by saturating the local dark matter density in our galaxy, $\rho_{LDM} \approx 0.4 \text{ GeV/cm}^3$ [61], with the total energy density of the oscillating axion/ALPs field, i.e. $\rho_{LDM} \approx m_a^2 a_0^2 / 2$. Assuming the QCD-axion coupling to the gluons and therefore an effective theta angle

$$\theta_a = \frac{a_0}{f_a} \approx \frac{\sqrt{2 \rho_{LDM}}}{m_a f_a} \approx \frac{\sqrt{2 \rho_{LDM}}}{0.5 m_a f_a} \approx 3 \times 10^{-19}.$$  

(2.22)

10The mode occupation numbers of dark matter bosons of mass $< 1 \text{ meV}$ suffice for the formation of a classical field.

11This assumes that the initial misalignment angle of the axion or ALP field in this light-mass scenario is tuned so small that the resulting ‘dark matter’ particles do not overclose the universe – see, e.g., [61] for more details.

12That means a resonance in an RF cavity in a strong magnetic field is excited by the inverse Primakoff effect.

13Starting with the QCD epoch ($\sim 10^{-4}$s after the Big Bang), the axion mass $m_a$ is constrained as $m_a \approx 0.5 m_* f_a / f_a$, where $m_*$ and $f_a$ are the pion mass and decay constant, respectively – see, e.g., Ref. [61] for more details.

14The to-be-measured value of the phase $\phi_0$ ensures that at the beginning of a measurement period, $t = t_0$, all spectral $\omega'$ components of the axion field, irrespective of their velocity $|\vec{v}| < v_{esc}$ (= the escape velocity from our galaxy), start coherently with the common phase $\cos(\phi_0)$ and stay approximately coherent as long as $|t - t_0| < \tau_a$. 

26
we would get from the naive formula (2.6) for the \( \bar{\theta} \)-induced nucleon EDM the following estimate of the axion-induced oscillating component of the nucleon EDM:

\[
d_{\text{osc}}(t) \sim 10^{-16} \cdot \frac{a(t)}{f_a} \sim 5 \cdot 10^{-35} \cos \left( \frac{1}{7} m_a c^2 (t - t_0) + \phi_0 \right) e \cdot \text{cm}.
\]  

(2.23)

The detection of an oscillating EDM of such an amplitude would be very demanding. In the case of an ALP, however, there is no strict relation between its mass \( m_a \) and its decay constant \( f_a \), such that mass regions with \( m_a < 0.5 m_a f_\pi / f_a \) and therefore effective ALPs angles with \( \theta_a > 3 \times 10^{-19} \) become accessible.\(^{15} \) In fact, first exclusion bounds in the domain of axion/ALPs mass (frequency) versus axion/ALP-gluon coupling strength have already been extracted from the recent neutron EDM measurements \[63\] and dedicated experiments applying nuclear magnetic resonance techniques or superconducting toroidal magnets are currently realised \[62, 64–67\].

In complete analogy to the neutron EDM experiment, the measurement/bounds of the proton (or deuteron) EDM by the frozen spin method in storage ring experiments can of course be analysed for slow oscillations, such that the neutron ALP-bounds can potentially be improved by the ratio of the projected sensitivity of the proton EDM measurement to the current neutron EDM limit, \( 3 \cdot 10^{-26} e \cdot \text{cm} \). But the advantage of the storage ring technique is really the search/scan for an oscillating EDM at the resonance conditions between the axion/ALP frequency and the \( g - 2 \) precession frequency of the storage ring. Such a resonance enhancement would allow to investigate an axion/ALP frequency range of \( \sim 1 \) nHz to \( \sim 100 \) MHz, where the lower limit is just due to the current bound on the spin coherence time while the upper bound is due to the spin-rotation frequency. Furthermore, the resonance method should by fiat be less affected by systematical uncertainties than the frozen spin one. And moreover, in a combined electric and magnetic storage ring (which is needed in the case of the deuteron or helion and maybe realised in the prototype scenario) effective radial electric fields in the centre-of-mass frame of the rotating particle can be achieved that are one or even two-orders of magnitude bigger than the presently realisable \( E \) fields in the laboratory. In this way, the projected sensitivity for oscillating EDM measurements by the resonance method may even reach the \( 10^{-30} e \cdot \text{cm} \) level.

**Synopsis**

Finally, let us emphasise that the physical reach of permanent proton EDM measurements of sensitivity \( \sim 10^{-28} e \cdot \text{cm} \) is competitive with or better than any other EDM measurement, while at a \( 10^{-29} e \cdot \text{cm} \) level the proton EDM measurements become our best hope for finding new sources of \( CP \) violation.

**References**


\(^{15}\)Only in the axion case, the estimate (2.6) constrains the value of the oscillating EDM, while in the ALPs scenario the coupling strength of \( a(t) \) to the nucleon is undetermined and does not depend on the ALP mass [60]. This arbitrariness can be taken into account by defining an effective ALP angle as \( \theta_a = c_a \theta_0 / f_a \), where \( c_a \) is unknown dimensionless parameter that in turn rescales the right hand side of (2.23).


28
Chapter 3

Historical Background

3.1 Beginnings at Brookhaven National Laboratory (BNL, USA)

The idea for using a storage ring to confine a charged-particle beam while testing it for the presence of an EDM grew out of the Brookhaven $g - 2$ experimental effort. Even at low sensitivity, the data from this experiment may be checked for effects that arise from an EDM. The results from BNL [1] and an even earlier CERN experiment [2] reported upper limits for the muon EDM in the $10^{-19}$ e·cm range. Discussions in the late 1990’s centred mostly on the muon experiment [3], but also considered the deuteron which has a similar magnetic anomaly to mass ratio.

A regular pattern of BNL meetings for discussion and planning developed. In 2004, a proposal for a storage ring search on the deuteron at the $10^{-27}$ e·cm level was submitted to the BNL Program Advisory Committee as Experiment 970. In light of the discrepancy between theory and experiment for the muon value of $g - 2$ [4], it was considered possible that contributions from triangle graphs associated with meson exchange in the deuteron would lead to an enhancement in the EDM of the deuteron and more favourable prospects for a search. However, the BNL PAC did not find the proposal sufficiently competitive with other smaller scale EDM searches to warrant the cost of constructing a new storage ring.

For a while, ring designs shifted to the development of resonant techniques to amplify and thus identify systematic errors [5]. But eventually these schemes were discarded as unworkable at the greater sensitivities needed, and attention returned to a more standard storage ring design.

Beginning in 2005, feasibility experiments were run at the KVI cyclotron facility in Groningen to measure broad range spin sensitivities for deuteron scattering on carbon near 100 MeV. These showed large analysing powers but also sensitivities to beam alignment errors that could not be cancelled with standard first-order analysis techniques [6]. In 2007, more definitive experiments were proposed for the COSY storage ring (Experiment 170) and approved for running. Tests began in 2008 leading to a final confirmation run in 2009 to demonstrate that, with a calibration of the sensitivity of the polarimeter to systematics, errors could be corrected to levels below one part in $10^5$ [6]. This was the first of what would become a series of beam studies to develop techniques needed for the EDM search.

In 2008, a second deuteron proposal was submitted to the BNL PAC. This time several improvements led to an anticipated sensitivity of $10^{-29}$ e·cm with up to a year of data collection [7]. This led to a technical review that was held in 2009 (see [8] for a Web site for the review). In the meantime, it was realised that a first experiment on the proton offered some technical advantages, including the ability to have counter-rotating beams travelling the same path in the same ring. This would optimise the cancellation of a large class of time-reversal conserving systematic errors. From this point on, proposals featured the proton rather than the deuteron. Work at COSY continued with the deuteron because of the investment already made in deuteron operation and the sense that any conclusions would apply to either proton or deuteron beams. Development continued at BNL and a second technical review was held in 2011, again with encouraging results (see BNL EDM site). In October of 2011 a full proposal was forwarded to the US DOE, but no formal evaluation was ever initiated. In collaboration with the US NSF, the two funding agencies decided to terminate all further work along these lines.

3.2 Continuation at the Forschungszentrum Jülich (FZJ, Germany)

First contributions to the storage ring EDM effort were made at FZJ in 2008-2009, when members of the BNL srEDM collaboration and scientists from the Groningen KVI started experiments together with
scientists from the Institute for Nuclear Physics (IKP) to investigate polarimetry issues at the cooler synchrotron COSY. It soon was realised that COSY [9] with its polarised proton and deuteron beams in the energy range required for srEDM establishes a unique and ideal facility to perform the required R&D.

The COoler SYnchrotron (COSY) is a worldwide unique facility for polarised and phase-space cooled hadron beams, which was utilised for hadron physics experiments until the end of 2014. Since then it has been used as a test and exploration facility for accelerator and detector development as well as for the preparation and execution of precision experiments (JEDI (Jülich Electric Dipole moment Investigations), TRIC (Time Reversal Invariance at COSY)). The COSY facility comprises sources for polarised and polarised protons and deuterons, the injector cyclotron JULIC (Jülich Light Ion Cyclotron), the synchrotron to accelerate, store and cool beams, and internal and external target stations for experimental set-ups.

$^3\mathrm{H}^- (D^-)$-ions are pre-accelerated up to 0.3 (0.55) GeV/c in JULIC, injected into COSY via stripping injection and subsequently accelerated to the desired momentum below 3.7 GeV/c. Three installations for phase-space cooling can be used: (i) a low-energy electron cooler (between 0.3 and 0.6 GeV/c), installed in one of the straight sections, (ii) stochastic cooling above 1.5 GeV/c, and (iii) a new high-energy electron cooler in the opposite straight section, which can be operated between 0.3 and 3.7 GeV/c.

Well-established methods are used to preserve polarisation during acceleration. A fast tune jumping system, consisting of one air-core quadrupole, has been developed to overcome depolarising resonances. Preservation of polarisation across imperfection resonances is achieved by the excitation of the vertical orbit using correcting dipoles to induce total spin flips. The polarisation can be continuously monitored by an internal polarimeter (EDDA); an additional polarimeter, making use of the WASA forward detectors, has recently been set up, and a further new polarimeter, based on LYSO-scintillators, is under development and will be installed in the ring in early 2019. For protons, a beam polarisation of 75% up to the highest momentum has been achieved. Vector and tensor polarised deuterons are also routinely accelerated with a degree of polarisation of up to 60%. Dedicated tools have been developed to manipulate the stored polarised beam and to precisely determine the beam energy.

In 2011, the JEDI (Jülich Electric Dipole moment Investigations) collaboration [10] was created, aiming to exploit COSY not only for the development of the key technologies for srEDM but also for performing a first direct EDM measurement for deuterons (“precursor experiment”). Since COSY is a conventional storage ring with magnetic bending, a dedicated insertion (“Radio-frequency (RF) Wien-filter”) must be used to be sensitive to an EDM. This latter project towards a proof-of-principle for srEDM is supported by an “Advanced Grant” of the “European Research Council” (2016 – 2021) [11].

Meanwhile, significant experimental progress has been made at COSY and elsewhere (see Appendix A, “Results and Achievements”). However, it has also become clear that between now (COSY, precursor experiment) and then (final clockwise, counter-clockwise all-electric EDM ring), an intermediate step (prototype, demonstrator) is required to test/demonstrate key issues, such as:

- Storage time of the beam (stochastic cooling)
- Spin coherence time
- Polarimetry
- Clock-wise (CW) and counter clock-wise (CCW) operation
- Effects of the magnetic moment

(see Chapter 13, “Road-map and Timeline”).

The prototype may, if equipped with magnetic elements in addition to electric deflectors, be used in the frozen-spin condition to determine an EDM limit for the proton (see Chapter 7, “The EDM Prototype Ring”).
3.3 Ongoing activity: the precursor experiment at COSY

During autumn 2018, the JEDI collaboration performed a first measurement of the deuteron EDM at COSY, with the analysis of the results in progress. In a pure magnetic storage ring such as COSY, an EDM will generate an oscillation of the vertical polarisation component. For a 970 MeV/c deuteron beam with the spin precession frequency of 120 kHz, a tiny amplitude is expected, e.g. $3 \times 10^{-10}$ for an EDM of $d = 10^{-24}$ e·cm. To allow for a build-up of the vertical polarisation proportional to the EDM, a radio-frequency (RF) Wien-filter [12] has to be operated. A prototype Wien-filter was successfully installed and operated in COSY in 2014. A new device with a stronger magnetic field (0.05 T·mm) was developed and constructed together with the Institut für Hochfrequenztechnik (IHF) at RWTH Aachen University and ZEA-1 in Jülich. This new RF Wien-filter was installed in COSY in May 2017 and a first commissioning run was successfully conducted in June 2017.

3.4 Charged-particle EDM Initiative and experience of the collaboration

In connection with the “Physics Beyond Colliders” (PBC) initiative of CERN and the “European Strategy for Particle Physics” (ESPP) update, a cooperation under the name “Charged Particle Electric Dipole Moment” (CPEDM) was formed in early 2017, comprising members of the srEDM and JEDI collaborations as well as new interested scientists from CERN in order to prepare the science case for a storage ring EDM search for the proton (deuteron and $^3$He) and the technical design study – in other words the current document.

The JEDI members of the Institut für Kernphysik (IKP) of the Forschungszentrum Jülich have a decade-long experience to design, to build and operate as well as to further develop accelerators: foremost JULIC and COSY, but also the polarised and unpolarised ion sources for protons and deuterons. IKP has also contributed significantly to the various versions of linear accelerators for spallation neutron sources and it has designed a superconducting linac, which was planned to replace JULIC as the injector for COSY. Recently, it has delivered the proton source for commissioning of the ELENA antiproton ring at CERN.

Unique experience is available to produce and accelerate polarised beams without polarisation loss and to manipulate them in COSY, to select polarisation states and to determine the degree of polarisation by the use of nuclear reactions with polarimeters, based on scintillator detectors. A huge expertise has been accumulated over the years to cool and store beams, to accelerate and decelerate them and to use them during energy ramping or at a fixed energy at internal target stations with thin solid, gas or pellet targets. It is also possible to provide (slow (resonant and stochastic) or fast) extracted beams to external target stations – this option was previously used for the TOF-spectrometer and is now exploited for all kinds of detector tests.

Electron cooling at low momenta (up to 600 MeV/c) has been used in COSY early on; more recently a high energy electron cooler (E_e < 2 MeV) has been installed and commissioned in the ring. Stochastic cooling is also used routinely in COSY (momentum range from 1.5 to 3.3 GeV/c); here new pick-up and kicker-devices have been developed at and implemented in COSY.

A group working at KAIST in South Korea (IBS Center for Axion and Precision Physics research, CAPP) has developed a large expertise in the use of SQUID magnetometers. A prototype EDM ring section has been constructed to investigate the cryogenic environment and magnetic sensitivity. This effort is in conjunction with the building of a magnetically shielded chamber to simulate conditions in an EDM beam line.

A group of scientists from CERN with enormous experience in accelerator design has joined the CPEDM project from the start. They already have made essential contributions to the study of electric deflection and to various kinds of systematic effects. Limiting the effects of systematics is the central issue in the success of the EDM storage ring project.
3.5 Further developments

Work is underway at COSY to develop electrostatic plates usable in a final EDM ring. An initial series of tests with half spheres demonstrated fields of 17 MV/m for stainless steel separated by 1 mm and 30 MV/m for aluminium separated by 0.1 mm. The next phase of the project will test a prototype electric field section 1-m long located in an existing dipole magnet with a large gap (ANKE, dipole 2) outside the COSY ring.

3.6 Summary

Summarising, it must be emphasised that in contrast to other EDM projects, e.g. for the neutron, the electron, the muon and others, which are pursued in many different places worldwide, for CPEDM, Europe will be in a unique position to design, construct and host such a project.

References

[11] ERC Advanced Grant (srEDM, 694340); see http://www.sredm-ercgrant.de
Chapter 4

Experimental Method

4.1 Introduction

The existence of a permanent Electric Dipole Moment (EDM) for fundamental particles or subatomic systems is still an open question in physics since such a quantity has never been detected. The EDM is a vector-like intrinsic property which measures the asymmetric charge distribution along its spin axis. Hence, an experiment to measure the latter often relies on the spin precession rate in an electric field. However, for charged particles, such a measurement cannot be made while maintaining the particle at rest since any applied electric field leads to acceleration. Instead, those fields can be provided as a part of a particle trap. For the experiment considered here, the trap is a storage ring with crossed vertical $\vec{B}_y$ and radial $\vec{E}_r$ fields that confine a beam of spin-polarised particles (e.g., protons, deuterons, etc.) into a design orbit (see Fig. 4.1). The Electric Dipole Moment ($\vec{d}$) couples to the electric fields while the Magnetic Dipole Moment ($\vec{\mu}$) couples to the magnetic fields so that, for a particle at rest, a spin precession will occur which is given by:

$$\frac{d\vec{S}}{dt} = \vec{d} \times \vec{E} + \vec{\mu} \times \vec{B},$$

(4.1)

In general, the MDM of subatomic particles is known to high precision and the aim of the proposed experiment is to determine the EDM part which leads only to much smaller spin rotations. Nevertheless, since the charged particle is subject to combined electromagnetic fields and therefore is not at rest, one needs to account for the kinematical effect that may alter its spin precession. For that reason, one shall invoke the Thomas-BMT equation [1] which gives the precession rate of the angle between the spin and momentum vectors in the inertial rest frame of the particle.

4.2 Spin evolution in electric and magnetic fields

In Chapter the T-BMT equation was introduced for generic $\vec{B}$ and $\vec{E}$ fields. The latter are defined in the laboratory frame while the spin is defined in the inertial rest frame of the particle. In a storage ring where the particle is being continuously deflected by the guiding electromagnetic fields to perform a closed orbit trajectory, it is convenient to rewrite the equation of motion in the non-inertial frame rotating with the velocity vector of the particle. A natural way to describe the rotation of the coordinate system is to use the Frenet-Serret frame attached to the reference orbit [2, 3] and therefore lying in the mid-plane of the accelerator as illustrated in Fig. 4.1. In that case, the angular velocity describing the rotation of the coordinate system is given by:

$$\vec{\Omega}_{cycl} = -\frac{\beta c}{\rho} \vec{u}_y,$$

(4.2)

where $\rho$ is the bending radius, $\beta$ the Lorentz factor and $\vec{u}_y$ is the unit vector perpendicular to the mid-plane of the ring. Writing the relativistic form of Newton’s second law for the reference particle in a perfect machine without any imperfections, and projecting it into the horizontal plane, it can be easily shown that:

$$\frac{1}{\rho} = -\frac{q}{m\gamma^2c^2} E_r + \frac{q}{m\gamma\beta c} B_y$$

(4.3)

\[\text{The dipole moment must be aligned with the only other vector quantity as a consequence of the Wigner-Eckart theorem.}\]

\[\text{Only the field components acting on a particle on the reference orbit in a perfect machine are taken into account to explain the basic idea of the measurement method: } E_r = E_0 \vec{u}_r \text{ and } B_y = B_0 \vec{u}_y \text{ where } \vec{u}_r \text{ is the unit vector pointing radially outwards, } \vec{u}_e \text{ is the unit vector co-linear with the velocity vector of the particle and } \vec{u}_y \text{ is the unit vector defined such that } \vec{u}_y = \vec{u}_e \times \vec{u}_r. \text{ Note that for the electric field to point inwards, } E_r < 0.\]
Now, making use of Eqs. (4.2) and (4.3), it results that the spin motion of the reference particle is given by the subtracted T-BMT equation:

\[
\frac{d\vec{S}}{dt} = \left( \vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}} \right) - \vec{\Omega}_{\text{cycl}} \times \vec{S},
\]

where

\[
\begin{align*}
\vec{\Omega}_{\text{MDM}} - \vec{\Omega}_{\text{cycl}} &= -\frac{q}{m} \left[ GB_y - \left( G - \frac{1}{\gamma^2 - 1} \right) \frac{\beta \times E_r}{c} \right] \quad (4.5) \\
\vec{\Omega}_{\text{EDM}} &= -\frac{\eta q}{2mc} \left[ E_r + eB_y \right]. 
\end{align*}
\]

In Eq. 4.4, \( \vec{S} \) is the spin unit vector in units of \( \hbar/2 \) (for fermions) defined in the Frenet-Serret frame of the reference particle, and \( t \) is the time in the laboratory frame of reference.

The dimensionless EDM parameter \( \eta \) is related to the electric dipole moment \( \vec{d} \) through

\[
\vec{d} = \eta \frac{q\hbar}{2mc} \vec{S},
\]

In addition, it is important to note that the form of the Thomas-BMT equation shown in Eqs. (4.4)–(4.6) does not include the effects of gravity. However, this will be described in the appendix D, Gravity and General Relativity as a 'Standard Candle' and has been studied by several authors [4–7].

### 4.3 The storage ring EDM search

The signal of an electric dipole moment (EDM) using the storage ring method relies on the direct observation of the rotation of the electric dipole and thus, the spin in the presence of an external electric field that is perpendicular to the axis of the particle spin [8]. The particles being studied are formed into a beam that is spin polarised, and the changes in the polarisation components are measured on the beam as a whole while it is confined in the ring. However, the MDM can also contribute to the polarization...
buildup in the same way that EDM does. Thus, the main idea of the storage ring EDM search (in a perfect machine) is to maintain the spin frozen along the momentum direction in order to nullify the MDM contribution and maximize the EDM signal buildup, hence the frozen spin concept that we discuss in the next section.

4.3.1 Frozen spin concept

To simplify the discussion, one shall assume that the particle is moving on the reference orbit in a perfect machine such that the only fields acting on it are the bending fields, $\vec{B}_y$ and $\vec{E}_r$ as illustrated in Fig. 4.1. Then, from Eq. (4.5), a general relationship between the fields can be established that sets the spin precession frequency due to the MDM (or $g$-2 precession) to zero in the Frenet-Serret frame of the particle:

$$G\vec{B}_y - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}_r}{c} = 0$$

(4.8)

and the radial E-field that is sensed by the EDM is given by:

$$E_r = c\frac{\beta\gamma^2 GB_y}{\beta^2\gamma^2 G - 1}$$

(4.9)

In other words, for each energy, there exits $(B_y, E_r)$ combinations such that the spin precession frequency due to the MDM equals the particle angular velocity. Thus, if the EDM contribution is disregarded and the initial condition begins with the spin parallel to the velocity, the spin will remain frozen in the horizontal plane along the momentum direction. However, in the presence of EDM, the spin will precess around the radial axis leading to a vertical spin component as sketched in Fig. 4.2.

Furthermore, if the anomalous magnetic moment $G$ of the particle is positive, then from Eq. (4.8), the frozen spin condition can be satisfied for an all electric ring and for one specific momentum that one generally refers to as the magic momentum $p_m$:

$$p_m = \frac{mc}{\sqrt{G}}$$

(4.10)

For the proton, this corresponds to a momentum $p_m = 700.740$ MeV/c i.e. to a particle kinetic energy of 232.8 MeV.

For particles with a negative anomalous magnetic moment such as deuterons for instance, there is no magic momentum and a combination of radial electric and vertical magnetic fields is necessary to achieve the frozen spin condition. In this case, the electric field must be pointing away from the centre of the ring ($E_r > 0$), thus reducing the bending of the beam from magnetic fields alone. This yields an increase of the ring circumference.

In Fig. 4.2, the frozen spin concept is illustrated where $\vec{v}$ is the particle velocity along the orbit, $\vec{B}$ and $\vec{E}$ are possible external fields (acting on a positively charged particle), and the spin axis is given by the purple arrow that rotates in a plane perpendicular to $\vec{E}$. If the initial condition begins with the spin parallel to the velocity, then the rotation caused by the EDM will make the vertical component of the beam polarisation change. This becomes the signal observed by a polarimeter located in the ring. This device allows beam particles to scatter from nuclei in a fixed target. The difference in the scattering rate towards the left and right sides of the beam is sensitive to the vertical polarisation component of the beam. Continuous monitoring by a pair of detectors, illustrated in blue in Fig. 4.2, will show a change in the relative left-right rate difference during the time of the beam storage if a measurable vertical spin component due to an ED, (or perturbations described in the next paragraph) is generated. A practical consideration is the need for an optimum polarimeter efficiency, which is the case for magic energy protons (see Chap. 12, Polarimetry).

Under realistic conditions, beam particles will execute transverse "betatron" and longitudinal "synchrotron" oscillations in an imperfect machine constructed with finite mechanical tolerances, positioning
Fig. 4.2: A diagram of the EDM experimental effect. A circulating beam particle (yellow) travelling counterclockwise in a storage ring has its initial polarisation parallel to the velocity. If the orbit is constrained magnetically with a $\vec{B}$ field down, then in the particle frame a radial inward electric field $\vec{E}$ is produced. The orbit may also be constrained by a radial $\vec{E}$ field created using electric field plates. If the EDM lies along the spin axis and is thus perpendicular to the electric field, then a precession as shown will be induced. This creates a vertical component to the polarisation that may be observed in the left-right asymmetry of scattering from a target (black) into detectors (blue) in the lower right part of the orbit.

errors of elements and stray fields from surrounding structures. Various effects can rotate the spin from the longitudinal into the vertical direction even without an EDM and may lead to systematic errors of the measurement. An example for the "frozen spin" fully electric proton EDM are residual magnetic fields. To mitigate the effect, the proposal includes installation of the ring in a state-of-the-art magnetic shielding reducing the residual field to about 1 nT. Even with such a shielding the residual radial magnetic field couples to the MDM and is expected to limit the sensitivity of the experiment to values well above $10^{-29}$ e.cm. Measures to further mitigate the effect due to the average radial magnetic field are described in section 4.3.2. A more thorough analysis of systematic effects is given in chapter 11.

The kinematic diagrams shown in Figs. 4.3 and 4.4 show the momentum and ring radius respectively as a function of the electric and magnetic fields available to fulfill the frozen spin condition for protons and deuteron beams. Pure electric rings lie along the horizontal axis. For the case of deuterons, no purely electric "frozen spin" solution exists which is consistent with the observation in Fig. 4.4 that none of the curves crosses the horizontal axis. The red dots in Fig. 4.3 labelled "pure electric ring" are for a realistic electric field of 8 MV/m corresponding to a bending radius of about 52 m. The red dots labelled prototype ring in Figs. 4.3 and 4.4 are motivated by the prototype described in chapter 8 and with a bending radius of 8.9 m. The energy is limited by the electric field around 7 MV/m; for protons, the "frozen" spin condition is fulfilled with 45.2 MeV kinetic energy and a magnetic field of 0.0326 T (see Fig. 4.3, both electric and magnetic field deflect in the same direction). For deuterons, the "frozen spin" condition would be fulfilled reversing the electric field, adding a magnetic field of 0.360 T and for kinetic energy of 164.4 MeV (indicated as red point in Fig. 4.4).

Figure 4.4 includes only the mixed-field prototype ring operating point for the deuteron at a much higher magnetic field than is required for the proton. There is no pure electric solution for the deuteron.

4.3.2 Dual beam operation

The large size of MDM effects compared to EDM effects also means that any storage ring experiment is sensitive to problems that might arise from issues such as fringe fields, component alignment, stray EM interference, etc., with the design and construction of the physical machine. One strategy to deal with these problems in general is based on the realisation that the EDM is time-reversal violating while the majority of the problems are time-reversal conserving. The experiment could be changed to a time reverse of itself by inverting the direction of all velocities, reversing all spins, and reversing all magnetic fields while maintaining the electric fields as is. In this case where the time-reversed beam travels inside
Fig. 4.3: Proton momentum $p$ (left) and storage ring bending radius $r$ (right), for different frozen spin combinations of electric and magnetic fields (the absolute value of the field is shown). For the pure electric ring the momentum is fixed to 0.7007 Gev/c.

Fig. 4.4: Deuteron momentum $p$ (left) and storage ring radius $r$ (right), for different frozen spin combinations of electric and magnetic fields.

the same machine as the initial experiment and is subject to all of the same imperfections as the original experiment, the results could be compared directly. In other words, addition of the measured rotations of the two counter-rotating beams will cancel all machine-related systematic imperfections such that the remaining part will correspond to the EDM signal (twice).

Nevertheless, if a residual radial magnetic field does not reverse between the two counter-rotating beams, this will yield a signal mimicking the EDM one. For the all-electric proton storage ring concept with a ring circumference $C = 500$ m, an average radial magnetic field as low as $B_r = 9.3$ aT will generate the same vertical spin component as the EDM signal of $10^{-29}$ e.cm. This is probably the most serious
systematic imperfection that needs to be corrected to reach the high sensitivity goal of the experiment. The first line of defence against such magnetic fields is shielding. State-of-the-art multilayer shielding with degaussing procedures can reduce the ambient field to the 1 nT level. Noting in addition that such a residual radial magnetic field does separate the orbits of the counter-rotating beams vertically, then the idea to remediate such an imperfection is to operate the machine with low vertical tune, i.e. with weak vertical focusing in order to maximize the separation between the two beams. The latter will be measured with ultra-sensitive SQUID magnetometers. For instance, with a vertical tune $Q_y = 0.1$, the same radial magnetic field of 9.3 aT leads to an average orbit separation of 5 pm. The measured vertical separation of the two counter-rotating beams will be reduced by an additional radial magnetic field to compensate. This method and, in particular the limitations, is further discussed in Chapter 10, "Sensitivity and Systematics".

### 4.3.3 General possibilities

Various categories of EDM storage rings are shown in Table 4.1. Of these, only the proton cases are seriously analysed in the present report. The deuteron and electron cases have been mentioned earlier in the report, but are not described in any further detail. The all-magnetic case is exploited to the extent possible in the COSY precursor experiments. But frozen spin is not possible with only magnetic bending and an effect is possible only because an RF Wien filter synchronised to the polarisation precession rate breaks the cancellation that prevents an EDM signal accumulation.

#### Table 4.1: General possibilities according to BMT equation.

<table>
<thead>
<tr>
<th>Field configuration</th>
<th>Particle type</th>
<th>G-factor</th>
<th>Kinetic energy (MeV)</th>
<th>Beams CW/CCW</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>all-electric</td>
<td>proton</td>
<td>+1.79285</td>
<td>232.8</td>
<td>concurrent</td>
<td>final ring, prototype required challenging polarimetry impractically short lifetime</td>
</tr>
<tr>
<td></td>
<td>electron</td>
<td>+0.00160</td>
<td>14.5</td>
<td>concurrent</td>
<td></td>
</tr>
<tr>
<td></td>
<td>muon</td>
<td>+0.00165</td>
<td>2991</td>
<td>concurrent</td>
<td></td>
</tr>
<tr>
<td>E/B combined</td>
<td>proton</td>
<td>+1.793</td>
<td>45</td>
<td>consecutive</td>
<td>compromised EDM precision E/B technological challenge must develop polarimetry</td>
</tr>
<tr>
<td></td>
<td>deuteron</td>
<td>-0.143</td>
<td>variable</td>
<td>consecutive</td>
<td></td>
</tr>
<tr>
<td></td>
<td>helium-3</td>
<td>-4.191</td>
<td>39</td>
<td>consecutive</td>
<td></td>
</tr>
<tr>
<td>all-magnetic</td>
<td></td>
<td></td>
<td></td>
<td>used for precursor no frozen spin possibility</td>
<td></td>
</tr>
</tbody>
</table>

Details of the ring design may be found in other chapters of this report: Chap. 7 describing the COSY precursor experiment based on deuterons, Chap. 8 for the proton EDM prototype ring and Chap. 9, for the all-electric proton EDM ring. The route toward the final ring, i.e. the all-electric proton EDM ring will be explained in the next chapter. In what follows, we discuss the experimental observable and the basic measurement sequence.

### 4.3.4 Experimental observable: beam bunch polarisations

As described in Ref. [9], the 232.8 MeV proton ring has a 500 m circumference and a confining electric field of 8 MV/m. The accumulation rate for a signal corresponds to a rotation of the polarisation according to

$$\Omega_{EDM} = \frac{2Ed}{\hbar}. \quad (4.11)$$

For an EDM of $d = 10^{-29}$ e·cm, the rate would be about $1.6 \times 10^{-9}$ rad/s.

The plan for an EDM-sensitive polarisation measurement is to record the horizontal asymmetry in the scattering of sampled protons from a carbon target at forward angles. At the energies where the EDM
search would be made, the interaction between the polarised protons and the carbon nucleus contains a large spin-orbit term. This gives rise in elastic scattering to an asymmetry between left and right-going particles when there is a vertical polarisation component present. For a complete description of polarisation observables and effects, see Tanifugi [10].

For spin-1/2 particles, this effect is described by the differential scattering cross section given in Eq. (4.12) with the angles defined in Fig. 4.5. The polarisation along any given axis is given in terms of the fraction of the particles in the ensemble whose spins, through some experiment, are shown to lie either parallel or anti-parallel to that axis. If these fractions are \( f_+ \) and \( f_- \) with \( f_+ + f_- = 1 \) for the two projections of the proton’s spin-1/2, the polarisation becomes \( p = f_+ - f_- \) and ranges between 1 and \(-1\). The scattering cross section \( \sigma_{\text{POL}} \) may be written in terms of the unpolarised cross section \( \sigma_{\text{UNP}} \) as

\[
\sigma_{\text{POL}}(\theta) = \sigma_{\text{UNP}}(\theta) \left(1 + p A_Y(\theta) \cos \phi \sin \beta \right)
\]

with the vertical component given by

\[
p_Y = p \cos \phi \sin \beta.
\]

The angles are defined with respect to a coordinate system shown in Fig. 4.5 in which a particle from the beam, travelling in the \(+z\) direction, is scattered by an absorber into the \(+x\) or “left” side of the \(xz\) plane. The scattering angle is \( \theta \). The polarisation direction, shown as the red arrow, is defined by the two polar coordinate angles \( \beta \) and \( \phi \). The polarisation effect reverses if the particles are detected at the same \( \theta \) on the \(-x\) or “right” side of the beam due to the \( \cos \phi \) dependence in Eq. (4.12). Thus this left-right asymmetry measures the vertical polarisation component \( p_Y \). The size of the signal is governed by the strength of the spin-orbit interaction, which gives rise to the asymmetry scaling coefficient \( A_Y(\theta) \), otherwise known as the analysing power.

![Fig. 4.5: The coordinate system for polarization experiments where the beam defines the z axis. The detector position in the xz plane corresponds to the scattering angle \( \theta \) which is used to determine the spin cross section as shown in Eq. (4.12). The angles defining the orientation of the positive polarization direction (see Eq. (4.12) are labelled in this diagram.](image)

In the case of the deuteron, which is spin-1, there are three fractions that describe the magnetic sub-state population, \( f_+ \), \( f_0 \), and \( f_- \) where \( f_+ + f_0 + f_- = 1 \). The two polarisations are vector,
\[ p_V = f_+ - f_- , \] and tensor \( p_T = 1 - 3f_0 \), which can range from 1 to \(-2\). If we are interested only in the EDM, then the vector polarisation suffices as a marker and the deuteron polarised cross section (Cartesian coordinates following the Madison Convention \([10]\)) becomes

\[
\sigma_{\text{POL}}(\theta) = \sigma_{\text{UNP}}(\theta) \left( 1 + \frac{3}{2} p_V A_Y(\theta) \cos \varphi \sin \beta \right) .
\]

(4.14)

Tensor polarisation is usually present to a small degree in polarised deuteron beams. There are three independent tensor analysing powers that each add another “\(p_T A\)” term to the equation above. Their effects may prove useful in polarisation monitoring or checking for systematic effects. Because this report explores the possibility of a proton storage ring, the deuteron spin dependence will not be further elaborated here.

In the energy range where we would like to run the EDM search, it happens that the spin-orbit interaction between light particles such as the proton and deuteron and the carbon nucleus provides a large analysing power \(A_Y(\sim 60\%)\) for beam particles that scatter elastically into the forward direction from nuclei in the target. If a target a few centimetres thick is positioned at the edge of the beam in such a way that beam particles may be extracted onto its front face, then all of the beam may be consumed while up to 1% of the particles scatter from a nucleus in the target and make their way to one of the forward detectors. This represents a very high efficiency for using the beam particles to search for any sign of a growing vertical polarisation component.

### 4.3.5 Basic measurement sequence

A typical single measurement sequence is outlined below with the aim of giving some notions of the overall approach. There are still many open questions, and it is clear that experience of operating, firstly a prototype, and subsequently the full ring will be required to firmly establish the procedures. Details of the beam preparation process and data taking may be found in the Polarimetry chapter 11.

- Several bunches with vertically polarized protons are injected CW and CCW into the storage ring.
- Beams must be injected into the ring in both directions in reasonably rapid succession. The polarisation begins perpendicular to the ring plane.
- Using an RF solenoid, the spins of the particles are rotated into the horizontal plane.
- Subsequently the beams are continuously extracted onto the target for \(\approx 1000\) seconds.
- The increase of the vertical polarisation is proportional to the EDM, and is measured via the left-right counting rate asymmetry in the detector (see the Polarimetry chapter 11).
- Averaging the polarisation measurements from the CW and CCW rotating beams cancels some of the systematic effects (e.g. some of geometrical phase effects). Other effects (e.g. residual radial magnetic fields) are determined from a spatial separation of the two beams (see chapter 10).

This sequence is repeated approximately \(10^4\) times per year of operation. Note that for a single store, statistical effects will be over two orders of magnitude larger than any EDM effect at the expected level of sensitivity.

### References


Chapter 5

Strategy

5.1 Introduction

The project to search for charged-particle electric dipole moments in storage rings has a strong science case, but at the same time it is facing demanding technological and metrological challenges. Moreover, it is obvious that such high-precision measurements will require commitments for a long period of time. In order to justify the significant expenditures for the ring(s), it will be inevitable to outline a clear plan (see Chapter 13, Road Map and Timeline) for moving towards the ultimate goal of an all-electric polarised proton EDM-facility with clockwise and counter-clockwise beams operating at the magic momentum: this must include not only the verification of all key technologies, but also the demonstration that the aimed-for sensitivities are feasible. This has already started with several polarised beam techniques meeting the EDM experimental requirements. It is now clear that the only viable way to continue this is to pursue a staged approach with a prototype ring as the essential demonstration milestone.

5.2 Starting point of the staged approach

The charged-particle EDM project is in an excellent position to start with, since a conventional (i.e., using magnetic deflection) storage-ring facility exists that provides all the required elements for R&D and will even allow a “proof-of-capability” measurement. COSY, the cooler synchrotron at the Institute for Nuclear Physics (IKP) of Forschungszentrum Jülich (FZJ) in Germany, is a storage ring for polarised proton and deuteron beams between 0.3 (0.55) and 3.7 GeV/c. Besides phase-space cooling (electron, stochastic), well-established methods are used to provide, manipulate and investigate stored polarised beams. Over the past decade, the JEDI (Jülich Electric Dipole moment Investigations) Collaboration has made significant progress using COSY as an EDM test facility (see: Appendix A, Results and achievements at Forschungszentrum Jülich). Currently, JEDI is conducting a precursor experiment (see below and Chapter 6, Precursor Experiment) to obtain a first directly measured EDM limit for the deuteron by exploiting a radiofrequency (RF) Wien filter in the ring. The experiment is sensitive to the EDM through its effect on the direction of the invariant spin axis of the ring. A first measurement has been conducted and is currently being analysed. Additional measurements are planned for the second half of 2019 and 2020.

5.3 Route toward the final ring

A prototype ring (see Chapter 7) offers new capabilities, in a small testing environment, that can address EDM design issues not accessible otherwise. These include electric field beam transport with the possibility to store two counter-circulating beams using ring lattice construction suitable for the final EDM experiment. With the addition of air-core magnetic bending, it becomes possible to “freeze” the beam polarisation along the direction of the beam velocity, thus making possible a more sensitive search for an EDM compared to the precursor experiment. Tests will show the limits on beam storage and the precision of beam monitoring and control. Most importantly, systematic effects that limit the sensitivity in EDM experiments may be studied directly along with efforts to mitigate them.

It is the large number of uncertainties, the most fundamental of which is coping with inevitable residual magnetic fields, that currently prevents a realistic full-scale ring design going beyond the previously published report [Ref, RSI paper]. (See Chapter 8, “All-electric proton EDM ring”). The final full-scale design will be an (essentially) all-electric, 233 MeV ring with simultaneously counter-rotating, frozen spin proton beams.
Fig. 5.1: Summary of the important features of the proposed stages in the storage ring EDM strategy.

With experience gained from the prototype ring, the information needed for a detailed design of the all-electric proton EDM ring (see Chapter 8) should be available. From prototype test results we expect to be able to justify the technology used and the sensitivity level to be achieved. Finally, detailed and realistic cost estimates will then be possible.

5.4 Science case beyond EDM

The rotation of the polarisation (precession of the spin vector) involved in an EDM search may also couple to any oscillating EDM associated with a surrounding axion field. Data from the EDM search may be scanned as has been shown in neutron and atomic EDM searches to be possible for any evidence of an axion. In addition, moving the EDM ring parameters away from the frozen spin condition enables a broader range search to be conducted.

It may also be possible to find conditions where the counter-rotating beams obey frozen spin requirements for different particle species, thus allowing a class of high precision comparisons of relative magnetic moments and EDMs, if they are observable. Thus the EDM ring will become a facility for different experimental programs with discovery potential at the frontier of new science.
Chapter 6

Precursor Experiment

6.1 Introduction

The first step in the staged approach is a set of "proof-of-capability" tests referred to as the "precursor experiment". It is performed at the Cooler Synchrotron COSY at Forschungszentrum Jülich in Germany, which is a magnetic storage ring providing polarised protons and deuterons in the momentum range 0.3 to 3.7 GeV/c, see Fig. 6.1.

![Fig. 6.1: The Cooler Synchrotron COSY at Forschungszentrum Jülich in Germany.](image)

6.2 Principle of the Measurement

In a magnetic storage ring the precession of the polarisation vector in the horizontal plane prevents a build-up of a vertical polarisation due to the EDM. The EDM just causes an oscillation of the vertical polarisation component with amplitude $\xi = \beta \eta /[2G]$. This signature is used in the muon $g - 2$ experiment to measure the muon EDM. For hadrons this method is less sensitive because $|G_{\text{hadron}}| \gg G_\mu$.

The precursor experiment is performed with deuterons at a momentum of $p = 970$ MeV/c. In this case the amplitude is only $3 \cdot 10^{-10}$ for an EDM of $d = 10^{-24}$ e.cm. 

To allow for a build-up of the vertical polarisation proportional to the EDM, a radio-frequency (RF) Wien-filter can be utilised [1, 2]. Such a device was developed and constructed, see Refs. [3–5]. It was installed in COSY in May 2017. Fig. A.5 shows a drawing of the Wien filter.

In order to obtain a build-up, the Wien filter has to be operated in resonance with the spin precession frequency $f_{\text{spin}}$. The resonance condition is given by

$$f_{\text{WF}} = f_{\text{rev}}|k + \nu_s|, \quad k \text{ integer},$$

(6.1)

1The dimensionless factor $\eta$ is related to the EDM $d$ via the relation $d = \eta \frac{e\hbar}{2mc} S$. 

46
where \( \nu_s = f_{\text{spin}}/f_{\text{rev}} \) is the spin tune, defined as the number of spin revolutions per turn. For the experiments at COSY, the Wien filter was operated at a frequency of \( f_{\text{WF}} \approx 871 \text{ kHz} \) which corresponds to \( k = -1 \). The revolution frequency is \( f_{\text{rev}} \approx 751 \text{ kHz} \). The integral magnetic field of the Wien filter is \( 0.019 \text{ Tmm} \) and the corresponding integral electric field amounts to \( 2.7 \text{ kV} \). A build-up is only observed if the relative phase \( \Phi \) between the fields of the Wien filter fields and the horizontal polarisation component match. The polarisation vector has to be parallel to the momentum vector in the Wien filter when the \( E \) and \( B \) fields are at their maximum.

The precursor experiment requires several additional prerequisites:

1. a long spin coherence time, [6]
2. a precise monitoring of the 120kHz precession in the horizontal plane, [7]
3. a feedback system controlling the relative phase of the polarisation vector and the Wien filter fields, [8]

which have all been achieved. More details are given in appendix A. As one example, we discuss the spin coherence time. Fig. 6.2 shows the normalised polarisation in the horizontal plane as a function of time or turn number. Even after 1000 s approximately 50% of the initial polarisation is left.

\[
\begin{align*}
\text{Fig. 6.2:} & \quad \text{left: Initially all spins point in the same direction (a). Difference in the precession frequency } f_{\text{spin}} \text{ lead to decoherence (b). Right: After optimisation a spin coherence time of the order of 1000 s was reached at COSY.} \\
\end{align*}
\]

\[6.3 \quad \text{Current Status}\]

With all these tools available, a first precursor test run was performed. The main operating parameters of COSY for the precursor experiment are shown in Tab. 6.1. The COSY ring indicating the main components used in the experiment is shown in Fig. 6.3.

\[\begin{tabular}{|l|c|}
\hline
COSY circumference & 183 m \\
\hline
deuteron momentum & 0.970 \text{ GeV/c} \\
\hline
\( \beta(\gamma) \) & 0.459 (1.126) \\
\hline
magnetic anomaly \( G \) & \( \approx -0.143 \) \\
\hline
revolution frequency \( f_{\text{rev}} \) & 750.6 kHz \\
\hline
cycle length & 100-1500 s \\
\hline
number of stored particles/cycle & \( \approx 10^9 \) \\
\hline
\end{tabular}\]

\[\text{Table 6.1: Values of the COSY operating parameters for the deuteron precursor EDM experiment.}\]

Fig. 6.4 shows the build-up rate \( \dot{\alpha} \) with \( \alpha = \dot{P}_{\text{vertical}}/P_{\text{horizontal}} \) of the vertical polarisation component as a function of the relative phase \( \Phi \). The expected sinusoidal shape is observed. To obtain this
data requires about 4 hours of beam time. For every single data point the relative phase $\Phi$ was set using
the feedback system. Systematic effects, like misalignments of ring elements, deviation from the design
orbit cause at this stage fake EDM-signals orders of magnitude larger than real EDM effect. These ef-
fects are under investigation. To get an idea about the statistical sensitivity the hypothetical signal of an
EDM of $d = 10^{-18}$ e.cm is indicated by the gray line. The statistical error of the measurement is of the
order of the symbol size, indicating that statistically one is sensitive to EDMs well below the $10^{-18}$ e.cm
level.

![Fig. 6.3: The COSY ring with the main components used in the precursor experiment.](image)

To get a deeper understanding of systematic effects, the invariant spin axis was varied. The in-
variant spin axis, $\hat{n}$, is defined as the rotation axis of the polarisation vector. In an ideal ring with no
EDM $\hat{n}$ points in the vertical $y$-direction. An EDM adds a radial $x$-component such that
$\angle(\hat{n}, \vec{e}_x) = \xi = \beta \eta/(2G)$. A Wien filter rotation around the longitudinal beam axis acts in the same direction,
$\angle(\hat{n}, e_z) = \Phi_{WF}$. The solenoidal field in the snake causes a tilt in the beam $z$-direction such that $\angle(\hat{n}, e_z) = \chi_{\text{sol}}/(2 \sin(\pi \nu_s))$ ($\chi_{\text{sol}}$ depends on the snake current). Thus, physically rotating the Wien filter by an angle $\Phi_{WF}$ around the beam axis and introducing a longitudinal magnetic field using the snake will introduce deliberate perturbations that cause changes in the build-up of a vertical polarization.

At a setting where the introduced perturbations cancel the imperfections of the COSY ring and the EDM effect, the build-up should vanish.

Fig. 6.4 shows the build-up for $\Phi_{WF} = 0$ and $\chi_{\text{sol}} = 0$. In Fig. 6.5 the so called resonance strength which is proportional to the amplitude of the observed oscillation in Fig. 6.4 is shown for various values of $\Phi_{WF}$ and $\chi_{\text{sol}}$ in a range between $-1.5^\circ$ and $1.5^\circ$. The surface is a fit to the data using an analytic expression for the build-up. The minimum of this graph gives the location of the invariant spin axis. In an ideal ring its location is $(\xi = \eta\beta/(2G), 0)$ in radial and longitudinal direction.

It should be noted that in total three "maps" were taken indicated by the different symbols in the plot. The fact that they give consistent results although taken several days apart indicates that the stability of COSY is sufficient to perform this kind of precision studies down to sensitivities corresponding to EDM values well below $10^{-18} \text{ e cm}$. Of course at this stage the deviation of the minimum from $(\xi, 0)$ is mostly attributed to systematic effects (e.g. misalignment of magnets and beam position monitors causing deviations from the design orbit). Work is going on to minimise these effects using beam based alignment and quantify them with the help of simulations.

Fig. 6.5: The resonance strength $\epsilon$ which is proportional to the amplitude of the observed oscillation in Fig. 6.4 ($\epsilon = \dot{\alpha}/(2\pi f_{\text{rev}})$), for various values of $\Phi_{WF}$ and $\chi_{\text{sol}}$. The surface is a fit to the data using an analytic expression for the build-up. The minimum of this graph gives the location of the invariant spin axis.
6.4 Outlook

In the first half of 2020 a second EDM run is planned by the JEDI collaboration. Prior to this run, beam based alignment procedures are performed in order to calibrate beam position monitors which in turn will lead to an improved orbit. This will likely reduce the shift of the invariant spin axis due to systematic effects.

In the same time simulations tools are developed (see Chapter 12) in order to estimate the contribution of systematic effects on the invariant spin axis. The goal is to perform with COSY a first EDM measurement with a precision similar to the one of the muon, i.e. $10^{-19}$ e.cm.

It should also be clear that gaining further orders of magnitude in precision is only possible with a dedicated storage ring using counter rotating beams where many systematic effects mentioned above cancel.

References


Chapter 7

The EDM Prototype Ring (PTR)

7.1 Introduction

7.1.1 Need for a prototype ring (PTR)

Intense discussions within the CPEDM collaboration have concluded that the final ring cannot be designed and built in one step (see Chapter 5, “Strategy”). Instead, a smaller-scale prototype ring PTR must be constructed and operated as the next step.

In the following Chapter 8, “All-Electric Proton EDM Ring”, the state of preparedness for a full-scale all-electric proton EDM of approximately 500 m circumference is discussed. Ideally containing only electric fields and no magnetic fields, this ring needs to be capable of storing 232.8 MeV frozen spin protons circulating in either clockwise (CW) or counter-clockwise (CCW) directions. Initially these beams will circulate consecutively. But, for best possible suppression of systematic EDM errors, the beams will later have to circulate concurrently.

As part of the preparation of the present report, the level of preparedness for constructing was studied in considerable detail, with the results distilled down to Table 8.2 in Chapter 8. There the “lacks of preparedness” are sorted by perceived “degrees of severity”.

Naturally, such a sorting cannot be arithmetically precise, but a kind of “triage” sorting is possible. Some all-electric EDM storage ring problems can be judged to be “show stoppers” which would definitively prevent EDM measurement from being accomplished. It is only because no relativistic all-electric storage ring has ever been designed and built that problems of this degree of severity cannot be ruled out by experience. Such problems have been colour-coded “red”.

Other problems, though obviously still in need of further refinement, have been colour-coded “green” to indicate that, based on wide experience that has been gained with polarized beams in magnetic storage rings, there is no need to be concerned about their performance in a full scale all-electric ring, neither in impairing beam performance, nor in limiting the precision of the EDM determination.

Finally there are cautionary problems, clearly between red and green in seriousness. These are coded “yellow”, for caution, in Table 8.2. The potential importance of these problems, in essentially all cases, is that they are capable of restricting the precision with which the proton EDM can be determined.

The primary basis for the conclusion that a prototype ring is needed is the presence of show-stopper entries in Table 8.2. These flags must certainly be cleared before serious full-scale planning can begin responsibly. Furthermore, the lack of experimental experience with electric rings prevents even any extrapolation from established experience.

Once all of the red flags have been cleared, serious design of a full-scale ring can be contemplated. Even then, a complete full-scale design will require that most, or perhaps all, of the yellow flags be cleared as well. These were the main predicates from which the proposed PTR program has been derived.

7.1.2 Considerations leading to two PTR stages

Goals for the PTR prototype ring have been constructed to correlate sensibly with these preparedness designations. In particular, two stages are planned. The goals of stage 1, after re-confirming beam control procedures that have already been developed at COSY, will be to turn all red flags in Table 8.2 at least to G(-) or Y(+). The goals of stage 2 will be more diverse. But their common thread will be to gain the experience needed to complete the design of the full-scale ring. This has to include acquiring information needed to predict the potential precision with which the proton EDM can be measured.
Certainly, as a prototype, the ring should be small and simple, and as inexpensive as possible. Yet the ring has to be designed to be capable of achieving its claimed goals. The primary goal of stage 1 is to demonstrate that performance routinely obtained in magnetic rings can be replicated in an all-electric ring. The goals for stage 2 mainly require frozen spin protons. (Except at the 232.8 MeV “magic” kinetic energy for which proton spins can be frozen in an all-electric ring) proton frozen spins require vertical magnetic field $B_z$ to be superimposed on the radial electric field $E_r$.

Several considerations went into the determination of kinetic energies for stages 1 and 2. To avoid new building costs, the ring circumference was constrained to not exceed 100 m. After allowing for adequate drift space for needed equipment, this led to a bending radius less than 9 m. For technical reasons the power supply voltages have been constrained to not exceed $\pm 200$ kV. A consequence of these requirements was that the proton kinetic energy should not exceed 30 MeV.

The proton polarimeter figure of merit is satisfactory at 30 MeV, but decreases with decreasing energy. As a result of these considerations, the 30 MeV nominal all-electric, stage 1, proton beam energy was adopted. (As it happens, electrons circulating under identical ring conditions will be approximately “magic”, meaning that their spins will be “frozen”. Except for the quite low efficiency of currently available electron polarimetry, this means that, in principle, the electron EDM can also be measured in the PTR.)

From 30 MeV proton energy for stage 1, the choice of 45 MeV for stage 2, frozen spin operation, followed almost automatically. In order to achieve the frozen spin condition for protons near this energy, approximately 1/3 of the bending shall be provided by magnetic elements. Since the magnet needs to be iron free (to avoid hysteresis and obtain the required reproducibility) air core magnets must be used. The required magnetic field is sufficiently low that this is not a serious constraint.

Up to this point in PTR design studies there has been no differentiation between all-electric, 30 MeV, stage 1 optics and 45 MeV, stage 2 frozen spin optics. The basic design has sufficient flexibility to meet both goals. In detail, of course, the working points and other details will be essentially different. Detailed lattice design and performance is described later, in Section 7.6.

### 7.1.3 Basic beam parameters and layout

During task force studies the ring evolved from a race-track to a square shape of various sizes. The present report describes the adopted “square ring”, having 8 m long straight sections. The basic proton kinematic data and field strengths are given in Table 7.1, and the ring layout is shown in Figure 7.1.

| Table 7.1: Basic beam parameters for the PTR ring |
|-----------------|--------|--------|---------|
| Kinetic energy | $E$ only | $E$, $B$ | unit |
| $\beta = v/c$  | 0.247  | 0.299  | MeV |
| $\gamma$ (kinetic) | 1.032  | 1.048  | |
| Momentum       | 239    | 294    | MeV/c |
| Magnetic rigidity $B \rho$ | 0.981  | T·cm |
| Electric field only | 6.67    | MV/m |
| Electric field $E$ (frozen spin) | 7.00    | MV/m |
| Magnetic field $B$ (frozen spin) | 0.0327  | T |

### 7.2 Goals for the 30 MeV all-electric PTR

The two primary goals for the 30 MeV stage can be expressed quantitatively. They are

1. to demonstrate the ability to store the $10^9$ polarized protons thought to be the minimum number
needed to be able to perform proton EDM measurements in a predominantly electric storage ring, and

2. as needed for reducing systematic error, the ability to produce two polarized beams, each with the same $10^{9}$ proton intensity, simultaneously counter-circulating in the same ring.

Technically, it would be sufficient for these goals to be achieved with unpolarised beams, since there is no reason to suppose that the storage capability depends in any way on the state of beam polarization. The proton intensity goal has been set conservatively low to avoid distractions associated with preserving polarization through the injection process—this can be perfected later, using well-understood experimental techniques.

Polarimetry already demonstrated in COSY will be sufficient to complete these goals. As in COSY, the spins will not be frozen, nevertheless the spin coherence time (SCT) can be determined. Also phase-locked spin control can be reconfirmed.

Secondary, qualitative goals for stage 1 therefore include replicating spin-control abilities in an all-electric ring, such as phase-locked loop stabilization of the beam polarization. This capability is required to provide input signals to the external correction circuits needed to manipulate the beam polarization. As in COSY, this capability does not require frozen-spin protons—it is enough for the spins to be “pseudo-frozen”; i.e. with spin tune equal to the ratio of two small numbers so that, viewed at fixed location, the polarization vector appears frozen.

Certain tertiary goals for stage 1 will also be needed to steer the upgrading of PTR for a more advanced second stage. But any such upgrades need to preserve the gross geometry of the ring. (Mainly to reduce cost, and speed progress) it seems prudent initially, to economize, with flexibility for later upgrades. Investigations in the first stage, can produce PTR modification possibilities needed to produce a more productive second stage. Some examples follow.

It is currently uncertain whether a completely cryogenic vacuum will be necessary. Connected to this issue is whether or not the beam emittance can be adequately controlled by stochastic cooling, and whether stochastic cooling adversely affects EDM experiments. Also connected with vacuum uncertainty is the possibility of a regenerative breakdown mechanism that could limit the proton beam current. Such a breakdown could commence with a temporarily free electron being accelerated toward
the positive electrode. Secondary electrons created on impact, would be immediately re-captured, but photons produced could strike the other electrode, producing secondary electron emission that could lead to regenerative failure. No such phenomenon has ever been observed in magnetic rings—but this is irrelevant, since there is no corresponding electron acceleration present. Some proton intensity limitations in non-relativistic rings seem consistent with such as interpretation. But no such limitation has been observed in electrostatic separators in either electron or proton high energy storage rings. Any such breakdown mechanism would presumably tend to be moderated by a superimposed magnetic field. But weak magnetic fields could be ineffective.

Magnetic shielding is another uncertain issue. There are well-understood (but expensive) passive magnetic shielding methods known, improving the shielding by at least one or two orders of magnitude. But they require detailed understanding of the apparatus, that can, realistically, be studied experimentally only in situ. Certainly magnetic shielding could be upgraded in the interval between stages. No active field control based on magnetic measurement is planned for stage 1, but could, optionally, be developed for stage 2.

The possibility of significant upgrading of positioning and alignment is also anticipated between stages 1 and 2. Ferrite kickers, assumed for stage 1, may need to be replaced by air core or electrostatic kickers for stage 2.

Greatly improved critical analysis of beam position monitor (BPM) performance is also expected of stage 1, for possible inclusion in stage 2. Similar investigations in the stability of basic mechanical and electrical parameters will be performed.

7.3 Goals for the 45 MeV combined E/B PTR

Stage 2 will be largely dedicated to the development of operational capabilities and identification of issues that need to be resolved before an eventual full scale ring design can be committed. The following goals are essential:

1. To lend confidence to an eventual full-scale EDM ring proposal, experimental methods are to be developed and demonstrated for measuring the proton EDM in a ring with superimposed electric and magnetic bending. Cost-saving measures in the prototype, such as room temperature operation, minimal magnetic shielding, avoidance of obsessively tight manufacturing and field-shape matching tolerances, are expected to limit the precision of the prototype ring EDM measurement. But data needed for extrapolation to the full scale ring has to be obtained from the PTR.

2. Demonstrate frequency domain control and measurement capability; for example a phase-locked spin wheel frozen spin beam control. (See Section 7.9)

3. Investigate, understand, and measure the general relativistic (GR) correction to the proton EDM measurement, which is expected to produce a “fake” EDM measured value of approximately $15 \times 10^{-29} \text{e-cm}$. This quantity can be calculated to better than one percent accuracy. This process is useful as a “standard candle”, whose measurement can provide a major physics motivation for stage 2. The factor of 15 by which the GR effect exceeds the nominal $10^{-29} \text{e-cm}$ effect provides a factor of 15 “cushion” in isolating systematic effects. The fundamental physical significance of this gravitation-dependent measurement is debatable. But any credible deviation from general relativity at atomic scales would be as alarming as any measurably large EDM. The fundamental experimental value for calibration purposes is also of debatable value; a fake EDM of far greater (more convenient) magnitude can be reliably mimicked using a Wien filter.

4. Finally, a first precise storage ring proton EDM measurement can be made. For various reasons, mainly due to cost-saving measures in PTR design, the achieved precision cannot, however, be expected to provide a significant test of the standard model. But information gained from this prototype measurement can be expected to produce specifications the nominal all-electric ring needs to meet to reach that goal.
7.4 Relation between PTR and the nominal all-electric ring

This section provides fine-grained technical details concerning the relation of the proposed prototype to the full scale ring. This is detail the average reader expects to exist for subsequent design refinement, but without the immediate need for such detail. Such readers may prefer to glance through this section only perfunctorily.

The details describe a four parameter lattice design for a complete family of stable all-electric storage rings, ranging from the PTR ring at the small radius, low energy end, to a full scale, large radius, high energy end. Especially for measuring the EDMs of particles other than protons, there are valid reasons for considering electric rings everywhere in this range. And, for the proton rings emphasized in this report, when comparing the results of different particle tracking programs, it is important for all assumed lattice parameter to be identical, even down to the fine-grained detail given here.

The structure of the prototype ring (PTR) has been obtained from the full-scale ring by down-scaling from the full scale Anastassopoulos et al. [1] design to 30/45 MeV, trying to keep the two designs as close as possible. After the down-scaling, mainly to make element lengths sensible for a low energy ring, small changes were then made to the PTR design before scaling back up to the full-scale ring. In this way, physical properties of the scaled-back-up full-scale ring and the Anastassopoulos et al. ring can be compared as in Table 9.1, in the full scale ring chapter. (As expected) agreement is quite good for all parameters, well within the ranges of parameter values of the various 2016 ring designs.

The skeletal PTR prototype lattice design is shown in Figure 7.2.

Fig. 7.2: Lattice layouts for proposed lattice half-cell (left) and full ring (right). The accumulated drift length is not enough for the ring to operate “below transition”. When scaling up to the eventual, full energy, all-electric ring, from four-fold to sixteen-fold symmetry, with drift lengths and bend lengths preserved (but bend angles four times less) the total circumference is to be about 500 m and operation will be well below transition.
In both ring designs, for flexibility, focusing is provided both by separated-function electric quadrupoles, and by (very weak) alternating-gradient, combined-function, electrode-shape focusing. (Current designs have favoured electric-quadrupole-only focusing).

It was decided that the scaling relation between prototype and full-scale ring would be performed by relating the ring super-periodicities in the ratio of 4 to 16, while leaving all lengths (except for straight section lengthening to be explained) within each super-period constant. Expressed as ring “shapes”, this scaling gives the prototype ring the appearance of a square with rounded corners (see Figure 7.2), while the full ring appears very nearly circular; (see Figure 8.1 of the nominal all-electric chapter). In this process the bend per super-period was reduced by an (integer) factor of 4. The values of the four main scaling parameters are shown as (upper-case) parameter values in Table 7.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>E_30MeV</th>
<th>EM_35MeV</th>
<th>EM_45MeV</th>
<th>EM_55MeV</th>
<th>E_233MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_NOMINAL [m]</td>
<td>9.0</td>
<td>9.0</td>
<td>9.0</td>
<td>9.0</td>
<td>40.0</td>
</tr>
<tr>
<td>L_LONG_Straight [m]</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>14.8</td>
</tr>
<tr>
<td>N_SUPER</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>M_NOMINAL</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The adopted scaling relations follow: the field index scales inversely with super-periodicity N_SUPER; with \( m = \pm M_{\text{NOMINAL}} \) being the field indices of the prototype ring, the scaling relation is \( m = \pm M_{\text{NOMINAL}} \times 4 / N_{\text{SUPER}} \). Lattice names are in the column headings. Minor scalings are indicated in Table 7.3.

**Table 7.3:** Minor geometric parameters: Theta, \( r_0 \), \( leh \), \( lss = 0.8 \) m, and \( llsh \) are, respectively, bend/half-period, bend radius, bend-half-length, short-straight-length, and long-straight-half-length. \( K^0 \) is proton kinetic energy and \( \pm m_{\text{fin}} \) are alternating field index values. Minor kinetic parameters: \( lq \) is quad length, \( q_F \) and \( q_D \) are quad strengths, \( gB_y/2 \) is half-gap width, \( Q_x \) and \( Q_y \) are tunes.

```
<table>
<thead>
<tr>
<th>Lattice name</th>
<th>K0 [MeV]</th>
<th>m_fin</th>
<th>Theta</th>
<th>r0 [m]</th>
<th>leh [m]</th>
<th>llsh [m]</th>
<th>lq [m]</th>
<th>qF/qD</th>
<th>circ. [m]</th>
<th>gBy2 [m]</th>
<th>Q_x/Q_y</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_30MeV</td>
<td>0.0300</td>
<td>0.010</td>
<td>0.175</td>
<td>9</td>
<td>3.53</td>
<td>2.60</td>
<td>0.2000</td>
<td>±0.01</td>
<td>83.7</td>
<td>0.035</td>
<td>1.768/0.093</td>
</tr>
<tr>
<td>E_45MeV</td>
<td>0.0450</td>
<td>0.010</td>
<td>0.175</td>
<td>9</td>
<td>3.53</td>
<td>2.60</td>
<td>0.2000</td>
<td>±0.01</td>
<td>83.7</td>
<td>0.035</td>
<td>1.350/0.093</td>
</tr>
<tr>
<td>E_233MeV</td>
<td>0.2328</td>
<td>0.025</td>
<td>0.176</td>
<td>40</td>
<td>3.93</td>
<td>7.00</td>
<td>0.2000</td>
<td>±0.0025</td>
<td>50.1</td>
<td>0.015</td>
<td>1.815/0.145</td>
</tr>
</tbody>
</table>
```

Detailed lattice descriptions (needed for computer processing) are contained in the following files, and identified in the tables by the first column entries (are available on request).

**EM_45MeV-con.xml:** “.xml” file containing all parameters (both symbols and their values) for a small (85 m circumference) proton EDM prototype ring, including (symbolic) parameters for scaling to the large (500 m circumference) all-electric proton EDM ring.

**EM_45MeV-nocon.xml:** Symbolic “.xml” file describing idealized lattice design.

**EM_45MeV.adxf:** Numerical “.adxf” file describing idealized lattice design.

**EM_45MeV.sxf:** Numerical “.sxf” file describing fully-instantiated lattice design (though without differentiated (i.e. individualized) parameter values.)

Initially, for both prototype and full-scale ring, the horizontal tune was expected to be just below 2.0 and the vertical tune less than 1.0, and tuneable to a value as low as 0.02. This ultra-low vertical tune was needed to reduce the vertical restoring force, to enhance the beam “self-magnetometry” sensitivity to beam displacement caused by radial magnetic field.

(As an aside, it can now be mentioned, that the doubly-magic EDM measurement method possibility avoids the need for ultraweak vertical focusing, allowing the focusing to be much stronger than
was initially anticipated. A very thorough and valuable 2015 study by V. Lebedev [3] analysed two frozen-spin all-electric designs, one very weak-focusing, the other stronger focusing. With ultra-low vertical tune no longer necessary, the scaled down PTR can be said to more nearly correspond to the stronger-focusing ring favoured there.)

For the full-scale ring the correspondingly smaller tune advance per super-period causes the focusing to be weaker. This is what permits the long straight sections of the full scale ring to be more than doubled, compared to the prototype (from 6 m to 14.8 m). This has the beneficial (perhaps even obligatory) effect, for the full-scale ring, of operating “below transition”. This ameliorates intrabeam scattering, as can be explained in connection with stochastic cooling. (Conversely, this is one respect in which the prototype ring optics is a not-quite-faithful prototype.) This choice was made to reduce the prototype size. Also, with the COSY ring as a candidate low energy injector ring, for reasons of beam bunch-to-bunch separation, the EDM prototype ring circumference of 91 m, exactly one-half of the COSY circumference, would be a natural choice.

7.5 Electric and magnetic bends

7.5.1 Electric part

The electrostatic deflectors consist (ideally) of two cylindrically-shaped parallel metal plates with equal potential and opposite sign. With the zero voltage contour of electric potential defined to be the center line of the deflector, the “ideal orbit” of the design particle stays on the center line. The electrical potential is defined to vanish on the center line of the bends, as well as in drift sections well outside the bends. So the electric potential vanishes everywhere on the ideal particle orbit. With the electric potential seen by the ideal particle continuous at the entrance and exit of the deflector, its total momentum is constant everywhere (even through the RF cavity, except during very weak acceleration needed to keep the beam centroid on the design axis).

There are restrictions on the minimum distance between deflectors. Recent candidate ring lattice studies have limited the horizontal good-field-region for stored particles to be 50 mm. This requires the minimum distance between electric deflector plates to be about 60 mm. The vertical beam size is several times larger than the horizontal. This imposes restrictions on the vertical dimensions of the flat part of the deflector too. Minimum vertical dimensions of the bending elements will be more than 100 mm. In order to minimize breakdown probability between the flat regions of the deflectors and move them to the edge, the shape of deflectors should follow Rogowski profiles at both vertical edges. The ends of individual deflectors need to be shaped to match the stray fields with subsequent deflectors.

The designed ring lattice requires electric gradients in the range 5-10 MV/m. This is more than the standard values for most accelerator deflectors separated by a few centimeters. Assuming 60 mm distance between the plates, to achieve such high electric fields we have to use high voltage power supplies. At present, two 200 kV power converters have been ordered for testing deflector prototypes. The field emission, field breakdown, dark current, electrode surface and conditioning are to be studied using two flat electrostatic deflector plates, mounted on the movable support with the possibility of changing the separation from 20 to 120 mm. The residual ripple of power converters is expected to be in the order of $\pm 10^{-5}$ at maximum 200 kV. This will lead to particle displacement on the order of millimeters. A smaller ripple or stability control of the system will be a dedicated task for investigations planned at the test ring facility.

7.5.1.1 Design of the electric part

The electric part of the ring can be considered a plate capacitor, whose distance parameter was determined from beam optics considerations. The 2D cross section is shown in Figure 7.3. The good-field region, or the region of interest (ROI), was specified to have dimensions 20 mm $\times$ 60 mm. The contours of the upper and lower edges of the plates were rounded according the Rogowski shape principle. Due to the
finite radius of curvature of the plates of about 8 m a field gradient is generated. Its magnitude can be estimated in the case of infinitely high capacitor plates, because in this case, the electric potential is purely logarithmic, and its gradient - the electric field - can be obtained analytically. For finitely high capacitor plates, this should still provide a good approximation.

\[ U(\rho) = U_i + (U_o + U_i) \cdot \frac{\ln(\rho/\rho_i)}{\ln(\rho_0/\rho_i)} \quad (7.1) \]

The corresponding electrical field in radial direction is given by:

\[ E_\rho(\rho) = -\frac{\partial}{\partial \rho} U(\rho) = -\frac{U_o - U_i}{\rho} \cdot \frac{1}{\ln(\rho_0/\rho_i)} \quad (7.2) \]

\[ U_i \] and \[ U_o \] are the potential values on the inner and the outer capacitor plates, respectively, with the corresponding values of the radii \[ \rho_i = 8.831 \text{ m} \] and \[ \rho_o = 8.891 \text{ m} \]. Here \[ U_i = -U_o = 210.2 \text{ kV} \].

Figure 7.4 shows the potential values and the electric field strength between the capacitor plates calculated with these parameters. There are two ways of dealing with the field strength gradient depicted in Figure 7.4. The accumulated effect may be compensated by electric quadrupoles outside the bending section. This solution is valid, and the required quadrupole strengths can be estimated from the figures given above. On the other hand, the gradient can be compensated locally by shaping the contour of the electrodes, giving the inner and the outer plate convex and concave shapes, with radii of curvature 8 m, respectively. Figure 7.5 suggests how the plates should be manufactured in order to provide this compensation.

The homogeneity profile of the electric field in the ROI is shown in Figure 7.6. The average value is about 7 MV/m, the same as predicted by the theoretical considerations leading to the results of Figure 7.4. The maximum relative difference of the electric field in the ROI is about \( 2.1 \times 10^{-3} \).
Fig. 7.4: Potential values and corresponding electric field strengths between the capacitor plates in the case of infinitely high capacitor plates. The average field strength is about 6.998 MV/m.

The geometry is not yet fixed but may be altered according to engineering needs. A guide to the expected homogeneity values on changing the distance and the height of the capacitor plates is depicted in Figure 7.6. The figure caption gives an example about how to read the graphics.

7.5.2 Magnetic part

The nominal magnetic bending field is vertical, \( B_y = B \cdot \hat{y} \). For the combined E/B-prototype ring a first design has been made based on the requirements on integrated electric and magnetic fields. Specifically the magnetic flux density of the magnet should be \( B = 32.65 \) mT and the corresponding electric field is \( E = 6.998 \) MV/m. The prototype ring comprises electric and magnetic units. The design is shown in Figure 7.7.

The electric bends were separately considered earlier. In this section we deal with the design of the magnets. The stray field of the magnet is investigated separately, because it determines the shape of the capacitors for the combined electric/magnetic design.

All the magnetic design simulations have been carried out using the programs Amperes (3D) and Magneto (2D) by IES (www.integratedsoft.com). For the electric field simulations the programs Coulomb (3D) and Electro (2D) by the same company were used.

7.5.2.1 Design of the normal conducting magnets

The required vertical flux density of 32.65 mT is small enough to envisage a solution with normal conducting, even air-cooled magnets. The magnets are designed according to the \( \cos \theta \)-scheme to ensure a high level of homogeneity of the magnetic field. In order to avoid detrimental magnetic fields from the return paths of the cables in the \( \cos \theta \)-dipole, even these have been distributed in a \( \cos \theta \)-fashion. This reduces the effective field in the region of interest, but the flux density is not very high anyway \( (B_y = 32.65 \) mT). The cross section of the \( \cos \theta \) magnet looks as depicted in Figure 7.8.

In this design the conductors have a cross section of 50 mm × 8.1 mm. No attempts have been made to ensure that conductors with this cross section are available on the market. In any case these dimensions will have to be chosen according to engineering requirements and availability. The rectangle in the center of Figure 7.8 represents the region of interest (ROI) with dimensions 20 mm × 60 mm. The average flux density in the ROI is 32.65 mT. Figure 7.9 shows the deviation from this value. It is less than 1 \( \mu \)T—so small that in reality the homogeneity will be dominated by manufacturing tolerances. This
Fig. 7.5: Introduction of concave and convex shapes of the electrodes as one possibility to reduce the gradient field due to the curvature. The inner (left) electrode has a concave shape depicted in the inset, whereas the outer one has a convex one. The corresponding arc can barely be distinguished from the straight line coming down from the ends of the curved sections, because the radius of curvature is about 8 m. The corresponding sagitta is only about 0.33 mm.

The contour plot is slightly asymmetric, because the magnet is not straight but follows a radius of about 8.8 m. This curvature introduces a gradient in the magnetic flux density, leading to a left-right asymmetry. This asymmetry has been reduced by the introduction of a slight rotation of the upper conductors and a reverse rotation of the lower ones by about 0.16° around the center of the arrangement, which cannot be perceived in Figure 7.8, because of the smallness of this angle.

The current density in the conductors is about 2.6 A/mm². For the present design the generated power amounts to about 43 kW at a current of 1053 A, corresponding to a voltage drop of about 41.0 V. This may be too high a value to rely on air cooling alone for the removal of the generated heat, but design studies have been carried out which show that the length of the conductors can be enlarged from 8.1 mm, thus reducing the current density and the heat load without compromising the field homogeneity. At present it seems reasonable to assume a water-cooled magnet. The mass of the copper conductors for a single magnet amounts to about 3000 kg. The magnet can be accommodated outside the vacuum tube.

7.5.2.2 Matching of magnetic and electric stray fields

A staged approach was agreed on to match electric and magnetic fields. A global matching of the electric and magnetic fields based on field integrals will suffice in the first stage. This requirement can easily be fulfilled from an engineering point of view and the design would already be well described at this point, but it will in the end not be sufficient to lower the EDM level to the anticipated values. For this purpose it will be necessary to ensure local matching of the magnetic and the electric field in a second stage.

Due to the fact that the global matching does not represent a major obstacle we are here concerned
Fig. 7.6: Variation (on a logarithmic scale) of the homogeneity of the electrical field strength as given by the difference of the maximum and minimum values on the circumference of the region of interest for a straight capacitor. Example: Without any change to the geometry the homogeneity is close to $10^{-2.7} = 2.1 \times 10^{-3}$. The enlargement of the (nearly) straight section of the plates by about 20 mm improves this value to $10^{-3.2} = 6.3 \times 10^{-4}$ (see horizontal arrow). A subsequent increase of the plate half distance by 12 mm deteriorates this value again to about $10^{-2.7}$ (vertical arrow).

with the task of locally matching electrical and magnetic field. Inside the magnet and inside the capacitor the fields are quite constant in amplitude, and their ratio can be chosen according to the requirements. In the stray field regions both fields reveal different decay lengths, because the magnetic field component is much larger in size than the electric one. For this reason, the magnetic stray field has a much larger decay length and the geometry of the electric capacitor has in some way to be adapted to the decay of the magnetic field. The decay of the magnetic field can hardly be changed, because the way the inner conductors are to be connected to the outer returning counterparts is more or less determined by the cross section shown in Figure 7.8.

Figure 7.10 shows how the magnetic field behaves along the central trajectory between adjacent magnets.

It is well known from electrostatics, that the electric field of a plate capacitor is inversely proportional to the distance of the plates for fixed potential difference. Several simulations for this study have shown that also locally the electric field follows this rule. More specifically, as long as the field plates are much higher than the gap distance the local electric field is indirectly proportional to the plate distance at this location. For this reason a flux density distribution like that shown in Figure 7.10 can be regarded as the inverse gap distance of a capacitor providing the same field behavior. From this consideration we can already draw the conclusion that it will be difficult to fulfill the requirement of locally matching the two fields at all locations on the trajectory because the magnetic field drops to very low values outside the magnet pairs, which would correspond to a very large capacitor gap. It may still be possible between the two magnets because the field reduction is given by a factor of 27 in Figure 7.10, which
would correspond to a gap distance of about $27 \times 60 \text{ mm} = 1620 \text{ mm}$. There may be a concern of how to accommodate such a large capacitor inside the vacuum tube, but it must be kept in mind that, where the gap increases considerably, there is no magnet to restrict this expansion and the beam tube may locally be much larger.

From Figure 7.10 it becomes clear that this solution is not viable if the magnetic field decreases even further, as in the region between two neighboring magnets in different quarters, as shown in this figure for the outermost distance values. In this case the question arises whether several such capacitors with step-wise decreasing potentials can be stacked along the trajectory to approximate the magnetic field decay in a step-wise fashion.

Figure 7.11 shows an example of this stacking principle for the field decay between magnets in different quarters. This figure shows the normalized electric and magnetic field obtained with numerical simulations. These normalized field values cannot be distinguished on this scale but the difference values (red curve) show small features in the overlap region (red curve) where two neighboring capacitors meet. In all there are 5 capacitors with decreasing potential differences, which require the same number of power supplies unless a solution with voltage dividers is chosen. The number of capacitors is dictated by the maximum expansion factor accepted, which for the example in this figure is about 1.9. This translates into a local distance of the capacitor of $60 \text{ mm} \times 1.9=114 \text{ mm}$.

Preliminary attempts have been made to reduce the amplitude of the red curve even further by letting the capacitors overlap slightly along the trajectory. This goal seems to be achievable, but should be pursued only after a thorough engineering of the design has been carried out. Figure 7.12 shows the expansion of the 5 overlapping capacitors in a cross section.

If larger expansion factors are acceptable, there may be space in the wide gap between the magnets to accommodate auxiliary devices like quadrupoles or beam position monitors. Figure 7.13 shows such an option with a larger capacitor gap at a coordinate of 1800 mm. The transition from one capacitor to the next takes place at the distance coordinates 850 mm and 1150 mm.
Fig. 7.8: Upper part of the cross section of the cos $\theta$ dipole, showing inner conductors (that dominate the field) as well as the return paths of the conductors (that return the currents without degrading the uniformity, at the small cost of reducing the field strength). The current direction is represented by the colour of the conductors. The beam tube is represented by the two concentric circles with an inner diameter of 300 mm. The outer diameter of the conductor circumference is 1148 mm. The ROI can be seen as a rectangle in the center, surround by two rectangles representing the electrodes. The field homogeneity in the ROI is shown in more detail in the next figure.

Fig. 7.9: Deviation of the flux density in vertical direction from the average specified value of 32.65 mT in the ROI. Enlarged view from Figure 7.8.
Fig. 7.10: Flux density between the two magnets in Figure 7.7 along the central trajectory within the ROI. In the center of the magnets a flux density of 32.65 mT is obtained whereas, midway between adjacent magnets, at $1.25 \times 10^4$ mm, the flux density drops to a value of about 1 mT.

Fig. 7.11: Normalized electric and magnetic field in the stray field region. The electrical field has been obtained by stacking 5 expanding capacitors at different potential, each starting again with a gap distance of 60 mm where its expanded predecessor ends up to a gap nearly double as large.
Fig. 7.12: Top view of the region between adjacent bend elements, showing capacitor plate separations expanding up to a maximum distance, before starting with a new capacitor at reduced potential. Matching Figure 7.11, the overall length is 2 m. The minimum separation values are approximately equal to the main bending field electrode separation. The various potential levels of the plates are indicated. There may be space to accommodate additional devices in the gap of the capacitor with the smallest potential difference. This example shown requires 5 capacitors, with over-all length of about two meters.

Fig. 7.13: Same as Figure 7.11, but with a larger expansion factor of about 2.95, yielding a larger space at a longitudinal coordinate of 1800 mm with a diameter of $60\text{ mm} \times 2.95 = 177\text{ mm}$. Only three capacitors are required in this case.
7.6 Ring design

The basic PTR geometric ring parameters have been given earlier in Table 7.1. As mentioned previously, to this point the ring optics for 30 MeV and 45 MeV have been taken to be identical. The bending, for example for 45 MeV protons, is done by eight 45-degree electric/magnetic bending elements. The acceptance of the ring is to be $10\pi \cdot \text{mm} \cdot \text{mrad}$ for $10^9$ particles. The lattice is based on fourfold symmetry, as shown in Figure 7.1. The bend elements consist of electric and magnetic bending. Pure electric bending can be used for 30 MeV protons but, for a (nominal maximum) proton energy of 45 MeV, superimposed magnetic and electric bending will be applied. The magnetic part of the bending has to be provided by a pure air coil magnet, to avoid hysteresis effects caused by using iron for the magnet. It will be possible to store both CW and CCW beams consecutively, but not concurrently.

The present design for the prototype is a “square” ring with four 8 m long straight sections. This has been the result of lattice studies using different shapes such as round or race-track shaped. The ring is shown in Figure 7.1. It consists of 4 unit cells each of them bending 90°. Each unit cell consists of a focusing structure F-B-D-B-F, where F is a focusing quadrupole, D is a defocusing quadrupole, and B is an electric/magnetic bending unit. The lattice is designed to allow a variable tune between 1.0 and 2.0 in the radial plane and between 1.6 and 0.1 in the vertical plane, as shown in Figure 7.14.

The straight sections have to house separate injection regions for clockwise (CW) and counterclockwise (CCW) beam operation. There will also be a quadrupole of type QSS in the centre of each of the straights, to provide additional tuning possibilities. The voltages on the electric bending plates will be limited to ±230 KV for technical reasons. The horizontal gap is determined by the horizontal beam size, which is determined by the maximum acceptance and the maximum beta function to be $2\beta_{x,\text{max}} = 2\sqrt{\alpha_{x,\text{max}}\beta_{x,\text{max}}} = 2\sqrt{10 \times 50} \approx 50 \text{ mm}$. With a safety factor of 1.2, the gap between the plates is then 60 mm, and the maximum electric field in the gap is $E_{\text{max}} = 2 \times 200/0.06 = 6.7 \text{ MV/m}$. The maximum vertical beta function determines the vertical beam size to be $2\beta_{x,\text{max}} \times 2\beta_{y,\text{max}} = 50 \times 90 \text{ mm}$, the field relative field homogeneity is specified to be better than $10^{-5}$. Ring element counts, geometry and other bend parameters are given in Tables 7.4 and 7.5.

Lattice flexibility is a goal for the design. The betatron working points can be varied over a large range as shown in Figure 7.14. A typical plot of the beta functions is given in Figure 7.15.

<table>
<thead>
<tr>
<th>Table 7.4: Geometry</th>
<th>Table 7.5: Bend elements, 45 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td># B-E deflectors</td>
<td>8</td>
</tr>
<tr>
<td># arc D quads</td>
<td>4</td>
</tr>
<tr>
<td># arc F quads</td>
<td>8</td>
</tr>
<tr>
<td>quad length</td>
<td>0.400 m</td>
</tr>
<tr>
<td>straight length</td>
<td>8.000 m</td>
</tr>
<tr>
<td>bending radius</td>
<td>8.861 m</td>
</tr>
<tr>
<td>electric plate length</td>
<td>6.959 m</td>
</tr>
<tr>
<td>arc length (45°)</td>
<td>15.7 m</td>
</tr>
<tr>
<td>circumference total</td>
<td>102 m</td>
</tr>
<tr>
<td>emittance $\epsilon_x = \epsilon_y$</td>
<td>1.0 $\pi \text{ mm-mrad}$</td>
</tr>
<tr>
<td>acceptance $\alpha_x = \alpha_y$</td>
<td>10.0 $\pi \text{ mm-mrad}$</td>
</tr>
<tr>
<td>Electric</td>
<td></td>
</tr>
<tr>
<td>electric field</td>
<td>7.00 MV/m</td>
</tr>
<tr>
<td>gap between plates</td>
<td>60 mm</td>
</tr>
<tr>
<td>plate length</td>
<td>6.959 m</td>
</tr>
<tr>
<td>total bending length</td>
<td>55.673 m</td>
</tr>
<tr>
<td>total straight length</td>
<td>44.800 m</td>
</tr>
<tr>
<td>bend angle per unit</td>
<td>45° m</td>
</tr>
<tr>
<td>Magnetic</td>
<td></td>
</tr>
<tr>
<td>magnetic field</td>
<td>0.0327 T</td>
</tr>
<tr>
<td>current density</td>
<td>5.000 A/mm²</td>
</tr>
<tr>
<td>windings/element</td>
<td>60</td>
</tr>
</tbody>
</table>
**Fig. 7.14:** On the left the horizontal betatron tune $Q_x$ and the vertical betatron tune $Q_y$ are plotted versus the strength of the QF quadrupole family; the quadrupole families QD and QSS are constant, while the QD quadrupole family QD is varied. The marked points are continued in the figure on the right.

**Fig. 7.15:** Beta functions and dispersion for a typical working point: $k_{QF}=0.05$, $k_{QD}=0.3$, $k_{QSS}=0$, $Q_x=1.73$, $Q_y=1.20$. 
7.7 Components

7.7.1 Beam Position Monitors (BPM)

There are about 20 beam position monitors located around the ring, as shown in Figure 7.16. A BPM is placed at the entrance and the exit of each bending unit. One BPM will be placed additionally close to the quadrupoles in the straight sections. The BPMs have to be mounted precisely and rigidly, as close as possible to the quadrupoles, to which they are accurately and rigidly attached.

A new type of BPM has been developed at the IKP of the Forschungszentrum Juelich. These pick-ups are presently in a development stage. The position resolution is measured to be $10 \mu m$ over an active volume of $100 \times 100 \text{mm}$ [11]. These Rogowski design BPMs are very attractive because of their short length of only 60 mm, and their anticipated accurate absence of systematic relative transverse displacement of forward and backward beams.

![Fig. 7.16: On the left are pictures of the Rogowski pick-up module. The inner diameter is 100 mm. Beam position monitor locations around the ring are shown on the right.](image)

7.7.2 Electric quadrupoles

The quadrupoles for PTR are characterized by aperture diameter 80 mm, powered at +/- 20 kV. We have simulated a design with a vacuum chamber of diameter 400 mm (Figure 7.17) on the left). The maximum pole tip potential is 30 kV to allow some margin for conditioning. A 3D design has been carried out. The calculated sextupole, octupole and higher harmonics of the integrated field seem very reasonable. The 3D integration model (Figure 7.17) on the right) suggests that the device can be built within the allocated 800 mm longitudinal length, but the (quite large) radial diameter is 620 mm.

Table 7.6: Calculated multipole content of electric quadrupoles. Index value 2 designates the fundamental quadrupole content.

<table>
<thead>
<tr>
<th>index</th>
<th>strength</th>
<th>index</th>
<th>strength</th>
<th>index</th>
<th>strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.145915</td>
<td>9</td>
<td>1.36256e-06</td>
<td>15</td>
<td>9.79269e-07</td>
</tr>
<tr>
<td>3</td>
<td>1.14093e-06</td>
<td>10</td>
<td>1.63810e-06</td>
<td>16</td>
<td>9.85316e-08</td>
</tr>
<tr>
<td>4</td>
<td>7.20433e-09</td>
<td>11</td>
<td>5.69516e-07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.18116e-06</td>
<td>12</td>
<td>1.07131e-07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.63343e-06</td>
<td>13</td>
<td>1.10359e-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.31927e-06</td>
<td>14</td>
<td>1.52276e-06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.7.3 RF solenoids

The vertical polarization of a stored beam can be rotated into the horizontal plane by the longitudinal field of an rf solenoid. As shown in Figure 7.18, the RF solenoid at COSY is a 25-turn air-core water-cooled copper coil with a length of 57.5 cm and an average diameter of 21 cm. It has an inductance of about 41 µH, and produces a maximum longitudinal RF magnetic field of about 1.17 mT$_{\text{rms}}$ at its center. The solenoid is a part of an RLC resonant circuit, which typically operates near 917 kHz at an RF voltage of about 5.7 kV$_{\text{rms}}$, producing a longitudinal RF field integral of 0.67 T-mm. Typical ramp-up times, from vertical to horizontal polarization, are about 200 ms.

Fig. 7.18: COSY LC-resonant RF solenoid. In COSY this element precesses the polarization vectors of all particle bunches identically. Its role in the PTR ring will be the same.

7.7.4 RF Wien filter

Outer part (left) and inner part (right) of the COSY waveguide RF-Wien-Filter are shown in Fig 7.19. Beam tuning manoeuvres described earlier in this chapter have employed small radial magnetic fields for applying small controlled torque to the beam polarization to control the spin wheel (explained further is Section 7.9). Such a radial magnetic field also causes an undesirable beam orbit perturbation. In some cases the applied radial magnetic field causes an acceptably small orbit perturbation. But, when this is not the case, the RF-Wien filter has to be used instead. One way of expressing the Wien filter
Fig. 7.19: Outer part (left) and inner part (right) of the COSY transmission line RF-Wien-Filter. In COSY this device emulates the spin precession caused by a deuteron EDM. In the PRT ring it can act identically on all bunches, or for precessing individual bunch spins, without influencing the other bunches [5].

"strength" is to give the spin wheel angular velocity caused per watt of power applied to the RF-Wien filter. In a COSY precursor RF Wien filter experiment a Wien filter magnetic field times length integral of $2 \times 10^{-6}$ T-m, caused a 0.16 Hz spin-wheel frequency $f_{\text{spin-wheel}} = \Omega_{\text{spin-wheel}}/(2\pi) = 0.16$ Hz. The power conversion was such that an RF power level of 1 kW provided a magnetic field times length integral equal to $1.6 \times 10^{-5}$ T-m. This calibration factor was deduced from an experiment using 0.97 GeV/c deuterons stored in COSY.

7.7.5 Vacuum

The requirement for the vacuum is mainly given by the minimum beam lifetime requirement of about 1000 s. The emittance growth in the ring caused by multiple scattering from the residual gas needs to be less than 0.005 mm-mrad/s. With the initial emittance assumed to be 1 mm-mrad the pressure will have increased to 5 mm-mrad within 1000 s. This requires partial pressures of less than $10^{-12}$ Torr for $N_2$ and $5 \cdot 10^{-11}$ Torr for $H_2$. The cooling rate for stochastic cooling should be better than $5 \cdot 10^{-3}$ mm mrad/s.

For such an ultra-high vacuum only cryogenic or NEG pumping systems can be used [10]. Bake-out must be foreseen for either cryogenic or NEG systems.

The usage of NEG systems introduces some problems: 1. The NEG material becomes saturated after several pump-downs; 2. The aging NEG material becomes brittle and leaves some dust in the vacuum vessel; 3. This storage ring is a prototype and a significant number of pump-downs will be part of the development program; 4. The high voltage system requires excellent vacuum.

A cryogenic vacuum system has also been considered for the PTR ring. The beam pipe would have to be a system of three concentric pipes. The inner shell would carry the liquid He. Next, in the outwards direction, is the 70 K pipe, while the outer shell would house super-insulation (and heating devices). To avoid these complications and expenses, it might be recommended to use the NEG based vacuum system. A system of NEG cartouches is presently under discussion.

7.8 Beam preparation

7.8.1 Design principles for bunch polarization patterns

Before describing proposed injection sequences, it is useful to establish some principles common to all or most schemes, whether for prototype ring or full-scale ring, single beam or dual beam injection.

Assuming the harmonic number is 80 for the full-scale ring, one can have a lop-sided fill with as many as 60 consecutive stable buckets filled and the other 20 empty. This allows the injection kicker
to be pulsed on for half a microsecond or so, which is comfortably long. For filling the other beam, the CCW one, the bunch train and kicker duration can be the same. Similar considerations apply to the prototype ring.

The stored bunches would then be too close to be acted on individually, so maybe one would prefer to have just 30 filled buckets, alternating with empty buckets. The spacing between bunches would be too close for single bunch injection or extraction, but it could be amply long for "tweaking" bunches individually.

A useful principle recognizes that the final ring is the "experiment" and the injection system is not. Any time spent in the final ring adjusting the bunch structure is time taken away from the experiment. So time taken to trim the spins after injection should be minimized. The responsibility for best arranging the bunch pattern is therefore delegated to the injection system. Minimum injection time would be achieved by injecting just two trains of prepared bunches, which could reduce the set-up time to as little as ten seconds or so.

Most of the following principles are intended to ensure the uniformity of all polarized bunch properties, at least to the extent possible, by assuring that all bunches are subject to identical injection treatment:

1. All spin flips should be performed in the low energy injection ring, where (at COSY) essentially 100 percent efficiency has been persuasively demonstrated [9].
2. During any single data collection sequence, there should be no change in the low energy source region. (Except for test purposes) this includes keeping identical bunch polarization. The basis for this constraint is to best maintain identical parameters for all bunches. (This constraint is not actually imposed from the point of view of minimizing the duration of the entire injection process. In fact, the time needed to change parameters for a subsequent train is expected to be only about 5 seconds.)
3. All injected bunches will have been pre-cooled in the low energy injection ring. In all cases, only vertically polarized bunches (all up, or all down) will be injected into the EDM ring.
4. Injection as close as possible to the magic frozen spin energy will be desirable, but the injected beam energy will always be off-energy by an amount great enough for the loss of beam polarization (after betatron and synchrotron equilibration, either by filamentation or by active damping of coherent oscillations) to be negligible.
5. Finally, and most importantly (not counting special polarimetry investigations) after all bunches have been populated with vertically polarized bunches, identical external fields will be applied to every bunch to bring all polarization orientations into their desired final injection state—i.e. the initial EDM measurement configuration state.

7.8.2 Pattern of bunch polarisation in RF buckets

The polarized bunch filling sequence can be described in general terms without having frozen the RF frequency or harmonic numbers. The same discussion can also apply to either a small prototype ring or the eventual full scale ring. In both cases preliminary commissioning will use just a single, say clockwise (CW) beam. However, since the sequential injection of simultaneously circulating beams does not greatly complicate the process, only dual beam injection and bunch polarization manipulation will be described here. It will be obvious below, which steps are to be skipped for single beam injection.

The longitudinal bunch patterns of counter-circulating beams in a predominantly electric EDM ring will be quite similar to the bunching pattern of first generation, single ring, electron-positron colliders like the Cornell Electron-Positron Storage Ring (CESR) or the DESY Doppel-Ring (DORIS). In all cases the RF timing has to be arranged so that all bunches, both CW and CCW, pass through the RF cavity (or cavities) at stable phases.

Assuming a single RF cavity, there will be a number of stable RF buckets, both CW and CCW,
equal to the harmonic number of the radio-frequency. Not all stable buckets will be filled. Single turn (or “kick”) injection will require the presence of pulsed kickers in the ring, whose turn-on and turn-off pulse-edge durations will have to be restricted to time intervals during gaps in the charge distributions of both CW and CCW beams. The length of each of these gaps has to be at least one (or higher integer multiple) of RF bucket lengths. We assume gaps and filled buckets alternate more or less uniformly around the ring.

Ideally, every bunch will have the same number of particles and be maximally polarized. But, for reasons of polarimetry, it is optimal for the polarization signs of adjacent bunches to alternate. When the injection phase has been (almost) completed in each of the beams the fill pattern will consist of regular repetitions in a single sequence: “up-polarized-bunch, gap, down-polarized-bunch, gap”. For the small PTR, two such sequences are planned—for a larger ring probably more.

In a final injection phase the bunch polarisations will be rotated, but, until this final injection phase, all bunch polarisations will be up or down, and bunches will be referred to as “up bunches” in “up buckets” or “down bunches” in “down buckets”. One could contemplate an “up bunch” being parked temporarily, for example, into a “down bucket” but, by an injection principle, this would not be favoured.

### 7.8.3 Direct beam injection into stable RF buckets

Injection will proceed in the following steps (for some of which there are optional procedures):

1. At some point a beam (cooled and at full energy) in the injection ring is selected for one injection path or the other. It consists of a train of uniformly-spaced, identical, vertically-polarized proton bunches—say “up bunches”.
2. All CW “up buckets” in the EDM ring are then filled by kick injection of a single train of appropriately spaced, timed, and cooled “up bunches” from the injection ring. For this injection phase the EDM ring energy will be slightly different, say higher, than the magic energy—just enough to prevent decoherence.
3. Next, with no change in ring energy, all CCW “up” buckets in the EDM ring are filled by kick injection of a single train of appropriately spaced, timed, and cooled “up bunches” from the injection ring.
4. For the next two steps, bunches identical to the previous train except for having been flipped into “down bunches”, and therefore, having all other properties the same (to the extent possible).
5. The previous two steps are then repeated, injecting “down bunches” from the injection ring.
6. After this sequence all “up buckets” and “down buckets” will have been properly populated. Up to this point, all bunch polarisations have been vertical, either up or down.
7. Then, by ramping the RF frequency down to the magic energy, uniformly and adiabatically, all bunch energies will have been tuned onto the magic energy. (Though all spins are still vertical, this no longer provides protection against decoherence).
8. Then, for a time interval that is an integral number of synchrotron oscillation periods, by applying an adjustable, uniformly distributed, radial, magnetic, $B_r$ trim field, all spin orientations will have been rolled through $\pi/2$ around the radial axis, ending up with longitudinally polarized bunches with alternating signs. Alternatively, this maneuver could be performed using a waveguide RF Wien filter.
9. Especially towards the ends of the previous two steps, both horizontal and vertical polarimetry will probably be needed to control the orientations of all bunches as intended.

### 7.8.4 Bunch fill pattern and kicker timing

Figure 7.20 contains a John Jowett space-time beam bunch evolution diagram. There are two CW and two CCW bunches. The space-time trajectories are helical, with time advancing to the right for the red bunches (with solid arrows); viewed from the left, these bunches are receding along CW orbits, with
Fig. 7.20: A modified “John Jowett beam bunch space-time plot” illustrating a pattern of counter-circulating bunches. For this example there are six stable buckets (for each beam direction) with two CCW blue bunches (separated by an empty bucket), two CW red bunches (separated by an empty bucket), and two empty gaps, available for single turn injection of a train of two (up or down) polarized bunches into one or the other beams. (This example is directly applicable to the EDM prototype ring bunch filling scheme explained in the text.)

time advancing from left to right. Blue bunches (with broken-line arrows) when viewed from the left, are approaching along CCW orbits, with time advancing from right to left. Though representing toroids, the plot is rendered in two dimensional by projection onto the plane of the paper. With advancing time, red and blue bunches “collide” (or, rather, pass harmlessly through each other) at the points indicated.

A kicker placed anywhere in PTR, pulsed with proper polarity, for a time conservatively shorter than 1/8 of a revolution period, can deflect any bunch without affecting any of the other three stored bunches.

7.9 Fundamenta1 physics opportunities for PTR

To explain the essential differences between stages 1 and 2 it is useful to expand language that is currently in common use.

Stage 1 discusses spin effects that are implicitly understood to imply in-plane precession, where “in-plane” implies precession in the (horizontal) plane of the accelerator. Eventually, if the beam polarization is frozen in this plane (where “frozen” implies “frozen relative to the particle trajectory) the EDM effect will accumulate monotonically. But, in stage 1, the proton spins cannot be frozen.

Stage 2 concentrates on “out-of-plane” precession, where “out-of-plane” refers to spin vector precession in the “vertical plane instantaneously tangent to the particle orbit”. It is precession into this plane, that is driven by a symmetry-violating effect such as a proton EDM, and that is the subject of the EDM measurement. In a paper discussing spin decoherence, Koop [4] introduced the “spin wheel” as a picturesque way of describing precession of the beam polarization vector in this “out-of-plane” plane. Though grammatically-dubious, this language is very helpful for visualizing the experimental investigations intended for stage 2. That this metaphorical language is due to Koop is of little importance, compared to Koop’s probably correct contention that, to the extent his “spin wheel” executes multiple revolutions, there is a strong suppression of spin decoherence, with a corresponding increase in the spin coherence time (SCT).

Regrettably, the magnitudes of the out-of-plane precessions due to the proton EDM, or the earth’s gravity, have to be expressed in units of nanoradian per second (nrad/s). The length of a run observing a single full turn of the spin wheel might take ten years. This means that, from an experimental point
of view, the Koop decoherence argument simply does not apply to any experiment in which the beam polarization is frozen in all degrees of freedom. However the Koop wheel picture remains as valuable as ever, as the rest of this introduction is intended to explain.

Expanding the imagery, if in-plane precession of the beam polarization vector is visualized as a propeller blade of a helicopter, and out-of-plane precession as a blade of a wind farm propeller, then, the remaining possible precession direction (azimuthal around the beam axis) can be visualized as the propeller of a propeller-powered airplane.

This remaining freedom, precession of the beam polarization around the beam axis, is driven by solenoidal (i.e. along the orbit) magnetic field acting on the proton MDM. Except for one phenomenon, this precession might seem to have negligible influence on the EDM experiment. (With further abuse of language) the airplane-propeller and helicopter-propeller precessions “do not commute”. This failure of commutation produces a “wind-farm propeller-like precession” which produces a spurious EDM signal; i.e. systematic error. (Important though this source of systematic error is, there is currently no plan (other than avoiding solenoidal fields) to study this effect in stage 2—the importance of this issue has to be addressed theoretically, for example by simulation. This is commonly referred to as “the geometric phase problem”.)

The main thrust of stage 2 is to study “out-of-plane” beam polarization precessions which, as just explained, can usefully be visualized as the rolling motion of a “spin wheel”. After this cartoonish description, a more technically informative discussion can be based on the matched-pair of graphs in Figure 7.21.

In each graph the horizontal axis is magnetic field (but note the huge difference in the scales) and the vertical axis describes the Koop wheel response (but note that angular velocity is plotted in (a) while the angular advance after a given time is plotted in (b)). The main point of these two graphs is that, in spite their vastly different scales, the slopes are determined by a single, truly constant, physical constant of nature—the magnetic dipole moment of the proton. (That this quantity is, itself, known to 10 decimal point accuracy, though interesting, is not the point here. It is the constancy.)

With the aid of a transmission line Wien filter, such as shown in Figure 7.19, and a frozen spin polarized proton beam, from data implied by the figure on the left of Figure 7.21, it will be possible to “calibrate” this slope, as a function of a single, externally-imposed current, which is, itself, experimentally reproducible to better than parts per million accuracy [6–8]. The ultimate EDM precision depends on either improving this accuracy, or on scheming to exploit it most effectively. Appropriately transformed to match the parameters of the experiment implied on the right, this calibration can be applied to determine the slopes on the right, in spite of the nine orders of magnitude difference of horizontal scales. The accuracy with which data implied by the figure on the right can be used to determine the proton EDM depends, primarily, on the precision with which the data points are determined, as indicated by their error bars and point locations—which have been chosen arbitrarily for the figure. Though just a cartoon, the fact that the two parallel lines on the right do not quite coincide, is intended to suggest the presence of errors in the extrapolations on the right. And there is another significant ambiguity in the figure on the right; the Δβ ranges may, or may not include the critical β = 0 point, at which the beams are truly frozen in all degrees of freedom. Though it is not obvious from the figures, the vast difference in horizontal scales “amplifies” this ambiguity.

One example of the “scheming most effectively” mentioned above, that can be developed using PTR, would be to exploit the wave-guide Wien filter to isolate just one of the bunches for phase-locking its spin wheel angle β. This would, of course, destroy any EDM information contained in this particular bunch. But the phase-locking would, to high precision, have no effect on the other bunch polarizations; they would still freely respond to the EDM torques (including spurious EDM-mimicking torques).

---

1With the exception of vertical positioning, which needs to be controlled to micro-meter accuracy, it is element positioning rigidity, current resettability, and time-independence of all parameters, more than absolute accuracy, that needs to be achieved.
Fig. 7.21: Graphs indicating dependence of spin wheel orientation angle $\beta$ on radial magnetic field $B_r$. The left-hand graph (a) is especially appropriate for spin wheel calibration, with large radial torques intentionally applied by stripline Wien filter (for example for calibration purposes). The right-hand graph (b) is especially appropriate for representing the dependence of change $\Delta \beta = \beta_{\text{end}} - \beta_{\text{beg}}$ (for example as the result of measuring unknown physically interesting torques during a long EDM or GR measurement run). For the graphs to be intelligible, the scales have to be unambiguously shown and the error bars need to be shown—here they are just order of magnitude estimates.

The prototype ring PTR can be used as a “dry run” prototype for investigating operational issues like this, to be applied later in the full-scale EDM ring. This is an example of the sort of investigations to be performed in stage 2.

For various technical reasons (mainly connected with minimizing the PTR cost) the EDM measurement just described is expected to provide a proton EDM upper limit only of about $10^{-26}$ e·cm. On the other hand, the out-of-plane precession induced by earth gravity, is calculated to mimic a proton EDM of $15 \times 10^{-29}$ e·cm. This “standard candle” data rate will therefore be 15 times greater than the nominal EDM rate, and the run duration needed to collect the same number of counts 15 times shorter. Because of its stochastic nature, the statistics-dominated “standard candle” run duration is correspondingly reduced, compared to the year-long data collection period assumed in the nominal proton EDM measurement plan. This vastly improved true and known data rate can be expected to permit systematic error investigations that would be impractical for an actual EDM measurement. Furthermore, the GR-predicted precession reversal upon magnetic field and beam direction reversal provides a further factor of two effective data rate improvement. (Depending on the precision with which magnetic field reversal can be controlled) this makes the goal of measuring the gravitational effect well worth pursuing in PTR.

7.10 Summary and Outlook
The concept of, and the need for, the prototype storage ring has been outlined in the previous sections to the best of our current understanding. The next stage is to produce a detailed design report which should
demonstrate the technical feasibility of a prototype storage ring. The plan of the CPEDM collaboration is to finalize this TDR in 2022 (see Chapter 13).

References


[3] V. Lebedev, Limitations on an EDM ring design, Report to the EDM collaboration meeting, Forschungszentrum, Juelich, Nov. 11, 2015


[10] H. Jagdfeld, F. Klehr, private communication, FZ-Juelich, 2018


Chapter 8

All Electric Proton EDM Ring

8.1 Introduction – BNL design

It has usually been assumed and, as far as we know it is still true, that the most sensitive proton EDM measurement will be made in a dedicated, precision, all-electric storage ring, in which clock-wise (CW) and counter clock-wise (CCW) beams are circulating concurrently at the “magic” kinetic energy of 232.8 MeV, for which the proton spins are “frozen”, for example pointing in the forward direction everywhere in the ring. Most recently, a design for this ring has been outlined in a publication of Anastassopoulos et al. [1], as the design had evolved from the more detailed earlier proposal [2]. Substantial analysis of (a quite similar earlier version of) this design has been produced by Lebedev [3]. Parameters for the Anastassopoulos et al. design are given in Table 8.1 (column “full scale”), and one quadrant of the full ring is shown in Figure 8.1. The present report does not attempt to replicate material in that publication in any substantive way. The purpose for any material copied is only for ease of comparison.

Planning for the prototype ring PTR began by down-scaling from the Anastassopoulos et al. design, by approximate factors of five in both lengths and kinetic energy. The down-scaling prescription is described in detail in the “EDM prototype” chapter. A result of the down-scaling is that, though the full-scale ring shown in Figure 8.1 looks “round”, the PTR ring shown in Figure 7.2 looks “square”. This is an artifact resulting from the scaling of lattice functions rather than the scaling of appearances.

After minor changes to match element lengths at the reduced beam energy, the adopted PTR dimensions were up-scaled, back to the full-scale size. Recalculated lattice parameters for the up-scaled ring are shown in Table 8.1. Agreement is quite good for all parameters, well within the ranges of parameter values of the various 2016 ring designs. For transverse optical properties this agreement follows more or less automatically from the scaling. For longitudinal dynamics the scaling is less transparent, since cavity frequencies and harmonic numbers do not scale automatically. However the well-established synchrotron oscillation formalism is expected to apply quite directly to both PTR and full-scale ring.

The only significant defect of the down-scaling has to do with sensitivity to intrabeam scattering. (For emphasis concerning this deficiency) the following paragraph is copied verbatim from Chapter 7.

“For the full-scale ring the correspondingly smaller tune advance per super-period causes the focusing to be weaker. This is what permits the long straight sections of the full scale ring to be more than doubled, compared to the prototype (from 6 m to 14.8 m). This has the beneficial (perhaps even obligatory) effect, for the full-scale ring, of operating “below transition”. This ameliorates intrabeam scattering, as can be explained in connection with stochastic cooling. (Conversely, this is one respect in which PTR is a not-quite-faithful prototype.) This choice was made to reduce the prototype size.”
Table 8.1: Lattice parameter comparison between a lattice up-scaled from the prototype PTR lattice (in the last column) to the the same parameters for full-scale all-electric EDM lattice (in the second to last column). Any differences between entries in these two columns lie well within the ranges of values for existing full-scale all-electric proton EDM rings.

<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>unit</th>
<th>full scale</th>
<th>PTR-scaled</th>
</tr>
</thead>
<tbody>
<tr>
<td>bending radius</td>
<td>$r_0$</td>
<td>m</td>
<td>52.3</td>
<td>47</td>
</tr>
<tr>
<td>electrode spacing</td>
<td>$g$</td>
<td>cm</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>electrode height</td>
<td>$d$</td>
<td>cm</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>deflector shape</td>
<td>$m$</td>
<td></td>
<td>cylindrical</td>
<td>$≈$cylindrical</td>
</tr>
<tr>
<td>electrode index</td>
<td></td>
<td></td>
<td>0</td>
<td>0.001</td>
</tr>
<tr>
<td>radial electric field</td>
<td>$E_0$</td>
<td>Mv/m</td>
<td>8.0</td>
<td>8.92</td>
</tr>
<tr>
<td>number long straights</td>
<td></td>
<td>m</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>long strght sec. leng.</td>
<td>$l_{ss}$</td>
<td>m</td>
<td>20.8</td>
<td>12.0</td>
</tr>
<tr>
<td>polarimeter sections</td>
<td></td>
<td>m</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>injection sections</td>
<td></td>
<td>m</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>total circumference</td>
<td>$C$</td>
<td></td>
<td>500.0</td>
<td>500.0</td>
</tr>
<tr>
<td>harmonic number</td>
<td>$h$</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>RF frequency</td>
<td></td>
<td></td>
<td>35.878</td>
<td>35.878</td>
</tr>
<tr>
<td>number of bunches</td>
<td></td>
<td></td>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>particles per bunch</td>
<td></td>
<td></td>
<td>2.5e8</td>
<td>5e8</td>
</tr>
<tr>
<td>mom. spread(not/cooled)</td>
<td></td>
<td></td>
<td>$±5e^{-4}/1e^{-4}$</td>
<td>$±5e^{-4}/1e^{-4}$</td>
</tr>
<tr>
<td>max. horz. beta func.</td>
<td>$\beta_{x,\text{max}}$</td>
<td>m</td>
<td>47</td>
<td>48</td>
</tr>
<tr>
<td>max. vert. beta func.</td>
<td>$\beta_{y,\text{max}}$</td>
<td>m</td>
<td>216</td>
<td>183</td>
</tr>
<tr>
<td>dispersion</td>
<td>$D$</td>
<td>m</td>
<td>29.5</td>
<td>46.1</td>
</tr>
<tr>
<td>horizontal tune</td>
<td>$Q_x$</td>
<td></td>
<td>2.42</td>
<td>1.75</td>
</tr>
<tr>
<td>vertical tune</td>
<td>$Q_y$</td>
<td></td>
<td>0.44</td>
<td>0.47</td>
</tr>
<tr>
<td>horz. emit.(not/cooled)</td>
<td>$\epsilon_x$</td>
<td>mm-mr</td>
<td>3.2/3</td>
<td>3/3</td>
</tr>
<tr>
<td>vert. emit.(not/cooled)</td>
<td>$\epsilon_y$</td>
<td>mm-mr</td>
<td>17/3</td>
<td>17/3</td>
</tr>
<tr>
<td>slip-factor</td>
<td>$\eta = \alpha - 1/\gamma^2$</td>
<td></td>
<td>-0.192</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 8.1: One quadrant of a full-scale, all-electric, frozen spin EDM storage ring. The total circumference is 500 m. Copying from the original caption, the deflector radius is 52.3 m and the plate pacing is 3 cm. The electric field is directed inward between the plates. The spin and momentum vectors are kept aligned for the duration of storage. A realistic lattice will include 40 bending sections separated by 36 straight sections 2.7 m long each, with electrostatic quadrupoles in an alternating gradient configuration, and four 20.8 long straight sections for polarimetry and beam injection. It will also include SQUID-based magnetometers distributed around the ring.

\[ k_1 = -3.3918 \text{ kV/cm}^2 \]
\[ k_2 = 4.1756 \text{ kV/cm}^2 \]
\[ k_3 = 3.7306 \text{ kV/cm}^2 \]
\[ k_4 = -3.2068 \text{ kV/cm}^2 \]

\[ E_{\text{bend}} = 8.016 \text{ MV/m} \]
8.2 Preparedness for the full-scale ring

Table 8.2 gives a long (but surely still incomplete) list of requirements that need to be satisfied before serious construction of a full scale EDM ring can begin. Each of these topics has been discussed in preparing this report, at least to the level of formulating criteria for assigning the “preparedness rankings” shown in this table. Though highly abbreviated in this table, most of the issues are expanded upon elsewhere in the report. The assignment of colour-coded scores is explained in the table caption. These scores are loosely correlated with the PTR prototype ring staging described in Chapter 8.

<table>
<thead>
<tr>
<th>Operations</th>
<th>Rank</th>
<th>Comment</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>spin control feedback</td>
<td>G(+)</td>
<td>COSY R&amp;D</td>
<td>App. A.1.3</td>
</tr>
<tr>
<td>spin coherence time</td>
<td>G(-)</td>
<td>COSY R&amp;D</td>
<td>App. A.1.2</td>
</tr>
<tr>
<td>polarimetry</td>
<td>Y</td>
<td>polarimetry is destructive</td>
<td>Chap. 11</td>
</tr>
<tr>
<td>beam current limit</td>
<td>R</td>
<td>enough protons for EDM</td>
<td>Sect. 7.2</td>
</tr>
<tr>
<td>CW/CCW operation</td>
<td>R</td>
<td>systematic EDM error reduction</td>
<td>Ref. [1]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GR gravity effect</td>
<td>G(+)</td>
<td>this paper, standard candle bonus stable, this paper</td>
<td>App. D</td>
</tr>
<tr>
<td>frozen spin fixed point stable?</td>
<td>G</td>
<td>may limit run duration</td>
<td>App. G.5.5</td>
</tr>
<tr>
<td>intrabeam scattering</td>
<td>Y</td>
<td>needs further study</td>
<td>Ref. [3]</td>
</tr>
<tr>
<td>geometric/Berry phase theory</td>
<td>Y</td>
<td></td>
<td>Ref. [4]</td>
</tr>
<tr>
<td>Components</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quads</td>
<td>G</td>
<td>e.g. CSR design</td>
<td>Chap. 9</td>
</tr>
<tr>
<td>polarimeter</td>
<td>G</td>
<td>COSY R&amp;D</td>
<td>Chap. 11</td>
</tr>
<tr>
<td>waveguide Wien filter</td>
<td>G</td>
<td>COSY R&amp;D precursor</td>
<td>App. A.1.5</td>
</tr>
<tr>
<td>electric bends</td>
<td>R(+)</td>
<td>sparking/cost compromise</td>
<td>App. A.1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physics &amp; Engineering</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cryogenic vacuum</td>
<td>Y</td>
<td>required?—cost issue only</td>
<td>Ref. [5]</td>
</tr>
<tr>
<td>stochastic cooling</td>
<td>Y</td>
<td>ultraweak focusing issue</td>
<td>Ref. [6]</td>
</tr>
<tr>
<td>power supply stability</td>
<td>Y(-)</td>
<td>may prevent phase lock</td>
<td>Chap. 7</td>
</tr>
<tr>
<td>regenerative breakdown</td>
<td>R(+)</td>
<td>specific to mainly-electric, not seen in E-separators</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EDM systematics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polarimetry</td>
<td>G(-)</td>
<td>COSY R&amp;D</td>
<td>Chap. 11</td>
</tr>
<tr>
<td>CW/CCW beam shape matching</td>
<td>Y</td>
<td>systematic error?</td>
<td>Chap. 10</td>
</tr>
<tr>
<td>beam sample extraction</td>
<td>Y</td>
<td></td>
<td>Chap. 11, App. K</td>
</tr>
<tr>
<td>control current resesetability</td>
<td>Y</td>
<td></td>
<td>Ref. [7]</td>
</tr>
<tr>
<td>BPM precision</td>
<td>Y(-)</td>
<td>Rogowski? Squids?</td>
<td>Chap. 7, Chap. 10</td>
</tr>
<tr>
<td>element positioning &amp; rigidity</td>
<td>Y(-)</td>
<td>must match light source stability</td>
<td>Ref. [8]</td>
</tr>
<tr>
<td>theoretical analysis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial B-field $B_r$</td>
<td>R</td>
<td>assumed to be dominant</td>
<td>Chap. 10 and refs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ref. [1]</td>
</tr>
</tbody>
</table>

Table 8.2: Status of colour-coded preparedness levels for the full-scale, all-electric ring. Green means “ready to break ground”, Yellow means “promising only”, Red means “critical challenge”. Plus (+) and (-) are to be interpreted as for college course grades. So the ranking, with most prepared first, is: +G-+Y-+R . Success in meeting prototype ring goals could amount, for example, to upgrading all scores to Y(+) or better.

Almost from its beginning in early 2018, an EDM feasibility task force realized that a small scale, inexpensive, prototype EDM ring would be needed, in order to investigate, experimentally, issues essential for an eventual, full scale EDM ring. At that time a unique full-scale lattice design had not yet been adopted. Different investigations were based on different lattice design files, but the differences
were slight enough to be insignificant, as regards scaling down to a prototype ring. It was further decided that the ring designs for the prototype ring and the final ring should be as closely identical as possible. With the frozen-spin proton kinetic energy being 233 MeV, prototype proton kinetic energies of 35, 45, and 55 MeV were considered. This scaling is described in full detail in Chapter 7.

A detailed list of needs and goals for the prototype ring is given in Chapter 7. In particular the procedure employed in scaling down the full scale ring (by a factor of roughly five in circumference) to the prototype ring is described and justified there in full detail. To check this scaling, the entries in the final column of Table 8.1 were calculated, for comparison with the Anastassopoulos et al. values given in the second-to-last column. Since the focusing is very weak in both cases, there is little reason to question the reliability of the scaling, as regards transverse optics. Of course, because of the different beam energies, there are substantial differences in the longitudinal dynamics. But, since this formalism is very well established in both cases, there is little reason to doubt this aspect of the scaling.

The most obvious need for building a prototype ring was the lack of significant experience with relativistic all-electric accelerators and especially storage rings. Of course this is due to the much more powerful bending that is possible with magnetic, rather than electric fields, and the resulting absence of all-electric role models. To make up for this deficiency, the electric fields have to be increased to a level that is limited by electrical breakdown. Experience in this area is largely based on high energy particle electrical separators. Just one, of many, but perhaps the most important, technical uncertainty has to do with the highest field, smallest bending radius, and hence the least expensive, all-electric ring that can be conservatively constructed and guaranteed to store 233 MeV protons.

The PTR staging can, to some extent, be correlated with the color-coded entries in Table 8.2. Red entries in that table represent critical challenges that would necessarily delay commencement of the full scale ring. They are also referred to as “quantitative goals” for PTR stage 1. The main goal of stage 1 is to remove the “R” flags from the table. This includes the “R(+)” associated with the electric bend/sparking compromise. This score was increased from “R” to “R(+)” only to acknowledge that, by increasing the ring radius sufficiently, sparking can be sufficiently suppressed. But this could lead to unacceptable cost increase.

Especially with the EDM precision being roughly proportional to proton beam current, the experimental determination of achievable beam intensity will be needed for any eventual full scale EDM ring design. Operational experience with electric rings has been very limited. There has been a significantly great accumulation of polarized beam experience, but all in magnetic rings, none in electric rings. In any case, another important goal for stage 1 is to remove the “R” flag associated with beam current limit.

Most of the entries in the table with yellow “Y” flags are to be studied in PTR stage 2. In Chapter 8 they are characterized as “qualitative goals”, at least partly to acknowledge their indefinite nature. For example there is one weakness in the PTR down-scaling, that will limit the extent to which prototype results can be reliably extrapolated to the full scale ring. (Along with well-understood residual vacuum growth) intrabeam scattering is expected to be a significant source of beam emittance growth, which, as well as increasing spin decoherence, can cause beam loss and limit run length. Full 3D equilibration of this growth source is only possible for “below transition” ring operation. This condition is met in the full scale Anastassopoulos et al. design, but not in the PTR design. (It would have required an approximate doubling of the ring circumference.) Investigating this issue will be one of the goals of the prototype ring.

The bottom (2-line) entry in Table 8.2 requires special explanation. This entry is not assigned a colour; it applies to the inevitable residual radial magnetic field average $\langle B_r \rangle$ after all efforts have been exhausted to trim it to its ideally zero value. This average $\langle B_r \rangle$ value is expected to dominate the proton EDM systematic error. But, because its value depends on the uncertain values of all other entries in the table, the $\langle B_r \rangle$ uncertainty cannot be compared directly to the other entries in the table—it depends on the accumulated effect of all the other values, and on their theoretical systematic error calculation.
8.3 New ideas

Of course one also expects investigations with a prototype ring to give rise to new ideas. In fact, the planning phase itself, now well begun, can motivate the development of new ideas. This study was no exception. By and large, though, to reduce the proliferation of speculative descriptions, the body of the present report concentrates mainly on fleshing out ring design and experimental methods as established in the first several months of the study.

Some of the main new ideas obtained during the preparation of this feasibility study, are described in appendices to this report. The titles of these appendices are prefixed with “New ideas:” to distinguish them from the preceding, more conventional, appendices. Also these appendices are introduced by brief abstracts.

References

Chapter 9

Electric Fields

9.1 Assumptions and boundary conditions

One proposal for the nominal all-electric EDM ring [1] is a fully electric strong focusing lattice, to obtain a 500 m circumference storage ring. It consists of 4 Long Straight Sections (LSS), to be used for injection of each beam (clockwise and counter clockwise) and two polarimeters. The long straight sections are linked with 10 cells, each containing 3 bending sections (bends, see Fig. 9.1), 2 Short Straight Sections (SSS) housing magnetometers and separate quadrupoles (quads). This lattice produces the most challenging requirements for the quadrupoles compared to a 'soft focusing' lattice, where at least some of the focusing is included in the bending elements.

An alternative soft focusing lattice [2] may use bends with soft focusing in the vertical plane. This will lead to bends with a field index between $m = 0.1$ and $m = 0.2$, which further increases the challenge for the bend design and manufacturing tolerance, while the quadrupole requirements would be less demanding with respect to the quads of the strong focusing lattice.

The fully electric machine imposes stringent requirements for the background magnetic field of less than $10^{-10}$ nT [1]. This requirement has an immediate impact on the construction of the ring elements and the materials that can be used. Austenitic stainless steels show a paramagnetic behaviour at room temperature and the relative magnetic permeability is typically in the order of $1.001 - 1.005$ for the fully annealed, fully austenitic grades. One must avoid the use of work hardened and/or welded components, where magnetic susceptibility could be higher as a function of the grade used. One could consider fully austenitic grades such as 316LN to avoid non-linear behaviour due to traces of ferromagnetic phases. Alternatively, titanium alloys could be used at likely higher cost, but the consequences of their relatively poor heat conduction is still to be studied in further detail. Depending on what approach will be retained to achieve the required vacuum level, poor heat conduction may complicate bake-outs or alternatively operation at cryogenic temperatures.

The required vacuum level is in the $10^{-11}$ mbar range. This implies that the equipment should be compatible with either bake-out at 200°C or 300°C, if the ring is to be operated at room temperature. Alternatively, it should be compatible with cryogenics cool down, in case that the ring is to be operated at cryogenic temperatures to avoid too many cold/warm transitions [3]. Running the electric devices at low temperatures may lead to a reduced voltage breakdown rate, but it requires the need for (a not yet existing design for) a cryogenic $>200$ kV feedthrough and possibly a bus bar at cryogenic temperatures. Both are substantial challenges to design and operate reliably.

The aim is to keep machine’s cross section below $1 \times 1 \text{ m}^2$ including the magnetic shielding that is needed to shield the background magnetic field (earth magnetic field and stray fields), and, as such, this has a direct impact on the design of the beam elements. All elements studied have a smaller cross section, but depending on the space needed for the magnetic shielding or the cryostats, this requirement may have to be revised.

9.2 Electrode material

The electrodes of the bending dipoles, as well as the quadrupoles, are large sized objects, and need to provide significant fields to achieve the required deflection. The high fields assumed for the ring need to be produced reliably with large electrode surfaces as well as 30 mm gaps. The High Voltage (HV) breakdown rate is expected to be in the order of 1/day for the entire machine, which is very hard to achieve with conventional electrode materials. The choice of the electrode material is also strongly influenced
by the vacuum requirements and the constraints that these impose on the materials. For example, coated aluminium is commonly used for large septa electrodes at CERN to operate up to 15 MV/m, but is incompatible with bake-out or cryogenics cool-down due to crack forming in the oxide coating of the electrode.

Stainless steel and titanium are compatible with the required vacuum conditions. Older literature demonstrated that titanium has a better Voltage Holding Capacity (VHC) than stainless steel [5, 6]. Operational experience with larger electrodes (of about 1 m) and similar gaps (30 mm) appears however limited to around 8 MV/m [7, 8]. Alternative electrode materials may be needed to achieve improved performance on similarly sized electrodes using similarly sized electrode gaps. In this respect, the work done on niobium electrodes [9] and TiN coated aluminium electrodes [10] using small electrodes, as well as TiN coated stainless steel electrodes [11, 12] using small electrodes and small gaps, are very encouraging. At CERN a campaign of breakdown conditioning and breakdown rates for various metals and alloys was done [13, 14] and demonstrates that there is a difference of more than an order of magnitude between the performance achieved in these small scale laboratory tests and the reported performance of large DC devices. This is a basis to expect that an increase of operational fields in the electric field devices to be used in the EDM ring may be possible compared to what is used for large DC electric field devices in accelerators so far.

One should not lose sight of the scaling laws for the voltage effect and, more importantly in our application, the area effect [15], where the VHC scales with the surface as:

\[
\text{VHC} = \sqrt{E \cdot U} \\
\text{VHC} = \frac{U}{\sqrt{d}} \propto A^{-\mu},
\]

where \(d\) is the distance between parallel plates, \(U\) the applied voltage, \(A\) the surface of the electrodes and \(\mu\) can be determined empirically.

### 9.3 Ring elements

#### 9.3.1 Main dipoles

**9.3.1.1 Strong focusing lattice main bending dipole**

In the strong focusing lattice, the focusing is entirely left over to the quadrupoles, and the dipoles do not focus in the vertical plane. The main dipoles of the strong focusing lattice use cylindrical electrodes, and an integrated field quality 1 ppm in \(\pm 20\) mm central Good Field Region (GFR) is requested [17]. Table 9.1 shows the principal requirements assumed for the dipoles.
Table 9.1: Main dipole parameters as assumed for the final the strong focusing lattice and as proposed for the strong focusing concept dipole.

<table>
<thead>
<tr>
<th></th>
<th>Strong focusing lattice assumption</th>
<th>Alternative proposed concept design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical length [m]</td>
<td>2.739</td>
<td>4.16</td>
</tr>
<tr>
<td>Equivalent length [m]</td>
<td>2.739</td>
<td>3.80</td>
</tr>
<tr>
<td>Required deflection [mrad]</td>
<td>52.36</td>
<td>78.54</td>
</tr>
<tr>
<td>Gap width [mm]</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Electrode height [mm]</td>
<td>200</td>
<td>280</td>
</tr>
<tr>
<td>Beam aperture ((a_x \times a_y) [mm^2])</td>
<td>30 \times 200</td>
<td>30 \times 200</td>
</tr>
<tr>
<td>Field homogeneity GFR (\not\propto 20 \text{ mm} \text{ ppm})</td>
<td>1</td>
<td>700</td>
</tr>
<tr>
<td>Main field [MV/m]</td>
<td>8</td>
<td>8.67</td>
</tr>
<tr>
<td>Voltage per polarity [kV]</td>
<td>(\pm 120)</td>
<td>(\pm 130)</td>
</tr>
<tr>
<td>Electrode radius in H [m]</td>
<td>52.3</td>
<td>48.4</td>
</tr>
</tbody>
</table>

Taking these requirements as a starting point an alternative design was developed, based on 2 instead of 3 bending dipoles per cell (see Table 9.1). This design minimises the required electric field in the dipoles as well as the impact of the end fields and as such improves the integrated field homogeneity.

To reduce the impact of end fields (see Fig. 9.2 right), a first quantification with 20 cm tall electrodes and a device with an equivalent length of 3.8 m (corresponding to 2 bends per cell) was simulated [13]. These first simulations show the integrated field homogeneity of the central GFR is \(7 \times 10^{-4}\), while for the full aperture of 30 \times 200 mm the integrated field homogeneity is \(4 \times 10^{-3}\) [19]. The main electric field is 8.67 MV/m, but the peak fields are around 10 MV/m. In Fig. 9.3 the fields at the horizontal and vertical mid-plane of the dipole are shown. Electrodes using Rogowski profile edges [18] should be explored to reduce the peak fields further. These simulation results seem to indicate that to reach the required homogeneity, the electrodes should be increased further in (vertical) size (see present design in Fig. 9.2 left), making the cross section requirement for the machine (to keep the cross section below \(1 \times 1\) metre) more difficult to respect.

A transfer matrix \(R\) (Eq. 9.3), based on the vector (Eq. 9.4) of the EDM deflector was computed using the 3D field map [19] and the beam dynamics code TRACK [20]. The deflector is very weakly focusing in the horizontal plane and essentially behaves like a drift region in the vertical plane. The dynamics is very linear inside a region of diameter 20 mm. Figure 9.4 shows the principal transfer matrix relations plotted as derived from the simulated field map of the proposed concept. The transfer matrix elements not plotted in Figure 9.4 are small, and not considered further.

\[
R = \begin{pmatrix}
R_{11} & R_{12} & R_{13} & R_{14} \\
R_{21} & R_{22} & R_{23} & R_{24} \\
R_{31} & R_{32} & R_{33} & R_{34} \\
R_{41} & R_{42} & R_{43} & R_{44}
\end{pmatrix}
\]  \hspace{1cm} (9.3)

\[
\begin{pmatrix}
X' \\
X \\
Y' \\
Y
\end{pmatrix} =
\begin{pmatrix}
\text{horizontal displacement} \\
\text{horizontal angular displacement} \\
\text{vertical displacement} \\
\text{vertical angular displacement}
\end{pmatrix}
\]  \hspace{1cm} (9.4)

9.3.1.2 Soft focusing main dipole

Additionally to the concept without vertical focusing, a concept for a bend with soft focusing in the vertical plane is being studied. Using quasi-spherical electrodes, this bends with a field index of \(m = 0.2\),
i.e. with a radius of 48.4 meter in the horizontal plane and 250 m in the vertical plane. The electrode curvature amount to just 24 µm at the top and bottom of the 200 mm tall electrodes. This highlights the need for very high manufacturing tolerances, both for the electrodes themselves, as well as the electrode fixation inside the vacuum vessel.

### 9.3.1.3 General main dipole considerations

Electrode manufacturing for both variants will be challenging. To avoid very heavy electrodes (80 kg if made of solid titanium), hollow electrodes manufacturing techniques, respecting the required tolerances, are to be developed. The mechanical strength of these electrodes needs to be designed taking into account the non-negligible force applied on them by the electric field to obtain the required field precision.

A mechanical concept was also developed for the dipole (see Fig. 9.1). The electrode supports are located close to the end of the central tank section. At this location the vacuum vessel is reinforced with external webs, to optimise its stability and to guarantee the electrode position is not affected by tank deformation due to vacuum forces. Three support feet will be mounted onto these support webs to allow precise alignment of the tank. To make sure the requested field quality of $10^{-6}$ could be reached
in the GFR of $\varnothing$ 20 mm, the electrodes need to be aligned parallel in the vertical plane with a precision better 0.3 $\mu$m, corresponding to 1.5 $\mu$rad. Therefore, the electrode supports should be adjustable (radial position, angle and height) to facilitate the electrode alignment during assembly, but this will be very substantial challenge. Upstream, the electrodes are longitudinally fixed to the electrode supports, while on the downstream end the fixation allows for longitudinal movement to limit stress on the ceramic insulators during bake-out or cool-down.

In principle, all dipoles will be powered in parallel to reduce the impact of errors provoked by the power converter stability. Conditioning may become challenging if it is to be done with all devices in parallel. This will be even more challenging, if the electrode materials used only have little margin with respect to the required electric field. Therefore, it is planned to disconnect each device and condition it individually. Since the electrode position is fixed, a two stage conditioning process could be envisaged. First, each polarity will be separately conditioned, mainly to condition the deflectors on the electrode supports and feedthrough. This would then be followed by bi-polar conditioning, to condition the principal electrode surfaces.

### 9.3.2 Quadrupole

The principal design assumptions [21] for the strong focusing lattice quadrupoles are shown in Table 9.2.

The present baseline lattice assumes quads of 400 mm physical length. Our studies however, have shown that the required field quality cannot be met with 400 mm long quads, partly due to the unrealistically high field gradient and unachievable VHC, as well as due to the effect of the end fields on the field quality. The field requirements can potentially be met with a 1 metre device (Fig. 9.5). First simulations [17] have shown the integrated field error of the 1 metre long (flange to flange) to be of the order of $1 \times 10^{-3}$. The maximum field on the electrodes should still be optimised, but it appears very difficult to keep this below 10 MV/m. Two quadrupole variants have been studied in further detail. The first is a fully symmetric variant that uses simplified round electrodes to facilitate manufacturing. The integrated field precision required seems however difficult to reach with cylindrical electrodes [3]. Therefore the second variant uses asymmetric hyperbolic poles: narrow gap poles in the horizontal plane will allow powering with a lower voltage of this pair, ultimately requiring only one large HV feedthrough. This facilitates integration as well as lowers the cost. The principal performance parameters of both
Table 9.2: Principal quadrupole parameters.

<table>
<thead>
<tr>
<th>Physical length [m]</th>
<th>Lattice assumption</th>
<th>Simplified 3D design</th>
<th>Asymmetric 3D design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent length [mm]</td>
<td>0.4</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Beam aperture ($a_x \times a_y$) [mm$^2$]</td>
<td>400</td>
<td>730</td>
<td>750</td>
</tr>
<tr>
<td>Equivalent length [mm]</td>
<td>30 × 200</td>
<td>30 × 200</td>
<td>30 × 200</td>
</tr>
<tr>
<td>Electrode length [mm]</td>
<td>n.a.</td>
<td>700</td>
<td>700 (top/bottom) 834 (left/right)</td>
</tr>
<tr>
<td>Field gradient (g) [MV/m$^2$]</td>
<td>50</td>
<td>27.4</td>
<td>26.66</td>
</tr>
<tr>
<td>Electrode voltage [kV]</td>
<td>±250</td>
<td>±137</td>
<td>±133/20</td>
</tr>
<tr>
<td>Main field on pole faces [MV/m]</td>
<td>~6</td>
<td>~3.8</td>
<td>~6.8</td>
</tr>
<tr>
<td>Quad focal length [m]</td>
<td>20.97</td>
<td>20.97</td>
<td>20.97</td>
</tr>
<tr>
<td>Field gradient homogeneity in GFR ≤20 mm</td>
<td>$1 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

variants are also shown in Table 9.2. The 2D field plots [19] for the quadrupole using the 3 different pole shapes (hyperbolic, round, and asymmetric) are shown in Fig. 9.6. The asymmetric quadrupole’s left/right electrode length is longer than the top/bottom electrode length, with the aim to approach as well as possible the corresponding iso-potential surface of an ideal symmetric quadrupole, (see Fig. 9.7). The need for at least 2 HV feedthroughs makes the requirement to keep the cross section below $1 \times 1 \text{ m}^2$ challenging, in particular when using a perfectly symmetric (horizontal vs. vertical) electrode design, where 2 large feedthroughs will be needed.

![Fig. 9.5: 1 meter long quad assembly (left), ideal symmetric quad (middle) and asymmetric quad (right).](image)

9.3.3 Injection equipment

Two long straight sections are dedicated to the injection of the two beams. To inject, the beam is deflected by an electrostatic septum followed by a fast-pulsed separator (fast deflector). The principal parameters of these devices are shown in Table 9.3 [21].

9.3.3.1 Injection septum

The septum and its (anode) support need to be curved to limit the gap width to 30 mm, while displacing the beam by 86 mm. The septum can be made of bent or segmented 1 mm thick titanium sheet. By keeping the gap limited to 30 mm, the operational voltage required is approximately 240 kV. The vertical
acceptance is reduced with respect to the vertical acceptance of the ring to make sure the septum remains vertically straight when subject to the mechanical force induced by the electric field. The solid cathode could be made of titanium.

9.3.3.2 Fast deflector

The fast deflector gap width is taking into account the beam sagitta using straight electrodes. The external electrode is installed so that at the exit the gap is 30 mm wide and the entrance gap width is 42.5 mm. This allows the operating voltage to be limited to 30.4 kV. The HV feedthroughs will have to be developed, since these are not commercially available. The feasibility of the fast deflector pulse generator still needs detailed study. In particular, the required rise and fall time feasibility are still to be confirmed. The pulse

Fig. 9.7: Rendering of the asymmetric quadrupole. The low voltage electrodes are longer than the high voltage pair (top/bottom).
Table 9.3: Principal injection element parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Septum</th>
<th>Fast deflector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical length [m]</td>
<td>3.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Equivalent length [m]</td>
<td>4.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Deflection angle [mrad]</td>
<td>57.34</td>
<td>10</td>
</tr>
<tr>
<td>Gap width $(a_x \times a_y)$ [mm$^2$]</td>
<td>30 × 200</td>
<td>42.5 × 200</td>
</tr>
<tr>
<td>Field [MV/m]</td>
<td>8.0</td>
<td>1.674</td>
</tr>
<tr>
<td>Electrode voltage [kV]</td>
<td>−240</td>
<td>± 30.4</td>
</tr>
<tr>
<td>Radius of curvature electrodes [m]</td>
<td>52.32</td>
<td>∞</td>
</tr>
<tr>
<td>$T_{\text{rise}}$ and $T_{\text{fall}}$ (0.2 % - 99.8 %) [µs]</td>
<td>∞</td>
<td>1.0</td>
</tr>
<tr>
<td>Capacitance per electrode [pF]</td>
<td>∼ 660</td>
<td>&lt; 500</td>
</tr>
</tbody>
</table>

generator can use semi-conductor switch stacks (MOSFETs most likely), but no commercially available switches have been identified.

9.4 Required R&D

To make sure that all requirements of the electric field elements are feasible, the following topics for further research are identified so far:

- Electrode material performance and their compatibility with bake-out or cryo cooling.
- Feasibility of required electrode alignment hence field precision for ring elements.
- Stable electrode fixation, allowing adjusting of the electrode position sufficiently precisely to obtain the required field quality.
- Feasibility of electrode manufacturing precision.
- Feasibility of the fast deflector pulse generator, in particular with respect to the required rise and fall times.

9.5 Summary

The present strong focusing lattice bend concept design achieves 700 ppm field homogeneity in the central GFR of $\varnothing$ 20 mm, which is worse than required. The electric field levels on the bend electrodes might be achievable with titanium, but alternative materials such as niobium or coated aluminium may provide more margin, and should reduce the spark rate. Further study of the voltage holding capacity (VHC) of large electrode materials is essential to make sure that the proposed elements can be operated at the required fields for extended periods while respecting the desired spark rate.

A design for the quad is under development, albeit with a physical length of 1 meter instead of the 400 mm assumed in the lattice. The asymmetric variant is supplied with 133/20 kV, with the maximum voltage close to the voltage used for the dipoles. The achievable integrated field gradient homogeneity in the GFR is for the time being insufficient, but by further optimising the electrode extremities it is expected that the requirement of $10^{-4}$ is reachable. The electric fields on the electrodes are compatible with the choice of titanium as electrode material. The cross-section of this asymmetric quad is somewhat smaller than a symmetric variant, facilitating integration within the planned cross section of the ring.

To allow the technical design of the electric field elements and in particular to determine the required mechanical tolerances for the dipoles and quadrupoles, such as electrode profile, electrode alignment etc., an analysis of these tolerances on the performance reach of the EDM storage ring should be performed. Since this is a problem with many input variables, one could use the Polynomial Chaos Expansion (PCE) method that was already successfully applied to determine the tolerances of an RF Wien filter [22].
For the injection of the beams into the storage rings, the feasibility of a curved septum followed by a fast deflector was studied. Both elements operate with conservative fields and voltages, although the feasibility of the fast deflector pulse generator is still to be studied further in detail with respect to the rise and fall time requirements.

References

Chapter 10

Sensitivity and Systematics

10.1 Statistical Sensitivity

The statistical error on the EDM $d$ is given by

$$\sigma_{\text{EDM}} = \sqrt{24 \beta_{pr} \frac{sh}{\sqrt{NfAPEr\tau}}} \tag{10.1}$$

with

$$\beta_{pr} = \begin{cases} \frac{G\gamma^2}{G+1} & \text{precursor experiment} \\ 1 & \text{prototype & final ring} \end{cases}$$

The parameters of Eq. (10.1) are described in Table 10.1. Eq. (10.1) assumes that the beam is constantly extracted on a target in order to measure the polarisation. It also assumes that the beam polarisation does not decohere during the measurement time $\tau$. Details how to arrive at Eq. (10.1) are given in Appendix C. The statistical error for the different stages are given in Tab. 10.1.

<table>
<thead>
<tr>
<th>parameter</th>
<th>pure magnetic ring &amp; Wien filter (precursor)</th>
<th>combined ring</th>
<th>all electric ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>polarisation $P$</td>
<td></td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>number of particles stored $N$</td>
<td>$10^9$</td>
<td>$2 \cdot 10^9$</td>
<td>$4 \cdot 10^{10}$</td>
</tr>
<tr>
<td>fraction of particles detected $f$</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>average analysing power $A$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>measurement duration $\tau$</td>
<td>1000 s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>electric/magnetic field $E, B$</td>
<td>$2.7 \text{ kV/m, } 19 \mu \text{T}$</td>
<td>$7.3 \text{ MV/m, } 0.03 \text{T}$</td>
<td>$8 \text{ MV/m, } -$</td>
</tr>
<tr>
<td>fraction of ring where fields are present $r$</td>
<td>$1/184$</td>
<td>0.55</td>
<td>0.65</td>
</tr>
<tr>
<td>$\sigma_{\text{EDM}}$/e cm $1\text{fill}$</td>
<td>$8.6 \cdot 10^{-21}$</td>
<td>$5.5 \cdot 10^{-26}$</td>
<td>$4.6 \cdot 10^{-21}$</td>
</tr>
<tr>
<td>$\sigma_{\text{EDM}}$/e cm $1\text{year}$</td>
<td>$8.6 \cdot 10^{-23}$</td>
<td>$5.5 \cdot 10^{-28}$</td>
<td>$4.6 \cdot 10^{-29}$</td>
</tr>
</tbody>
</table>

Table 10.1: The statistical uncertainty for the three different stages proposed. Note that in the executive summary factor $r$ was omitted.

10.2 Systematic Effects

Any phenomena other than an EDM generating a vertical component of the spin limit the sensitivity, i.e. the smallest detectable EDM, of the proposed experiment. Such systematic effects may be generated by unwanted electric fields due to imperfections of the focusing structure as misalignments of components, by magnetic fields penetrating the magnetic shielding or generated inside the shield e.g. by the beam itself or the RF cavity, or gravity. A combination of several such phenomena or a combination of an average horizontal spin and one of these phenomena may as well lead to such systematic effects. This chapter describes the present stage of the understanding of systematic effects limiting the sensitivity of the experiment concentrating on the measurement of the EDM in an electrostatic "frozen spin" ring [1,2], which is considered the present baseline proposal. Nevertheless, many of the mechanisms described are relevant for other proposals as a hybrid ring with electric bending magnetic focusing [3] and the "double magic" ring [4].

93
Studies on systematic effects have been carried out and are underway by several teams of the CPEDM collaboration to further improve the understanding of basic phenomena to be taken into account and estimate the achievable sensitivity. Note that these studies are still underway and the present report is a snapshot aiming at describing the present understanding. The preliminary conclusion is that achieving the sensitivity target of $10^{-29}$ e·cm is very challenging and will probably not be possible with the present baseline fully electrostatic "frozen spin" synchrotron.

### 10.2.1 Recap of the proposal

![Fig. 10.1: Sketch of the proposal to measure the proton EDM in a frozen spin "magic energy" electrostatic ring](image-url)

The basic idea of the proposal to measure the proton EDM in an electrostatic ring [1, 2] is depicted in Fig. 10.1. Bunches, represented by red and blue arrows, of protons polarized in longitudinal direction are circulating in an electrostatic ring. The bending electric field pointing towards the inside is represented by green arrows. Bunches circulating clockwise (CW) are represented by blue arrows and bunches circulating counter-clockwise (CCW) by red arrows. The direction of the arrows indicates the polarization. For the case sketched in Fig. 10.1, both the CW and the CCW beam have bunches polarized parallel to the direction of movement and opposite to the direction of movement. Such a bunch structure is advantageous to reduce some of the systematic effects reducing the sensitivity of the experiment, but on the other hand is difficult to generate. The signature of an electric dipole moment \( \vec{d} \) (aligned with the spin of the particles and, for the rotation indicated in the sketch parallel to the spin), is a rotation of the spin into the vertical direction.

The basic equation, used for most of the consideration presented, is the "subtracted form of the Thomas-BMT equation" and is the difference between the angular frequency \( \vec{\Omega}_s \) of the spin rotation and an angular frequency \( \vec{\Omega}_p \) of the rotation of the direction of movement of the particle. With the choice of a vanishing longitudinal component of the angular frequency describing the rotation of the direction of movement.

---

1Some proposals [5] foresee as well bunches polarized in radial direction circulating simultaneously with the bunches polarized parallel or anti-parallel to the direction of movement. Such bunches allow to quantify and, possibly with appropriate feedback systems, to reduce some systematic effects.
the particle\(^2\), this quantity is given by:

\[
\Delta \Omega = \dot{\Omega}_s - \dot{\Omega}_p = -\frac{q}{m} \left[ G \vec{B}_\perp + (G + 1) \frac{\vec{B}_\parallel}{\gamma} - \left( G - \frac{1}{\beta^2 \gamma^2} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2} \left( \frac{\vec{E}_\perp}{c} + \frac{1}{\gamma} \frac{\vec{E}_\parallel}{c} + \vec{\beta} \times \vec{B} \right) \right]
\]

where \( q \) and \( m \) are the charge and mass of the particle, \( \vec{B} \) and \( \vec{E} \) the magnetic and electric field, \( \beta \) and \( \gamma \) the relativistic factors and \( \vec{\beta} \) a vector with length \( \beta \) and a direction parallel to the velocity. \( \vec{B}_\parallel = (\vec{\beta} \cdot \vec{B}) \vec{\beta}/\beta^2 \) and \( \vec{E}_\parallel = (\vec{\beta} \cdot \vec{E}) \vec{\beta}/\beta^2 \) denote the longitudinal components in direction of the velocity of the magnetic and electric fields. \( \vec{B}_\perp = \vec{B} - \vec{B}_\parallel \) and \( \vec{E}_\perp = \vec{E} - \vec{E}_\parallel \) are the components of the magnetic and electric field perpendicular to the direction of movement\(^3\). The quantities \( G \) and \( \eta \) describe the magnetic dipole moment, which is in general well known and the electric dipole moment to be measured.

For the case of protons \( G = 1.79285 \). Note that for a proton EDM of \( d_s = 10^{-29} \text{e}\cdot\text{cm} \), which is often quoted as expected sensitivity of the proposed facility, \( \eta \) is as low as \( \eta_s = 1.9 \times 10^{-15} \).

In a fully electrostatic machine installed inside a perfect magnetic shielding to reach \( \vec{B} = 0 \) and without EDM, a particle spin aligned with the direction of the movement will rotate together with the particle velocity, i.e. fulfil the "frozen spin" condition, if:

\[
\beta \gamma = \beta_m \gamma_m = \frac{1}{\sqrt{G}}
\]

where \( \beta_m \) and \( \gamma_m \) denote the "magic" relativistic factors. For protons \( \beta_m = 0.598379 \) and \( \gamma_m = 1.24811 \). With the "magic" relativistic factors one obtains the magic proton momentum \( p_m = \beta_m c \gamma_m m = 700.74 \text{MeV}/c \) and the "magic" kinetic energy \((\gamma_m - 1)mc^2 = 232.792 \text{MeV} \). Note that real "magic" relativistic factors are obtained only for positive values of the quantity \( G \), as is the case for protons. Thus, a purely electrostatic ring fulfilling the "frozen spin" condition is not possible for particles with negative \( G \) such as deuterons.

An electric dipole moment described by a non-zero \( \eta \) generates a rotation of the spin from the longitudinal direction into the vertical direction. In an electrostatic ring with circumference of \( C = 500 \text{m} \), which is about the minimum required with the given beam energy and in order keep feasible maximum electric field strength, the angular frequency for \( \eta = \eta_s \) is about 1.6 mrad/s. This small vertical spin rotation has to be detected by precise polarimetry in order to identify the particle EDM.

Additional ingredients to the "magic energy" proton EDM measurement concept are (i) to simultaneously circulate polarized beams in both clockwise (CW) and counter-clockwise (CCW) direction, (ii) to operate the synchrotron with a very weak vertical tune \( Q \equiv 0 \) and up to \( Q_V = 0.44 \) with some variants even foreseeing to periodically vary \( Q_V \) by say about \( \pm 10\% \) and (iii) to use a measurement of the vertical separation of the two counter-rotating beams to estimate the average radial magnetic field, which causes the most important systematic measurement error. Furthermore, the average horizontal spin will be continuously monitored with the polarimeters. A feedback loop will be implemented to bring the measured horizontal spin back to zero. Note that this feedback

\(^2\)The transverse component of the angular frequency describing the rotation of the particle direction \( \vec{t} = \vec{v}/|\vec{v}| \), with \( \vec{v} \) the particle velocity, is given by \( \Omega_{p,\perp} = \vec{t} \times (d\vec{t}/dt) \). An arbitrary longitudinal component \( \Omega_{p,\parallel} = \kappa \vec{t} \) can be added such that the angular frequency describing the rotation of the particle direction is given by \( \Omega_p = \vec{t} \times (d\vec{t}/dt) + \kappa \vec{t} \). It is easy to show that \( d\vec{t}/dt = \Omega_p \times \vec{t} \) for any value of the free parameter \( \kappa \). Here \( \Omega_{p,\perp} = 0 \) is assumed. Nevertheless, the longitudinal components of \( \Omega_p \) and of \( \Delta \Omega \) have to be interpreted with care and is the topic of discussions on-going at present.

Considerations presented in this chapter implicitly assume a coordinate system attached to the trajectory (or rather the closed orbit) of the particle. The rotation of this coordinate system can be described by a unique angular frequency \( \Omega_p \) with an appropriate choice of a (small) longitudinal component. This is somewhat inconsistent with the choice \( \Omega_{p,\parallel} = 0 \) made here. Studies are on-going and will be published soon.

\(^3\)Using these definitions for the longitudinal and perpendicular field components and \( \Omega_p = (q/\gamma m) \left( (\vec{\beta} \times \vec{E}/c) / \beta^2 - \vec{B}_\perp \right) \), it is simple to show the this equation is consistent with the Thomas-BMT Eq. 1 in chapter.
system must be able to correct horizontal spin component of both the CW and the CCW beam. Thus, for simultaneous operation with CW and CCW beams two parameters, for example the RF frequency acting on the beam energy and a small vertical magnetic field, have to be used by this feedback loop correcting small horizontal spin components.

Note that individual particles have energies slightly different from the "magic" energy and consequently their spin rotates in the horizontal plane away from the longitudinal direction. Thus, it is important that an RF system is present and the beam is bunched such that the particles execute synchrotron oscillations. During about half of the synchrotron oscillation, the particle will have an energy higher than the "magic" energy such that the spin rotates faster than the direction of movement. During the other half of the synchrotron oscillation below the "magic" energy condition, the spin rotation is slowed down. Thus, the synchrotron oscillation lead to periodic rotations of the spin in the horizontal plane, which average to zero over long periods, but introduce horizontal spin components with opposite signs in the head and the tail of a bunch.

10.2.2 Sources for systematic Errors and general Comments

Any phenomenon other than an EDM of the particle generating a rotation of the spin into the vertical direction generates signals, which can be misinterpreted and lead to systematic errors of the measurement. Effects considered so far and contributing either alone or in combination with other effects to such a rotation are:

- Magnetic fields: Even small magnetic fields around $\Delta B = 1 \text{nT}$ penetrating state-of-art multilayer shielding after degaussing procedures may lead to spin rotations into the vertical plane, which are orders of magnitude larger than the ones due to smallest EDM, one would like to be able to detect. In particular, an average radial magnetic field as low as $B_s = 9.3 \text{aT}$ for $C = 500 \text{m}$ circumference machine generates the same vertical spin component as an EDM of $10^{-29} \text{e}\cdot\text{cm}$.
- Imperfections of the electrostatic machine: Typical imperfections of electrostatic synchrotrons are misalignments of bends and quadrupoles or mechanical imperfections of components (e.g. small errors of the spacing between electrodes of bends alter the electric field and as consequence the deflection), which lead to deformations of the so-called closed orbit, i.e. the average transverse offset of the circulating beam and, in consequence, to local shifts of the kinetic energy of the particle directly impacting spin rotations as described in Eq. (10.2). Combination of several such imperfections can lead to a rotation of the spin from the longitudinal into the vertical direction.
- Gravity: gravity leads to a spin rotation from the longitudinal direction into the vertical direction of $44 \text{nrad/s}$ for protons [6–9]. Nevertheless, the phenomenon does not mimic an EDM in the sense that the spin rotations due to gravity correspond to an EDM of opposite sign for the CW and CCW beam. This effect is unrelated to other sources of systematic effects and will not be treated any more.
- Average horizontal spin component: an average horizontal component of the spin, which may not be seen by the polarimeter due to an asymmetry or even the result of a feedback loop aiming at rotating the spin in the horizontal plane into the longitudinal direction may lead to a generation of vertical spin combination with vertical closed orbit perturbations.
- Cavity misalignment and closed orbit perturbation (offset of the transverse beam position) at the location of the cavity: The azimuthal magnetic field of the cavity is a special case of magnetic fields, which (i) creates strong effects already with small offsets between the beam position and the center of the cavity and (ii) has a strong gradient.

Phenomena possibly generating systematic measurements errors compromising the sensitivity of the experiments, which have not yet been studied in detail, are:

- Betatron oscillations and different beam emittances of the two counter-rotating beams: studies
described here are for particles following the "closed orbit" and betatron oscillation are not yet taken into account.

- Inhomogeneous beam distributions: a small vertical polarisation, which is different for particles at the center of the bunch and particles executing large synchrotron and/or betatron oscillations, could be generated by the beam preparation process. If particles with large oscillation amplitudes tend to be intercepted by the polarimeter earlier than particles from the center of the bunches, the average observed vertical spin changes over a measurement cycle even in the absence of an EDM.
- Electromagnetic field generated by other particles in the same bunch or by particles of the beam rotating in opposite direction.
- Electromagnetic field generated by the interaction of the circulating beams with the surrounding vacuum chambers (image currents etc.).

For numerical evaluations, the $C = 500$ m circumference strong focusing lattice [10] will be used. This lattice has been optimised to obtain beam-lifetimes close to requirements with Intra Beam Scattering (IBS) and foreseen intensities and, amongst all proposals, is the closest to a ring, which could be constructed.

10.2.3 Radial magnetic Field leading to a systematic Error proportional to the Perturbation

Horizontal or "radial" magnetic fields are the only perturbation generating a rotation of the spin from the longitudinal into the vertical direction, which is directly proportional to the perturbation. There are two major sources for magnetic fields not generated by the beam itself and acting on the beam, which have as well different impacts on the measurement: residual static magnetic fields penetrating the shielding and magnetic fields from the RF cavity.

10.2.3.1 Residual Magnetic Field penetrating the magnetic Shielding

Typical residual magnetic fields inside state-of-the-art multi-layer shielding with degaussing procedures are around $\Delta B = 1$ nT, which is about eight orders of magnitude larger than horizontal magnetic fields $B_s = 9.3$ aT generating the same effect than the EDM sensitivity aimed for in typical proposals [1,2]. The average of the radial magnetic field around the ring circumference will be somewhat smaller than $\Delta B$, but still orders of magnitude larger than $B_s$ (The radial magnet field will vary strongly over a distance comparable to the transverse size of the shielding, which is expected to be around 1 m.) Assuming optimistically that the circumference $C = 500$ m can be divided into 500 sections with a length of 1 m with about constant field and no correlation of the field between different sections, one comes to an RMS value of the transverse field of about $\Delta B_s / \sqrt{500} \approx 45$ pT. Note that static average horizontal fields coupling to the known proton magnetic moment mimic an EDM in the sense that the contributions from the two counter-rotating beams do not cancel for the final result.

An essential ingredient of the proton EDM measurement proposals in an electrostatic ring is to operate the machine with weak vertical focusing, such that horizontal magnetic fields lead to a vertical separation of the two counter-rotating beams, which is measured with ultra-sensitive SQUID based pick-ups. Note that for the strong focusing lattice proposal [2,10] with a vertical tune of $Q_V = 0.44$, an average horizontal magnetic field $B_s$ leads to an average vertical separation of the two counter-rotating beams of $\Delta y_s \approx 0.26$ pm. The measured beam separation will be compensated by additional magnetic fields generated by electrical currents inside the shielding as much as possible with the achievable measurement

$^4$Disregarding gravity [6–9] mentioned already and well understood and not a concern for the EDM measurement.

$^5$Other proposals foresee weaker vertical focusing and lower tunes [1]. An average horizontal magnetic field $B_s$ with a vertical tune of $Q_V = 0.1$ would give a vertical separation of the beams of $\Delta y_s \approx 5$ pm. However, with the foreseen intensities they feature IBS growth rates not compatible with typical assumptions on the machine cycle length of around 1000 s; optimising a machine with such a low vertical to obtain IBS rates compatible with expected cycle lengths and intensities leads to excessive vertical beam sizes.
accuracy. The average vertical beam separation after correction over the full duration of the experiment still may have to be used to reduce the systematic measurement error. Even after averaging over several pick-ups installed around the ring and extensive averaging over durations comparable to the machine cycle and, further, averaging over many machine cycles, the determination of the remaining average horizontal magnetic field will be very challenging and, likely, prevent the experiment from reaching the sensitivity aim $d_s = 10^{-29} \text{e} \cdot \text{cm}$. The following effects may compromise the sensitivity of the experiment:

- Limited accuracy of orbit difference measurements even with averaging over many pick-ups and over long durations.

- Observation of orbit difference only at discrete positions around the circumference: even with the assumption that the focusing is perfectly constant around the circumference, the average of the orbit difference measured by a finite number of equally spaced pick-ups is in general slightly different from the average [10]. A rough estimate for the strong focusing ring proposed [1,2,10], where the pick-ups not perfectly spaced\(^6\), leads to the conclusion that this effect limits the uncertainty of the final result to an EDM value about four orders of magnitude higher than $d_s \approx 10^{-29} \text{e} \cdot \text{cm}$. The effect can be mitigated in theory by an optimized spacing of the orbit difference pick-ups and a modulation of the vertical tune [2,11]. The feasibility of the latter implies that the working point has to regularly cross betatron resonances, which is delicate and may lead to unacceptable beam losses.

- Wanted and unwanted variations of the Twiss betatron functions around the circumference: In general, the transverse focusing is not homogeneous around the circumference. Even the "smooth focusing" lattices feature field free straight sections without focusing at all. In consequence, the so-called Twiss betatron functions vary around the circumference. Thus, the effect of a local horizontal magnetic field on the average orbit separation, which depends on the local betatron function, will depend on the position. A rough estimate based on the strong focusing lattice proposal leads to the conclusion that this effect limits the uncertainty of the final result to an EDM value about five orders of magnitude higher than $d_s \approx 10^{-29} \text{e} \cdot \text{cm}$. The effects can be mitigated by designing a lattice with small variations of the vertical betatron functions. Note that these considerations triggered the proposal of a hybrid ring [3] with electric fields bending the beam and magnetic quadrupoles for focusing. The situation is even more delicate for a realistic ring with "beta beating", i.e. unwanted and unknown variations of the betatron function w.r.t. the lattice design due to unknown focusing errors. Careful studies assuming realistic focusing errors and realistic procedures to quantify and correct the resulting betatron beating are required to assess the effect and the implication on the achievable sensitivity.

- Coupling of the betatron motion between the two transverse planes: Unavoidable skew quadrupolar components due to mechanical imperfections (for example rotation of quadrupoles around the longitudinal axis, electrodes of bendings not being perfectly parallel ..) couple the betatron oscillation in the two transverse planes. A horizontal separation between CW and the CCW beam due to residual vertical magnetic fields at the location of such skew quadrupolar components will generate different vertical deflections for the two counter-rotating beams. The resulting vertical separation between the two counter-rotating beams is misinterpreted as the signature of a horizontal "radial" magnetic field and leads to a systematic measurement error.

### 10.2.3.2 Magnetic Fields due to the Cavity

Typical azimuthal magnetic field of RF cavities are orders of magnitude higher than the ones relevant for a study of systematic errors of a proton EDM measurement. Even in case of perfectly aligned cavity, individual particles will "see" horizontal magnetic fields and spin rotation into the vertical (and the horizontal) direction. However, the effect to the final result of the EDM measurement will be strongly

---

\(^6\)This lattice with four-fold symmetry and, each quarter consisting out of 5 arc cell and one straight section cell, has 36 orbit difference pick-ups adjacent to quadrupoles in arcs only.
suppressed (i) due to cancellation of the effect for different particles of a bunch crossing the cavity with different transverse positions and/or (ii) for one particle crossing the cavity gap each turn with different betatron phases and transverse positions.

The situation is different for an offset of the electrical center of the RF cavity with respect to the vertical closed orbit of say \( \Delta y = 100 \mu m \). The integrated horizontal field seen by the CW beam in ring operated below transition due to a cavity operated with harmonic \( h \) and peak RF voltage \( V_{RF} \) is \( \Delta Bdl = \frac{\pi \beta m c C}{eC} h V_{RF} \Delta y \). Inserting parameters \( V_{RF} = 6 \) kV and \( h = 100 \) for the strong focusing EDM ring proposal one obtains \( \Delta Bdl = 0.75 \) nT m, which has to be compared with the integrated field around the circumference \( B_s C = 4.7 \) fT m generating the same rotation of the spin into the vertical direction than an EDM of \( d_s = 10^{-29} \) e·cm. Thus, an offset of \( \Delta y = 100 \mu m \) between the electrical center of the cavity and the vertical closed orbit leads to a rotation of the spin into the vertical direction a factor \( 1.6 \times 10^5 \) larger than the effect for a proton EDM \( d_s \). As the direction of the magnetic field is inverse for CW and CCW beams, the effect does not mimic EDM in the sense that contributions from the two counter-rotating beams to the final result cancel each other in a perfect measurement set-up. Nevertheless, this cancellation relies on a measurement of the vertical spin build up with high precision for both beams, which may be very challenging\(^7\).

10.2.4 Second order Effects

Several cases of effects, where two different perturbations as e.g. residual vertical and longitudinal magnetic fields penetrating the shielding generate a vertical spin component, will be described in this section. These phenomena are second order effects in the sense that the resulting vertical spin for small perturbations is proportional to the square of the perturbation (if both the vertical and the longitudinal magnetic field in the example are increased by a factor \( k \), the resulting vertical spin is increased by a factor \( k^2 \). All consideration reported apply strictly speaking for beam particles following the closed orbit of the ring and not executing any betatron oscillations.

Several but not all of the effects described below have been reported and interpreted in terms of geometric phase effects.

10.2.4.1 Rotation of the spin from the horizontal into the vertical Direction by a vertical Slope of the Orbit inside Bendings

A geometrical interpretation of the effect rotating the spin from the horizontal into the vertical direction, which has been described in the past \([12]\), is sketched in Fig. 10.2. If the "frozen spin" condition is fulfilled, the rotation of the spin and the direction of the trajectory are described by the same angular frequency vector \( \vec{\omega}_s = \vec{\omega}_p \), which is pointing downwards with a small longitudinal component \( \omega_{s,z} = \frac{\beta m c}{\rho} y_{co} \) with \( y_{co} = \frac{dy_{co}}{ds} \) the slope of the vertical orbit and \( \rho \) the curvature radius. This yields, even if the "frozen spin" condition is fulfilled, to a build up of the vertical spin of \( \frac{ds_y}{dt} = \omega_{s,z} s_x = \frac{\beta m c}{\rho} y_{co} s_x \). The vertical spin generated over one turn given by:

\[
\Delta s_y = \int_0^C ds \frac{y_{co}'(s)}{\rho(s)} s_x(s) . \tag{10.4}
\]

\(^7\)Another mitigation measure in case of imperfections of the polarity measurement to be discussed is a feedback loop detecting spin rotations of the CW and CCW not compatible with EDM (not "mimicking" EDM) and correcting them for example acting on the vertical closed orbit at the location of the RF cavity. Note that there are other effects described in the next section generating as well spin rotations of the two counter-rotating beams which are not compatible with an EDM and would be corrected by such a feedback loop.
10.2.4.2 Horizontal spin and non-zero average Slope of vertical Orbit inside Bend

The average horizontal spin of the particles will be monitored continuously using the polarimeter and a feedback loop mentioned in section 10.2.1. A small asymmetry of the polarimeter may lead to a non-zero horizontal component of the spin $\Delta s_x$. Spin rotations in the horizontal plane, which cancel over one turn, may as well lead to non-zero average horizontal spin even if the horizontal spin vanishes at the location of the polarimeter. With the help of Eq. (10.4) and using a nomenclature different from the one in reference [12] giving the same result, the average rate of change of the vertical spin becomes:

$$\frac{ds_y}{dt} = \frac{2\pi \beta m c}{C} <y'_{co} > s_x$$

with $N_b$ and $L_b$ the number and length of bends and $<y'_{co}>$ the average slope of the vertical closed orbit inside bends.

An average of the horizontal spin $\Delta s_x = 0.1$ mrad and a vertical misalignment of one vertically defocusing quadrupole at the transition from an arc to a straight section of "strong focusing" lattice ($N_b = 120$ and $L_b = 2.7$) by $\Delta y = 0.1$ mm leads to an average slope of $<y'_{co}> = -8.2 \cdot 10^{-8}$ rad and a vertical spin build up of $-18 \mu$rad/s. As the average horizontal spin of the two counter-rotating beams may not be correlated (independent polarimeter for CW and CCW beam with different uncorrelated asymmetries), the systematic EDM measurement error due the effect considered here cannot be reduced by counter-rotating beams. However, even an imperfect polarimeter together with a feedback acting on bunches polarized parallel to the movement and opposite to the movement of the same (say CW) beam would generate the same horizontal residual spin; the effect on the final EDM result from these bunches with opposite polarization cancels. Note that residual horizontal magnetic fields will generate smaller vertical orbit distortions and, thus, generate a smaller effect than typical misalignments of quadrupoles or tilts of electric bends.

A thorough investigation of the effect requires an simulation of a machine with realistic imperfections and a correction scheme based on (imperfect) position pick-ups and correctors.

Another mitigation measure proposed for some of the schemes is to foresee bunches with horizontal polarization [5] in addition to the bunches with longitudinal polarization to measure and, possibly, correct a rotation from the horizontal into the vertical direction.
Vertical spin due to horizontal and vertical quadrupole Displacements and Orbit Distortions in both planes

A case considered in the past [13] are simultaneous transverse offsets of electrostatic quadrupoles in vertical and horizontal direction. To better understand the mechanism generating a vertical spin component, a special case with two quadrupoles located at opposite positions in the ring and misaligned with transverse offset in both transverse planes. The sign of the transverse offsets for the two quadrupoles are opposite. Such transverse offsets by \( \Delta x = \Delta y = \pm 0.1 \text{ mm} \) of two quadrupoles located in the center of (opposite) straight section of the strong focusing ring proposal results in the closed orbits (1st order contributions taken into account only) shown in Fig. 10.3. The energy of a particle following the closed orbit \( x_{co} \) inside a bend is in general different from the magic energy due to the non-zero electric potential. This energy offset leads to a rotation of the spin around a vertical axis. The resulting small horizontal spin of a proton polarized parallel to its momentum circulating in CW direction is given by:

\[
s_x = \int ds \frac{2}{\gamma m} x_{co}(s) \rho(s) \ .
\]

Using Eq. (10.4), the vertical spin obtained over one revolution is given by:

\[
\Delta s_y = \int_0^C ds y'_{co}(s) \rho(s) s_x(s) \ .
\]

The functions required to compute the resulting vertical spin build up are plotted in the lower image of Fig. 10.3. The result can be interpreted in terms of a geometric phase effect as it is the result of two rotation, one around a vertical axis and the other around a longitudinal axis, which are out of phase. For the case, based on the strong focusing lattice, described, one obtains an average build up of the vertical spin of \( \frac{ds_y}{dt} = -4.5 \mu\text{rad}/s \). The effect to be expected in a realistic machine can only be estimated by thorough simulations with realistic assumptions for misalignments of components and closed orbit correction. Note that this effect does not mimic an EDM in the sense that the contributions from the CW
and the CCW beam to the final result cancel. Nevertheless, a fast rotation from the longitudinal into the vertical plane may be challenging for the polarimeter as the build-up of vertical spin has to be measured with very high relative precision. One may as well consider a feedback system to correct spin rotations (from this and other effects that do not mimic EDM) into the vertical plane not compatible with an EDM for example by acting on the vertical position at the location of the RF cavity.

Note that the effect can not be cured by using a "weak focusing" lattice as a horizontal offset and rotations around the longitudinal axis result in the same phenomena, but in addition a more direct rotation from the longitudinal into the vertical direction, which will be described in the next section.

### 10.2.4.4 Electric Bend with horizontal Offset and a Rotation around the longitudinal Axis

![Fig. 10.4: Misalignment of two pairs of electric bends around the center of opposite arcs.](image)

An electric bend with a horizontal offset $\Delta x$ induces an electric potential at the location of the reference orbit and, in consequence, moves the kinetic energy of a beam particle from the "frozen spin" condition. An additional rotation of the same bend around the longitudinal axis by an angle $\alpha$ leads to a vertical electric field component. The consequence is a non-zero radial component of $\Delta \vec{\Omega}$ and a rotation of the spin from the longitudinal direction into the vertical direction, which differs from the rotation of the direction of movement. In reality, the situation is slightly more complicated as the misalignment of the bends affect as well the closed orbits in both planes such that (i) the closed orbit gives an additional contribution to the shift of the kinetic energy from the "frozen spin" condition and (ii) the effect described in the previous section 10.2.4.3 has to be taken into account as well. For numerical evaluations we consider again the strong focusing lattice with (for symmetry reasons) two electric bends on either side of the center of opposite arcs misaligned by $\Delta x = \pm 0.05$ mm and $\alpha = \pm 0.05$ mrad. The net rotation of the spin from the longitudinal into the vertical direction taking both effects into account is given by:

\[
\begin{align*}
  s_x(s) &= \int_{s_0}^{s} ds \frac{2 x_{cc}(\hat{s}) - \Delta x(\hat{s})}{\gamma_m} \frac{1}{\rho^2(\hat{s})} \\
  \Delta s_y &= \int_{0}^{C} ds \frac{2 x_{cc}(s) - \Delta x(s)}{\gamma_m} \frac{1}{\rho^2(s)} \alpha(s) + \int_{0}^{C} dy_{cc}(s) \rho(s) s_x(s).
\end{align*}
\]  

(10.8)
One finally obtains $\frac{d\theta}{dt} = -5.45 \mu\text{rad/s}$. Again, this effect does not mimic EDM as contributions from the CW and the CCW beam to the final result cancel.

10.2.4.5 Vertical Spin build up from a Combination of magnetic and vertical field Components generating closed Orbit Deformations in both Planes

Either a vertical magnetic and electric field or a horizontal magnetic and electric field lead to orbit deformations in both transverse planes and, in turn, to a build-up of a vertical spin component in a way similar to the mechanism described in section 10.2.4.3. However, the cases presented here mimic EDM in the sense that the contributions from the CW and CCW to the final EDM value do not cancel.

For the case with vertical electric and electric field components, the vertical magnetic field contribute as well to the rotation of the spin in the horizontal plane. Thus, stronger vertical spin build is expected than for horizontal magnetic field components and this case is treated here. An integrated vertical magnetic field of $\pm \Delta B_y = 1 \text{nTm}$ at the location of quadrupoles located in the center of opposite straight sections in the strong focusing lattice is assumed. These quadrupoles are vertically misaligned by $\pm 0.1\text{mm}$. The resulting orbit distortions as well the spin rotation $s_x$ in the horizontal plane and the derivative of the vertical closed orbit are plotted in Fig. 10.5. The vertical spin generated over one turn is given by Eq. (10.4) and evaluates to $\Delta s_y = 8.5 \cdot 10^{-15} \text{rad}$. The resulting vertical spin build-up rate for this probably optimistic case is $3.1\text{nrad/s}$, which is almost a factor two larger than the one due to an EDM of $d_s = 10^{-29} \text{e}\cdot\text{cm}$.

10.2.5 Summary

The phenomena considered as potential limitations to reaching the target sensitivity of $d_s = 10^{-29} \text{e}\cdot\text{cm}$ are summarized in Tab. 10.2. Static (not due to cavity with a vertical offset) horizontal (radial) magnetic fields are expected to be the main source of systematic errors and to limit the achievable sensitivity to a value larger than this target. In addition, several second order effects, where two distinct imperfections of the real machine w.r.t. the perfect design case lead to a spin rotation from the longitudinal to the vertical direction, have been considered. Higher order effects as well as betatron and synchrotron oscillations have not been taken into account and are expected to give smaller contributions to systematic effects.

For most second order phenomena, only simple special cases with sometimes optimistic assumptions have been considered aiming at understanding the underlying mechanisms. This has to followed by more realistic studies with positioning errors of all elements, realistic orbit correction scenarios and
<table>
<thead>
<tr>
<th>First order effects</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Static horizontal magnetic field</td>
<td>Mimics EDM (no cancellations between contributions from CW and CCW beam on final result). Effect due to typical magnetic fields inside state-of-the-art shielding about eight orders of magnitude larger than effect due to smallest EDM to be detected; expected to be the main limitation to achievable sensitivity even with orbit separation measurement to estimate (and correct) average horizontal magnetic field.</td>
</tr>
<tr>
<td>Horizontal magnetic field due to cavity offset</td>
<td>Does not mimic EDM, but fast rotation of spin requiring high precision polarimetry and/or feedback.</td>
</tr>
<tr>
<td>Gravity</td>
<td>Effect about factor 30 larger than the one due to $d_s = 10^{-29}$ e.cm. Does not mimic EDM (no cancellations between contributions from CW and CCW beam on final result) and, thus, not expected to limit the sensitivity of the experiment.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second order effects</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal spin and non-zero average slope of vertical orbit inside bends</td>
<td>Depends on polarimeter properties of the two rings, contribution mimicking EDM and incompatibility with sensitivity goal $d_s = 10^{-29}$ e.cm likely. Mitigation by bunches polarized in forward and backward direction? Mitigation by additional bunches with horizontal polarization?</td>
</tr>
<tr>
<td>Horizontal and vertical offsets of electric quadrupoles</td>
<td>Does not mimic EDM, large effects expected, high precision polarimeter and/or feedback required.</td>
</tr>
<tr>
<td>Electric bends with simultaneous horizontal offset and rotation around longitudinal axis</td>
<td>Does not mimic EDM, large effects expected, high precision polarimeter and/or feedback required.</td>
</tr>
<tr>
<td>Static vertical and longitudinal magnetic fields</td>
<td>Does not mimic EDM, moderate effect probably not limiting sensitivity.</td>
</tr>
<tr>
<td>Vertical magnetic field from cavity and static longitudinal magnetic field</td>
<td>Mimics EDM, effect small and not expected to limit sensitivity.</td>
</tr>
<tr>
<td>Magnetic and electric fields generating orbit deformations in both planes</td>
<td>Mimics EDM, worst case with vertical magnetic and vertical electric field probably rules out to reach the sensitivity goal of $d_s = 10^{-29}$ e.cm.</td>
</tr>
</tbody>
</table>

Cross-checked with simulations. Some effects do not mimic EDM in the sense that the contributions from the CW and CCW beam on the final results cancel. Nevertheless, there are cases leading to a vertical spin build-up several orders of magnitude faster than the smallest EDM to be measured. This requires either to measure the vertical polarization with high accuracy or to implement a feedback system reducing the effect (could be based on any of the effects generating such a spin rotation as for example bends with horizontal offsets and rotations around the longitudinal axis).

The optimum operational scenarios depend on the main source for systematic errors of the experiments. In case, the main contribution comes from the average horizontal magnetic field (after the implementation of mitigation measures), operation with simultaneous CW and CCW beams is important, but the filling patterns of the two rings is not critical. If second order effects generate a significant contribution to systematic effects, the filling pattern of the two beams becomes important. The optimum operational scenario to control systematic effects would be to have both the CW and the CCW beam with part of the bunches polarized in forward direction, part of the bunches polarized in backward direction.
and some of the bunches with horizontal polarization. However, a filling scenario to generate such a bunch pattern is not available.

References


[9] A. Laszlo and Z. Zimboras, Quantification of GR effects in muon g-2, EDM and other spin precession experiments, Class. Quantum Grav. 35 (2018) 175003.


Chapter 11

Polarimetry

11.1 Introduction to Polarimetry

The quantum nature of the nucleon or nucleus requires that any electric dipole moment (EDM) be aligned with the spin axis. Thus the experimental connection with the EDM is found through the preparation of beams with spin polarisation and the measurement of small spin polarisation changes that may be interpreted as evidence for interactions that are signatures of an EDM. Fortunately, polarised beams and the measurement of nuclear spin polarisation through strong interaction processes are both mature technologies and capable of high precision. It is also fortunate that polarimeters for spin polarisation measurements are at their most sensitive in the range of beam energies where storage ring technology for spin manipulation also works well. This chapter describes the polarisation measurements planned for the EDM search in detail.

The chapter begins with a short review of polarisation terminology as codified in the Madison Convention for protons (spin-1/2) and deuterons (spin-1). It then moves to a summary of the polarisation measurements of the beam as it proceeds through the preparation and acceleration process. The rest of the chapter deals with the polarimetry planned for the EDM storage ring itself, showing new technology developed for the calorimeter detectors and arrangements for making polarisation measurements with high efficiency and precision. Much of this chapter is devoted to ways to handle systematic error problems and limits on the use of counter-rotating beams for identifying the time-reversal violating EDM.

11.2 Polarimeter Spin Formalism

The plan for an EDM-sensitive polarisation measurement is to record the horizontal asymmetry in the scattering of protons or deuterons from a carbon target at forward angles. At the energies where the EDM search would be made, the interaction between the polarised particles and the carbon nucleus contains a large spin-orbit term. This gives rise in elastic scattering to an asymmetry between left and right-going particles when there is a vertical polarisation component present.

For spin-1/2, the polarisation along any given axis is given in terms of the fraction of the particles in the ensemble whose spins, through some experiment, are shown to lie either parallel or anti-parallel to that axis. If these fractions are $f_+$ and $f_-$ for the two projections of the proton’s spin-1/2, the polarisation becomes $p = f_+ - f_-$ which ranges between 1 and $-1$ with $f_+ + f_- = 1$. The scattering cross section $\sigma_{POL}$ may be written in terms of the unpolarised cross section as

$$\sigma_{POL}(\theta) = \sigma_{UNPOL}(\theta)[1 + pA_Y(\theta)\cos(\phi)\sin(\beta)]$$

(11.1)

with the vertical polarisation component

$$p_Y = p \cos(\phi)\sin(\beta)$$

(11.2)

and where the angles are defined with respect to a coordinate system shown in Fig. 11.1 below.

The left-right asymmetry measures the vertical polarisation component $p_Y$. The size of the signal is governed by the strength of the spin-orbit interaction, which gives rise to the asymmetry scaling coefficient $A_Y(\theta)$, otherwise known as the analysing power. The left-right asymmetry arises from the $\cos(\phi)$ dependence of the cross section on the azimuthal angle of the polarisation. If two identical detectors are placed symmetrically about the $z$ axis and their rates are $L$ and $R$, then the asymmetry is given by
In the case of the deuteron, which is spin-1, there are three fractions that describe the magnetic sub-state population, \( f_+ \), \( f_- \), and \( f_0 \). The two polarisations are vector, \( p_V = f_+ - f_- \), and tensor \( p_T = 1 - 3f_0 \) which can range from 1 to -2. If we are interested only in the EDM, then the vector polarisation suffices as a marker and the deuteron polarised cross section (Cartesian coordinates) becomes

\[
\sigma_{\text{POL}}(\theta) = \sigma_{\text{UNPOL}}(\theta) \left[ 1 + \frac{3}{2} p_V A_Y(\theta) \cos(\phi) \sin(\beta) \right]
\] (11.4)

Tensor polarisation is usually present to a small degree in polarised deuteron beams. There are three independent tensor analysing powers that each add another \( p_T A \) term to the equation above. Their effects may prove useful in polarisation monitoring or checking for systematic errors.

### 11.3 Beam Preparation

The essential spin-related parts of the EDM storage ring injector beam line are shown in Fig. 11.2. These components are site-independent in the description below. The diagram contains a polarised proton source with its associated low energy polarimeter, spin rotation and proton acceleration equipment, a trip through the a storage ring for a phase space reduction through electron cooling and bunching, and suitably located polarimeters that confirm that all this works and that calibrate the polarisation of the proton beam. This section summarises the polarisation features.

High intensity, pulsed polarised proton sources suitable for use on colliders have reached a mature state with adequate beam (\( 10^{12}/\text{pulse} \)) for a storage ring EDM search [1]. The version currently in operation at RHIC may be considered as a model. Brookhaven has a crew of a few members whose job is to maintain, operate, and improve the ion source.

The source creates a high-brightness proton beam in a high-efficiency extraction system before it is neutralised in hydrogen gas to make a well-collimated atomic beam. From there it converges into a pulsed He ionizer that makes a low-emittance proton beam within a strong axial magnetic field. Inside
the high-field region, polarised electrons are added to the protons from an optically pumped rubidium vapour. The neutral atoms proceed to a Sona transition that transfers the electron polarisation to the protons [2]. The atoms are given an additional negative charge in a sodium vapour and extracted at 35 keV to form a beam for subsequent acceleration. Either state of polarisation along the magnetic field axis is possible. The polarisation is in excess of 80%. As an alternative to ionisation, an atomic beam polarimeter is present that is capable of measuring the atomic polarisation. This allows tuning of the source parameters without requiring acceleration of the beam to higher energy.

For transport through the storage ring, the polarisation direction must be perpendicular to the ring plane (aligned with the ring magnetic fields). To accomplish this prior to acceleration, electrostatic plates bend the proton beam without spin precession. This produces a sideways polarisation that passes through a solenoid where it rotates into the vertical direction. Initial acceleration is then provided by a linear accelerator such as an radio-frequency quadrupole or a drift tube linac. Once the protons reach an energy where nuclear scattering can yield high spin sensitivities, a carbon-target polarimeter becomes feasible. One should be installed along the beam line so as to verify that the ionisation, spin rotation, and first acceleration have not altered the polarisation. Measurements of the proton-carbon analysing power [3], shown in Fig. 11.3 over a range of angles and energies between 60 and 70 MeV, indicate large values near one at angles less than 60° that are practical for mounting monitor detectors (usually plastic scintillators). This involves the construction of a small scattering chamber. The target may be a thin foil of carbon mounted on a movable ladder.

The passage of the beam through the storage ring is critical for two reasons. First, the in-plane polarisation in the EDM storage ring has a polarisation lifetime that is improved if the phase space of the beam is made as small as possible [4]. This may be achieved while also using this ring for the second purpose, to accelerate the proton beam to the energy of 232.8 MeV before injection into the final storage ring. During the acceleration process, the proton energy will pass through the $G\gamma = 2$ imperfection resonance. This resonance, which is driven by magnetic field errors in the ring, is often strong enough to depolarise the beam. Usually, the remedy is to make the resonance even stronger by introducing briefly an imperfection in the form of a vertical steering bump that causes the polarisation to completely flip sign as it passes through the resonance. A crucial step in the setup of the beam is to tune the bump so that the maximum polarisation survives acceleration. For this, another polarimeter is needed just past extraction.

Fig. 11.2: Block diagram showing the main components of the injector beam line that are related to spin manipulation and measurement.
Fig. 11.3: Contour map of the analysing power for proton scattering from carbon in the energy range from 20 to 84 MeV [3]. The ridge in the 40° to 60° range is particularly well suited for monitoring beam polarisation.

from the storage ring. Again, scattering from carbon, shown here in Fig. 11.4 at 250 MeV [5], provides scattering angles with very large analysing powers that may, in fact, be used as the polarisation standard for this experiment. A simple foil target and scintillation detectors are again appropriate.

Fig. 11.4: Measurements of the angular distribution of the analysing power for proton scattering from carbon at a beam energy of 250 MeV [4]. Note that both the first and second interference peaks show very large analysing power values. The second peak is close enough to one that it may serve as a calibration standard for the subsequent use of the beam.

There need to be two paths by which the injection of the beam is made into the EDM ring in order to have both clockwise (CW) and counter-clockwise (CCW) beam circulating during the measurement. The switch between these two paths should be relatively rapid so that the requirement that the two beams be as identical as possible is easier to The critical elements are shown in Fig. 11.5.
Fig. 11.5: Diagram of elements essential for spin handling during injection. Beams must be injected into the ring in both directions in reasonably rapid succession. It should be possible to have polarisation along any direction. The polarisation begins perpendicular to the ring plane. A solenoid (up to 2 T·m strength and likely superconducting) is capable of rotating the polarisation by at least 90° in either direction. This is followed for the beam line to either side of the EDM ring by a bending magnet whose angle is 40.22°. This makes the plane of the polarisation parallel to the beam direction so that the changes are forward and back. A second solenoid rotates this polarisation into the horizontal plane. Some time will be needed for ramping these solenoids if it is desired to have a variety of directions within one beam store. It is assumed that a complete spin flip will be made at the ion source.

11.4 Main Ring Polarimeter Design Goals

The goal for the EDM search makes certain requirements on the polarimeter system, including both target and detectors along with the associated data acquisition system.

- The system must make efficient use of the beam particles. A polarisation sensitivity at the level of one part per million requires capturing \(10^{12}\) usable polarimeter events, a process that may require many months of data taking. For this, we have explored the use of thick targets located at the edge of the beam at COSY. Particles that enter the front face will be lost from the beam, but the thickness (17 mm in tests) enhances the probability of scattering into one of the polarimeter detectors. The goal would be to achieve an efficiency near 1%. Eventually, most of the beam is used up hitting this target.
- At the same time the analysing power \(A_Y\) should be as large as possible. Values in excess of 0.5 are available for optimal choices of the detector acceptance for either protons or deuterons.
- The method of choosing which events to include in the data set should be relatively insensitive to the choice of cuts so that small changes have a minimal effect on the measured asymmetry. In the case of deuterons, it may be important to insert a range absorber ahead of the trigger detector so that most of the breakup proton flux is removed before being processed in the data acquisition system. Data acquisition firmware that digitises pulse shape and has a high throughput may make this requirement less stringent. To ensure a proper early-to-late asymmetry difference, the trigger threshold must be stable over time.
- In the EDM search, the left-right asymmetry carries the information on the EDM. At the same time a monitor is needed for the magnitude of the beam polarisation. Such a measurement requires that we rotate the polarisation periodically from its frozen spin orientation into the sideways direction. Alternatively, some bunches may be loaded into the EDM ring with a sideways orientation. In this
configuration, the polarisation is measured through the down-up asymmetry in the scattering. Thus a full azimuthal acceptance is needed in the polarimeter detectors. If these detectors are segmented, some elements near 45° lines may be used for both left-right and down-up measurements, thus increasing the useful efficiency of the device.

- For the counter-rotating beams (CW and CCW, see below), one block target may serve for the measurement of polarisation for both beams. This implies that a set of detectors be located in both directions from the target. Backscatter from the target is not expected to be a problem, even with a $10^{-6}$ sensitivity requirement on the measured asymmetry. Proton-carbon elastic scattering data indicates such cross sections are down by eight orders of magnitude [5].
- Extraction of beam onto the block target at COSY has usually been achieved by heating the beam to enlarge the phase space in the plane where the target is located. Horizontal and vertical heating may be operated independently, creating the opportunity for two independent polarimeter locations on the EDM ring. More than one polarimeter is useful as a check against systematic errors.
- Studies undertaken in 2008-2009 demonstrated that the sensitivity of the polarimeter to systematic errors (rate and geometry changes) may be calibrated. With the use of positive and negative polarisation states, such a calibration can be used to remove the effects of the systematic errors. Such a technique thus becomes an important requirement.

### 11.5 Implementation of the Polarimeter

While in principle, the polarisation may be deduced from an absolute measurement of the cross section alone, experience with polarisation measurements strongly favours the use of both a left and right detector simultaneously. In addition, polarised ion sources can provide beam in either a positive or a negative polarisation state, and the use of both states for the experiment is recommended. In the EDM storage ring, the beam injection plan is to fill the ring with both CW and CCW beams, allow them to come to equilibrium in a coasting state without bunching, then impose bunching. The beam is vertically polarised and both CW and CCW beams are filled using a single polarisation state from the ion source. Once bunched into the final pattern, an RF solenoid with multiple harmonics of the bunched beam frequency will be used to precess the bunch polarisations into the ring plane with alternate bunches polarised in opposite directions. The higher harmonic portions of the RF solenoidal field will be optimised so that all parts of each bunch are polarised in the same direction following the rotation. Once in plane, the orientation of the polarisation in the bunches is maintained using feedback. The feedback system also rotates the polarisations so that the spin alignment axis is parallel to the beam velocity, thus creating the frozen spin condition. The orientation is then maintained by nulling the down-up asymmetry from a continuous polarisation measurement.

For the rotation of the polarisation into the ring plane, the solenoid must carry a complex waveform with several harmonics. The goal is to have the amount of rotation, given by the solenoid strength, be the same across the length of each bunch. Outside of the bunch, the solenoid is not constrained. Figure 11.6 below shows fits with 3 and 4 harmonics that have been adjusted to best reproduce a level equal to one over half of the time ($-\pi/4$ to $\pi/4$) as indicated by the faint red line. The lower plot is an enlargement of this critical region. Variations are less than 3% for 3 harmonics and less than 0.5% for 4. The series converges rapidly. Particles within the bunch will sample various areas of this plot as they undergo synchrotron oscillations of different amplitudes, so the size of the differences with unity tell us something about the residual vertical polarisation in the beam at the end of this process. The expansion used here contains only odd harmonics and in this way provides two flat-tops. This rotates alternate bunches in opposite directions into the ring plane. So with an even number of bunches we obtain a condition with half of the bunches having one polarisation and the other half with the opposite. Thus the RF solenoid runs on a harmonic that is half the bunch number. (Note that in this scheme each bunch sees the same incremental rotation every time it passes the solenoid because the field is the same. This works because at the frozen spin condition with a zero spin tune, there is no rotation of the in-plane component of the
polarisation as the bunch goes around the ring. It is clearly important to inject the beam at the proper momentum.

**Fig. 11.6:** Two curves showing the ability to achieve a flat function over an interval where the beam pulse exists and whose polarisation must be rotated uniformly into the ring plane. The upper panel shows the whole curve, the lower panel shows an enlargement of the top of the curve. Inclusion up to harmonics 3 and 4 yields larger and smaller variations.

If the polarisation states are $+$ and $-$, and the detectors L and R, then the EDM asymmetry, which is the product of the polarisation and the analysing power, may be given by the “cross ratio” formula

$$
\epsilon = \frac{r - 1}{r + 1} \quad \text{where} \quad r^2 = \frac{\sigma_{L+}\sigma_{R-}}{\sigma_{L-}\sigma_{R+}}
$$

This formula has the advantage that it cancels to first order common errors that depend on differences in the acceptance of the left and right detector systems and differences in the integrated luminosities for the plus and minus polarisation states. At the precision required for an EDM search, higher order errors still affect this formula for the asymmetry and must be removed, as will be discussed later.

Detectors above and below the beam (“up” and “down”) are sensitive the horizontal $x$-component of the polarisation. With frozen spin, this should vanish. Thus any non-zero value implies that the match between the polarisation and velocity rotation rates is not perfect. As had been demonstrated at COSY [6], such information may be fed back to a suitably sensitive adjustment, such as the rf cavity frequency, to correct the misalignment. This needs to be done continuously during the EDM experiment. In addition, at regular intervals the polarisation should be rotated into the sideways direction (or allowed to precess through a full circle) to provide a monitor of the polarisation magnitude. The accumulation of the EDM signal goes as the time integral of this magnitude.

For tensor polarised deuterons, it is possible to utilise a comparison between the scattering rates in different polar angle ranges as a beam polarisation monitor, if the tensor polarisation is made intentionally large. This eliminates the necessity for periodic rotations of the polarisation away from the direction of the beam. However, a tensor polarisation may also generate a left-right (EDM-like) asymmetry if there is a misalignment between the polarisation axis and the velocity. This systematic error will be discussed later.
The necessity to monitor continuously both the vertical and horizontal (sideways) polarisation components during the experiment places a premium on polarimeter efficiency, the fraction of particles that scatter usefully into the detectors divided by the number that are removed from the beam. A sensitivity requirement for the EDM asymmetry of $10^{-6}$ implies $10^{12}$ recorded and useful events, which may necessitate as much as a year of running time. Polarimeters used in double scattering experiments [7, 8] show that for proton and deuteron beams of a few hundred MeV, efficiencies of 1-3% have already been achieved using thick (few centimetre of solid material) targets. This makes these energies an ideal choice for the EDM ring. (Higher energies imply larger storage rings and additional construction costs.)

11.6 Choice of analysing reaction

Work with highly-efficient double-scattering polarimeters at these energies has concentrated on the elastic scattering channel at forward angles ($5^\circ$ to $16^\circ$) as the best choice for polarimetry. Essentially all polarimeters have employed carbon as the target material. The angular distribution represents an interference pattern created by scattering from opposite sides of the nucleus. This angle range typically encompasses one full analysing power oscillation for deuterons and half of an oscillation for protons. At angles less than $5^\circ$, Coulomb scattering takes over from the nuclear and the analysing power quickly goes to zero. This region should be avoided.

![Graphs showing cross section, analysing power, and special figure of merit for proton elastic scattering on $^{12}$C at 250 MeV bombarding energy.](image)

In Fig. 11.7 is the cross section and analysing power angular distribution for proton scattering from carbon at 250 MeV beam energy [5]. The Coulomb-nuclear interference region lies inside $5^\circ$. Beyond this angle, the cross section arises almost exclusively from nuclear scattering. The elastic scattering channel shown here dominates all other reactions inside about $15^\circ$. With a positive spin-orbit interaction and an attractive nuclear field at the surface of the nucleus, there is a strong sensitivity to the polarisation of the incident proton that results in a positive analysing power. Both the cross section and the analysing power show an oscillation pattern that reflects interference from opposite sides of the nucleus.
The relative merits of various parts of the angular distribution for polarimetry purposes is usually evaluated through the use of a figure of merit: $\text{FoM} = \sigma A^2$. This is shown in the bottom panel with a factor of $\sin(\theta)$ included to adjust for the falling solid angle near $0^\circ$. This leads to a clear peak in the $\text{FoM}$. Beyond about $17^\circ$ the analysing power is falling through zero and little additional information is available. A similar peak characterises deuteron scattering, but it is a few degrees more narrow.

Because the forces leading to the large positive analysing power are a property of the nuclear surface and have nothing directly to do with any reactions that might take place (so long as the energy transfer is much smaller than the beam energy), similar features exist also for a large number of other direct reaction channels. So there is no particular requirement that the detector be capable of resolving the elastic scattering group exclusively, which requires high resolution in the measurement of the elastic scattering (or other charged particle) energy. This simplifies polarimeter design. The critical feature then becomes the choice of an acceptance that maximises the figure of merit, and how stable this acceptance (and trigger threshold) is over time.

Efficient double-scattering polarimeters for protons have been built and used successfully between 100 and 800 MeV with a carbon target and simple polarimeter detectors consisting of thin plastic scintillators [9]. McNaughton’s summary plot shown below in Fig. 11.8 illustrates that the best analysing power falls almost exactly at 232.8 MeV where the proton EDM experiment may run with an all-electric ring. Most proton polarimeters have used carbon as the target because of its ease of handling, wide interference pattern period, and large forward cross section. All targets in this mass region of the period table tend to give similar results. The all-electric frozen spin energy is marked in red.

These considerations lead to a simple conceptual design for the EDM polarimeter that is shown schematically here in Fig. 11.9.

The main detectors consist of an energy loss detector that identifies the particle followed by a total energy calorimeter. In many cases, proton polarimeters have used only the $dE/dx$ detector. For the deuteron, the main background will be protons from deuteron breakup. These protons have almost no sensitivity to beam polarisation, and every effort should be made to eliminate them from the trigger rather than relying on post-detector processing. Various groups [7, 8] have successfully employed iron absorbers ahead of the scintillator system. If the absorber is appropriately designed, the event trigger from the scintillators may be optimised for large figure of merit and small sensitivity to scintillator gain drifts.

In order to make more precise models of any EDM polarimeter, data base runs have been made at

![Fig. 11.8: A collection of operating point analysing powers for proton-carbon polarimeters at intermediate energies [9]. The curve is a guide to the eye. The red line marks the magic energy of the EDM search.](image)
Fig. 11.9: Schematic layout showing the important components of an EDM polarimeter. The beam goes from right to left, and passes through a thick carbon target. Scattered particles first encounter a tracking detector that traces rays back to the target. Next an absorber removes unwanted events. Lastly, a $\Delta E$ and $E$ detector pair identify the energy of the particles of interest along with the particle type.

COSY for deuterons at a variety of energies between 170 and 380 MeV. A similar run for protons was completed in the fall of 2018. The analysis of these data is in progress. Figure 11.10 shows the results of a deuteron data base run at the KVI Groningen at 110 MeV.

The upper left panel shows a 2-D representation of the events recorded at $27^\circ$. Clear bans for protons, deuterons, and tritons appear. The coloured regions indicate places to include in the polarimetry (green) and places to avoid (magenta). The proton band shows a large contributions from deuteron breakup that has almost no spin dependence. On the right are two panels for deuterons and protons individually. The regions are marked there as well. The proton distribution from breakup is large. This should be mostly eliminated if absorber material were installed ahead of the detector system.

11.7 Target operation in a storage ring

Before our COSY investigation, there was no information on highly efficient polarimeter operation in a storage ring. So we undertook tests to see if a thick target could be operated in this environment while still allowing the beam to circulate. What worked was placing a square-cornered block about 3 mm from the beam centre line. Various schemes were tested to bring the beam to the target slowly, extracting the beam over an extended period of time. We found that it is better to move the beam than the target, since the beam moves smoothly. A steering bump changes the beam path length, creating a problem for maintaining the spin tune (needed for frozen spin). So most of the COSY runs have made use of white noise heating applied through a set of strip-line plates that enlarges the beam through phase space growth. The white noise is applied over a narrow frequency range around one of the betatron oscillation harmonics, and this couples well to the beam.

Extraction of the beam using white noise appears to be a two-step process, based on tilted beam studies [10]. The mean distance from the surface closest to the beam near the point where the particles penetrate the leading face of the target block is about 0.2 mm. This is much larger than the change per turn in the position of the beam. The first step in the extraction process is an encounter with a microscopic ridge on the close face of the target block. This induces a betatron oscillation in the particle, which often
Fig. 11.10: Panels showing as sample from a broad range data base run taken at the KVI-Groningen. The deuteron energy was 110 MeV. Scattering from a carbon target was observed at 27°. Particle type is distinguished in the $\Delta E$ by $E$ upper-left panel. Energy for particles emerging as protons or deuterons are shown in the two right-side panels. Areas outlined in green have significant analysing power and could be used for polarimetry. Areas marked in purple have a low spin sensitivity and should be avoided.

survives to continue around the storage ring. On some subsequent pass, that oscillation takes it far enough from the beam centre that it impacts the front face of the block. With a 0.2 mm typical distance from the leading corner to the point of impact, many of the perturbed particles penetrate the entire carbon block and therefore have a maximal probability of undergoing a scattering event. This model is confirmed by the observation that the efficiency of the COSY carbon block target is consistent with Monte Carlo calculations that assume a full interaction with the target [10].

The main disadvantage of this target arrangement is that it favours particles that are in the halo of the beam. Below $10^9$ deuterons/fill at COSY, there appears to be no issue that is associated with this as the polarisation lifetime measurements show a smooth depolarisation curve, as expected. At higher beam currents, structures appear in the time dependence of the polarisation that indicate more complex histories in bringing the beam to the target. Modelling of the time dependence confirms this.

Carbon block target thicknesses at COSY were typically 17 mm with a density of approximately 1.7 g/cm$^3$. As the target is made thicker, the energy loss of particles in the target increases. Modelling of the response must therefore consider changes in the cross section and analysing power angular distributions with changing particle energy. These changes, plus considerations of beam alignment, probably restrict carbon block thicknesses to less than 5 cm. This thickness is enough, however, to achieve efficiencies on the order of 1%.

11.8 Development of calorimeter detectors for an EDM polarimeter

For some time, Irakli Keshelashvili and his group at COSY have been utilising dense LYSO crystals as the EDM polarimeter’s calorimeter detector. They have developed $3 \times 3$ cm$^2$ modules that include a silicon photo-multiplier as a readout (see Fig. 11.11). The resolution for stopped 270-MeV deuterons is
approximately 1%.

**Fig. 11.11**: A sample LYSO crystal showing the parts with labels.

These modules will ultimately go into a larger volume array (see Fig. 11.12) that surrounds the beamline.

**Fig. 11.12**: Model view of the detector system inside the LYSO-based polarimeter. The scattered beam expanding from the target is shown in red. The LYSO-crystal calorimeter detectors appear in light blue in the segmented arrangement likely to be used for left-right and up-down asymmetry measurements. Just in front of the calorimeter is an outline sketch of two layers of triangular-shaped scintillation detectors. SiPM light collectors located on the ends of the bars are not shown. All particle tracks penetrate both vertical and horizontal layers. Energy sharing between neighbouring scintillators allows for a more precise position determination. The shutter assembly in front allows for an absorbing layer to be imposed in front of the detectors to remove unpolarised background events. A mechanical system will open and close the shutter leaves.

A 48-module mock-up has been tested and will then be moved to the ANKE target area at COSY for installation on the beam line and further testing at the beginning of 2019. A block target will be a part of the installation.

Initial tests of the LYSO modules have been made at 93, 196, 231, and 267 MeV with a deuteron beam in the external beam “Big Karl” area at COSY. An overlay of a preliminary series of spectra is shown in Fig. 11.13. The energies listed in the figure take into account losses from windows and upstream trigger detectors. The four energies were measured in different setups and relative gains have not been reconciled.
11.9 Use of the polarimeter to maintain frozen spin

For EDM data runs of about 1000 s, the precision with which the polarisation must be maintained parallel to the proton velocity is about one part in $10^9$ over the course of a beam store. Prior to storing the beam, the value of the spin tune cannot be known to this level of precision, so a feedback mechanism must be used to maintain alignment. One such mechanism was tested at COSY [6]. The analysis of the polarimeter data for in-plane polarisation yields a magnitude and phase for each time interval (1-4 s in duration, for example). A scheme was developed to provide very precise changes to the frequency of the RF cavity controlling the beam based on a running analysis of the polarimeter data as it was acquired.

Figure 11.14 shows an initial situation in which the spin tune is not matched to the rotation of the beam. This result is a slope with time for the phase data. At two times, a signal and its opposite were sent to the RF signal generator requesting a change in frequency. This was immediately reflected in a change in the slope of the phase, which is a measure of the spin tune relative to an assumed value.

In another test, shown in Fig. 11.15, a change was sent to the RF generator and then quickly
reversed, so that the spin tune remained the same after as before. But the pulse caused the phase itself to shift. With the changes calibrated, the figure shows steps of about 1 rad resulting from a series of such pulses sent to the RF generator.

**Fig. 11.15:** Phase of the rotating in-plane polarisation relative to a standard clock reference as a function of time in the store. Periodically a pulse is sent to the COSY rf generator that makes a step in the revolution frequency and then quickly reverses it. This pulse makes a step in the phase. Once calibrated, these steps can be tuned to be about 1 rad.

Figure 11.16 is an example of how this might look in a realistic situation. The upper curve is the corrected phase and the lower curve shows the time sequences of changes made to the rf cavity frequency in order to maintain that level of phase reproducibility. The average deviation, indicated by the grey band, is ±0.21 rad. This level of control is adequate for the EDM experiment.

**Fig. 11.16:** (top) Measurements of the phase of the rotating in-plane polarisation with reference to an external clock as a function of time. These measurements are being used to correct the COSY revolution frequency in real time so that the phase remains stable at zero (arbitrarily chosen), beginning at 89 s. The grey band indicates the RMS deviations in the phase. (bottom) Depiction of the actual changes generated by the feedback system and sent to the RF frequency generator as a function of time.

This technology is essential for maintaining the frozen spin condition needed to observe an EDM. In must be in place and operating as soon as the beam is injected into the main storage ring.

119
11.10 Correction of rate and geometry errors in the polarimeter

A cross-ratio analysis of data from a polarimeter, as described in the requirements section above, cancels most first-order errors. In a storage ring, the beam itself is continuously changing with time in both intensity and geometric placement, so higher-order effects need to be addressed. This is particularly true if sensitivities approaching $10^{-6}$ need to be probed.

![Diagram](image.png)

**Fig. 11.17:** Measurements with the EDDA detector of the left-right asymmetry as a function of the angle error in mrad (down triangles) or the position error (up triangles). Various features of the measurements are marked, including some interpretations through model parameters (usually logarithmic derivatives) from the fits through the data.

In 2008 and 2009, the EDDA detector system, then used as a polarimeter for COSY, was calibrated for geometric and rate error sensitivity. The beam was scanned horizontally in both angle and position. The effects of rate were also present in the data as the rate changed with the time in the store. An example of a piece of the geometric data for the left-right asymmetry is seen in Fig. 11.17. Measurements of a number of polarisation observables were made with five different polarisation states. Angular deviations (down triangles in mrad) and position variations (up triangles in mm) were recorded. As seen in the figure, the effects are large and clear. In the same set of data, changes due to the data acquisition rate were also recorded. A model of all of the error effects was constructed in terms of the logarithmic derivatives of the cross section and analysing power as geometric parameters, and these parameters as well as other factors including rate changes were used to reproduce the data, as shown. The free geometric variable in the model was taken to be the angle deviation from a straight beam. The model was sufficiently robust that it could predict effects for any of the measured polarisation observables within the errors in the observable measurements.

In the geometric case, Fig. 11.17 shows different effects for angle and position changes. These could be reconciled provided an effective distance to the detector was assumed, and this became one of the fitting parameters. If this substitution works well, then it can become the basis for reducing the geometry effects to a single parameter. The quality of this result is shown in Fig. 11.18. Measurements
of the left-right asymmetry correction are overlaid for both angle and position, and shown to lie along a similar slope.

![Graph of left-right asymmetry vs. index parameter](image)

**Fig. 11.18:** Changes to the left-right asymmetry as a function of an index parameter (defined below). The index parameters is tied to either position or angle variations of the beam on target. The overlap of these two sets of data into one universal line indicates that a single index parameter is capable of correcting both types of errors.

We chose to recast this relationship in terms of an index parameter $\phi$:

$$\phi = \frac{s - 1}{s + 1} \quad \text{where} \quad s^2 = \frac{\sigma_{L+}\sigma_{L-}}{\sigma_{R+}\sigma_{R-}}$$

(11.6)

This quantity is available experimentally in real time. Thus independent of the cross ratio or any other polarisation observable, a correction may be applied. The model is used to calculate the correction, such as a change in position along the sloped line in Fig. 11.18. This can be applied to any polarisation observable. A term,

$$W = \sum \sigma_i,$$

(11.7)

is also available for the counting rate. An example is shown in Fig. 11.19. The measurements of a beam with a constant polarisation is given by the red data. The time dependence is an error that depends on the data rate as it creates pile-up effects in the detectors. Correction of that error yields the blue data. But these data are still not right because of a geometric misalignment. The final correction leads to the black data, with are constant in time to better than one part in $10^5$, which is statistically limited. If the calibrations are known in advance, such corrections may be made in real time during the experiment, a feature that will be essential in maintaining the polarisation pointing along the velocity through feedback.
The example in Fig. 11.17 of calibration data for five polarisation states is linear in the case of the left-right asymmetry. Higher-order effects appear as curvatures of various ranks, which may also be parameterised using powers of the logarithmic derivatives. The combination of all of these properties of the model-based calibration and driver-term corrections makes it possible to extract a signal as small as $\delta \epsilon = 10^{-6}$ reliably from a series of time-dependent asymmetry measurements.

### 11.11 Polarimeter rotations, energy loss, and deuteron tensor polarisation effects

The polarimeter must be set up so that the coupling between horizontal asymmetries and vertical asymmetries, as established by the location of the ring plane, is as small as possible. Such a correlation is measured by stepping the polarisation direction (registered as a phase in the feedback circuit) around the in-plane circle and comparing vertical and horizontal asymmetries as shown in Fig. 11.20 for $t = 0$ s. Imagined data points for the correlation with an incomplete cancellation are indicated by plus signs. As time proceeds through the beam store, any EDM effect will cause the left-right asymmetry ($\epsilon_{L-R}$) to rise with time, taking the correlation with it. This allows in principle for a separation of these effects, but it must be remembered that for a single store, the statistics on each of the data points will be over two orders of magnitude larger than any EDM effect at the expected level of sensitivity. The correlation cancellation is likely to be incomplete because of the large running time needed to establish the correlation.

For an off-centre block carbon target, the simple down-up raw asymmetry may be of the order of 0.2. Since left-right sensitivities as small as $10^{-6}$ may be involved in the EDM signal itself, cancellation of these polarimeter rotation effects to a similar degree must also be arranged, either by a mechanical adjustment in the polarimeter detector acceptance or cancellation through terms in the systematic error calibration described earlier. The risk, for example, is that energy loss due to collisions with background gas (or the polarimeter target) will populate lower energy particle orbits, causing on average a drift away from the frozen spin condition that increases with time. While continuous polarisation measurements are used to correct the frozen spin condition, errors that tend in one direction may produce a bias in the result, and would appear in the figure as data points no longer symmetrically distributed about the
A correlation plot similar to the figure has also been suggested for the elimination of other effects, such as the slow vertical polarisation growth associated with a residual sideways magnetic field or vertical electric field in the EDM ring. In that case, the horizontal axis of the plot will be some measurement of the sideways field, such as that indicated by the SQUID readout proposed for the proton ring. Because all of these effects operate at the same level or higher than any EDM signal, a comprehensive analysis must be done at the end for all of the data. Magnetic field errors are apt to appear as changes from store to store while polarimeter rotation effects always appear within a store and are not as likely to change over time.

A similar effect arises in addition for the deuteron beam since, in most polarised ion sources, it is impossible to eliminate completely a tensor polarisation component in the beam. One is often present at the percent level. (Indeed, there may be arguments for having a large tensor polarisation since it may be monitored as a continuous measure of the beam polarisation through the $T_{20}$ analysing power without having to periodically rotate the polarisation axis of the beam into the sideways direction.) A rotation of the tensor polarisation axis to either the left or right will directly generate a left-right asymmetry through the $T_{21}$ analysing power. This analysing power is not directly driven by the spin-orbit force, so its values for most forward angle polarimeter geometries are typically less than a few percent.

Systematic errors of due to polarimeter rotation and $T_{21}$ may be detected in a running experiment by looking for a correlation between the down-up asymmetry driving the feedback circuit to hold the frozen spin condition and the left-right asymmetry that in principle carries the EDM signal. In the deuteron case, left-right asymmetry sensitivity through either an effective polarimeter rotation or $T_{21}$ sensitivity may be separated in a calibration using a series of in-plane polarisation directions and looking at the nature of the correlation with rotation angles up to $\pi/2$. A linear relationship indicates a polarimeter rotation effect while a dependence that goes as the sine of twice the in-plane rotation angle indicates a $T_{21}$ sensitivity. These two effects will in general have different sizes or slopes for small rotation angles.

### 11.12 Time-reversed experiment

The EDM violates the symmetries of parity conservation and time reversal. In the case of time reversal, the direction of rotation of the beam around the ring would be changed and all polarisations and magnetic fields would have the opposite sign. Since this is a physically realisable experimental condition, it has been suggested that it be a part of the protocol for the EDM search. In the case of the proton with a
positive anomalous magnetic moment, the condition of frozen spin may be realised with only an electric field. This field remains the same under time reversal, thus it should be possible to operate the storage ring with both beams (CW and CCW) at the same time. This offers the chance to compare beam locations, profiles, intensities, and polarisations in order to verify that they are, in fact, identical. A second polarimeter would need to be installed in the ring in order to capture measurements of the reversed-direction polarisation. Some economies of construction and the use of only one extraction mechanism favour a design in which the two polarimeter detector schemes are located on either side of a single block target. Measurements made to large scattering angles of elastic proton scattering from carbon [5] show a drop of eight orders of magnitude of the cross section between the forward scattering angles used for polarisation measurements and similar backward scattering angles. This should be enough to suppress any interference with small changes being measured through the forward scattering asymmetry.

Essentially all systematic error effects that give rise to an EDM-like signal (changing vertical component of the polarisation over time) are time-reversal conserving. This would appear as a rising signal for both CW and CCW cases while the EDM signal would rise in one instance and fall (go negative) in the other. So any unsuppressed systematic error could be cancelled by subtracting the CW and CCW measurements.

Since the measurement (for small angles of vertical rotation of the polarisation) is one of a continuously rising effect, let us denote scattering to the left as:

\[ \sigma_{\text{POL}} = \sigma_{\text{UNPOL}} [1 + (S + E) pA] \] (11.8)

where \( S \) is the rate of rise due to remaining systematics and \( E \) is the rate of rise due to the EDM. The simple left-right asymmetries for CW and CCW become:

\[ \epsilon_{\text{CW}} = \frac{L - R}{L + R} = (S + E) pA \quad \text{and} \quad \epsilon_{\text{CCW}} = (S - E) pA \] (11.9)

so \[ \frac{1}{2} (\epsilon_{\text{CW}} - \epsilon_{\text{CCW}}) = EpA . \] (11.10)

This subtraction works only to the extent that \( pA \) values for both CW and CCW are well calibrated. If we define \( p \) and \( A \) to be the average values for CW and CCW, and we define \( \delta p \) and \( \delta A \) to be the difference between the calibrated and the actual values of the polarisations and analysing powers, then, when expanded to first order:

\[ \frac{1}{2} (\epsilon_{\text{CW}} - \epsilon_{\text{CCW}}) = EpA + SpA \left( \frac{d}{p} + \frac{a}{A} \right) . \] (11.11)

This means that the systematic contribution to the EDM signal can be suppressed only to the extent that the unknown fractional errors in the CW – CCW polarisation and analysing power differences are small enough to render the systematic error negligible compared to the EDM signal.

In the case of the beam polarisation, this introduces the requirement that the CW and CCW beams in the experiment be filled using the same polarisation state from the ion source. Likewise, care must be taken in the construction of the polarimeters and the setup of their detector readout to ensure that the effective analysing powers are also as identical as possible. This puts a premium on other efforts to reduce the systematic error contribution initially.

References


Chapter 12

Spin Tracking

12.1 Introduction
Spin tracking simulations of the complete EDM experiment are crucial to explore the sensitivity of the planned storage ring EDM searches and to investigate the systematic limitations. Existing spin tracking programs have been extended to properly simulate spin motion in presence of an electric dipole moment. The appropriate EDM kicks and electric field elements (static and RF) have been implemented and bench-marked. Furthermore, a symplectic description of fringe fields, field errors, and misalignments of magnets has been adapted and verified. For a detailed study during particle storage and spin build-up of an EDM signal, a large sample of particles must be tracked for billions of turns. This is a challenging task because it requires beam and spin tracking for about $10^9$ turns\(^1\).

12.2 Simulation Programs
Given the complexity of the task, and in order to ensure the credibility of the results, various simulation programs using different algorithm are upgraded and bench-marked with the required accuracy and efficiency:

- **COSY Infinity** [1], based on map generation using differential algebra and the subsequent calculation of the spin-orbital motion for an arbitrary particle including fringe fields of elements. COSY Infinity and its updates are used including higher-order nonlinearities, normal form analysis, and symplectic tracking. COSY Infinity contains elements to simulate $E \times B$ elements (static and RF).
- **COTOBO (COSY Toolbox)** [2] has been developed to perform the simulations, based on a C++ interface for COSY Infinity. The usability of ROOT [3] enables a fast and easy way to analyse the simulation results.
- **MODE** (Matrix integration of Ordinary Differential Equations) [4, 6] is a software package that provides nonlinear matrix maps building for spin-orbit beam dynamics simulation. MODE mathematical model is based on Lorentz and Thomas-BMT equations that are expanded to Taylor series up to the necessary order of nonlinearity including fringe fields of elements. The numerical algorithm is based on matrix presentation of Lie propagator.
- **Bmad** [7] has various tracking algorithms, including Runge-Kutta and symplectic (Lie algebraic) integration. Routines for calculating transfer matrices, emittances, Twiss parameters, dispersion, coupling, and fringe field contributions are also included. Bmad, by interfacing with the PTC tracking code [8], can, for example, compute Taylor maps to arbitrary order and do normal form analysis.
- **Homemade integrating program** [11], solving equations of particle and spin motion in electric and magnetic fields using Runge-Kutta integration. The programs models spin-orbital motion including fringe fields in elements. The algorithm used in the code is by several orders of magnitude slower than codes based map generation using differential algebra. Therefore, the program was predominantly used to investigate short-time phenomena and for bench-marking the other codes.
- **Simulation code for numerical Integration of beam and spin motion** [12] is a very simple but general approach and integrate the equation of motion as well as the T-BMT equation numerically. Standard algorithms like the fourth order Runge-Kutta algorithm are compared to newer ones and great emphasis is placed on the modular implementation in C++ for maximal flexibility.

\(^1\)Corresponds to measurement time of 15 minutes on the circumference of the COSY lattice.
Particle and spin tracking programs have been bench-marked and simulation results compared between different simulation codes and to beam and spin experiments at COSY to ensure the required accuracy of the results [2, 10, 13].

12.3 Status and Plans
Different possible scenarios for EDM measurements have been investigated to explore the sensitivity. In a first step the resonant method [14, 15] has been developed to be able to perform an EDM measurement at COSY. In parallel detailed studies have been carried out to explore the sensitivity of the deuteron precursor experiment at COSY [16–18]. In this context two different approaches have been investigated to perform deuteron EDM measurements in dedicated storage rings:

- The frozen-spin method [19], where the electrostatic and magnetic bending fields in a storage ring are adjusted according to the particle momentum in such a way that the longitudinally polarised spins of the particle beam are kept aligned (frozen) with their momenta.
- The quasi-frozen-spin method [21, 22], where the anomalous magnetic moment of the particles has to have a small negative value like for deuterons. In this case electric and magnetic field deflectors can be spatially separated. The spins oscillate around the momentum direction in the horizontal plane back and forth every time the particle passes through a magnetic or an electrostatic field. The spin oscillations of individual particles compensate each other with respect to the momentum vector in the magnetic and electrostatic part of the ring.

Different examples of spin tracking results for deuteron EDM storage rings utilising various lattice configurations are published in [21–25].

The CPEDM consortium started a new initiative to design a prototype EDM (Electric Dipole Moment) storage ring [26], with predominantly electric bending. Operated at proton beam energies between 30 and 50 MeV, the main purpose of this ring will be to carry out R&D work related to a final 233 MeV frozen-spin proton EDM ring. Recently spin tracking simulation started to support this development for dedicated proton EDM rings [28–30].

12.4 Spin tracking Simulations for Deuteron and Proton EDM Measurements
As described before, several simulation programs are utilised to simulate the vertical polarisation build-up for Deuteron and Proton EDM measurements induced by field and alignment errors of magnets and compared in detail to the polarisation build up assuming different EDM magnitudes.

12.4.1 Precursor Experiment for Deuterons at COSY
To be able to simulate the polarisation build up for the precursor experiment applying the resonant method [14, 15], time-dependent transfer maps have been implemented in COSY Infinity [2] to investigate the sensitivity of the precursor experiment using an RF Wien filter. This device provides superimposed electric and magnetic RF field such that they do not influence the particles’ trajectory but lead to an additional rotation of the spin around the magnetic field of the device. Thus, this so called Wien Filter will change the invariant spin axis. In order to determine the polarisation build up due to the electric dipole moment, it is necessary to know the orientation of the invariant spin axis. Since the particle and spin motion is perturbed by imperfections of the storage ring magnets, shifts, tilts and rotations can be superimposed to study randomised sets of magnet misalignments [2, 5]. The resulting closed orbits can be corrected by the orbit correction system to suppress false spin rotations via the magnetic moment.
12.4.1.1 EDM Build-up with misaligned magnets

Different magnitudes of the standard deviation of the Gaussian distributed quadrupole shifts between 1 \( \mu \)m and 1 mm have been simulated. For each of these misalignments a tracking simulation has been performed using different EDM magnitudes. The Wien filter field’s phase has been locked to the situation of maximum buildup. This results in the shown buildup for different RMS values of the vertical orbit displacements at the quadrupoles in 12.1 [2].

As long as the EDM contribution to the polarisation buildup is significantly larger than the buildup introduced by misalignments of magnets, both effects can be experimentally separated. For a randomised error with a standard deviation of 0.1 mm, the RMS value of the displacements is around 1 mm. In this case, the contribution to build up from misalignments of magnets and EDM is in the same order for \( \eta = 10^{-4} \). This corresponds to an EDM of roughly \( 5 \cdot 10^{-19} \) cm.

Results of benchmarking concerning changes in tune and chromaticity as well as driven oscillations of the polarisation vector can be found in Ref. [9].

12.4.1.2 Determination of the invariant spin axis

In order to determine the polarisation build up due to the electric dipole moment, it is necessary to know the orientation of the invariant spin axis. One current challenge for the precursor experiment is the lack of knowledge of the radial component of the invariant spin axis \( \vec{n} \) that cannot be measured. A possible solution is its determination by simulating the COSY lattice and performing spin tracking. The EDM as well as misalignments of lattice elements are affecting the particles trajectory and therefore the spin
motion and tilt the invariant spin axis.

Fig. 12.2: Due to a permanent EDM the invariant spin axis tilts into horizontal direction the angle $\xi_{EDM}$.

In case of an ideal ring and a vanishing EDM the invariant spin axis always points in vertical direction as the spin precesses in the horizontal plane. In the presence of an EDM the invariant spin axis is tilted in the horizontal direction by the angle $\xi_{EDM}$ as sketched in figure 12.2. This angle is directly proportional to the magnitude of the EDM and can be written as

$$\tan(\xi_{EDM}) = \frac{\eta \beta}{2G}. \quad (12.1)$$

In order to determine the invariant spin axis, the spin of the reference particle is tracked for $N$ turns resulting in an ensemble of spin vectors $\vec{s}_j$ where $j \in \mathbb{N}$ and $j \in [1, N]$ [20]. For each possible configuration of three spin vectors ($\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$) an invariant spin axis $\vec{n}_i$ is calculated as follows.

$$\vec{u}_i = \vec{s}_{2,i} - \vec{s}_{1,i} \quad (12.2)$$

$$\vec{v}_i = \vec{s}_{3,i} - \vec{s}_{1,i} \quad (12.3)$$

$$\vec{n}_i = \frac{\vec{v}_i \times \vec{u}_i}{|\vec{v}_i \times \vec{u}_i|} \quad (12.4)$$

Figure 12.3(left) shows a schematic description of the calculation. The invariant spin axis is then calculated as the average of all $\vec{n}_i$ vectors. Spin tracking is done using the software library Bmad. Figure 12.3(right) shows the spin distribution after tracking through the COSY lattice including the misalignments of dipoles and quadrupoles as well as an illustration of individual spin vectors $\vec{s}_j$, the invariant spin axes $\vec{n}_i$ and the average invariant spin axis $\langle \vec{n} \rangle$. 

129
12.4.2 Proton EDM Storage Ring

After starting to design a prototype EDM storage ring [26], operated at proton beam energies between 30 and 50 MeV, spin tracking simulation are performed to investigate the sensitivity of such a ring for EDM measurements. Spin tracking simulation are also carried out for several groups to simulate spin motion for dedicated EDM rings for 233 MeV frozen-spin protons [10, 27–30].

12.4.2.1 Simulations using a Runge-Kutta integration method

The spin is determined from the T-BMT equation describing the precession rate of the angle between the spin and momentum vectors of a relativistic particle in the presence of electromagnetic fields. The latter have to be evaluated at each location of the particle. Thus, the T-BMT equation is coupled to the equation of motion for which, in general, a closed form solution cannot be obtained. Given the high sensitivity aimed, precise numerical simulations are necessary and bench-marking with analytical estimates can help understand the major systematic effects. For instance, an average radial magnetic field of a few aT yields a vertical spin buildup similar to an EDM signal level of $10^{-29} e.cm$. Thus, a precise knowledge of the field at each integration step is crucial in order to determine its impact on the spin dynamics. In addition, due to the coupling between the different spin components which induces additional phases, the rapid oscillatory behaviour of the spin has to be finely resolved. In what follows, one discusses several examples where the considered ring lattice is based on the strong focusing one that was proposed by V. Lebedev to achieve the beam requirements suitable for EDM: the simulated ring consists of 4 superperiods each containing 5 FODO cells. There are 6 electric bends per FODO cell characterised by 8 MV/m radial electric field for 3 cm plate separation. In the interface between the bending and the straight sections, the hard edge model was assumed, which means that the electric fields are constant everywhere within the element and drop abruptly to zero at the edges. Nevertheless, the energy change of the particle was taken into account. This is a particularly useful model to simplify the analysis. In what follows, one discusses some selected cases of lattice imperfections yielding a vertical spin buildup. Further details regarding some of the numerical simulations and their comparison with the analytical estimates can be found in [30]. In particular, it appears that the established formalism which is based on the Bogoliubov-Krylov-Mitrpolski method of averages [31] is very useful to calculate the spin precession rates at the observation point, i.e. at the location of the polarimeter. In addition, it enabled the
calculation of the geometric phases as discussed below.

### 12.4.2.2 Misalignment of focusing elements

In the case of misplacement of lattice elements, such as electric quadrupoles, orbit distortions can occur leading to a vertical spin build-up [28]. The latter can exhibit a linear and/or quadratic increase with time, depending on the amplitude of the perturbation. Example of tracking simulations for the all-electric ring, in the case where one defocusing quadrupole was misaligned by several micrometers are shown in fig 12.4. A particle with an energy spread of $\Delta p/p = 10^{-5}$ was tracked on the perturbed closed orbit and its spin is recorded after each turn completion. Good agreement was achieved between the tracking simulations and the analytical estimates and it is shown that the quadratic increase is due to two contributions: a longitudinal spin precession mainly caused by the vertical slope in the electrostatic deflectors and a linear radial spin buildup due to the displacement from the magic energy so that $s_y \propto y' \Delta p/p$. In the limit where $\Delta p/p = 0$, the quadratic increase vanishes and one obtains a linear increase due to higher order terms.

### 12.4.2.3 Geometric phases

The next tracking simulation example considered is that of the geometric phases, also referred to as the Berry phases [32]: In the case where the parameters of the system are varied slowly such that the value of the particle coordinates end the same as they started and if the average perturbations are balanced within one revolution, then the non trivial phase picked-up by such perturbations is called the Berry phase. Such effects, due to the non-commutation of spin rotations around different axes, can dominate if the beam energy is very close to the magic one. Let’s assume that the beam is injected at the magic energy and that the lattice has alternating magnetic field imperfections. Such an imperfection is represented by the presence of both vertical and longitudinal magnetic fields which are 90 degrees out of phase as illustrated in fig 12.5. In particular, it can be seen that the radial spin is rotated into the vertical plane by means of
Fig. 12.5: Spin and orbit evolution for a lattice with alternating magnetic field imperfections: a vertical magnetic field $B_y$ yields a horizontal spin component which is rotated into the vertical plane by means of a longitudinal field component $B_z$. The closed orbit of the perturbed motion is shown in blue and the particle motion is clockwise starting from Point A.

the longitudinal magnetic fields such that the leading term of the vertical spin buildup is given by [30]:

$$\frac{ds_y}{dt} \approx \frac{1}{c\beta_C} \left( \frac{e}{m} \right)^2 \left( G + \frac{1}{\gamma} \right) \frac{1 + G}{\gamma} B_y L_y B_z L_z$$  \hspace{1cm} (12.5)

$$\approx 5.92 \times 10^5 B_y L_y B_z L_z$$  \hspace{1cm} (12.6)

Thus, assuming integrated field perturbations such that $B_y L_y = B_z L_z = 1$ nT.m, this yields a spin precession rate of $\approx 5.92 \times 10^{-4}$ nrad/s which is well below the EDM signal level. Fig 12.6 shows a comparison of the tracking simulations with the first order analytical estimate of the spin buildup due to the Berry phases where one can see a good agreement of both estimates. In addition, it is important to note that such an effect can be cancelled by using two counter-rotating beams and taking the difference of the signals.
Fig. 12.6: Vertical spin buildup from tracking simulations and comparison with the analytical estimate given by Eq. (12.6).

References


[29] Julien Michaud (Laboratoire de Physique Subatomique & Cosmologie IN2P3 (CNRS), France, PhD thesis at Universite Grenoble Alpes, to be published (2019).


Chapter 13

Roadmap and Timeline

13.1 CPEDM Strategy

As emphasised above this challenging project needs to proceed in stages that are also outlined in Fig. 13.1:

1. COSY will continue to be used as long as possible for the continuation of critical R&D associated with the final experiment design. An important requirement is to test as many of the results as possible with protons where the larger anomalous magnetic moment leads to more rapid spin manipulation speeds.

2. The precursor experiment will be completed and analysed. Some data will be taken with an improved version of the Wien filter with better electric and magnetic field matching.

3. The next stage is to design, fund, and build a prototype ring to address critical questions concerning the features of the EDM ring design. At 30 MeV, the ring with only an electric field can store counter-rotating beams, but they are not frozen spin. At 45 MeV with an additional magnetic field, the frozen spin condition can be met. But the magnetic fields also prevent the CW and CCW beams from being stored at the same time. Even so, an EDM experiment may be done with these two beams used on alternating fills.

4. Following step 3, the focus will be to create the final ring design, then fund and construct it.

5. Once the ring is ready, the longer term activity will be to commission and operate the final ring, improving it with new versions as the systematic errors and other experimental issues are understood and improved.

<table>
<thead>
<tr>
<th>1</th>
<th>Precursor Experiment</th>
<th>dEDM proof-of-capability (orbit and polarization control; first dEDM measurement)</th>
<th>Magnetic storage ring</th>
<th>Polarized deuterons</th>
<th>d-Carbon polarimetry</th>
<th>Radiofrequency (RF) Wien-filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Prototype Ring</td>
<td>pEDM proof-of-principle (key technologies, first direct pEDM measurement)</td>
<td>- High-current all-electric ring</td>
<td>- Simultaneous CW/CCW op.</td>
<td>- Frozen spin control (with combined E/B-field ring)</td>
<td>- Phase-space beam cooling</td>
</tr>
<tr>
<td>3</td>
<td>All-electric Ring</td>
<td>pEDM precision experiment (sensitivity goal: $10^{-29}$ e cm)</td>
<td>- Frozen spin all-electric (at $p = 0.7$ GeV/c)</td>
<td>- Simultaneous CW/CCW op.</td>
<td>- B-shielding, high E-fields</td>
<td>- Design: cryogenic, hybrid,...</td>
</tr>
</tbody>
</table>

Ongoing at COSY (Jülich) 2014 → 2021
Ongoing within CPEDM 2017 → 2020 (CDR) → 2022 (TDR)
Start construction > 2022
After construction and operation of prototype > 2027

Fig. 13.1: Summary of the important features of the proposed stages in the storage ring EDM strategy.

Future scientific goals may include conversion of the ring to crossed electric and magnetic field operation so that other species besides the proton could be examined for the presence of an EDM. Analysis of the data may be made for signs of axions using a frequency decomposition and investigation of counter-rotating beams with different species used in novel EDM comparisons.

The prototype ring and the CPEDM stages are host-independent. If the prototype is built at COSY, it would take advantage of the existing facility for the production of polarised proton (and deuteron)
beams, beam bunching, and spin manipulation. COSY itself could be used for producing electron-cooled beams. It may also be built at another site (e.g., CERN) provided that a comparable beam preparation infrastructure is made available. In either case, the lattice design will mimic that of the high-precision ring in order to test as many features as possible on a smaller scale.

### 13.2 Timeline

As shown, a staged approach is pursued with step-1 (“Precursor Experiment”) currently ongoing. This is partially funded by an ERC Advanced Grant, which runs until September 2021. The next stage (step-2, “Prototype Ring”) has started last year (2017) and a CPEDM task force is working on the “Conceptual Design Report” (CDR, due in 2020) and will subsequently finalise the “Technical Design Report” (TDR, ready in 2022). If funding can be secured, construction could start beyond 2022. Currently, about 5 years are foreseen for building and operation of the prototype. Only after that does it seem conceivable to design, build and operate the final ring (step-3, “All-electric Ring”).

A more detailed timeline is presented in Fig. 13.2. See the caption for details.

---

**Fig. 13.2:** This “Timeline” follows the anticipated evolution of the storage ring EDM project through several events (numbered) and stages. At present (2019), experimental work will continue with COSY to look into feasibility issues regarding electron-cooling, begin development of a search for axion-like particles, and continue to refine the precursor experiment as a first measurement of the deuteron EDM. Meanwhile, a long “Yellow report” is being prepared in CERN format to outline the plans for a prototype ring and the eventual construction of a full scale all-electric ring to measure the proton EDM. Later in the year, the Helmholtz Research Association (HGF) will begin the strategic evaluation process for the research program “Matter and the Universe” (MU) for the next “Program-oriented Funding” (PoF) period that will start at the beginning of 2021. Also in parallel, work will begin on a Conceptual Design Report (CDR), followed by a Technical Design Report (TDR), for the creation of a small electric (and later electromagnetic) storage ring to answer feasibility questions about the design and use of such rings for EDM searches. If support continues during PoF 4, then the efforts with the COSY machine will switch to a development with polarised proton beams that duplicates what has been achieved for deuterons (red band). Other types of research in related symmetries also continues (green band). Work will also start for the construction of the electric version of the prototype ring (orange). Commissioning with first beam at 30 MeV starts in 2025 to demonstrate high intensities and counter-rotating two-beam operation. A second version with magnetic bending added to enable frozen spin operation begins in 2028 at 45 MeV. As new feasibility studies with the prototype come to fruition, work starts with a CDR/TDR for the proton EDM experiment. This project will be commissioned in the mid-2030s.
Appendices
Appendix A

Results and achievements at Forschungszentrum Jülich

This appendix describes results and achievements obtained up to now. It comprises results obtained the C0oler SYnchrotron COSY at Forschungszentrum Jülich and of the Jülich Theory group. Activities and achievements like polarimetry and spin tracking are described in dedicated chapters.

A.1 Results and achievements at COSY

For most of the studies the parameters listed in Tab. A.1 were used.

<table>
<thead>
<tr>
<th>COSY circumference</th>
<th>183 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron momentum</td>
<td>0.970 GeV/c</td>
</tr>
<tr>
<td>$\beta(\gamma)$</td>
<td>0.459 (1.126)</td>
</tr>
<tr>
<td>magnetic anomaly $G$</td>
<td>$\approx -0.143$</td>
</tr>
<tr>
<td>revolution frequency $f_{\text{rev}}$</td>
<td>750.6 kHz</td>
</tr>
<tr>
<td>cycle length</td>
<td>100-1500 s</td>
</tr>
<tr>
<td>number of stored particles/cycle</td>
<td>$\approx 10^9$</td>
</tr>
</tbody>
</table>

Table A.1: Values of the COSY operating parameters for most of the studies reported in this appendix.

A.1.1 High precision spin tune measurements

Although not directly connected to the EDM measurement in a dedicated storage ring using the frozen spin method, the measurement of the the fast 120 kHz precision of the polarisation vector around the magnetic guiding field in the horizontal plane is an import step in understanding and controlling spin precession in a storage ring.

In an ideal planar magnetic storage ring, the spin tune $-\nu_s$ defined as the number of spin precessions per turn is given by $\nu_s = \gamma G \approx -0.16$ ($\gamma$ is the Lorentz factor, $G$ the gyromagnetic anomaly). At $p = 970$ MeV/c, the deuteron spins coherently precess at a frequency of about 120 kHz in COSY. The spin tune was deduced from the up-down asymmetry of deuteron-carbon scattering. In a time interval of 2.6 s, the spin tune was determined with a precision of the order $10^{-8}$, and to $10^{-10}$ for a continuous 100 s accelerator cycle [1], see Fig. A.1.

To appreciate this high relative precision of $\sigma_{\nu_s}/\nu_s \approx 10^{-9}$ in a 100 s cycle, a comparison to the equivalent quantity in the muon $g - 2$ measurement is helpful. Here the precision reached is about $\sigma_{\nu_s}/\nu_s \approx 10^{-6}$ per year, i.e. a ppm measurement of $a = (g - 2)/2$ in one year. The three order of magnitude higher precision in a much shorter time is mainly explained by the fact that the cycle length is much larger (100 s compared to 600 \mu s).

Note that a spin rotation due to an electric dipole moment of $d = 10^{-24}$ e cm for one turn is given by $\nu_s = \frac{vmc^2}{es} = 5 \times 10^{-11}$. This means that with the statistical precision $\sigma_{\nu_s} = 10^{-10}$ reached in a single cycle of 100 s one is statistically sensitive to EDM of the order of $10^{-24}$ e cm even with an accelerator not constructed for this purpose. Of course additional rotations due to the magnet moment due to imperfections of the storage ring are orders of magnitude larger and have to be understood.
Fig. A.1: Upper plot: The phase of the polarisation vector in the horizontal plane evaluated close to spin revolution frequency of the polarisation vector using a Fourier analysis over $10^6$ turns. Middle: spin tune change from obtained from two consecutive phase measurements. Lower: spin tune change obtained from a parabolic fit in the upper plot.

A.1.2 Long horizontal polarisation lifetime

To reach the desired statistical precision, a spin coherence time of the order of 1000 s has to be reached. A rough estimate shows that this is not a simple task. A momentum spread of $\Delta p/p \approx 10^{-5}$ corresponds to $\Delta \gamma/\gamma \approx 2 \cdot 10^{-6}$. Since the spin tune is given by $\gamma G$, after $\approx 10^6$ turns (i.e. $\approx 1$ s) the polarisation is lost in the horizontal plane. Using a bunched beam, first order effects in $\Delta p/p$ can be cancelled and the spin coherence time reaches a few seconds.

Using a combination of beam bunching, electron cooling, sextupole field corrections, and the suppression of collective effects through beam current limits a deuteron beam polarisation lifetime of 1000 seconds in the horizontal plane of the magnetic storage ring COSY could be reached [2]. The result is displayed in Fig. A.2.

A.1.3 Feedback and control of polarisation

The precise measurement of the horizontal spin precession together with long spin coherence times allowed us to set up a polarisation feedback system. In a dedicated ring its role is to maintain the polarisation vector always (anti-) parallel to the momentum vector to maximise the statistical sensitivity.

The use of feedback from a spin polarisation measurement to the revolution frequency of a 0.97 GeV/c bunched and polarised deuteron beam in the Cooler Synchrotron (COSY) storage ring has been realised in order to control both the precession rate ($\approx 120$ kHz) and the phase of the horizontal polarisation component. Real time synchronisation with a radio frequency (rf) solenoid made possible the
rotation of the polarisation out of the horizontal plane, yielding a demonstration of the feedback method to manipulate the polarisation, see Fig. A.3. In particular, the rotation rate shows a sinusoidal function of the horizontal polarisation phase (relative to the rf solenoid), which was controlled to within a one standard deviation range of $\sigma = 0.21$ rad, see Fig. A.4. The minimum possible adjustment was 3.7 mHz out of a revolution frequency of 753 kHz, which changes the precession rate by 26 mrad/s [3]. Such a capability meets a requirement for the use of a dedicated storage rings for EDM measurements.

A.1.4 Invariant spin axis measurements

Another application of the precise spin tune measurement is the measurement of the invariant spin axis. An extended paper entitled "Spin tune mapping as a novel tool to probe the spin dynamics in storage rings" describes this in detail [4]. It is motivated by the fact that precision experiments, such as the search for electric dipole moments of charged particles using storage rings, demand for an understanding of the spin dynamics with unprecedented accuracy. New methods based on the spin tune response of a machine to artificially applied longitudinal magnetic fields, which is called "spin tune mapping", has been developed. The technique was experimentally tested in 2014 at COSY and, for the first time, the angular orientation of the stable spin axis at two different locations in the ring has been determined to an unprecedented accuracy of better than 2.8 $\mu$rad.

A.1.5 Radio-Frequency Wien filter for spin manipulation

In a pure magnetic storage ring like COSY, an EDM will generate an oscillation of the vertical polarisation component. For a 970 MeV/c deuteron beam with the spin precession frequency of 120 KHz, a tiny amplitude is expected, e.g., $3 \cdot 10^{-10}$ for an EDM of $d = 10^{-24}$ cm. To allow for a build-up of the vertical polarisation proportional to the EDM, a radio-frequency (RF) Wien-filter has to be operated. Such a device was developed and constructed, see Fig. A.5. This RF Wien-filter was installed in COSY in May 2017. A first commissioning run was successfully conducted in June 2017. [5, 6]. Fig. A.5 shows a drawing of the Wien filter. During the 2018 test run it was operated with magnetic (electric)
Fig. A.3: The left right asymmetry, proportional to the vertical polarisation as a function of time. Initially the polarisation is pointing upwards (red points) or downwards (black points) depending on injection. At $t \approx 88$ s the polarisation is flipped into the horizontal plane with the help of the rf-solenoid. The polarisation vector starts to precess in the horizontal plane. At $t \approx 116$ s the solenoid is switched on again. The fact that vertical polarisation raises is a proof that the feedback systems is working.

Fig. A.4: Upper: The phase as a function of time with feedback off (blue) and on (red). The read points stay within a RMS of 0.21 rad (grey band). Lower: Adjustments of the COSY frequency.
Fig. A.5: Left: Design model of the RF Wien filter showing the parallel-plates waveguide and the support structure. 1: beam position monitor (BPM); 2: copper electrodes; 3: vacuum vessel; 4: clamps to hold the ferrite cage; 5: belt drive for 90° rotation, with a precision of 0.01° (0.17 mrad); 6: ferrite cage; 7: CF160 rotatable flange; 8: support structure of the electrodes; 9: inner support tube. The axis of the waveguide points along the z-direction, the plates are separated along x, and the plate width extends along y. During the EDM studies, the main field component $E_x$ points radially outwards and $H_y$ upwards with respect to the stored beam. Right: Photograph of Wien filter installed in COSY.

Field integrals of 0.019 Tmm (2.7kV). First results obtained with this device will be discussed in the next subsection.

A.1.6 Measurements of deuteron carbon and proton carbon analysing powers

The way to measure the vertical polarisation proportional to the EDM is to scatter deuterons or protons elastically off a carbon target. To achieve high accuracy the analysing should be large and should be known with small uncertainties. A series of measurements were performed. Fig. A.6 shows the analysing power of deuteron carbon scattering for various beam energies as a function of the polar angle of the deuteron in the lab system. Data using a polarised proton beam were also taken. The analysis is going on.

A.1.7 Orbit control

Systematic errors for EDM measurement occur for example due to magnet misalignments and orbit offsets. At COSY many new devices and procedures could be tested and implemented to improve the orbit. First of all, an automatised orbit control system was implemented which allows one to correct the orbit in real time. This system reduces the orbit correction procedure from about 10 hours to less than one hour. As an example Fig. A.7 shows the result of an orbit after correction. The RMS of the horizontal (vertical) orbit is 1.46 (0.90) mm.

A.1.8 Beam Based Alignment

Beam based alignment is a procedure to verify that the beam passes through the centre of a quadrupole. A off-centre path through a quadrupole results in a deviation of the beam. Modifying the quadrupole strength, this deviation can be measured. From a surveying procedure the quadrupole position is known to approximately 0.2 mm. Using the beam based alignment procedure the position of the BPM relative to the quadrupoles could be determined. Fig. A.8 shows preliminary results. For 12 (of the total 56) quadrupoles offset of the BPMs of a few millimetres were found. These offset can now be corrected. This should result in a orbit closer to the design orbit and will reduce the systematic error of the precursor experiment.
**Fig. A.6:** Reconstructed vector analyzing power for deuteron beam energies of (from top to bottom) 380 MeV, 340 MeV, 300 MeV, 270 MeV, 235 MeV, 200 MeV and 170 MeV. The curves are subsequently offset by 0.4 for better readability. The statistical errors are indicated by the black error bars on the data points. The systematic error is shown as red regions.

### A.1.9 Beam Position Monitor

New devices, so called Rogowski coils were built and tested in COSY. The Rogowski coils consist four segments (up-right, down-right, down-left, up-right). A time varying beam induces a voltage in the four coils. Combining the four voltages the beam position can be determined. Fig A.9 shows a photograph of a coil installed in COSY and the principle setup of the coils. First calibration measurements show that the accuracy is less than 100 μm can be reached, see Fig. A.10.

### A.1.10 Electrostatic and combined Deflector development

The future measurements of the EDM at COSY storage ring require development of a prototype of electrostatic or combined electromagnetic beam bending element. In case of proton beam and magic momentum of 701 MeV/c all elements of such ring can be electric, but in all other cases existence of the magnetic fields is obligatory. The electrostatic deflectors should consist of two parallel metal plates under equal potential of a different sign. Equal electric potential seen by the particle at the entrance and
Fig. A.7: COSY orbit measurement. The upper plot shows the vertical (red) and horizontal (blue) orbit as a function of the longitudinal position in COSY. The desired orbits are shown in gold and green (both coincide with the $x = y = 0$ line) for the vertical and horizontal orbit, respectively. The RMS of the horizontal (vertical) orbit is 1.46 (0.90) mm. The plot in the centre shows the steering magnet currents applied for the correction.

Fig. A.8: Offset of the beam at the position of various quadrupoles. Since the quadrupoles are aligned to 0.2 mm, these values can be used to calibrate the beam position monitors (BPMs).

at the exit of the deflector will not change the total momentum of the particle. This puts restrictions on the minimum distance between deflectors. Recent possible ring lattices studies limit the good-field-region for stored particles to be 40 mm. It leads to the minimum distance between electric deflector plates to be about 60 mm. The vertical beam size is several times larger than horizontal one impose restrictions on the vertical dimensions of the flat part of the deflector too. Minimum transverse dimensions of the bending elements will be more than 100 mm. In order to minimise breakdown probability between the flat regions of the deflectors and move it to the edge, the shape of deflectors should follow Rogowski profile on both vertical ends. The end caps of individual deflector should be made to couple the stray fields with subsequent deflectors. Designed ring lattice require electric gradients in the order of 5-10 MV/m. This is more than the standard values for many accelerator deflectors located at a distances of a few centimetres apart. Assuming 60 mm distance between the plates, to achieve such high electric fields we have to use high voltage power supplies. At present, two 200 kV power converters are ordered.
Fig. A.9: Photograph of a Rogowski coil installed in COSY and schematic of the Rogowski coil setup.

Fig. A.10: The left figure shows the difference of the measured positions and the prediction from a model for different regions in the $x - y$ plane as indicated by the right figure.

for testing deflector prototypes. The field emission, field breakdown, dark current, electrode surface and conditioning should be studied using two flat electrostatic deflector plates, mounted on the movable support with possibility to change distance between 20 and 120 mm. The residual ripple of power converters in the order of $1 \times 10^{-5}$pp at maximum 200kV will lead to particle displacement on the order of millimetres. A smaller ripple or stability control of the system will be a dedicated task for investigations planned at the test ring facility.
A.1.11 "Spin-Offs"

This subsection just lists a number of publications that were initiated by the studies for an storage ring EDM measurement but also have application in other areas.

1. Polynomial Chaos Expansion method as a tool to evaluate and quantify field homogeneity of a novel waveguide RF Wien filter [5]
   A full-wave calculations demonstrated that the waveguide RF Wien filter is able to generate high-quality RF electric and magnetic fields. In reality, mechanical tolerances and misalignments decrease the simulated field quality, and it is therefore important to consider them in the simulations. In particular, for the electric dipole moment measurement, it is important to quantify the field errors systematically. Since Monte-Carlo simulations are computationally very expensive, we discuss here an efficient surrogate modelling scheme based on the Polynomial Chaos Expansion method to compute the field quality in the presence of tolerances and misalignments and subsequently to perform the sensitivity analysis at zero additional computational cost.

2. Computational framework for particle and spin simulations based on the stochastic Galerkin method [7]
   An implementation of the polynomial chaos expansion is introduced as a fast solver of the equations of beam and spin motion of charged particles in electromagnetic fields. We show that, based on the stochastic Galerkin method, our computational framework substantially reduces the required number of tracking calculations compared to the widely used Monte Carlo method.

3. Control of systematic uncertainties in the storage ring search for an electric dipole moment by measuring the electric quadrupole moment [8]
   Measurements of electric dipole moment (EDM) for light hadrons with use of a storage ring have been proposed. The expected effect is very small, therefore various subtle effects need to be considered. In particular, interaction of particle’s magnetic dipole moment and electric quadrupole moment with electromagnetic field gradients can produce an effect of a similar order of magnitude as that expected for EDM. This paper describes a very promising method employing an RF Wien filter, allowing to disentangle that contribution from the genuine EDM effect. It is shown that both these effects could be separated by the proper setting of the RF Wien filter frequency and phase. In the EDM measurement the magnitude of systematic uncertainties plays a key role and they should be under strict control. It is shown that particles’ interaction with field gradients offers also the possibility to estimate global systematic uncertainties with the precision necessary for an EDM measurement with the planned accuracy.

   Azimuthal asymmetries play an important role in scattering processes with polarised particles. This paper introduces a new procedure using event weighting to extract these asymmetries. It is shown that the resulting estimator has several advantages in terms of statistical accuracy, bias, assumptions on acceptance and luminosities compared to other estimators discussed in the literature.

5. Amplitude estimation of a sine function based on confidence intervals and Bayes’ theorem [10]
   This paper discusses the amplitude estimation using data originating from a sine-like function as probability density function. If a simple least squares fit is used, a significant bias is observed if the amplitude is small compared to its error. It is shown that a proper treatment using the Feldman-Cousins algorithm of likelihood ratios allows one to construct improved confidence intervals. Using Bayes’ theorem a probability density function is derived for the amplitude. It is used in an application to show that it leads to better estimates compared to a simple least squares fit.

   The tensor polarisation of particles and nuclei becomes constant in a coordinate system rotating with the same angular velocity as the spin, and it rotates in the laboratory frame with the above angular velocity. The general equation defining the time dependence of the tensor polarisation is
An explicit form of the dynamics of this polarisation is found in the case when the initial polarisation is axially symmetric.

A.2 Results and achievements from the Jülich/Bonn theory group

The IKP-3/IAS-4 at the Forschungszentrum Jülich together with the theory group at the Helmholtz-Institut für Strahlen- und Kernphysik (HISKP) at the University of Bonn – both headed by Ulf Meißner – have performed a number of benchmark calculations for the EDMs of proton, neutron and light nuclei using chiral effective nuclear field theory (chiral perturbation theory and its extension to few-baryon systems) and lattice QCD simulations.

This project on hadronic electric dipole moments started with the diploma thesis of Konstantin Ottnad (HISKP) on electric dipole form factors of the neutron in chiral perturbation theory in the year 2009 [12]. His work culminated in a publication [13] that analysed the QCD $\theta$-angle induced EDMs of the neutron and proton to third order in $U(3)_L \times U(3)_R$ baryon chiral perturbation theory, in a covariant and by the number of colours ($N_c$) extended version. A new upper bound\(^1\) on the vacuum angle, $|\theta| \lesssim 2.5 \cdot 10^{-10}$ was given and the matching relations for the three-flavor representation to the SU(2) case was derived. These relations still comprise today’s $\theta$-induced EDM predictions for the neutron and proton in chiral perturbation theory.

In 2012 IAS-4/IKP-3 extended the above work to the QCD $\bar{\theta}$-term-induced electric dipole moment (EDM) of the deuteron, where the genuine two-nucleon contributions of the $P$- and $T$-violating form factor $F_3$ of the deuteron was calculated in the Breit frame of this nucleus using chiral effective field theory up to and including next-to-next-to-leading order [14]. In particular, it was found that the difference between the deuteron EDM and the sum of proton and neutron EDMs corresponds to a value of $(0.54 \pm 0.39) \times 10^{-16}$ e.cm. Both the nucleon-nucleon potential and the transition current contributions were calculated, where the $CP$- and isospin-violating $\pi NN$ coupling constant $g_1^\theta$ was identified as the source of the dominating contribution to the uncertainty. The role that the vacuum alignment plays for the generation of $g_1^\theta$ was outlined and an estimate of the additional and previously unknown contribution to $g_1^\theta$ was derived from a resonance saturation mechanism involving the odd-parity nucleon resonance $S_{11}(1535)$.

In the same year Guo (HISKP) and Meißner calculated the electric dipole form factors and moments of the ground state baryons in chiral perturbation theory at next-to-leading order [15]. It was shown that the baryon electric dipole form factors at this order depend only on two combinations of low-energy constants. This was used to derive various relations for the baryon EDMs that are free of unknown low-energy constants which can be used to cross-check future lattice QCD results. Thus for a precision extraction from lattice QCD data, the next-to-leading order terms have to be accounted for. Akan (HISKP), Guo and Meißner revisited in 2014 the above work by investigating finite volume corrections to the $CP$-odd nucleon matrix elements of the electromagnetic current, which can be related to the electric dipole moments originating from strong $CP$ violation in the continuum, in the framework of chiral perturbation theory up to next-to-leading order taking into account the breaking of Lorentz symmetry [16]. A chiral extrapolation of the recent lattice results of both the neutron and proton electric dipole moments was performed in addition.

In 2014 Jan Bsaisou (IKP-3/IAS-4) finished his PhD thesis at the university of Bonn on electric dipole moments of light nuclei in chiral effective field theory [17].\(^2\) Starting from the QCD $\bar{\theta}$-term and the set of $P$- and $T$-violating effective dimension-six operators, he presented a scheme to derive the induced effective Lagrangians at energies below $\Lambda_{\text{QCD}} \sim 200$ MeV within the framework of chiral perturbation theory (ChPT) for two quark flavors – applying the formulation of Gasser and Leutwyler. It was shown

\[^1\]The estimate is modulo the unknown contributions of the contact interactions needed to removed the infinities of the one-loop calculations.

\[^2\]Part of this work was documented in the prior publication [14].
that the differences among the sources of $P$ and $T$ violation manifest themselves in specific hierarchies of coupling constants of $P$- and $T$-violating vertices. He computed the relevant coupling constants of $P$- and $T$-violating hadronic vertices which are induced by the QCD $\theta$-term with well-defined uncertainties as functions of the parameter $\tilde{\theta}$. The relevant coupling constants induced by the effective dimension-six operators were given as functions of so far unknown Low Energy Constants (LECs) which can not be determined by ChPT. Estimates of the coupling constants from Naive Dimensional Analysis (NDA) proved to be sufficient to reveal certain hierarchies of coupling constants. The different hierarchies of coupling constants translated into different hierarchies of the nuclear contributions to the EDMs of light nuclei. In this way he could calculate within the framework of ChPT the two-nucleon contributions to the EDM of the deuteron up to and including next-to-next-to leading order and the two-nucleon contributions to the EDMs of the helion ($^3$He nucleus) and the triton ($^3$H nucleus) up to and including next-to-leading order. These computations comprised thorough investigations of the uncertainties of the results from both the $P$- and $T$-violating and conserving components of the nuclear potential. Quantitative predictions of the nuclear contributions to the EDMs of the deuteron, the helion and the triton induced by the QCD $\tilde{\theta}$-term as functions of $\tilde{\theta}$ with well-defined uncertainties were presented, while the EDM predictions for the effective dimension-six sources were given as function of the unknown LECs with NDA estimates. Several strategies to falsify the QCD $\theta$-term as a relevant source of $P$ and $T$ violation were presented, whereby a suitable combination of measurements of several light nuclei and, if needed, supplementary lattice QCD input could be used. He demonstrated how particular effective dimension-six sources can be tested by EDM measurements of light nuclei – with supplementary Lattice QCD input in the future.

While the above thesis discussed strategies to separate the various dimension-six EDM operators individually, the IAS4-/IKP-3 publication by Dekens et al. [18], using information from this thesis and from the paper by Dekens and de Vries [19] on the renormalisation group running of the dimension-six sources for $P$ and $T$ violation, showed that the proposed measurements of the electric dipole moments of light nuclei in storage rings would put strong constraints on models of flavor-diagonal $CP$ violation [18]. This analysis was exemplified by a comparison of the Standard Model including the QCD theta term, the minimal left-right symmetric model, a specific version of the so-called aligned two-Higgs doublet model, and, “en passant”, a minimal supersymmetric extension of the Standard Model. Again by using effective field theory techniques it was demonstrated to what extent measurements of the electric dipole moments of the nucleons, the deuteron, and helion could discriminate between these scenarios and how measurements of electric dipole moments of other systems relate to light-nuclear measurements. In particular, the focus was on the most important $P$, $T$-violating hadronic interactions that appear in each of the scenarios, especially on the $P$, $T$-violating pion-nucleon interactions and the nucleon EDMs. It was demonstrated that chiral effective field theory is a powerful tool to study the observables of light nuclei and that measurements of light-nuclear EDMs can be used to disentangle different underlying scenarios of $CP$ violation.

The EDM predictions of IAS-4/IKP-3 up to the year 2014 were summarised in Ref. [20], and a consistent and complete calculation of the electric dipole moments of the deuteron, helion, and triton by chiral effective field theory was given in Ref. [21]. The $CP$-conserving and $CP$-violating interactions were treated on equal footing and the $CP$-violating one-, two-, and three-nucleon operators were considered up to next-to-leading-order in the chiral power counting. In particular, for the first time EDM contributions induced by the $CP$-violating three-pion operator were calculated. It was found that effects of $CP$-violating nucleon-nucleon contact interactions are larger than those predicted in previous studies involving phenomenological models for the $CP$-conserving nucleon-nucleon interactions. The results which apply to any model of $CP$ violation in the hadronic sector can be used to test various scenarios of $CP$ violation. In particular, the implications on the QCD $\theta$-term and the minimal left-right symmetric model were demonstrated. Furthermore, in Ref. [22] the underlying scheme was presented to derive – within the framework of chiral effective field theory - the list of parity- and time-reversal-symmetry-violating hadronic interactions that are relevant for the computation of nuclear contributions.
to the electric dipole moments of the hydrogen-2, helium-3 and hydrogen-3 nuclei. The scattering and Faddeev equations required to compute electromagnetic form factors in general and electric dipole moments in particular were documented there in addition.

In 2015 Shindler, Luu and de Vries (IAS-4/IKP-3) proposed a new method to calculate electric dipole moments induced by the strong QCD $\bar{\theta}$-term [23]. The authors based their method on the gradient flow for gauge fields which is free from renormalisation ambiguities.\(^3\) The method was tested by computing the nucleon electric dipole moments in pure Yang-Mills theory at several lattice spacings, enabling a first-of-its-kind continuum extrapolation that is theoretically sound.

In the same year Guo et al. (2015) [25] presented an entirely dynamical calculation of the electric dipole moment of the neutron on the lattice. They computed the electric dipole moment $d_n$ of the neutron from a fully dynamical simulation of lattice QCD with 2 + 1 flavors of clover fermions and nonvanishing $\theta$-term. The latter was rotated into a pseudoscalar density in the fermionic action using the axial anomaly. To make the action real, the vacuum angle $\theta$ was taken to be purely imaginary. The physical value of $d_n$ was obtained by analytic continuation ($d_n = -3.9(2)(9) \times 10^{-16}$ e.cm) and an upper bound on the QCD theta angle ($|\theta| \lesssim 7.4 \times 10^{-11}$) was presented.

In 2016 Meißner and de Vries reviewed the progress in the theoretical description of the violation of discrete space-time symmetries in hadronic and nuclear systems [26]. They focused on parity-violating and time-reversal-conserving interactions which are induced by the weak interaction of the Standard Model, and on parity- and time-reversal-violating interactions which can be caused by a nonzero QCD theta term or by beyond-the-Standard Model physics. Especially, they reviewed the development of the chiral effective field theory extension that includes discrete symmetry violations and discussed the construction of symmetry-violating chiral Lagrangians and nucleon-nucleon potentials and their applications in few-body systems. In their review of the parity- and time-reversal violation, of course information from the above mentioned HISKP and IAS-4/IKP3 publications was used, but also results of three recent publications coauthored by the IAS-4 member de Vries were integrated: the first on the constraint of the neutron EDM on the value of the $CP$-and isospin-violating pion-nucleon coupling constant $g_1$ in the case of dimension-6 interactions [27], the second on the extension to SU(3) chiral perturbation theory and the update on the determination of the $CP$-violating isospin-conserving pion-nucleon coupling constant $g_\theta^\pi$ [28], and the third on direct and indirect constraints on the complete set of anomalous $CP$-violating Higgs couplings to quarks and gluons originating from dimension-6 operators [29].

In 2017 Wirzba, Bsaisou and Nogga [30] gave an update on the predictions of Refs. [21, 22], especially by extending the computation of the relevant matrix elements of the nuclear EDM operators in the deuteron case to the N4LO level of chiral effective field theory. Furthermore, they incorporated a review about the underlying principle that the existence of a nonzero EDM of an elementary or composite particle (in fact, of any finite system) necessarily involves the breaking of a symmetry, either by the presence of external fields (i.e. electric fields leading to the case of induced EDMs) or explicitly by the breaking of the discrete parity and time-reflection symmetries in the case of permanent EDMs.

In a series of publications, a collaboration including a current and two former members of IAS-4/IKP-3 refined the method of Ref. [23] by extending it from the calculation of EDMs induced by the strong QCD $\bar{\theta}$-term [31] to include the dimension-6 Weinberg term [32], and the quark-chromo EDM operator [33]. This work accumulated in Ref. [34] where the electric dipole moment of the nucleon induced by the QCD theta term was calculated in the gradient flow method with $N_f = 2 + 1$ flavors of dynamical quarks corresponding to pion masses of (700, 570, and 410) MeV which are used by performing an extrapolation to the physical point based on chiral perturbation theory. The calculations applied 3 different lattice spacings in the range of $0.07 \text{ fm} < a < 0.11 \text{ fm}$ at a single value of the pion mass, to enable control on discretisation effects. Also finite size effects were investigated using 2 different volumes. A novel technique was applied to improve the signal-to-noise ratio in the form factor.

\(^3\)In fact, their method was already documented in the publication [24] in a more broader context.
calculations. The very mild discretisation effects observed suggested a continuum-like behaviour of the nucleon EDM towards the chiral limit. Under this assumptions the results read $d_n/\bar{\theta} = -1.86(59) \cdot 10^{-16}$ e.cm and $d_p/\bar{\theta} = 1.5(2.2) \cdot 10^{-16}$ e.cm. Assuming the theta term is the only source of $CP$ violation, the experimental bound on the neutron electric dipole moment limits was predicted as $|\bar{\theta}| < 1.61(51) \cdot 10^{-10}$.

References

Bonn University, April 2014.


Appendix B

Mitigation of background magnetic fields

The EDM signal can be mimicked by magnetic fields in several different ways. The most critical effect comes from a static radial magnetic field, requiring a cancellation down to attoTesla level. Static longitudinal magnetic field has a similar effect, but with a few orders of magnitude more flexible restriction. Moreover, several configurations of alternating magnetic fields result in EDM-like spin precession too. We studied each of these scenarios and proposed solutions to cancel the effect.¹

Throughout this section, static and alternating fields refer to the particle’s rest frame. As an example, the earth’s field is alternating in the particle’s rest frame even if it is purely static in the lab frame.

B.1 Static magnetic field configurations

B.1.1 Static radial magnetic field

As Figure B.1 shows, the static radial magnetic field should be kept at attoTesla level to avoid the systematic error (assuming \( d_p = 10^{-29} \text{ e·cm} \)). This is obviously not possible with magnetic shielding alone. It should be measured and compensated actively. Our proposal is to measure the relative position of the counter-rotating beams, proportional to the average radial magnetic field. For the all-electric baseline ring, an attoTesla level field splits the counter-rotating beams vertically by picometers. The split beams induce a magnetic field in the horizontal direction. The magnitude of this field \( (B_x) \) can be measured by a magnetometer/gradiometer at a few cm horizontal distance (Figure B.2).

![Fig. B.1: Assuming \( d_p = 10^{-29} \text{ e·cm} \) for the baseline ring, 10 MV/m electric field and 17 aT magnetic field result in similar spin precession in the vertical plane.](image)

We are planning to use SQUID-based beam position monitors (BPM) to measure \( B_x \). In order to suppress the environmental noise, the vertical motion of the beams will be modulated at 1-10 kHz by means of the quadrupoles. The typical white noise of the DC SQUIDs at that range is less than 1

¹ This appendix was authored by Y.K. Semertzidis and S. Haciomeroglu of the Center for Axion and Precision Physics Research, KAIST, South Korea.
Bx due to the split beams in such a case is given as

\[ B_x(t) = \frac{\mu_0 I \Delta y}{\pi r^2} 2A \cos(\omega_m t) \]  

(B.1)

with the beam current \( I \), vertical split \( \Delta y \), horizontal distance between the pickup loop and the beams \( r \), and the modulation amplitude and frequency \( A \) and \( \omega_m \) respectively. Putting \( I = 10 \text{ mA}, \Delta y = 0.5 \text{ pm}, A = 0.1 \) and \( r = 2 \text{ cm} \) into Eq B.1 gives \( B_x \approx 1 \text{ aT} \cos(\omega_m t) \).

As a reference, with an array of 8 SQUIDs of \( 10^{-15} \) T sensitivity at 1 Hz bandwidth (1 fT/√Hz), it requires \( 2 \times 10^5 \) seconds of averaging to achieve SNR > 1 as \( B = 10^{-15} \text{ fT}/\sqrt{8 \times 2 \times 10^5} = 0.8 \text{ aT} \).

---

**Fig. B.2:** A magnetometer can pick up the magnetic field of horizontal direction, that is induced by the vertically split counter-rotating beams.

---

**B.1.1.1 Preliminary tests with SQUID-based BPM**

SQUID-based magnetometers can measure magnetic field variations with unprecedented noise level below 1 fT/√Hz. This is why they became the best candidates for the beam position monitors in the pEDM experiment. In addition to high resolution, the SQUID-based magnetometers have sufficient bandwidth and compact size that allows using multi sensor arrays placed along the beam trajectory inside a super-conductive shielding structure.

Figure B.3 shows the 3D drawing of the BPM. It will operate in vacuum at 4 Kelvin. They are positioned on the horizontal plane to measure the vertical split.

The BPM works inside a magnetically shielded room (MSR). The data transfer between the SQUIDs and the computer is done via fiber lines to minimise electromagnetic noise. Figure B.4 shows the picture of the first prototype.

The operation frequency was chosen to be around 1-10 kHz for minimising the external noise. This will be achieved by modulating the vertical tune \( Q_y \) as mentioned above.

We conducted preliminary tests with a setup having a similar SQUID electronics but a different design of pickup loop geometry and Dewar. The Dewar and the eight SQUID gradiometers are shown
in Figure B.5. They were originally designed at KRISS/Korea for biomagnetic applications. It has 8 axial wire-wound first-order gradiometers positioned along a bottom line inside a fiberglass Dewar. Each gradiometer has 20 mm diameter and 50 mm baseline and bonded to the double relaxation oscillation (DROS) SQUID current sensor. DROS SQUIDs have a large flux-to-voltage transfer coefficient that minimizes the contribution of the direct read-out electronics noise. The white noise of the gradiometers is about $3 \text{ fT}/\sqrt{\text{Hz}}$ at frequencies above 1 Hz.

For these measurements with long time averaging, the magnetic field was generated by two parallel traces of a 100 $\mu$m separation on a PCB, and carrying opposite currents of 100 $\mu$A. The applied current was a 300 Hz sinusoidal AC, corresponding to around 200 fT amplitude field at the pickup loop location.

The measurements showed more than two orders of magnitude suppression of the white noise from the gradiometers and the SQUID read-out electronics (Figure B.6). This corresponds to $\approx 25 \text{ aT}/\sqrt{\text{Hz}}$ with 5-hour averaging. This indicates very high long-time stability and low intrinsic fluctuation level in the instrument that includes all cryogenic and semiconductor, both analog and digital electronics.

The real design proposed for the experiment (Figure B.4) includes 16 magnetometers with two-turn 17 mm diameter pick-up coils bonded to DROS SQUIDs. It allows us to expect more than 3 times lower white noise floor, i.e. about 8 aT/$\sqrt{\text{Hz}}$, after 5 hour averaging. For the further noise decrease we expect using single-chip integrated magnetometers similar to ML12 reported in [2] but with chip size $24 \times 24 \text{mm}^2$. Such magnetometers have white noise below 0.2 fT/$\sqrt{\text{Hz}}$ at frequency above 1 kHz.

In the hybrid ring design, the compensation of the radial magnetic field does not have to be so strict, because the magnetic focusing mechanism makes a partial cancellation. According to the simulations,
Fig. B.4: The first prototype of the BPM. The three layers of the dewar and the LHe tank are covered with aluminized mylar. The partially inserted half cylinder below the LHe tank has housings for the SQUIDs. The other half was not inserted for easier visibility.

the restriction releases by five orders of magnitude.

B.1.2 Static longitudinal magnetic field

A static longitudinal magnetic field can appear in the presence of an electric current passing through the horizontal plane at the inner side of the ring. For instance, 25 mA current passing through the center of the ring induces $B_L \approx 1$ nT. In such a case, the spin precession $s_V$ on the vertical plane becomes

$$s_V(t) = \frac{e g B_L}{2m \gamma \omega_a} \left[ \cos(\omega_a t) - 1 \right]. \quad (B.2)$$

where $g$, $e$, and $m$ are the magnetic anomaly, electric charge and mass of the proton, $\gamma$ is the relativistic Lorentz factor and $\omega_a$ is spin precession rate on the vertical plane. Note that $s_V$ becomes quadratic if $\omega_a$ has a constant nonzero value. Figure B.7 shows the spin components on the horizontal ($s_r$) and vertical ($s_V$) directions in the presence of 50 pT longitudinal and vertical magnetic fields. While the vertical magnetic field does not affect $s_V$ directly, it has an indirect effect via $\omega_a$. As seen in the plot, $s_V$ grows much faster compared to the EDM signal.

B.1.2.1 Eliminating the effect of the longitudinal magnetic field

One needs to have a 1 fT level average magnetic field in the vertical and longitudinal directions to reduce the effect to the level of the EDM signal (nrad/s).

The spin precession rate on the vertical plane is

$$\omega_R = \frac{e g}{m 2\gamma} B_L s_R. \quad (B.3)$$

with $s_R$, the horizontal spin component. As the equation shows, the effect of $B_L$ enhances proportionally with $s_R$. This effect can be exploited by using a radially polarised test bunch. According to Equation B.3, the spin precession rate from $B_L = 1$ fT is $\omega_R = 2.2 \times 10^8 \times 10^{-15} = 220$ nrad/s without any contribution from the EDM as $\vec{s} \times \vec{E} = 0$ Monitoring that bunch with the polarimeter, its $\omega_R$ can be frozen by applying an inverse longitudinal magnetic field with 1 fT resolution.
Fig. B.5: Time-averaging measurements were done with a setup having the same electronics but different Dewar (Left) and the gradiometer (Right) designs.

Fig. B.6: 100 µA current was applied through the parallel traces, resulting 200 fT on the pickup. The noise at 1s is a few fT/√Hz, consistent with the 3 fT/√Hz sensitivity of the SQUIDs. The noise decreases down to 25 aT/√Hz after 5 hours of averaging.

B.1.3 Static vertical magnetic field
As seen in the above section, the static vertical magnetic field does not have a direct effect on $s_B$. But it enhances the effect of the longitudinal field. It can be cancelled similar to the static radial magnetic field case. But this time the field requirement is much more flexible.

B.2 Effect of alternating magnetic fields and the geometric phases
We have studied the major configurations of the magnetic field in a continuous ring. In each case, we have simulated pairs of 1 nT fields at perpendicular directions with different phases. In some cases
Fig. B.7: The spin components as simulated for 1 ms storage time with a magic particle in an electric ring. Because of the short storage time compared to one cycle of $\omega_a$, $s_R$ changes linearly, and $s_V$ approximates to a quadratic function (See Equation B.2). Left: 50pT vertical magnetic field causes $\omega_a \approx 12.5$ mrad/s on the horizontal plane. Right: Having linear dependence on $\omega_a$, $s_V$ has quadratic dependence on time. Combination of 50pT longitudinal and vertical static magnetic fields grows the vertical spin component up to 67 prad , matching well with the analytical estimation.

Table B.1: Summary of the major independent magnetic field configurations. $\langle \omega_r \rangle$ is the average spin precession rate on the vertical plane. Each simulation was done with 1 nT magnetic field strength.

<table>
<thead>
<tr>
<th>Field $B_R$</th>
<th>AC Phase</th>
<th>$\langle \omega_r \rangle$ [rad/s]</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>n/a</td>
<td>0.18</td>
<td>Measurement and active cancellation with BPMs</td>
</tr>
<tr>
<td>DC $B_L$</td>
<td>n/a</td>
<td>$&lt; 5.5 \times 10^{-6}$, proportional to $\omega_a$</td>
<td>Current to be limited to $&lt; 1$mA and DC $B_V$ to be avoided</td>
</tr>
<tr>
<td>DC $B_V$</td>
<td>n/a</td>
<td>0</td>
<td>Can be avoided with BPM similar to $B_R$ case</td>
</tr>
<tr>
<td>$B_V$ &amp; $B_L$</td>
<td>$90^\circ$</td>
<td>$9 \times 10^{-9}$</td>
<td>CW/CCW cancel</td>
</tr>
<tr>
<td>$B_R$ &amp; $B_V$</td>
<td>$0^\circ$</td>
<td>$3.5 \times 10^{-9}$</td>
<td>CW/CCW average out</td>
</tr>
<tr>
<td>$B_R$ &amp; $B_L$</td>
<td>$0^\circ$, $90^\circ$</td>
<td>$&lt; 10^{-10}$</td>
<td>CW/CCW average out</td>
</tr>
<tr>
<td>$B_V$ &amp; $B_L$</td>
<td>$90^\circ$</td>
<td>$&lt; 10^{-10}$</td>
<td>CW/CCW average out</td>
</tr>
<tr>
<td>$B_V$ &amp; $B_L$</td>
<td>$0^\circ$</td>
<td>Negligible</td>
<td></td>
</tr>
</tbody>
</table>

we have seen the spin growing much faster than the EDM signal, like in the case of longitudinal and vertical magnetic fields with $90^\circ$ phase difference ($B_V$ & $B_L$, $90^\circ$ of Table B.1). Some configurations are harmless as they average out themselves. Some of them cancel out thanks to the counter-rotating beam design.

Table B.1 summarises all of the studied cases, including static (DC) and alternating field configurations. According to our studies, the effect of the magnetic field can be kept under control by means of

- SQUID-based BPMs for static radial magnetic field
- less sensitive BPMs for static vertical magnetic field
- a radially polarised test bunch for the static longitudinal magnetic field
- counter-rotating beams.
While the coupling between the magnetic fields of perpendicular directions is harmless in a continuous ring, the coupling between the beta function and some multipoles of an alternating radial magnetic field splits the beams the same way as a static radial magnetic field. Our simulations show that the magnetic field must be smooth down to 1 pT level to avoid this systematic error. As will be seen in the next section, we have shown that the magnetic field along the shielding prototype is smooth at the level 10 pT within the storage time. Another one or two orders of magnitude can be gained by flipping the quadrupole signs between runs.

We have also proposed the hybrid ring design to avoid this problem. Magnetic focusing in the baseline ring compensates the external fields effectively, suppressing the above-mentioned systematic error significantly.

### B.3 Magnetic shielding

We are considering the magnetic shielding for keeping the beam more stable in the presence of large transient fields. We have designed a prototype, collaborating with P. Fierlinger’s group at TUM/Germany (Figure B.8). It contains two layers of magnifer, a high permeability material for low frequency shielding. High frequency shielding requires a material with high conductivity, like aluminium. The shielding factor of the system is approximately 500 at low frequencies.

![Magnetic shielding prototype](image)

**Fig. B.8:** The magnetic shielding prototype was developed in collaboration with Fierlinger Magnetics, a Germany-based company. It contains two layers of high permeability material, separated by ≈ 10 cm. The thickness of each layer is 1mm. The length is approximately 2.5 m.

The working principle of the magnifer relies on the domain structure inside it. The direction of magnetisation is uniform in these small regions, separated with the so-called domain walls. External magnetic field can move the domain walls, changing the total magnetisation of the material. The shielding structure gets magnetised over time because of this effect. Demagnetisation (or degaussing) is a commonly used method to avoid it. It is basically conducted by application of alternating field with a decreasing amplitude. This has an effect similar to shaking, which randomises the domain magnetisation over the material. The red cable in Figure B.8 is used for applying a current for degaussing. Our studies showed that the uniformity of the cables along the material matters for the degaussing performance at the inner layer, but not at the outer. Therefore unlike the outer one, the inner magnifer layer has uniformly distributed degaussing cables.
B.3.1 Residual field

There are several key factors for the performance of degaussing. First of all, the amplitude of the applied magnetic field should be large enough to saturate the material. The cycles should be slow enough to let the domain movement. \( \approx 10 \text{ Hz} \) for this prototype. The last steps of degaussing should be smooth enough for more evenly distributed domain configuration. At the end, the material would still have a nonzero magnetisation which results in the so-called “residual field” inside the shielded volume. Figure B.9 shows the residual field measurement inside the prototype after degaussing. As seen, 1 nT field can be easily achieved with two-layer shielding after degaussing.

![Fig. B.9: Residual field measurement inside the prototype. The x-axis is the longitudinal position of the fluxgate sensors. The field is larger at the edges due to the caps of the prototype, which will not be used when installed at the ring.](image)

B.3.2 Time stability of the residual field

Time stability of the residual field becomes critical especially when the beta function of the beam is not uniform. Coupling between the varying beta function and the magnetic field moves the beam vertically, mimicking a DC radial magnetic field. We proposed to change the polarity of the quadrupoles to cancel this effect. According to our simulations, this requires a stable residual field along the ring to \( < 100 \text{ pT} \) level.

We tested the prototype inside our magnetically shielded room (MSR) as seen in Figure B.10. In the tests we used only the outer layer of the prototype. Then, after degaussing it, we measured the magnetic field inside the prototype. Figure B.11 shows the field at three locations along the axis separated by 70 cm. The measurement lasted almost 25 hours. The variation of the field is mainly related to the temperature. It decreases overnight and increases after the sunrise. Of course the stability during the whole day is irrelevant in the pEDM experiment. Rather, we are interested with the stability within one storage or two.

Figure B.12 zooms the morning period of Figure B.11, where the temperature changes the most rapidly. According to the plot, the change in 20 minutes is around 100pT. For the effect mentioned above, the beta function varies at different locations in the ring. Therefore one needs to look at the correlation between different points. If the field changes the same way, then the beta function will not vary the same way with the magnetic field and the effect will be smaller.
Figure B.13 shows the difference between two sensors at the same region with Figure B.12. The distance between those two points is 1.4m. The residual field changes by 10 pT over the period of 20 minutes.

Fig. B.10: Time stability measurements were done inside the MSR.

To sum up, we have a prototype to prevent the effects from transient magnetic fields in the pEDM experiment. Its residual field is as low as 1 nT with good time stability and field uniformity along the cylindrical axis. The time stability within 1.4 meter is measured to be $\approx 10$ pT within 20 minutes.

B.4 Summary

Our studies show that we can keep the static magnetic field under control in an alternating gradient, all-electric ring. The active and passive cancellation of the magnetic field requires several components including counter-rotating beams, beam position monitors, a test bunch with horizontal polarisation and magnetic shielding. The tests that we have made with the SQUID-based BPMs and the magnetic shielding prototype yielded promising results.

Alternating magnetic fields are harmless in a continuous all-electric ring. But the coupling between the beta function and the alternating radial magnetic field causes a vertical split similar to the static radial magnetic field. This can be suppressed by flipping the quadrupoles at every run and keeping the residual field uniformity at 10 pT level. But, the hybrid ring is a more efficient solution to this problem. Overall, according to our simulations, the hybrid ring has more flexible requirement for the field cancellation in all scenarios.
**Fig. B.11**: Magnetic field is measured over the course of 25 hours. The sensors were located at several locations along the prototype. The dominant reason for the change of the field is the temperature.

**Fig. B.12**: The protons will be stored for 20 minutes in the ring. Therefore the stability of the field at that period is important. Zooming in the depicted 20 minute region of Figure B.11, one sees that the field change is \( \approx 100 \) pT. Note that this is the worst period of the measurement, where the temperature changes rapidly.

**Fig. B.13**: The field at different locations around the ring are quite correlated. The difference between the field 140cm apart change together in the measurements. The difference is \( \approx 10 \) pT level.
References


Appendix C

Statistical Sensitivity

C.1 Statistical error on EDM

The spin motion, relative to the momentum vector, is governed by the BMT-equation

\[
\frac{d\vec{S}}{dt} = (\vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}}) \times \vec{S}
\]  

(C.1)

\[
\vec{\Omega}_{\text{MDM}} = -\frac{q}{m} \left[ G\vec{B} + \left(G - \frac{1}{\gamma^2} - 1\right) \frac{\vec{v} \times \vec{E}}{c^2} \right]
\]  

(C.2)

\[
\vec{\Omega}_{\text{EDM}} = -\frac{\eta q}{2mc} \left[ \vec{E} + \vec{v} \times \vec{B} \right]
\]  

(C.3)

with \( \vec{d} = \eta \frac{q}{2mc} \vec{s} \), and \( \vec{\mu} = 2(G + 1) \frac{q}{2m} \vec{s} \). For this discussion \( \vec{B} \) and \( \vec{E} \) denote a vertical magnetic and a radial electric field, respectively.

For a pure electric ring the angular precession frequency due to the EDM is given by

\[
\vec{\Omega}_{\text{EDM}} = \frac{\eta q}{2mc} \vec{E}.
\]  

(C.4)

One finds

\[
\vec{\Omega}_{\text{EDM}} = \frac{dE}{\hbar}
\]  

(C.5)

Using the numerical values of Tab. C.1 for protons (i.e. \( s = 1/2 \)) one arrives at

\[
\vec{\Omega}_{\text{EDM}} = 2.4 \cdot 10^{-9} \text{s}^{-1}.
\]  

(C.6)

| \( N \) | \( 4 \cdot 10^{10} \) | \( 2 \cdot 10^{10} \) CW, \( 2 \cdot 10^{10} \) CCW | particles per fill |
| \( E \) | \( 8 \text{ MV/m} \) | electric field |
| \( T_{\text{cyc}} \) | \( 1000 \text{s} \) | cycle length |
| \( P \) | 0.8 | polarisation |
| \( A \) | 0.6 | analysing power |
| \( f \) | 0.005 | detection efficiency |
| \( s \) | 1/2, 1 | spin quantum number |
| \( d \) | \( 10^{-29} \text{ cm} \) | EDM |

Table C.1: Parameters used to evaluate the statistical error.

To evaluate the statistical error we discuss three different scenarios:

1. There is only a precession due to the EDM, i.e. one observes only the initial linear rise of the polarisation vector because \( \vec{\Omega}_{\text{EDM}} \tau \ll 1 \) The polarisation is continuously measured indicated by the points in the graph below:
2. As for scenario 1), but half of the beam is extracted at $t = 0$, the other half is extracted at $t = \tau$:

3. In this scenario, the precession is dominated by systematic effects, one observes thus many oscillations during the cycle of length $T_{\text{cyc}}$:

In all three cases it is assumed that the EDM is extracted from the difference and sums of the polarisations of the CW and CCW measurements.

1) Assuming a polarisation vector initially along the momentum vector, we get

$$\dot{P}_\perp = \Omega_{\text{EDM}} P = \frac{dE}{\hbar} P,$$

resulting in

$$d = \frac{\hbar \dot{P}_\perp}{EP}.$$  \hspace{1cm} (C.7)
In general the variance on the slope parameter $s$ of a straight line is

$$V(s) = \frac{\sigma^2}{N_{\text{points}}V(t)}$$

where $\sigma$ is the error on each individual point. $N_{\text{points}}$ is the number of points entering the fit and $V(t)$ is the variance of the points along the horizontal axis (i.e. time in the EDM case). For evenly distributed values in time, one has $V(t) = T_{\text{cyc}}^2/12$. The slope in the EDM measurement is just $\dot{P}$. The error on one polarisation measurement, determined from the azimuthal distribution of events, is

$$\sigma^2_P = \frac{2}{(N_f/N_{\text{points}})A^2}.$$

The variance on the slope $\dot{P}$ is, thus

$$\sigma^2_{P\perp} = \frac{2}{((N_f)/N_{\text{points}})A^2} \frac{12}{N_{\text{points}}T_{\text{cyc}}^2} = \frac{24}{(N_f)(AT_{\text{cyc}})^2}.$$

From eq. C.7 we find

$$\sigma_{\text{EDM}} = \frac{s\hbar}{EP} \sigma_{P\perp}$$

which results in

$$\sigma_{\text{EDM}} = \frac{s\hbar}{EP} \frac{\sqrt{24}}{\sqrt{N_fAT_{\text{cyc}}}}$$

$$= \frac{s\hbar}{\sqrt{24N_fAT_{\text{cyc}}}}$$

(C.11)

Here it is assumed that there is no decoherence during $T_{\text{cyc}}$.

2) Taking only two measurements at $t = 0$ and $t = \tau$, the slope is determined by

$$\dot{P}_\perp = \frac{P(T_{\text{cyc}}) - P(0)}{T_{\text{cyc}}}.$$

Using

$$\sigma^2_{P(0)} = \sigma^2_P(T_{\text{cyc}}) = \frac{4}{(N_f)A^2}$$

one finds

$$\sigma^2_{P\perp} = \frac{2}{T_{\text{cyc}}^2} \frac{4}{(N_f)A^2} = \frac{8}{(N_f)(AT_{\text{cyc}})^2}.
$$

resulting in

$$\sigma_d = \frac{s\hbar}{EP} \sigma_{P\perp}$$

$$= \frac{s\hbar}{EP} \frac{\sqrt{8}}{\sqrt{N_fAT_{\text{cyc}}}}$$

$$= \frac{\sqrt{8s\hbar}}{\sqrt{N_fAPET_{\text{cyc}}}}$$

(C.15)

3) According to Ref. [1] the error on the frequency is given by

$$\sigma^2_\Omega = 2 \frac{24}{(N_f)(APT_{\text{cyc}})^2}$$
Using the relation
\[ \Omega_{\text{EDM}} = \frac{dE}{s\hbar} \]

one finds for the error on \( d \)
\[ \sigma_d = \sqrt{48} \frac{s\hbar}{\sqrt{NfAPET_{\text{cyc}}}}. \]

### C.2 Precursor Experiment

For the precursor experiment the build-up of the vertical polarisation is given by
\[ P_\perp = n \frac{\eta\beta}{4G} \frac{e}{mc^2} \left( \frac{G+1}{\gamma^2\beta^2} \right) ELP. \]

(C.16)

One finds the following expression for the error on the EDM:
\[ \sigma_{\text{EDM}} = \left| \hbar s \left( \frac{2G\gamma^2}{G+1} \right) \frac{U}{LE} \right| \frac{\sqrt{24}}{\sqrt{NfAP\tau}}. \]

(C.17)

To arrive at eq. C.7 the number of turns \( n \) were replaced by the time of the measurement \( \tau \) times the revolution frequency \( f_{\text{rev}} \), \( n = \tau f_{\text{rev}} \). The revolution frequency can be expressed as \( f_{\text{rev}} = \beta c/U \), where \( U \) is the circumference of the ring.

Using the following parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>(-0.14)</td>
</tr>
<tr>
<td>( s )</td>
<td>1</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.13</td>
</tr>
<tr>
<td>( E \cdot L )</td>
<td>2.7 kV</td>
</tr>
<tr>
<td>( \tau )</td>
<td>1000s</td>
</tr>
<tr>
<td>( P )</td>
<td>0.8</td>
</tr>
<tr>
<td>( A )</td>
<td>0.6</td>
</tr>
<tr>
<td>( N )</td>
<td>(10^9)</td>
</tr>
<tr>
<td>( f )</td>
<td>0.005</td>
</tr>
<tr>
<td>( U )</td>
<td>184m</td>
</tr>
</tbody>
</table>

one arrives finally at
\[ \sigma_{\text{EDM}}(1\text{fill}) = 8.6 \cdot 10^{-21} \text{e cm}, \]

per fill of 1000s.

### C.3 Summary

The statistical error on the EDM \( d \) is given by
\[ \sigma_{\text{EDM}} = \alpha \beta_{\text{pr}} \frac{s\hbar}{\sqrt{NfAP\tau ET_{\text{cyc}}}}. \]

with
\[ \beta_{\text{pr}} = \begin{cases} \frac{2G\gamma^2}{G+1} & \text{precursor experiment,} \\ 1 & \text{prototype & final ring} \end{cases} \]

The factor \( \alpha \) depends on the way the polarisation is measured and on the spin coherence time \( \tau \). The factor \( \tau \) added is the fraction of the ring equipped with \( E \)-field (or Wien filter, in the case of the precursor.
experiment). In section C.1 it was assumed that the polarisation is constant over $T_{cyc}$. If $\tau \approx T_{cyc}$, the average polarisation is smaller by the factor

$$\int_0^{T_{cyc}} \frac{e^{-t/\tau}}{T_{cyc}} dt = 1 - e^{-1} \approx 0.63$$

assuming an exponential decrease. The error is increased accordingly. The factor $\alpha$ is listed in Tab. C.2 for the various cases.

For the best (worst) case in the table the errors on $d$ in units of $e$ cm, using the values in Tab. C.1 for the final ring are:

<table>
<thead>
<tr>
<th></th>
<th>one cycle</th>
<th>one year ($10^4$ cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.6 \times 10^{-27}$</td>
<td>$2.6 \times 10^{-29}$</td>
<td></td>
</tr>
<tr>
<td>$1 \times 10^{-26}$</td>
<td>$1 \times 10^{-28}$</td>
<td></td>
</tr>
</tbody>
</table>

References
Appendix D

Gravity and General Relativity as a ‘Standard Candle’

The Thomas-BMT equation including General Relativity corrections reads [1–3]:
\[
\frac{d\vec{S}}{dt} = \left(\tilde{\Omega}_{\text{MDM}} + \tilde{\Omega}_{\text{EEDM}} + \tilde{\Omega}_{\text{GRgeo}}\right) \times \vec{S},
\]
where, in the Frenet-Serret coordinate system whose axis orientation is determined from the local particle motion [4]. \( \tilde{\Omega}_{\text{MDM}} \) refers to the angular velocity from the magnetic dipole moment minus the cyclotron angular velocity, \( \tilde{\Omega}_{\text{EEDM}} \) to the one from the electric dipole moment, and \( \tilde{\Omega}_{\text{GRgeo}} \) to the angular velocity of the geodetic (de Sitter) minus the corresponding angular velocity for the particle revolution.\(^2\)

\[
\tilde{\Omega}_{\text{MDM}} = -\frac{q}{m} \left[ GB - \frac{\gamma G}{\gamma + 1} \vec{\beta} \cdot \vec{B} \right] - \left(G - \frac{1}{\gamma^2 - 1}\right) \frac{\vec{\beta} \times \vec{E}}{c},
\]
\[
\tilde{\Omega}_{\text{EEDM}} = -\frac{\eta q}{2mc} \left[ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{\beta} \left( \vec{\beta} \cdot \vec{E} \right) + c\vec{\beta} \times \vec{B} \right],
\]
\[
\tilde{\Omega}_{\text{GRgeo}} = -\frac{\gamma}{\gamma^2 - 1} \vec{\beta} \times \vec{g}.
\]

Here \( \vec{g} \) is the gravitational acceleration at the Earth’s surface – for further definitions see Ref. [1] and Chap. 4. Furthermore, according to [1], \( \vec{E} \) and \( \vec{B} \) in Eqs. (D.2) and (D.3) have to be replaced by \( \vec{E} + \vec{E}_g \) and/or \( \vec{B} + \vec{B}_g \), respectively, where \( \vec{E}_g \) and/or \( \vec{B}_g \) are focusing fields compensating the gravitational downwards pull on the beam particles of mass \( m \) and velocity \( c\vec{\beta} \),

\[
\vec{E}_g = \gamma \left( 1 + |\vec{\beta}|^2 \right) m\vec{g} = \frac{2\gamma^2 - 1}{\gamma} m\vec{g}.
\]

This follows from the storage-ring lattice condition for the closed orbit:

\[
\frac{2\gamma^2 - 1}{\gamma} m\vec{g} + q \left( \vec{E}_g + c\vec{\beta} \times \vec{B}_g \right) \equiv 0,
\]
e.g. the upwards pointing vertical electric field for a pure electric ring reads

\[
\vec{E}_g = (\vec{E}_g \cdot \hat{y}) \hat{y} = \frac{2\gamma^2 - 1}{\gamma} \frac{m}{q} (-\vec{g}),
\]
while the gravity-compensating radially inwards/outwards pointing magnetic field for a counterclockwise/clockwise beam would be

\[
\vec{B}_g = (\vec{B}_g \cdot \hat{x}) \hat{x} = (2\gamma^2 - 1) \frac{\gamma}{\gamma^2 - 1} \frac{m}{q} \frac{\vec{\beta} \times \vec{g}}{c} = \frac{2\gamma^2 - 1}{\sqrt{\gamma^2 - 1}} \frac{m|\vec{g}|}{cq} \frac{\vec{\beta} \times \vec{g}}{c}.
\]

\(^1\)Deviating from the local coordinate system used in Chap. 4, here the right-handed, beam-comoving coordinate system \((x, y, z)\) is defined by the unit vectors \( \hat{z} = \vec{\beta}/|\vec{\beta}| \equiv \vec{\beta}, \hat{y} = -\vec{g}/|\vec{g}| \equiv -\hat{g} \) and \( \hat{x} = -\hat{z} \times \hat{y} = \vec{\beta} \times \hat{g} \), i.e. the unit vector \( \hat{y} \) is always pointing opposite to the gravitational acceleration \( \vec{g} \). Thus for a clockwise beam we have \( \hat{x} = \hat{r} \), while for a counterclockwise beam \( \hat{x} = -\hat{r} \), where \( \hat{r} \) is the outside-pointing radial unit vector inside the storage-ring plane.

\(^2\) \( \tilde{\Omega}_{\text{GRgeo}} \) is calculated from the difference between the gravity-induced ‘spin-orbit’ precession around a radial axis in the Earth gravitational field, \( \Omega_{\text{LS}} \) (the de Sitter precession aka the geodetic effect) [5–7], and the particle revolution around the same axis in the Earth’s gravitational field, \( \Omega_{\text{rev}} \), cf. [1]:

\[
\tilde{\Omega}_{\text{LS}} = \frac{2\gamma + 1}{\gamma + 1} \frac{\vec{\beta} \times \vec{g}}{c}, \quad \tilde{\Omega}_{\text{rev}} = \frac{1 + \beta^2}{\beta^2} \frac{\vec{\beta} \times \vec{g}}{c} = \frac{2\gamma^2 - 1}{\gamma^2 - 1} \frac{\vec{\beta} \times \vec{g}}{c}, \quad \tilde{\Omega}_{\text{GRgeo}} = \tilde{\Omega}_{\text{LS}} - \tilde{\Omega}_{\text{rev}}.
\]
with

\[ c\beta \times \vec{B}_g = (2\gamma^2 - 1) \frac{\gamma}{\gamma^2 - 1} \frac{m}{q} \left( \beta (\vec{\beta} \cdot \vec{g}) - \vec{g} (\vec{\beta} \cdot \vec{g}) \right) = \frac{2\gamma^2 - 1}{\gamma} \frac{m}{q} (-\vec{g}). \tag{D.9} \]

1. Note that for the case of the frozen-spin (fs) condition, \(1/(\gamma^2 - 1) := G\), in an all-electric ring we have [7]

\[ \Omega^{B=0,fs}_{\text{GR}} = - \frac{\gamma}{\gamma^2 - 1} \frac{\vec{\beta} \times \vec{g}}{c} \bigg|_{\text{fs}} = - \frac{\vec{g}}{c} G \sqrt{\frac{1+G}{G}} \frac{1}{\sqrt{1+G}} = - \frac{\vec{g}}{c} \sqrt{\frac{G}{c}} \vec{\beta} \times \vec{g}, \tag{D.10} \]

which agrees with the earlier result of Orlov, Flanagan, and Semertzidis [8]. Thus the geodetic effect of general relativity would induce a ‘fake’ proton EDM value of, e.g.,

\[ d_p^{GR} \approx 2.88 \cdot 10^{-27} \text{e cm} \quad (\text{i.e. } \eta^{GR}_d \approx 2.74 \cdot 10^{-14}) \text{ corresponding to } E_y = 10 \text{ MV/m}, \]
\[ d_p^{GR} \approx 2.88 \cdot 10^{-27} \text{e cm} \quad (\text{i.e. } \eta^{GR}_d \approx 2.74 \cdot 10^{-13}) \text{ corresponding to } E_y = 1 \text{ MV/m}, \]

where \(E_y\) is the mean radial component of the electric field, and could therefore serve as a standard source or ‘standard candle’ for EDM measurements in frozen-spin all-electric storage rings, while the gravity-compensating fields just correspond to \(E_g \approx 0.173 \mu \text{V/m} \) or \(B_g \approx 0.967 \text{fT}\). Such tiny focusing fields are automatically generated by a minuscule orbit displacement by the Earth gravity pull.

2. If the radial component \(B_x = \hat{x} \cdot \vec{B}\) of the magnetic field is identically zero, the \(\vec{F}_g\) compensating field only arises from the vertical electric field \(E_y = \hat{y} \cdot \vec{E}\) and therefore we would have as the gravity-induced contribution to the angular velocity \([1, 7]\)

\[ \Omega^{E_y=0}_{\text{GR}} = \Omega_{\text{GRgeo}} - \frac{q}{m} \left( G - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}_g}{c} = \frac{1 - G(2\gamma^2 - 1)}{\gamma} \frac{\vec{\beta} \times \vec{g}}{c}. \tag{D.11} \]

Obviously, in the frozen-spin case, \(1/(\gamma^2 - 1) = G\), this result will become the one of Eq. (D.10) and of Ref. [8].

3. If the vertical electric field \(E_y = \hat{y} \cdot \vec{E}\) is identically zero, the \(\vec{F}_g\) compensating field only arises from the radial magnetic field \(B_x = \hat{x} \cdot \vec{B}\) and therefore we would find as the gravity-induced contribution to the angular velocity \([1, 4, 7]\)

\[ \Omega^{E_y=0}_{\text{GR}} = \Omega_{\text{GRgeo}} - \frac{q}{m} G \vec{B}_g = - \frac{\gamma}{\gamma^2 - 1} \left( 1 + G(2\gamma^2 - 1) \right) \frac{\vec{g}}{c}. \tag{D.12} \]

If the frozen-spin condition \(1/(\gamma^2 - 1) = G\) of the all-electric ring is inserted, the result of Eq. (D.12) is enhanced by a factor \((3 + G)\) in comparison to Eq. (D.10), i.e.

\[ \Omega^{E_y=0,fs}_{\text{GR}} = - \frac{\vec{g}}{c} (3 + G) \sqrt{G} \vec{\beta} \times \vec{g}. \tag{D.13} \]

4. In a mixed ring with \(E_y \neq 0 \neq B_x\) using

\[ \kappa \equiv \frac{c\beta B_x}{E_y} \approx \text{const.}, \tag{D.14} \]

we can derive from the storage-ring lattice condition (D.6),

\[ \frac{m|\vec{g}|}{q} \frac{2\gamma^2 - 1}{\gamma} = E_y + c\beta B_x = E_y(1 + \kappa), \tag{D.15} \]

footnote text
the following expression for the gravity-induced angular velocity [7]:

\[ \vec{\Omega}_{EB}^{GR}(\kappa) = \left\{ -\frac{\gamma}{\gamma^2 - 1} \beta \frac{g}{c} \left( c\beta B_x + \left( 1 - \frac{1}{G\gamma^2 - 1} \right) \beta E_y \right) \right\} \hat{\beta} \times \hat{g} \]

\[ = \left\{ -\frac{\gamma}{\gamma^2 - 1} - \frac{q}{m|\vec{g}|} \frac{G}{\beta^2} \left( c\beta B_x + E_y - \frac{1}{\gamma^2} \left( 1 + \frac{1}{G} \right) E_y \right) \right\} \frac{\hat{\beta} \times \hat{g}}{c} \]

\[ = -\frac{\gamma}{\gamma^2 - 1} \left\{ 1 + (2\gamma^2 - 1) \left( G - \frac{G + 1}{\gamma^2(1 + \kappa)} \right) \right\} \frac{\hat{\beta} \times \hat{g}}{c} \]

\[ = \frac{1}{1 + \kappa} \left( \vec{\Omega}_{Bx = 0}^{GR} - \kappa \vec{\Omega}_{Ey = 0}^{GR} \right). \tag{D.16} \]

Of course, one recovers the expressions (D.11) and (D.12) from (D.16) if one simply inserts \( \kappa \to 0 \) or \( \kappa \to \infty \), respectively, while by applying the “frozen-spin value”, \( 1/(\gamma^2 - 1) = G \), we would get the general form \( -\hat{\beta} \times (\hat{g}/c) \sqrt{G(1 + (3 + G)/\kappa)}/(1 + \kappa) \).

Note that the contributions (D.10)–(D.13) and (D.16) switch sign if a counterclockwise beam is replaced by a clockwise one. This clearly separates these contributions from any (MDM-term induced) fake-EDM signal when a radial magnetic field points, for both beams, in the same direction – either in the outward \( (\hat{r}) \) or in the inward radial \( (\hat{r}) \) direction. In fact, if the scenario \( E_y = 0 \) can be realised (or the value of \( \kappa \) can be determined in the general case (D.16) by some means), the lattice orbit condition (D.6) ensures that \( B_x \) of each of the beams is determined, in the average, by Eq. (D.15), respectively. Thus the extraction of the gravity-induced spin rotation from the half-sum/half-difference of counterclockwise and clockwise beams – assuming that the horizontal spins of the beams point in the opposite/same direction\(^4\) – would determine the orbit-averaged value of the effective radial magnetic field which then could be used to correct the EDM signal.

References


\(^4\)Here, the qualifier “opposite/same spin direction” in the frozen-spin scenario refers to the setting that the components of the horizontal spin in beam direction of the clockwise and counterclockwise case agree/differ in sign.
Appendix E

Additional Science Option: Axion Search

E.1 Concept of Search for Axion-like Particles

The theoretical prediction of a neutron electric dipole moment based on QCD is given by $|d_n^\theta| = \theta_{QCD} \cdot 10^{-16}$ e·cm [1]. However, the most sensitive experimental result [2], $d_n = 2.9 \times 10^{-26}$ e·cm (90% C.L.) for the neutron, sets a very strict upper limit on the parameter $\theta_{QCD}$. Since there is no natural explanation for the extremely small value of $\theta_{QCD}$, this is sometimes referred to as the strong CP problem.

The axion originated from a new symmetry postulated by Roberto Peccei and Helen Quinn to solve the strong CP problem in QCD physics [3]. The small parameter is explained by a dynamic scalar field that maintains the symmetry. The particle associated with this field is the axion. Furthermore, if the axion is very light, it interacts so weakly that it would be nearly impossible to detect in conventional experiments. But it would be an ideal dark matter candidate as it interacts gravitationally with the matter around it.

The axion couples to gluons, fermions, nucleons, etc. This coupling induces an oscillating electric dipole moment (EDM) in nucleons [4, 5]. This may be expressed as:

$$d_n = 1.2 \times 10^{-16} \frac{a(t)}{f_a} \cdot e \cdot \text{cm} = 1.2 \times 10^{-16} \frac{a_0 \cos(m_a t + \phi_a)}{f_a} \cdot e \cdot \text{cm}$$

(E.1)

where $a(t)$ is the axion/dark matter field, $m_a$ is the axion mass, and $f_a$ is the axion decay constant. $\phi_a$ is an unknown local phase that we will need to consider later.

Three conditions must be met in order to consider using the horizontally polarised deuteron beam at COSY to search for axions. First, the effects of the axion must be coherent across a large spatial range so that, as the beam circulates, it remains under the influence of a single axion. This will also mean that all of the deuterons in the beam will show the oscillating EDM property simultaneously. Thus, an electric dipole moment parallel to the polarisation (average orientation of the deuteron spins) may be used to test for the presence of an axion. Second, this interaction must remain present in the COSY experimental hall for a time long enough for the beam to respond. Crossing an axion resonance in a scanning search would likely require a few seconds. Any axions in the neighbourhood of earth are likely bound to the Milky Way galaxy and thus there is a lower limit on how quickly they will vanish from view. Third, the density of axions must be high enough that the chance of observing one is substantial during the time that the store is under way. Estimates [4, 5] based on the confinement of the axion to our region of the Milky Way galaxy suggest that these coherence requirements are met at the frequency where we would make a feasibility study ($\sim 630$ kHz) with a quality factor $Q$ for the axion’s oscillation exceeding $10^6$.

The experiment to search for an axion would consist of a series of runs in which the revolution frequency of the machine is changed continuously in a slow ramp [6]. Measurements of the polarisation components would be made during the ramp. If the in-plane polarisation precession frequency happens to match the axion frequency, then a resonance between the two will cause the vertical component of the polarisation to undergo a jump proportional to the ratio of the size of the oscillating EDM to the speed of the ramp. A comparison of polarisation asymmetries collected during non-ramped times at the beginning and ending of the scan would suffice to quantify the size of any suspected change.

Experimental signals based on a sub-atomic EDM depend on a torque about an electric field along the radial direction in a storage ring that lifts the polarisation direction out of the ring plane, giving it a small and rising vertical component. Despite the large electric field that exists in the beam frame from the magnets that confine the deuteron beam to the COSY ring, the continuous rotation of the in-
plane polarisation relative to the beam velocity makes it impossible for any static EDM signal to become large enough through a $d \times E$ torque to observe directly. Progress is cancelled by retreat whenever the projection of the EDM on the tangential direction reverses. But if there is an oscillating EDM that varies at the same frequency as the rotating polarisation, then a vertical component of the polarisation will start to grow. A proposal was accepted by the COSY Program Advisory Committee to develop and describe techniques that could be used in such an axion search and to quantify the sensitivity for reasonable operating conditions. The plan is to start with the deuteron momentum of $p = 0.97$ GeV/c where there is COSY experience with the preparation of a horizontally polarised beam with a long polarisation lifetime [7].

### E.2 Technical Considerations for an Axion Search

Accumulation of the vertical component polarisation signal depends on the alignment of the polarisation along the direction of the beam velocity with the maximum of the value of the oscillating EDM. This alignment is controlled by the axion phase $\phi_a$. If these two oscillations are out of phase by $\pi/2$, then no accumulation will occur. The plan to overcome this difficulty is to operate COSY on the fourth harmonic (for which hardware already exists), producing four circulating bunches in the ring at the same time. If an RF solenoid is used to precess the polarisation from the vertical direction (as it is upon injection into the ring) into the horizontal plane, then the resulting laboratory-frame polarisation pattern of the four beams in the ring is shown in Fig. E.1 for $f_{\text{SOL}} = f_{\text{REV}}(1 + G\gamma)$ where $G$ is the deuteron’s magnetic anomaly and $\gamma$ is the usual relativistic parameter.

![Fig. E.1: Laboratory-frame polarisation directions of the four beam bunches as seen from above the plane of the storage ring. The labels show the order (A, B, C, D) in which they were generated by the RF solenoid operating on the $1 + G\gamma$ harmonic.](image)

This pattern features two directions (A and D) that are nearly orthogonal. This means that the experiment carries sensitivity to both components of the phase of the oscillating EDM (sine and cosine). The remaining two polarisation directions may be used to verify that the amplitude of any prospective axion signal varies in a sinusoidal pattern around the circle in Fig. E.1 in a manner consistent with the two phase components present in the A and D directions. In addition, there are pairs of polarisation directions that are nearly opposite. This provides an opportunity to use them as opposite polarisation states in a “cross ratio” which would serve to reduce or eliminate first-order errors in the scanning process due to geometric or rate-dependent systematic errors that can develop during the beam store [8]. Bunch B may be compared with the average of A and C, and bunch C may be compared with the average of B and D.

One way to search for an axion-like particle is to vary the polarisation rotation rate continuously.
while monitoring the vertical polarisation. If the frequency of rotation happens to match the axion frequency at some time during the scan, the resonance condition will create a jump in the polarisation, as shown in Fig. E.2.

![Figure E.2](image_url)

**Fig. E.2:** A calculation of the resonance crossing with a scan rate of 0.5 Hz/s. The strength of the oscillating EDM is $1.6 \times 10^{21}$ e·cm. Within the span of less than one second, this causes a jump of $-0.75$ in the $p_Y$ component of the beam polarisation (assumed to initially be completely polarised in the ring plane).

A practical scheme for producing such scans would require that the range of the scan is not be so large that it passes outside the acceptance of the storage ring. In addition, the frequency of the RF solenoid that initially precesses the polarisation from the vertical to the horizontal direction must be adjusted to match the $1 + G\gamma$ spin tune resonance. The easiest way to organise the scan is to vary the revolution frequency of the beam. Critical magnetic ring components, such as the dipole magnets, would be programmed to follow.

### E.3 Initial Tests with Beam

In December, 2018, there was an opportunity to switch COSY to operate on the $h = 4$ harmonic. At that time, the RF solenoid was running on the $1 - G\gamma$ harmonic and the sextupole magnets along with electron cooling had been set for long in-plane polarisation lifetime. In Fig. E.3 there is a representation of the count rate in the WASA detector as a function of time in the store (horizontal) and position around the ring (vertical).

The four beam bunches show clearly after 80 s following a period of electron cooling. At this time the RF solenoid frequency was associated with the $1 - G\gamma$ harmonic. This yields a different pattern of polarisation directions compared to Fig. E.1. In the laboratory frame we have:

Like the pattern shown in Fig. E.1, this pattern also presents bunches A and D with polarisation directions that are nearly perpendicular. So this pattern also suffices to detect the axion for any value of the axion phase. But the other two polarisation directions, B and C, lie in the same quadrant. Their polarisation directions are similar, and any axion signal will tend to have a similar signature as A and D. Thus we cannot use these signals in a cross ratio treatment to eliminate systematic errors in the measurements of the asymmetry.

Experimental verification of these polarisation directions depends on measurements made with a polarimeter located at one spot on the COSY storage ring. It will see the four bunches sequentially at different time. Given that the polarisation continues to rotate in the ring plane, this leads to a different set of directions measured at the polarimeter. Since the polarisation is rotating at about 630 kHz, it is most useful to consider expressing this polarisation as a magnitude and a phase with respect to a starting
Fig. E.3: Count rate in the polarimeter as a function of time in the store (horizontal) and position around the circumference of COSY (0 to $2\pi$). Extraction of the beam onto the WASA polarimeter begins at 90 s. Prior to 80 s the beam is being electron cooled. There are four horizontal ridges corresponding to the four beam bunches.

Fig. E.4: Directions of the in-plane polarisations in the laboratory frame for the case of an RF solenoid operated on the $1 + G\gamma$ harmonic. The labels follow the scheme of Fig. E.1. The opening angle for adjacent pairs is shown as $28.8^\circ$.

time that is the beginning of data acquisition. Normally, trimming the fields in the ring, especially the sextupole components, is very useful in maintaining the size of the IPP. Then the important question is a measurement of the phases for the four beam bunches. An example is shown in Fig. E.5.

The match (red lines) with a prediction consistent with Fig. E.4 is good (see caption). The pattern on phases shows three angular separations of 1.822 rad and a final separation of 0.817 rad. This set of unequal gaps indicates that phase A is uniquely identifiable as the bunch synchronised with the maxima in the 871 kHz RF solenoid pattern at $t = 0$ (start of solenoid operation). Like the phase pattern in Fig. E.5, the angular separations in Fig. E.4 are also the same ($28.8^\circ$) except for the separation between bunches D and A, which is much larger.

In the case of the $1 + G\gamma$ harmonic recommended for this process, the pattern of three wide and one narrow angular separation in the polarimeter measurements changes to three narrow and one wide different in the phase pattern. The narrow angle is $1.32^\circ$ and the wide angle is $2.32^\circ$. This leads to a separation angle for the polarimeter measurement of $201.6^\circ$ between successive beam bunches in Fig. E.1.
Fig. E.5: Measurements of the polarisation phases for the four beam bunches in a test run made in December, 2018. The phase, measured along the horizontal axis, is shown as a function of time in the store. The phase is relative to a calculation of the polarisation direction based on an assumed value for the spin tune frequency ($f_{\text{REV}} G\gamma$) that yields a prediction of the phase at any moment in time during the store. A perfect match between the prediction and the measurements yields phase values that remain constant with time. The numbers on the curve correspond to the four bunches (A through D). Along the left-hand axis, a diagram using red lines shows the predicted relative phase separations that corresponds to the polarisation pattern shown in Fig. 4. Given the value of $G\gamma$, the separation of the phase lines should be either 1.822 rad (for pairs A-B, B-C, and C-D including wrapping through $2\pi$) or 0.817 rad (for pair D-A). This diagram gives a good account of the phase separations as measured.

In the scan for the axion, different axion phases are distributed with a sinusoidal dependence on the axion phase, as shown in Fig. E.6.

The different polarisation jumps is one of the features that distinguishes the detection of an axion from the observation of a machine resonance. In the case of the machine resonance, there is no phase and all four bunches should observe the same polarisation jump. The distribution of polarisation directions as shown in Fig. E.1 ensures the signals will appear with opposite signs for some pairs of directions.

Machine resonances must also appear at frequencies related the value of $G\gamma$ through

$$G\gamma = \ell + m\nu_X + n\nu_Y + k\nu_{\text{SYNC}} \quad (E.2)$$

where $\ell$, $m$, $n$, and $k$ are integers and the tunes ($\nu$) are connected to horizontal (X) and vertical (Y) betatron oscillations as well as synchrotron oscillations [9]. Smaller integers generally indicate stronger resonances. These checks should allow for the separation of axion signals from other effects.

E.4 Immediate Plans

A running period started 1 April 2019 with COSY for the purpose of testing the feasibility of creating a 4-beam setup and a frequency ramp with properties appropriate for conducting an axion search. The setup includes previously developed conditions for long IPP lifetime, which requires electron cooling as well as trimming the ring fields with sextupole components such that the X and Y chromaticities are simultaneously set to zero [7].

The new features begin with the four-bunch setup. The bunches must be well separated spatially so that there is no significant transfer of beam particles from one bunch to the next. This would tend
to depolarise the bunches, as the pattern in Fig. E.1 requires nearly opposite polarisation directions for neighbouring bunches. Polarisation measurements would ensue to check that the understanding of the relative phases between bunches is correct. This would constitute a confirmation of the patterns shown in the previous section.

The next step in the preparation would be the creation of conditions for ramping the machine revolution frequency to make frequency scans possible. Speeds would be slow, perhaps 0.1 Hz/s. Storage times of 150 s during the ramp means a frequency step of 15 Hz per scan. Since there are nearly opposite polarisation directions represented in the laboratory polarisation pattern, only one polarisation state is needed from the ion source. The data from one scan cannot be directly combined with another since the relative phase may change, even if the same axion is present. This value depends as well on the start time for the RF solenoid, and this cannot be synchronised with the axion phase. Multiple scans of the same frequency range are advisable since at any given time, an axion may not be present. Ramping the magnetic field of the COSY ring along with the frequency is required in order to maintain the circumference of the orbit. This allows the spin tune \( (G\gamma) \) to be known from the revolution frequency. However, the development of the software for dealing with IPP also provides for a direct measurement of the spin tune at any time during the process [10], and this will act as a confirmation that the machine conditions are being maintained.

With each scan, a comparison of the vertical polarisation component difference between the beginning and the end of the run is needed to determine whether or not there is evidence for a polarisation jump during the scan. The statistics of this comparison may be improved if there are times of no ramping before and after the actual ramp. Some threshold (such as 2 or 3 standard deviations) must be chosen. If passed, the scan should be repeated to determine whether or not it was an outlier. Once identified, additional scans are needed in order to have the statistics to determine the time location of the polarisation jump with precision.

Initial results from this development period are expected to be modest in terms of both the sensitivity and frequency range covered.
References
Appendix F

New ideas: Hybrid Scheme

Abstract
This appendix examines the possible replacement of electrical quadrupoles by magnetic quadrupoles for performing the focusing in the full scale ring, which is then referred to as a “hybrid ring”. Because alternating gradient magnetic focusing is used, simultaneous CW and CCW storage continues to be possible, while still allowing for moderately strong vertical focusing, along with the simultaneous CW and CCW storage needed for canceling important systematic errors. This promises to greatly reduce the contribution of radial magnetic field uncertainty to the EDM systematic error. 1

F.1 Experimental Method using a hybrid ring lattice

Simultaneous storage in clock-wise (CW) and counter-clock-wise (CCW) allows for the cancellation of important systematic errors [1, 3]. In combined electric and magnetic fields, e.g., the deuteron ring [2], it is not possible to store the beams CW and CCW simultaneously and much of the systematic error work was geared towards fixing potential problems due to that fact. The all-electric ring allows for it, however the main potential systematic error is large (a consequence of the large sensitivity on the proton EDM) and the required level to know the radial B-field around the ring is at the 10 aT level. High precision SQUID-based BPMs have been developed to be able to detect the required signal caused by the splitting of the counter-rotating beams [3, 4]. In order for the method to have high sensitivity to the potential systematic error, the vertical focusing strength is kept low, making it rather difficult to handle. A hybrid ring, in which alternating magnetic focusing is used, allowing simultaneous CW and CCW storage, allows for strong vertical focusing, and simultaneous CW and CCW storage for canceling important systematic errors [5].

The counter-rotating beams do not actually go through the same places everywhere, due to the fact that the vertical focusing includes magnetic focusing. Therefore, those beams may not exactly cancel those systematic errors at all places. However, we have shown that it is possible to use the same magnetic quads with flipped field directions (opposite sign currents) and on average the particles do follow the same trajectories. This idea seems to work very well, eliminating completely the radial B-field issue. In addition, the vertical dipole E-field effect is cancelled completely in CW and CCW injections as is the effect of gravity. The suggested working lattice is shown in Figure F.1, which is a modification of the lattice shown in the paper [4] describing the all-electric storage ring method, but this time the electric quadrupoles are replaced with corresponding magnetic ones. Figure F.2 shows the vertical beta-function of the CW and CCW stored beams, and Figure F.3 the corresponding for the horizontal. Flipping the sign of the currents in the magnetic quadrupoles will produce symmetric beta-functions for the CW and CCW beams.

However, it is always possible that some electric focusing will be present somewhere in the ring. This focusing and/or defocusing could originate from the bending electric field plates, which produce the required radial E-field. One or both plates could be misaligned, readily producing a vertical dipole, but also a quadrupole or even higher multipole E-fields. There could also exist induced charges (image charges) from any horizontally placed metals around the lattice, the tune shift and tune spread effects

1 This appendix was authored by Y.K. Semertzidis and S. Haciomeroglu of the Center for Axion and Precision Physics Research, KAIST, South Korea.

179
due to high beam intensities, etc. Some of those systematic errors we may be able to detect, e.g., by modulating the voltage on the bending E-field plates or control them by using beam bunch intensities of various strengths. At the end of the experiment, however, we need to have high confidence regarding the origin of the effect. Here we are suggesting using a number of runs with different vertical magnetic focusing strengths in order to differentiate between a systematic error and a genuine EDM signal.

Fig. F.1: A detail of the storage ring lattice is shown here with focusing and defocusing quadrupoles (shown as \(k_3\) and \(k_4\)). The bending sections, including the short straight sections, have a length of 10.417 m, three sections assembled as one unit. The long straight sections are 20.834 m long with a quadrupole (shown as \(k_4\)) in the middle and two half-length quads (shown as \(k_1\)) at both ends. The values of the magnetic quadrupole strength are: \(k_1 = 0.1 T/m\), \(k_2 = -0.1 T/m\), \(k_3 = -0.1 T/m\), \(k_4 = 0.1 T/m\). The vertical tune, when running with these quadrupole strengths, is \(Q_y = 0.67\), while the horizontal tune is \(Q_x = 1.73\).

The total effect, i.e. the vertical spin precession rate, is going to be in a functional form:

\[
R_V = R_{\text{EDM}} + R_{\text{Br}} \times \frac{Q_{\text{Backgr}}^2}{\zeta \times Q_{\text{Magnetic}}^2 + Q_{\text{Backgr}}^2 + \ldots}
\]

where \(R_V\) is referring to the total vertical spin precession rate, \(R_{\text{EDM}}\) refers to the portion due to the particle EDM, \(Q_{\text{Backgr}}^2 = f(Q_{\text{Electric}}^2, Q_{\text{Image Charge}}^2, Q_{\text{Beam Intensity}}^2, \ldots)\) corresponds to the square of the tuning due to non-magnetic effects, \(Q_{\text{Magnetic}}^2\) is the square of the tune due to the magnetic quads, \(Q_{\text{Electric}}^2, Q_{\text{Image Charge}}^2, Q_{\text{Beam Intensity}}^2\) are the square of the tunes due to the electric quads, the forces due to induced charges, and the forces due to the beam intensity, correspondingly. \(R_{\text{Br}}\) refers to the vertical spin precession rate due to the radial B-field. The point is that a net radial B-field can create a vertical spin precession, which can only be canceled exactly by another B-field; in this case we assumed it to be the magnetic focusing. Magnetic focusing can essentially eliminate this systematic error provided that it is the only source focusing the beam. Figure F.4 shows the average vertical offset of the stored beam as a function of the radial B-field multipole whose amplitude is always kept at 1 pT. Figure F.5 shows the vertical spin precession rate under the same conditions. A genuine EDM signal for \(10^{-29} e \cdot \text{cm}\) is larger than 1 nrad/s, and therefore much larger than the above background signal. However, if on one of the magnetic quadrupoles we add an overlapping electrical quadrupole with a strength of 1 kV/m², then we get the much larger spin precession rate of 0.4 nrad/s, for \(N = 4\) harmonic case of the radial B-field. This effect will be further and effectively suppressed by applying varying levels of magnetic field focusing, as described in the section below.
Experimental Approach. We apply a series of B-field focusing strengths, from weak to stronger ones to probe the EDM effect. With magnetic focusing the main systematic error is the out-of-plane dipole electric field, which is cancelled by CW and CCW beam storage as in the deuteron storage ring EDM experiment. Since simultaneous CW and CCW storage is possible in the current configuration, then most of the issues related to E-field direction stability go away. In addition, any focusing effect from the electric field plates or any other sources is sorted out by running the experiment at different alternating magnetic focusing strengths as shown in Figure F.6. Here, an additional electric focusing exists together with a DC ($N = 0$) radial magnetic field around the ring with strength of 1 pT. The electric focusing is originated by shaping all the bending plates, producing a vertical focusing with a field index of $m = 0.1$. 

---

**Fig. F.2:** The vertical beta-function values around the ring for CW and CCW operations. They flip sign when the magnetic quadrupoles are running with the opposite sign and therefore the counter-rotating particles on average trace the same paths.

**Fig. F.3:** The horizontal beta-function values around the ring for CW and CCW operations.

**Fig. F.4:** The average vertical beam offset when only magnetic focusing is used, as a function of the radial B-field multipoles ($N$-values). The amplitude of the background radial B-field is always kept at 1 pT, while the quadrupole strength is kept at ±0.1T/m.
The spin precession rate equation, when expanded, can be written as

\[ R_V = R_{\text{EDM}} + R_{Br} Q_{\text{Backgr}}^2 P_{m1} - R_{Br} Q_{\text{Backgr}}^4 P_{m1}^2 + \ldots \]

with \( P_{m1} = 1/(\zeta \times Q_{\text{Magnetic}}^2) \), showing clearly that for a large magnetic focusing tune, i.e., \( P_{m1} \to 0 \), the spin precession rate corresponds to the EDM signal. Hence, the DC offset in Figure F.6 corresponds to the EDM signal and the obtained value is consistent with the simulations. In Figure F.6, the spin precession rate corresponds to \( 10^{-28} \, e \cdot cm \) EDM level to prove the principle of the method. It will be advantageous to keep the spin precession rate lower by adding much stronger magnetic focusing cases and keep the electric focusing below the \( m = 0.01 \) level. The method will work best, requiring less leverage, when the magnetic focusing is dominating all other focusing effects. In a similar way, we can prove that the sextupole vertical electric field cancels with CW and CCW storage, etc., provided that the beam emittances are the same to an adequate level. From our simulations we infer that the SQUID-based BPMs resolution requirements are relaxed by several orders of magnitude over the lattice where electric focusing is used, which is a major breakthrough. The new requirements are a well-shaped quadrupole magnetic field in the ring, so that the center of the CW and CCW beams overlap within 100 nm at all magnetic quadrupole strengths, using the SQUID-based BPM signals. In addition, the ring needs to be flat (absence of corrugation) to 100 nrad, which we achieve by a combination of mechanical alignment, beam-based alignment and by using bunches polarized in the radial direction. A summary of the main systematic errors in the experiment with hybrid fields (electric bending and magnetic focusing) and their current remediation plan is given in Table F.1.

F.2 Systematic errors

F.3 Conclusions

The hybrid ring, where the radial E-field bends the stored beam, and an alternating B-field provides focusing allows for simultaneous CW and CCW storage eliminating the most important systematic error source. The experiment will also run at various magnetic focusing strengths to eliminate possible electric focusing sources, etc. In addition, the counter-rotating beams will sense any misalignment better than needed as well as the spin precession of a beam with a radial spin direction. The method needs to be studied by an independent group, which should take less than six months to complete.
Fig. F.6: The vertical spin precession rate as a function of the $P_m = 1/Q_y^2$ when the background effect is due to a combination of a DC ($N = 0$) radial magnetic field around the ring with strength of 1 pT and a large electric focusing effect of the bending plates. The bending plate focusing corresponds to an (electric) vertical focusing field index of $m = 0.1$. The fit result is from a first order polynomial. The DC-offset corresponds to the EDM precession rate, which in this case is $-1.9 \times 10^{-8}$ rad/s, consistent within the estimated errors to the input EDM value corresponding to $-4.1 \times 10^{-8}$ rad/s.

Table F.1: Main systematic errors and their remediation when hybrid fields (electric bending and magnetic focusing) are used.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Remediation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial B-field.</td>
<td>Magnetic focusing.</td>
</tr>
<tr>
<td>Radial B-field when other then</td>
<td>Varying magnetic focusing and fit for the DC offset in the vertical precession rate.</td>
</tr>
<tr>
<td>magnetic focusing is present.</td>
<td></td>
</tr>
<tr>
<td>Dipole vertical E-field.</td>
<td>CW and CCW beam storage.</td>
</tr>
<tr>
<td>Corrugated (non-planar) orbit.</td>
<td>Observe CW vs. CCW beam split with magnetometers, e.g., SQUID-based BPMs [3].</td>
</tr>
<tr>
<td></td>
<td>Probe with stored beams with their spins frozen in the radial direction [2].</td>
</tr>
<tr>
<td>RF cavity misalignment</td>
<td>Vary the longitudinal lattice impedance to probe the effect of the cavity’s vertical angular misalignment. CW and CCW beams cancel the effect of a vertically misplaced cavity. [4]</td>
</tr>
</tbody>
</table>

References

Appendix G

New ideas: Doubly Magic EDM Measurement Method

Abstract

This appendix discusses “doubly-magic trap” operation of storage rings with superimposed electric and magnetic bending, allowing spins in two beams to be frozen (at the same time, if necessary), and their application to electric dipole moment (EDM) measurement. Especially novel is the possibility of simultaneous storage in the same ring of frozen spin beams of two different particle types. A few doubly-magic cases have been found: One has an 86.62990502 MeV frozen spin proton beam and a 30.09255159 MeV frozen spin positron beam (with accuracy matching their known magnetic moments) counter-circulating in the same storage ring. (Assuming the positron EDM to be negligibly small) the positron beam can be used to null the worst source of systematic EDM error – namely, the existence of unintentional and unknown average radial magnetic field \(< B_r >\) which, acting on the MDM, causes spurious background spin precession indistinguishable from foreground EDM-induced precession. The resulting measured proton minus positron EDM difference is then independent of \(< B_r >\). This amounts to being a measurement of the proton EDM.

Most doubly-magic features can be tested in one or more “small” EDM prototype rings. One promising example is a doubly-magic proton-helion combination, which would measure the difference between helion (i.e. helium-3) and proton EDM’s. This combination can be used in the near future for EDM measurement in a small, 10 m bending radius ring, using only already well-understood and proven technology. In the standard model both EDM’s are negligibly small. Any measurably large difference between these EDM values would represent “physics beyond the standard model”.

G.1 Introduction

G.1.1 Major previous EDM advances

Comparably important EDM advances that have been made in the recent past can be listed: The storage ring “frozen spin concept” according to which, for a given particle type, there can be a kinetic energy for which the beam spins are “frozen” in a storage ring—for example always pointing along the line of flight, Farley et al. [1]; The recognition of all-electric rings with “magic” frozen spin kinetic energies (14.5 MeV for electrons, 233 MeV for protons) as especially appropriate for EDM measurement, Semertzidis et al. [2]; The “Koop spin wheel” mechanism, in which a small radial magnetic field \(B_r\) applied to an otherwise frozen spin beam causes the beam polarisation to “roll” around a locally-radial axis [3] (systematic precession around any axis other than this would cancel any accumulating EDM effect); Spin coherence times long enough for EDM-induced precession to be measurably large, Ewersmann et al. [4]; “Phase-locking” the beam polarisation, which allows the beam polarisation to be precisely manipulated externally, Hempelmann et al. [5].

\(^1\)This appendix, very slightly changed here, was originally distributed by Richard Talman as arXiv article 1812.05949, [physics.acc-ph], submitted 14 Dec 2018.
G.1.2 Koop spin wheel

By design, the only field components in the proposed ring would be the radial electric component $E_x$, and ideally-superimposed magnetic bending would be provided by a vertical magnetic field component $B_y$. There also needs to be a tuneable radial magnetic field $B_r \equiv B_x$, both to compensate any unintentional and unknown radial magnetic field and to control the roll-rate of the Koop spin wheel.

For a “Koop spin wheel” rolling around the radial $x$-axis, unpublished notes from a Juelich lecture by I. Koop [6] provide formulas for the roll frequencies (expressed here in SI units, with $B\rho$ in T.m),

$$\Omega_{Bx} = -\frac{1}{B\rho} \frac{1 + G}{\gamma} cB_x, \quad \text{and} \quad \Omega_{EDM}^{Bx} = -\frac{1}{B\rho} \left( \frac{E_x}{c} + \beta B_y \right).$$

(G.1)

$G$ is the anomalous magnetic moment, $\beta, \gamma$ are relativistic factors. $\Omega_{EDM}^{Bx}$ is the foreground, EDM-induced roll frequency. $\Omega_{Bx}^{Koop}$ is a roll frequency around the same radial axis, caused by a radially magnetic field $B_x$ acting on the MDM. $cB\rho = pc/(qe) \equiv pc/(Ze)$ is the standard accelerator physics specification of storage ring momentum. The factor $\eta$ expresses the electric dipole moment $d = \eta \mu$ in terms of the magnetic moment $\mu$ of the beam particles.

G.1.3 Proposed EDM measurement technique

The proposed EDM measurement technique starts by measuring and nulling

$$\Omega_{Koop}^x = \Omega_{EDM}^x + \Omega_{Bx}^x \rightarrow 0$$

for the spin wheel of a secondary beam. The secondary beam is then dumped and, with no change of ring conditions whatsoever, the matching frozen spin primary beam is stored. Since the primary beam is subject to the same radial magnetic fields as the secondary beam, its $\Omega_{EDM}^x$ roll rate will then provide a direct measurement of the primary beam EDM $d$.

Previously one will, of course, also have followed Koop in minimising $\langle B_x \rangle$, by measuring the differential vertical separation of the two beams, which is similarly proportional to $\langle B_x \rangle$.

G.1.4 Polarimetry assumptions

Ultimate EDM precision may depend on resonant polarimetry, probably based on the Stern-Gerlach interaction [14] [15] [16]. Meanwhile, impressive beam polarisation control has been achieved using polarimetry based on left-right scattering asymmetry of protons or deuterons from carbon [5], and much more progress will undoubtedly be made with this method. Any prototype EDM ring to be built in the near future will need to rely initially on this form of scattering asymmetry polarimetry.

G.2 Orbit and spin tune calculation

G.2.1 Terminology

Fields are “cylindrical” electric $E = -E_0 \hat{x}r_0/r$ and, superimposed, uniform magnetic $B = B_0 \hat{y}$. The bend radius is $r_0 > 0$. Terminology is needed to specify the relation between electric and magnetic bending: Cases in which both forces cause bending in the same sense will be called “constructive” or “frugal”; Cases in which the electric and magnetic forces subtract will be referred to as “destructive” or “extravagant”. There is a reason for the “frugal/extravagant” terminology to be favoured. Electric bending is notoriously weak (compared to magnetic bending) and iron-free (required to eliminate hysteresis) magnetic bending is also notoriously weak. As a result an otherwise-satisfactory configuration can be too “extravagant” to be experimentally feasible.

The design particle has mass $m > 0$ and charge $qe$, with electron charge $e > 0$ and $q = \pm 1$ (or some other integer). These values produce circular motion with radius $r_0 > 0$, and velocity $v = v\hat{z}$,
where the motion is CW (clockwise) for $v > 0$ or CCW for $v < 0$. With $0 < \theta < 2\pi$ being the cylindrical particle position coordinate, the angular velocity is $d\theta/dt = v/r_0$.

To limit cases we consider only electrons (including positrons) protons, deuterons, tritons, and helions; that is e-, e+, p, d, t, and h. The circulation direction of the so-called “master beam” (of whatever charge $q_1$) is assumed to be CW or, equivalently, $p_1 > 0$. The secondary beam charge $q_2$ is allowed to have either sign, and either CW or CCW circulation direction.

G.2.2 Fractional bending coefficients $\eta_{\text{E}}$ and $\eta_{\text{M}}$

(In MKS units) $q\varepsilon E_0$ and $q\varepsilon \beta c B_0$ are commensurate forces, with the magnetic force relatively weakened by a factor $\beta = v/c$ because the magnetic Lorentz force is $q\varepsilon v \times B$. Newton’s formula for radius $r_0$ circular motion can be expressed using the total force per unit charge in the form

$$F_{\text{tot}} = \frac{\beta p c}{e} E_0 + q \beta c B_0,$$

coming from the cross-product Lorentz magnetic force, the term $q \beta c B_0$ is negative for backward-travelling orbits because the $\beta$ factor is negative. The “master” beam travels in the “forward”, CW direction. For the secondary beam, the $\beta$ factor can have either sign. For $q = 1$ and $E_0 = 0$, formula (G.3) reduces to the standard accelerator physics “cB-phi=pc/e”. For $E_0 \neq 0$ the formula incorporates the relative “effectiveness” of $E_0/\beta$ and $cB_0$.

Fractional bending coefficients $\eta_{\text{E}}$ and $\eta_{\text{M}}$ are then defined by

$$\eta_{\text{E}} = \frac{r_0}{p c / e} \frac{E_0}{\beta}, \quad \text{and} \quad \eta_{\text{M}} = \frac{r_0}{p c / e} \frac{c B_0}{\beta},$$

neither of which is necessarily positive. They satisfy $\eta_{\text{E}} + \eta_{\text{M}} = 1$.

G.2.3 Spin tune expressed in terms of $\eta_{\text{E}}$ and $\eta_{\text{M}}$

With $\alpha$ being the angle between the in-plane component of beam polarisation and the beam direction, the “spin tune” is defined to be the variation rate per turn of $\alpha$, expressed as a fraction of $2\pi$. Spin tunes in purely electric or purely magnetic rings are given by

$$Q_{\text{E}} = G - \frac{1}{\gamma^2 - 1} \gamma^2 = G - \frac{G + 1}{\gamma}, \quad Q_{\text{M}} = G\gamma,$$

with superimposed fields, the spin tune can be expressed in terms of the fractional bending coefficients,

$$Q_{\text{S}} = \frac{d\alpha}{d\theta} = Q_{\text{E}} \eta_{\text{E}} + Q_{\text{M}} \eta_{\text{M}}.$$

G.2.4 The “magic energy” condition.

Superimposed electric and magnetic bending permits beam spins to be frozen “frugally”; i.e. with a ring smaller than would be required for all-electric bending. The magic requirement is for spin tune $Q_{\text{S}}$ to vanish;

$$Q_{\text{S}} = \eta_{\text{E}} Q_{\text{E}} + (1 - \eta_{\text{E}}) Q_{\text{M}} = 0.$$

Solving for $\eta_{\text{E}}$,

$$\eta_{\text{E}} = \frac{G}{G + 1} \gamma^2, \quad \eta_{\text{M}} = 1 - \frac{G}{G + 1} \gamma^2.$$

For example, with proton anomalous moment $G_p = 1.7928474$, trying $\gamma = 1.25$, we obtain $\eta_{\text{E}} = 1.000$ which agrees with the known proton 233 Mev kinetic energy value in an all-electric ring. For protons in the non-relativistic limit, $\gamma \approx 1$ and $\eta_{\text{E}}^{\text{NR}} \approx 2/3$. The magic electric/magnetic field ratio is

$$\frac{E}{cB} = \frac{\beta \eta_{\text{E}}}{\eta_{\text{M}}} = \frac{\beta G \gamma^2}{1 + G(1 - \gamma^2)} = \frac{G \beta \gamma^2}{1 - G / \beta^2 - \gamma^2}.$$

186
G.2.5  Wien filter spin-tune adjustment

Superimposed electric and magnetic bending fields allow small correlated changes of $E$ and $B$ to alter the spin tune without affecting the orbit. Being uniformly-distributed, appropriately matched electric and magnetic field components added to pre-existing bend fields can act as a (mono-directional) “global Wien filter” that adjusts the spin tune without changing the closed orbit. Replacing the requirement that $\eta_E$ and $\eta_M$ sum to 1, we require $\Delta \eta_M = - \Delta \eta_E$, and obtain, using the same fractional bend formalism, for a Wien filter of length $L_W$ the spin tune shift caused by a Wien filter of length-strength product $EL_W$ is given by

$$\Delta Q_s^{W} = - \frac{1}{2\pi} \frac{1 + G}{\beta^2 \gamma^2} \frac{EL_W}{mc^2/e}. \quad (G.9)$$

For “global” Wien filter action, $L_W$ is to be replaced by $2\pi r_0$.

G.3  “MDM comparator trap” operation

G.3.1  Dual beams in a single ring.

This section digresses temporarily to describe the functioning of dual beams in the same ring as a “spin tune comparator trap”. A “trap” is usually visualised as a “table-top apparatus”. For this appendix “table-top radii” of 10, 20, or 50, meters (or rather curved sectors of these radii, expanded by straight sections of comparable length) are considered.

Gabrielse [8] has (with excellent justification) boasted about the measurement of the electron magnetic moment (with 13 decimal point accuracy) as “the standard model’s greatest triumph”, based on the combination of its measurement to such high accuracy and on its agreement with theory to almost the same accuracy. Though other magnetic moments are also known to high accuracy, compared to the electron their accuracies are inferior by three orders of magnitude or more. One purpose for a spin-tune-comparator trap would be to “transfer” some of the electron’s precision to the measurement of other magnetic dipole moments (MDM’s). For example, the proton’s MDM could perhaps be determined to almost the current accuracy of the electron’s.

Different (but not necessarily disjoint) co- or counter-circulating beam categories include different particle type, opposite sign, dual speed, and nearly pure-electric or pure-magnetic bending. Cases in which the bending is nearly pure-electric are easily visualised. The magnetic bending ingredient can be treated perturbatively. This is especially practical for the 14.5 MeV electron-electron and the 233 Mev proton-proton counter-circulating combinations.

Eversmann et al. [4] have demonstrated the capability of measuring spin tunes with high accuracy. By measuring the spin tunes of beams circulating in the same ring (not necessarily simultaneously) the MDM’s of the two beams can be accurately compared.

G.3.2  Sensitivity to imperfections

So far only perfect apparatus has been considered. Here we comment on imperfections. The main attribute to be claimed for the spin tune comparator will be its relative insensitivity to imperfections. Whatever validity is claimed will come from a combination of (1) basing parameter determinations only on frequency measurement, (2) accurate knowledge of the MDM’s, and (3) on the degree to which the spatial orbits of co- or counter-circulating beams are constrained to be identical to high accuracy. Also important will be the degree to which the ratio of electric to magnetic field is constant around the ring.
Master beam and potential secondary beams on the same design orbit

Fig. G.1: Examples of “secondary beams” designed to have the same design orbits as a (shaded) beam 1 “master beam”. Electric and magnetic force strengths are crudely represented by the lengths of their (bold-face) vectors. This figure is limited to very-relativistic (VR) electrons (of either sign) and not-very-relativistic (NVR) protons (of either sign). CW and CCW orbits are identical, except for traversal direction. For stable beam circulation the sum of electric and magnetic forces has to be centripetal. This condition is violated in case (a); the centrifugal electric force exceeds the centripetal magnetic force.

(To be shown shortly) radial positioning errors are not a serious concern but requiring the design orbits to be accurately planar (i.e. lying in a single horizontal plane) markedly improves the MDM (and later the EDM) measurement accuracy.

The reason for controlling vertical orbit excursions to better accuracy than horizontal has to do with spin precession control. Let us assume that element positions are established initially to ±100 micrometer accuracy horizontally, and ±10 micrometer accuracy vertically. Corresponding angular precision tolerances of about one-tenth milliradian horizontally and one-hundredth milliradian vertically will also be assumed.

Quoting G. Decker from 2005 [9] “Submicron beam stability is being achieved routinely at many of these light sources in terms of both AC (rms 0.1 - 200 Hz) and DC (one week drift) motion.” For fairly-smooth orbits, if the orbits are that close at all BPM locations, they will be almost that close everywhere. With both spin tunes accurately measured, and their MDM’s known, the average circumference uncertainty will be dominated by spin tune measurement inaccuracy, which could correspond to 11 decimal point circumference accuracy.

In any case it is the circumference differences rather than the individual circumferences that will govern the accuracy of the spin tune comparator. After nulling all BPM differences, the CW and CCW circumferences will then be equal to about 13 decimal places.

With revolution period known “perfectly” from RF frequency measurement, and average velocity known “perfectly” from frozen spin and accurately known MDM, even the absolute circumference value will be known to high accuracy.

G.3.3 Spin tune invariance and spin tune comparator trap precision

By Eqs. (G.5) spin tunes \( Q_E \) and \( Q_M \) depend only on \( G \) and \( \gamma \) but not on bend radius \( r_0 \). This implies, for planar orbits, that spin tunes are conserved constants of the motion, independent of horizontal steering
errors—assuming, of course, that components stay rigidly fixed in place.

But (because of commutativity failure for rotations around non-parallel axes) vertical steering errors prevent the spin tune formulas from being universally valid conservation laws. Even so, from up-down symmetry, one expects the change $\Delta Q_S$ in spin tune caused by a vertical deflection angle $\Delta y'$ to be proportional to $\Delta y'^2$. By limiting the magnitudes of vertical deflection angles $\Delta y'$ to be less than, say $10^{-7}$, one can expect the spin tunes $Q_E$ and $Q_M$ to be independent of lattice errors to, e.g. 14 decimal place accuracy. Knowing the spin tunes and $\gamma$ values of both beams precisely, and knowing the MDM of the particles in one of the beams, allows the MDM of particles in the other beam to be determined to high accuracy.

This is how a “spin tune comparator trap” can compare MDM’s precisely. Parameter tolerances for EDM measurement will be comparable to those discussed in the previous section.

G.4 Secondary beam solutions

G.4.1 Analytic formulation

Assume the parameters of a frozen spin master beam have already been established. As well as fixing the bend radius $r_0$, this fixes the electric and magnetic bend field values $E_0$ and $B_0$. A further constraint that needs to be satisfied for secondary beam operation is implicit in the equations already derived. To simplify the formulas we make some replacements and alterations, starting with

$$\frac{p c}{e} \rightarrow p, \quad \text{and} \quad \frac{m c^2}{e} \rightarrow m,$$

(G.10)

The mass parameter $m$ will be replaced later by, $m_p$, $m_d$, $m_{\text{tritium}}$, $m_e$, etc., as appropriate for the particular particle type. These changes amount to switching the energy units from joules to electron volts and setting $c = 1$.

The number of ring and beam parameters can be reduced by forming the combinations

$$E = q E_0 r_0, \quad \text{and} \quad B = q c B_0 r_0.$$  

(G.11)

After these changes, the closed orbit condition has become

$$p^4 - 2B p^3 + (B^2 - E^2) p^2 - E^2 m^2 = 0,$$  

(G.12)

an equation to be solved for secondary beam momentum $p$. Any solution meets the requirement for spin tune comparator functionality, but not yet, in general, the doubly-magic, vanishing-spin-tune condition.

Any stable secondary beam orbit has to satisfy this equation but, because the electric and magnetic field values have been squared, not every solution of the equation has electric and magnetic field values that match the signs or magnitudes of the field values $E_0$ and $B_0$ constrained by the primary beam. So solutions of Eq. (G.12) have to be culled for consistency. The bending force has to be centripetal and consistent with bending in a circle of radius $r_0$.

By construction the already-established existence of a stable master beam implies the existence of a real, CW (i.e. $p_1 > 0$) solution of the equation, say with mass $m = m_1$. We look for other stable solutions, say with mass $m = m_2$ and momentum $p_2$, for which there are no parameter changes whatsoever, neither in $E_0$ nor $B_0$, nor in the sign or magnitude of the bend radius of curvature.

For spin tune comparator functionality, satisfying Eq. (G.12) is sufficient for finding compatible dual beam parameters, including determining their spin tunes to the high precision with which the anomalous magnetic moments are known.

If anti-protons, anti-deuterons, or other anti-baryons were experimentally available, the flexibility provided by Eq. (G.12) would be especially useful. The TCP combination of time, charge, and parity symmetry transformations would then provide TCP-matched solutions of the equation. But the only
available negative particle is the negative electron, so TCP invariance applies usefully only to beam combinations containing an electron or a positron beam.

Limiting particle types to positron, proton, deuteron, tritium, and helions, a fairly comprehensive list of promising “doubly-magic candidate” solutions has been produced, satisfying these requirements, including the requirement that the master beam satisfy the magic beam condition.

For EDM measurement functionality the further constraints to be met are severe. With parameters established and set such that the “master beam” is magic, the only remaining free parameter is the secondary beam energy. Doubly magic solutions are sought by varying this energy (always constraining the primary beam to satisfy the spin condition G.7). As well as meeting the vanishing spin tune condition, the energy also has to be such that beam production and handling is practical, and high quality polarimetry is available.

G.5 Three practical doubly-magic solutions

G.5.1 Promising doubly-magic solutions

Several doubly magic beam pairs have been discovered. For this appendix just three cases are considered. Their parameters are given in Table G.1. Details are given in the figure caption and case by case explanations are given in the sequel.

Eq. (G.12) has been solved with MAPLE to produce Table G.1. (Intended only for checking derived results, and otherwise unreliable) the numerical anomalous magnetic moment values used have been:

\[
G_{\text{[positron, e+]}} = 0.00115965218076 \\
G_{\text{[proton, p]}} = 1.79284735650 \\
G_{\text{[helion, h]}} = -4.18396274016 
\] (G.13)

<table>
<thead>
<tr>
<th>r0 m</th>
<th>KE1 GeV</th>
<th>E0 V/m</th>
<th>B0 T</th>
<th>(\eta_E)</th>
<th>KE2 GeV</th>
<th>pc2</th>
<th>QS2</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>CW p</td>
<td>0.2328</td>
<td>8.386e+06</td>
<td>1.6e-08</td>
<td>1</td>
<td>CCW p</td>
<td>0.2328</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CW p</td>
<td>0.2328</td>
</tr>
<tr>
<td>(b)</td>
<td>PERTURBED DOUBLY-MAGIC PROTON-PROTON (original) HOLY GRAIL option</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>CW p</td>
<td>0.08663</td>
<td>6.355e+06</td>
<td>0.016</td>
<td>0.766</td>
<td>CCW e+</td>
<td>0.03009</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(c2) DOUBLE-MAGIC POSITRON-PROTON (inverse of (c1))</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>CW e+</td>
<td>0.03009</td>
<td>6.355e+06</td>
<td>-0.016</td>
<td>4.155</td>
<td>CCW p</td>
<td>0.08664</td>
</tr>
<tr>
<td>(c1) DOUBLY-MAGIC HELION-PROTON (new) HOLY GRAIL option</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>CW h</td>
<td>0.03924</td>
<td>5.265e+06</td>
<td>-0.028</td>
<td>1.351</td>
<td>CCW p</td>
<td>0.03859</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(q2) DOUBLE-MAGIC PROTON-HELION (inverse of (q1))</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>CW p</td>
<td>0.03859</td>
<td>5.265e+06</td>
<td>0.028</td>
<td>1.6958</td>
<td>CCW h</td>
<td>0.03924</td>
</tr>
</tbody>
</table>

Table G.1: Beam-pair combinations for the three EDM experiments discussed in this appendix; master beam entries on the left, secondary beam on the right. “(b)”, “(c1)”, etc. are case labels. Dual rows allow either particle type to be designated “master beam”. Candidate beam particle types are “e+”, “p”, “d”, “t”, “h” labelling positron, proton, deuteron, triton, and helion rows. Bend radii, particle type, and kinetic energies are given in the first three columns. There is no fundamental dependence of spin tune \(Q_s\) on \(r_0\), but \(r_0\) values have been chosen to limit \(|E_0|\) to realistic values. Bend radii choices of 10 m, 20 m, and 50 m result from the compromise between reducing ring size and limiting electric field magnitude. \(r_0\) can be increased beneficially except for cost in all cases, but not necessarily decreased. Master beam spin tunes are always exactly zero. Spin tunes of secondary beams are given in the final column. In all cases they are close enough to guarantee they can be tuned exactly to zero. Further, case by case, explanations are given in the text.
Example (b) is perturbatively close to the already-known, singly-magic, all-electric solutions for protons. Examples (c1) and (c2) are doubly-magic solutions with positron and proton beams; the dually tabulated cases make the point that either beam can be interpreted as being the “master beam”. Example (q1) and (q2) show doubly-magic solutions with proton and helion beams.

G.5.2 Perturbative variant of all-electric (original) holy grail ring.

Case (b) in Table G.1 represents the all-electric frozen-spin proton ring which, up to now, has been implicitly anticipated to be the ultimate apparatus for measuring the proton EDM. With its detailed features not yet understood this ring has been christened as the “holy grail” ring. Not intentionally pejorative, this language has been intended to acknowledge the significant uncertainties concerning the detailed properties of such a ring. In the table this name has been changed to “(original) HOLY GRAIL” ring, so as to leave available the name “(new) HOLY GRAIL” ring, for the ultimate EDM ring proposed in case (c1).

In fact, case (b) is already a more realistic representation of the all-electric ring in the sense that some residual non-vanishing vertical magnetic field will be inevitable, even in an all-electric ring. This will require simultaneously-frozen-spin beam energies to have slightly different energies in all cases.

With distributed electric and magnetic fields, using Eq. (G.9) to describe the performance of the entire ring as a Wien filter, it will not be difficult to meet the doubly-magic condition, even in the presence of extraneous weak vertical magnetic field. In itself, this would not justify distributed magnetic field, however, as the same trimming could be done with a short local Wien filter.

However the “perturbative” solutions (available also for all-electric electron, triton, and carbon 13 frozen spin rings) are very robust in the sense that the superimposed magnetic field can be varied over a large range while preserving the doubly-magic capability. This opens up the possibility of investigating systematic EDM errors by varying the magnetic bending fraction by a large factor.

This robust property applies uniquely to perturbations away from an all-electric ring. (In this case only) the structure of Eq. (G.12) guarantees that there is a continuum of doubly-magic solutions in the vicinity of the all-electric condition. With counter-circulating beams of the same particle type, if the bending is frugal for one beam it is necessarily extravagant for the other. But, since the sign of \( \eta_M \) reverses at the all-electric point, the continuity of solutions of Eq. (G.12) guarantees the existence of a continuum of doubly-magic solutions in this vicinity. This is the justification for attaching “perturbed” to the name of case (b). This opens the possibility of reducing EDM systematic errors by acquiring data in configurations with substantially different magnetic fields.

There is a complication concerning RF frequency, in that slightly different beam velocities will cause either slightly different orbits or slightly different revolution periods. For slow particles, such as protons, this may require running on different harmonics of a single RF cavity. For positrons, because they are fully relativistic, this would probably be impractical, and the orbits would have to differ slightly. This RF issue is addressed explicitly below in the discussion of proton-helion case (q1).

G.5.3 Proton-positron solution—the (new) holy grail.

From the point of view of greatest promise for ultimate fundamental physics discovery, case (c1) (with equivalent case (c2)), for proton and positron beams, seems to be the most promising case. It enables measurement of the difference between a master beam containing protons and a secondary beam containing positrons.

Cancelling the Koop wheel roll rate of the secondary beam containing positrons cancels the radial magnetic field (under the assumption that the positron EDM is negligibly small). This allows the primary beam Koop wheel roll rate to serve as a measurement of the proton EDM.

As well as providing a clean, frequency difference measurement of the proton EDM, the beams
can circulate simultaneously. Because positron and baryon velocities differ by an order of magnitude, it is probably impractical for the acceleration to be provided by harmonics of a single RF cavity; dual RF systems will be needed.

A major impediment in this case is the low analysing power of existing polarimetry methods for electrons (of either sign). To remove the “holy grail” qualification in this case will require the development of resonant electron polarimeter. This limitation is discussed further below. Achieving non-destructive, high analysing power electron polarimetry seems likely to be the only remaining major impediment to using EDM measurement to test the “standard model” of particle physics.

G.5.4 Helion-proton solution, JEDI-capable option.

Like the doubly-magic baryon-positron pair solutions, doubly-magic, different-type baryon-baryon pairs can be used to obtain EDM differences. A doubly-magic triton/proton solution has been found, but it requires electric fields that are probably unachievable, even in the largest ring currently under consideration.

However, by fortuitous accident of their anomalous magnetic moments, there is a doubly-magic helion/proton solution (q1) (with equivalent (q2)) that needs only a small ring. (The development of a polarised helion beam at BNL is described by Huang et al. [18].) For this case radius \( r_0 \) has been taken in the table, in round numbers, to be 10 m. But (with electric field increased by 10 percent) this case is intended to match the 9 m bending radius EDM prototype ring described elsewhere in this present CERN yellow report.

The (q1) case has a CW, frugal bending solution for protons as master beam, with a CCW, extravagant bending helion beam as secondary beam. Carbon scattering asymmetry polarimetry will presumably be used for both beams.

With a single RF cavity, to account for the different proton and helion velocities, the RF harmonic numbers can be 107 and 180, resulting in revolution period fractional difference of \( 3 \times 10^{-6} \).

What makes this doubly-magic proton-helion option exciting is that, in the near future, using only currently-established experimental techniques, an upper limit for the EDM of baryons can be substantially reduced from current limits, possibly even to a level capable of demonstrating “physics beyond the standard model”.

G.5.5 Stability of the 233 MeV all-electric fixed point

An important motivation for building an EDM prototype ring followed from the observation that inevitable magnetic field contamination will cause an all-electric ring to be an unrealisable idealisation. Stated more succinctly, it is impossible to construct a ring with exact time reversal symmetry. (Especially considering the extreme narrowness of the frozen spin condition) this observation has the important consequence that beams, identical except for direction, cannot counter-circulate simultaneously. This threatens to make a truly all-electric ring unsatisfactory. Note, though, that the word identical was italicised in the previous sentence. This leaves open the possibility of non-identical beams counter-circulating simultaneously. The following discussion expands upon this possibility.

This section defines “frozen-spin, fixed point stability” to be general enough to assure that counter-circulating proton beams can be simultaneously frozen, even if some fraction of the bending is magnetic rather than electric. With vanishing magnetic field this is assured by time reversal symmetry. The problem is that any magnetic field present in the ring, no matter how small, will destroy the time-reversal symmetry property that a truly all-electric ring guarantees. For brevity the treatment of only the most serious failure of time reversal symmetry will be described. Formally speaking, all that is known is that, for protons, the ring has a fixed point near 233 MeV. But it is not automatically known whether this fixed point is stable or unstable.
It was shown theoretically in an appendix to the original Courant and Snyder alternating gradient paper [19], that magnetic ring orbits are guaranteed to have a stable periodic closed orbit fixed point, with the consequence that there is an ellipsoidal region of six dimensional phase space in which particles can circulate indefinitely. The following argument, essentially a continuation of the earlier “perturbed doubly-magic proton-proton” sub-section, shows that the all-electric ring is similarly stable, even in the presence of (sufficiently weak) superimposed magnetic field.

Assume, for example, the presence of a small, unintentional, vertical magnetic bending field perturbation $\Delta B_y$, superimposed on an otherwise-ideal all-electric ring. Such a perturbing field would provide constructive bending for one beam and destructive bending for the other. This would violate the time reversal symmetry of the apparatus, which might seem to prevent both beams from having frozen spins at the same time. It needs to be shown that this is not the case.

The leading effect of the $\Delta B_y$ perturbation would be, after RF capture and bunching, to reduce the average momentum of one beam and increase that of the other. Would neither beam, therefore, have the “magic” momentum needed for frozen spins? In fact, as in the prototype ring, the spin tune can vanish even in the presence of some magnetic bending. So at least one of the beams, let us say the lower momentum beam, can be adjusted to have frozen spins.

What about the other beam, the beam with higher momentum? Most succinctly stated, the frozen spin condition is for the spin tune to vanish. Plotted as a function of beam momentum, the spin tune therefore has to change sign as the beam momentum passes smoothly through the magic condition. Fortunately the electric/magnetic frozen spin bending requirement has the same dependence on momentum; if the electric and magnetic bending fractions have to be constructive (as they do for protons for momentum below the all-electric magic value), then they have to be destructive for momentum above the all-electric value. As just explained, for the higher momentum, oppositely directed, proton beam, the electric and magnetic forces do, indeed, combine destructively—as needed. As a result, with the available degrees of freedom, if one beam satisfies the frozen spin condition, parameters can be varied such that the spins of the other beam are frozen as well. In fact, in the small $B_y$ limit, if one beam is magic, then, automatically, the other beam is magic also.

The conclusion is that the “all-electric” ring does not, in fact, have to be all-electric for both beam polarisations to be frozen. In the present language, the ring is “doubly magic”.

Of course, a price has to be paid. With the two beams having different (average) momenta, yet bunched longitudinally by the same RF cavity, the two beam circumferences will have to be (slightly) different. A correction for the effect of this imperfection would then have to be dead-reckoned theoretically, or handled some other way. Such corrections need to be applied in every precise experiment. And this correction will surely be small compared to other anticipated errors.

Continuing the fixed-point stability proof, the prototype ring accepts the existence of a $B_y = 0.03$ T magnetic bending perturbation that is many orders of magnitude greater than will be likely in the full-scale ring, yet freezes the beam spin nevertheless. This is not at all ideal for EDM determination, because an inevitable consequence of intentionally applying a strong $B_y$ bending field in the prototype ring will be an unintentional $\Delta B_r$ radial magnetic field, that can be expected to be much greater than whatever radial magnetic field will be present in the nominal, all-electric ring. By being “all-electric” the full-scale ring will be optimal, at least in this respect.

A useful way of understanding the effect of such an added $\Delta B_y$ magnetic field, is to realise that the entire ring can be regarded as a distributed Wien filter. For protons of velocity $v_p$ there is a matching radial electric field $\Delta E_r = v_p \Delta B_y$ field which, applied to the bending electrodes, exactly restores one or the other of the circulating beam design orbits, while doubling the perturbing effect on the other. This procedure effects all bunches identically. As explained in the prototype chapter, to tune individual bunches requires a local, stripline-based Wien-filter.

Once one has accepted the existence of an “unintentional” $B_y$ magnetic field (perhaps at the micro-
Tesla level) one is tempted to consider the intentional inclusion of a $B_y$ magnetic field (perhaps at the milli-Tesla level, or even at the 0.03 T level of the prototype ring.) (While the deuteron combined E/B ring was still in serious contention, the inevitable presence of a strong magnetic field was occasionally pitched as a “feature”, rather than as a “bug”, because it provided another variable degree of freedom that could be used to refine the EDM determination.)

But the price of intentionally stronger $B_y$ is correspondingly greater. Greater momentum difference between counter-rotating beams could lead to unacceptably large orbit differences that could only be avoided by dual RF cavities, or running on adjacent harmonic numbers of the single cavity. Neither approach is attractive, but either could, perhaps be implemented.

G.6 Gravitational effect EDM calibration

Various authors [11] [12] [13] have pointed out that general relativity (GR) introduces effects that could be measurably large in proposed EDM rings. László and Zimborás [10] calculate the GR influence on storage rings designed for EDM measurement. The GR effect mimics the EDM effect. Mistaken attribution to proton EDM produces a spurious proton EDM value of approximately $3 \times 10^{-28}$ e-cm. This is about thirty times greater than the precision anticipated for the (original) holy grail ring and not inconsistent with an Orlov, Flanagan, Semertzidis [13] estimate. It is an accuracy that should be achievable with a small EDM prototype ring.

The GR effect has two main ingredients, both essentially classical.

– One is a “toy-top-like precession” caused by the earth’s (uniform) gravitation field applying torque to the particle angular momentum. More accurately, the torque causing out-of-plane precession is applied by ring focusing electric fields acting on the particle MDM. Long time beam survival guarantees the absence of average vertical force.

– The other is a secular “Foucault-pendulum-like” precession of the angular momentum during repeated transits of a closed circular path.

Once under control, the GR signal will serve as a valuable calibrator of the EDM detection apparatus. The absolute level of this calibration signal will be at the optimistic (i.e. large EDM value) end of the range of plausible “physics beyond the standard model”.

G.7 The need for non-destructive resonant polarimetry

Arthur Schawlow, co-inventor of the laser, is credited with the advice to “Never measure anything but frequency”. Though not emphasised up to this point, this principle is implicit in the present paper. Though this advice is often accepted, its basis is rarely explained.

In our case the EDM signal at the end of an hour-long run may be an EDM-induced beam polarisation angular difference of, say, a milliradian, between initial and final beam polarisation orientations. Expressed as a fraction of a complete revolution of the beam polarisation, this is $10^{-3}/(2\pi)$. For any single run this angular shift is likely to be comparable with the difference uncertainty of destructive polarimetry initial and final orientation measurements. (Then by averaging over, say, one thousand runs, the statistical error can be reduced by a factor of thirty or so.)

Consider the same hour-long run with non-destructive resonant polarimetry, assuming, for the moment, the polarimeter natural resonant frequency to be the same as the beam revolution frequency. When sensed instantaneously, the resonator phasor angular advance from run beginning to run end is likely to approximately match the $10^{-3}/(2\pi)$ difference of the previous paragraph, with “phase noise” having yielded approximately the same uncertainty. But (absent other sources of low frequency noise) after non-destructive averaging the resonator phase for few-minute intervals at both beginning and end, the per-run phasor angular advance can be determined with far less uncertainty than is possible with destructive scattering asymmetry.
This has not yet included two other factors that favour resonant polarimetry. One of these factors is that the whole beam is measured at both beginning and end. With destructive polarimetry, at best, orientation of only half of the beam is measured at run beginning; the other half of the beam is measured at the end.

The other advantage of resonant polarimetry would be that, in practice, the resonant polarimeter frequency will be in the GHz range, 1000 times higher than the revolution frequency. Generally speaking, absolute precision seem to increase inerobably as technological advances allow processing at ever higher frequencies. But it would not be legitimate to therefore claim a 1000 times higher precision, without having acquired a deeper understanding of the issues. In our case, for example, at every instant of time there will be a significantly large spread of particle revolution frequencies, more or less centred on a frequency that is known with exquisite accuracy from the known beam magnetic moments. Without having a clear understanding of the fluctuations and averaging it is hard to refine the determination of the phase precision of resonant polarimetry.

Regrettably, the entire discussion of resonant polarimetry up to this point has been “counting chickens before they’ve hatched”. Resonant polarimetry has never, in fact, been demonstrated to be practical. However, theoretical calculations (admittedly due largely to the present author) based on the Stern-Gerlach interaction, have shown that the regular passage of bunches of polarised electrons through a cavity should produce detectably-large cavity excitation [14] [15] [16]. The latter two of these references describe, in considerable detail, experiments being planned to test both transverse and longitudinal polarimetry, using a polarised electron linac beam in the CEBAF injection line at the Jefferson Laboratory in Newport News, Virginia. Within a few years tests like these should have resolved the issue concerning the practicality of Stern-Gerlach polarimetry for electrons.

The proton’s magnetic dipole moment is three orders of magnitude smaller than the electron’s. In the absence of noise background a proton Stern-Gerlach signal reduced by this factor, would still be detectably large but, without extremely narrow band lock-in detection, the proton polarimetry signal is likely to be swamped by noise. This makes phase-locked-loop proton beam polarisation control based on resonant polarimetry likely to fail, even if resonant electron polarimetry has been demonstrated to succeed. This is my expectation.

It is this expectation that makes the doubly-magic proton-positron combination for measuring baryon EDM’s seem especially important. With a positron beam phase-locked to resonant Stern-Gerlach polarimeters (both transverse and longitudinal) the Koop wheel manipulations, so optimistically assumed in the present paper, should, indeed be extremely precise for the positron beam.

By exploiting the known relation between positron and proton MDM’s, it should then be possible to freeze the co-rotating proton spins just by controlling the positron beam spin tune and phase. With the frequency and phase of the proton beam magnetisation then known to such high precision, the frequency filtering of a proton beam Stern-Gerlach resonator can be selective to reject the noise which would, otherwise, prevent the accurate resonant determination of the magnetisation signal.

Only when non-destructive positron polarimetry has been successfully demonstrated will it be legitimate to remove the “holy grail” designation from the case (c1) positron-proton doubly-magic EDM ring design, to make the discovery of physics beyond the standard model likely.

G.8 The EDM measurement campaign

The majority of my work in the storage ring EDM area for the last several years has been performed during, and in connection with, my stays at the IKP Institute for Nuclear Physics of Forschungszentrum, Juelich.

During 2018, in response to a CERN invitation, an EDM task force at the IKP laboratory has been performing a feasibility study of measuring electric dipole moments, especially of the proton. A full report is due by the end of the year. The initial motivation for building a small prototype EDM ring was
to demonstrate the ability to store enough protons to enable an EDM measurement in a storage ring with predominantly electric bending. A preliminary report was issued after the first quarter of 2018 [17]. The present appendix has been coordinated with this task force planning.

As well as developing long term planning, an important thrust of the task force has been to advocate the immediate development of designs for a “small” EDM prototype storage ring. The doubly-magic design should have a major impact on motivation. This design eliminates the need to use the vertical separation of counter-revolving beam orbits to suppress radial magnetic field. Previous EDM designs have required excruciatingly small vertical betatron tune in order to enhance this “self-magnetometry” sensitivity to vertical beam separation of counter-circulating beams. The correspondingly weak focusing was expected to set a small limit on the proton beam intensity.

The doubly-magic EDM ring design transfers this self-magnetometry responsibility to a secondary frozen spin beam (with the admitted cost of measuring EDM differences rather than absolute EDM values). Elimination of the need for ultraweak focusing should enable the beam current intensities to be limited only by previously-encountered understood effects. This will permit the storage ring to have much stronger, alternating gradient focusing, which can be expected to increase the achievable proton beam current substantially.

Another motivation for building a small prototype EDM ring has been to develop and demonstrate the performance of instrumentation and procedures that will be needed for a subsequent larger ring. These applications are implicit in the examples of Table G.1. Especially relevant is the doubly-magic combination of case (q1), which can be used to measure the difference of proton and helion EDM’s. This can be done using carbon scattering polarimetry of the type that has been developed, and is already in service, in the Juelich COSY ring. As already stated, any miserably large difference between proton and helion EDM’s would constitute physics beyond the standard model.

Important contributions by my EDM collaborators need to be acknowledged, especially to Sig Martin and Helmut Soltner for detailed discussions of implementation practicalities. Acknowledgements are also due to Maxime Perlstein for insisting on a less confusing treatment of the orbitry, to Eanna Flanagan and Andras Laszlo for communications concerning general relativistic effects, and to Andreas Wirzba for conveying and explaining a GR analysis by Kolya Nikolaev.

References
[3] I.A. Koop, Asymmetric energy colliding ion beams in the EDM storage ring, Paper TUPWO040, in Proceedings of IPAC2013, Shanghai, China, 2013. When beginning to prepare the present paper, though aware of the “Koop spin wheel”, we had not realised that the shared beam EDM approach, not including the doubly-magic possibility, nor the possibility of electron or positron secondary beam, nor the spin tune comparator functionality, had been proposed by Koop five years earlier.


[17] JEDI EDM task force “Easter Report”, unpublished internal report, Institute for Nuclear Physics of Forschungszentrum, Juelich, Germany, April 30, 2018


Appendix H

New ideas: Spin Tune Mapping for EDM Searches

Abstract
The appendix describes an EDM measurement method that uses the Wien filter to produce spin phase advance in the same plane in which the MDM-induced spin precession occurs. For protons at 30 MeV (non-frozen spin) this plane is the horizontal plane of the ring. For protons at 45 MeV with “frozen spin” this plane is orthogonal to the radius of the ring, where the MDM-induced spin precession is produced by horizontal magnetic and vertical electric fields of the ring lattice imperfections. The EDM can then be extracted by ultra-precise determination of the shift of the spin precession frequency when the sign of the MDM component is reversed between running the beam clockwise and counter-clockwise in the ring. An important virtue of the method is that, in the case of a pure electric ring (not necessarily frozen spin), it is free of the background from imperfect magnetic fields of the ring lattice and also allows to protect against the presence of external magnetic fields. Also, because frozen spin operation is not required, the method can be used to measure the deuteron EDM. For searches of proton EDM in the prototype EDM storage ring, sensitivity $\approx 2.2 \cdot 10^{-24} e \cdot cm$ at beam energy 30 MeV can be reached.

H.1 Introduction
Interaction of MDM with vertical electric imperfection fields in the pure electric storage ring creates the tilt of invariant spin axis $c = c_y + c_{xz}^{MDM}$ away from vertical direction $e_y$, where $c_{xz}^{MDM} \perp e_y$ and $c_y \parallel e_y$, also $c_y \approx 1$. Projection $c_{xz}^{MDM}$ is a function of azimuthal angle – it depends on which point in the ring the invariant spin axis is viewed at. The reason for that is non-commutativity of spin rotations in the imperfection fields. Reduction of imperfection fields implies that all elements of the ring are precisely aligned relative to common vertical axis which becomes a normal vector to the planar beam orbit. Then $c_{xz}^{MDM} \rightarrow 0$ at every point of the ring.

Interaction of EDM with electric field in the ring tilts the invariant spin axis towards X-axis (X-axis is pointing against the radius of the ring). This tilt is an indication of EDM signal. For pure electrostatic storage ring the tilt angle $\xi_{EDM}$ due to EDM is defined as

$$\tan \xi_{EDM} = \frac{\eta \beta}{2(1 - \beta^2(1 + G))}$$

(H.1)

where $G$ is anomalous magnetic moment of particle, $\eta$ is related to EDM. For protons with kinetic energy $T=30$ MeV, $\xi_{EDM} \approx 0.4\eta$.

H.2 The mixing of EDM signal with systematic background from MDM
In non-ideal storage ring, the tilt due to EDM adds up to the tilt induced by MDM (up to a first order expansion in small $\xi_{EDM}$) and EDM signal mixes with systematic effects of MDM spin rotation in imperfection fields:

$$c = c_y + c_{xz}^{MDM} + \xi_{EDM} e_x.$$  

(H.2)

Experimental determination of the orientation of the invariant spin axis was performed at COSY [1]. The method was based on the observation of the most precise quantity measured presently at COSY
at $10^{-10}$ level for 100 seconds of the beam cycle – a spin tune [2]. Two static solenoids, one in each straight section of the ring, were acting as artificial imperfections which induced the change of the spin tune when powered on. The change of the spin tune was predicted by the theoretical model. The unknown parameters were the tilts of the invariant spin axis towards Z-axis (which points along the momentum), $c_z$, at the spots where the solenoids were located. Sensitivity to the angular direction of the invariant spin axis was achieved.

Determination of $c_z$ projections with this method requires the use of static Wien filters with transverse horizontal magnetic fields ($\vec{B} = \vec{e}_x B$). Such Wien filter rotates the spin around X-axis by constant angle each turn and changes the spin tune. Presently at COSY, there are two Wien filters that can work with horizontal orientation of B-field, but both of them are radio-frequency devices. Running such RF Wien filter on beam revolution frequency allows it to perform as a static one. The time when RF field reaches its maximum should be synchronized with the time when the bunch is passing through the Wien filter. However, the measurement of $\xi_{\text{EDM}} \vec{e}_x$ separately from the direction of $\vec{c}_{\text{MDM}}$ would still be not possible for COSY (see [6]). The use of two Wien filters would provide information about azimuthal dependence of the sum $\vec{c}_{\text{MDM}} + \xi_{\text{EDM}} \vec{e}_x$. This will give an input to the model of the ring which should be based on the precise knowledge of the fields and beam orbit. Then variations of $\vec{c}_{\text{MDM}}$ from one point to another can be predicted and compared with measured ones, at the same time $\xi_{\text{EDM}}$ will be unknown parameter which needs to be determined.

### H.3 Advantage of electrostatic rings

The advantage of pure electrostatic machine is that two countercirculating beams can be stored simultaneously. It allows to control the unwanted magnetic fields in the ring by observing the relative separation of closed orbits for clockwise (CW) and counterclockwise (CCW) beams. Then if unwanted, non-reversible magnetic fields are removed, closed orbits become equal and following relations are true:

\begin{align}
\vec{c}_{\text{cw}} &= \vec{\xi_{\text{EDM}}} \vec{e}_x \\
\vec{c}_{\text{ccw}} &= -\vec{\xi_{\text{EDM}}} \vec{e}_x 
\end{align}

As it was already explained in previous section, $\vec{c}_{\text{MDM}}$ is a function of azimuthal angle, therefore this property depends on where in the ring the $\vec{c}_{\text{cw}}$ and $\vec{c}_{\text{ccw}}$ are viewed at – it should be the same point for both CW and CCW bunches.

Eqs. H.3 – H.4 are also true for any storage ring operating at a non-frozen spin, be it pure magnetic, pure electric or hybrid electric and magnetic ring, assuming correct expression in Eq. H.1 for $\xi_{\text{EDM}}$.

If the condition $\vec{c}_{\text{cw}}(\xi_{\text{EDM}} = 0) = -\vec{c}_{\text{ccw}}(\xi_{\text{EDM}} = 0)$ can be guaranteed by making CW and CCW beams equal, then Eqs. H.3 – H.4 would allow an extraction of the EDM signal from the sum of measured $\vec{e}_x$-projections of $\vec{c}_{\text{cw}}$ and $\vec{c}_{\text{ccw}}$. In the sum the systematic effects of MDM spin rotations related to the imperfections of electrostatic ring lattice are cancelled. Hence the prototype electrostatic EDM ring opens a unique opportunity to test the principle of separating the EDM signal from the MDM systematic effect using simultaneously countercirculating beams with non-frozen spin.

### H.4 The effect of the Wien filter on beam and spin

Measurement of $\vec{e}_x$-projection of invariant spin axis by observation of spin tune perturbations demands the use of static Wien filter with horizontal transverse spin rotation axis $\vec{w} = \vec{e}_x$. But zero Lorentz force condition for the fields of the Wien filter can only be fulfilled for one direction of the beam:

\[ \vec{E} + \vec{\beta} \times \vec{B} = 0. \]
In order to fulfill zero Lorentz force condition for opposite beam direction, magnetic field in the Wien filter should change the sign:
\[ \vec{E} + (-\beta) \times (-\vec{B}) = 0. \]
\[ (H.6) \]

The change of magnetic field direction can be achieved by making it an RF field that oscillates at the beam revolution frequency, in a similar way as proposed in section \( H.2 \). Electric field should remain constant for every turn.

There are two points in the ring where CW and CCW bunches are always diametrically opposite to each other and where they intersect, as shown on Fig. \( H.1 \). Azimuthal position of this points is controlled by RF cavity. Then the Wien filter should be installed at the point where the CW and CCW bunches are diametrically opposite to each other on every turn, so that after half of the revolution period, either CW or CCW bunch enters the Wien filter (see Fig. \( H.2 \)).

The ideal Wien filter has exactly crossed E and B fields matched to a zero Lorentz force and rotates the spin around X-axis. Consider now the case of the imperfect Wien filter with horizontal magnetic and vertical electric fields which are not strictly orthogonal to each other and the ratio between E and B does not exactly match the zero Lorentz force condition. Such Wien filter will steer the beam vertically and it can have other components of spin rotation axis, \( \vec{w} = \vec{e}_x w_x + \vec{e}_y w_y + \vec{e}_z w_z \), besides \( w_x \approx 1 \). The Wien filter changes closed orbit, which leads to the change in the direction of the invariant spin axis at the point where the Wien filter is installed. This change adds up to the effect of systematic MDM spin rotations \( e_{\text{MDM}}^{xz} \). For both countercirculating beams these additions are equivalent if the changes of closed orbits are also equal. This will be the case if magnetic field precisely reverses the direction between the appearances of CW and CCW beams at the Wien filter. Then Eqs. \( H.3 - H.4 \) remain valid.

The axis of spin rotation in the Wien filter for CW beam in comparison to that of CCW beam is exactly opposite because of B-field reversal:
\[ \vec{w}^{\text{cw}} = -\vec{w}^{\text{ccw}}. \]
\[ (H.7) \]

**H.4.1 Spin tune shift by Wien filter and EDM**

The analysis presented here is assuming that the beam in the storage ring has the energy away of the “frozen spin” condition. It is bunched and polarization of the bunch is in horizontal plane. Continuous measurement of time-dependent horizontal polarization \( P_x = \sum_{i=1}^{N} S_{ix}^i \) allows to determine the spin tunes of CW and CCW bunch (\( N \)-number of particles). The sextupole fields are set up to provide at least \( \tau = 1000 \) seconds of spin coherence time.
The change of the spin tune \( \Delta \nu_s \) produced by the spin kick \( \psi \) in the Wien filter, is given by:

\[
\cos \pi (\nu_s + \Delta \nu_s) = \cos \pi \nu_s \cos \frac{\psi}{2} - \vec{c} \cdot \vec{w} \sin \pi \nu_s \sin \frac{\psi}{2}
\] (H.8)

The difference of scalar products \( \vec{c} \cdot \vec{w} \) for CW and CCW beams gives:

\[
\vec{c}_{cw} \cdot \vec{w}_{cw} - \vec{c}_{ccw} \cdot \vec{w}_{ccw} = 2w_x \sin \xi_{EDM}
\] (H.9)

Then the difference of the spin tunes for CW (\( \nu_{cw} = \nu_s + \Delta \nu_{cw} \)) and CCW (\( \nu_{ccw} = \nu_s + \Delta \nu_{ccw} \)) bunch is proportional to EDM tilt angle and spin kick of Wien filter, while the effects of MDM spin rotations cancel:

\[
\nu_{cw} - \nu_{ccw} = \frac{1}{\pi} \xi_{EDM} \psi
\] (H.10)

Time dependence of transverse horizontal projection of polarization is measured (see Fig. H.3). The spin tunes of CW and CCW bunches are determined, each one should depend quadratically on \( \psi \). In order to control the time-dependent systematic effects within a beam cycle, the phase shift \( \Delta Q \cdot t = 2\pi(\nu_{cw} - \nu_{ccw})f_{rev}t \) between spin oscillations of CW and CCW beam can be monitored (here \( f_{rev} = \beta c/U \) - a revolution frequency). Statistical sensitivity to the EDM is given by:

\[
\sigma(|\vec{d}|) = h\gamma^2 |\vec{d}| \frac{1 - \beta^2(G + 1)}{G + 1} \frac{U \sqrt{12}}{EL \sqrt{N} \tau} AP \tau
\] (H.11)

Sensitivity is inversely proportional to the electric field integral \( EL \) in the Wien filter.
Systematic effects that are coming from external magnetic fields (such as magnetic field of Earth) lead to $\vec{c}^{\text{cw}}(\xi_{\text{EDM}} = 0) \neq -\vec{c}^{\text{cw}}(\xi_{\text{EDM}} = 0)$ for both points in the ring (see Fig. H.1) where the Wien filters could be installed. Moreover, due to different direction of the external magnetic field at every element of the ring in the particle rest frame and because of non-commutativity of spin rotations, $\vec{c}^{\text{cw}}_1(\xi_{\text{EDM}} = 0) + \vec{c}^{\text{cw}}_2(\xi_{\text{EDM}} = 0) \neq \vec{c}^{\text{cw}}_2(\xi_{\text{EDM}} = 0) + \vec{c}^{\text{cw}}_1(\xi_{\text{EDM}} = 0)$, where indices 1 and 2 distinguish between two points in the ring for location of Wien filters. If the external magnetic field is weak and orbit separation between CW and CCW beams is not measurable, then the crosscheck of the measurement $\nu^\text{cw}_s - \nu^\text{ccw}_s$ with second Wien filter for the same beam can provide another control of such effects. Inequality $\nu^\text{cw}_s - \nu^\text{ccw}_s \neq \nu^\text{cw}_{s1} - \nu^\text{ccw}_{s1}$ indicates that systematic effects are present, while in the ideal case, the r.h.s. of Eq. H.10 should be the same for the measurement with every Wien filter and independent of where in the ring it is installed.

The EDM limit of $\sigma_p \approx 2.2 \cdot 10^{-24} e \cdot \text{cm}$ can be achieved over the year of measurements, if the new technique is applied at the prototype EDM storage ring for protons at 30 MeV. Integral field 0.0005 Tm at the maximum peak of B-field is required in the Wien filter together with 37.5 KV of constant integral electric field. Polarization $P = 0.8$ and beam intensity $10^9$ particles per fill is assumed. Detection efficiency $f = 0.0014$ and analyzing power $A = 0.47$ can be achieved with multifoil carbon polarimeter for protons [4].

In order to minimize the effects of synchrotron oscillations which lead to non-compensated Lorentz force for head and tail particles in long bunches, it is advisable to have a flat-top pulsed B-field.

### H.5 Spin wheel at the beam energy of frozen spin

In a special case when proton energy is such that "frozen spin" condition is met, vertical component of invariant spin axis $c_y \vec{e}_y$ vanishes in Eq. H.2 and evolution of vertical polarization should be measured. If no imperfection fields are present in the ring, EDM aligns the invariant spin axis with X-axis: $\xi_{\text{EDM}} = \pi/2$ and $\vec{c} = \vec{e}_x$, while the spin tune becomes $\nu_s = \frac{1}{2} \eta \gamma \beta$.

When the Wien filter that is described in section H.4 works in the ring at "frozen spin", it allows to align the invariant spin axis with X-axis in the presence of imperfection fields. That leads to the "Spin Wheel" (see [3] and section 7.9) for both CW and CCW bunches that has frequency proportional to the spin kick of the Wien filter. Then the difference of the "spin wheel" tunes is:

$$\nu^\text{cw}_s - \nu^\text{ccw}_s = \eta \beta \gamma$$ (H.12)

---

**Fig. H.3:** Time dependence of horizontal spin projection $S_x$ for CW (red) and CCW (blue) particle. Black curve denotes projection $S_x$ for either CW or CCW particle in case EDM is zero or Wien filter is switched off.
where $\eta$ is directly proportional to EDM, $\beta$ and $\gamma$ -Lorentz factors. The wheel frequency is 10 Hz for the mentioned E- and B-field integrals at proton kinetic energy of frozen spin $T \approx 232$ MeV in the "nominal all electric storage ring" (see chapter 8) and 153 Hz for protons at $T \approx 45$ MeV in the prototype EDM ring (PTR) where combined E- and B-fields in the deflectors are used to freeze the spin. In the latter case, either CW or CCW beam is stored in consecutive beam cycles.

H.6 Other possibilities for EDM measurements with countercirculating beams

H.6.1 An option with two RF Wien filters in the prototype EDM ring

Development and construction of the Wien filter described in section H.4 requires time and resources. There is another option to perform the EDM measurement at non-frozen spin energies of the beam in electrostatic ring. It is based on the method [5] described in chapter 6. Combined effect of EDM and MDM on the spin motion in the ring is given by Eq. H.2. Small vertical spin oscillations produced from horizontal components of $\vec{c}$ are resonantly excited by the spin kicks in the radiofrequency Wien filter (it has vertical spin rotation axis $\vec{w} = \vec{e}_y$). That leads to much greater amplitude of $S_y$ oscillations which becomes accessible for polarimetry. The frequency of $S_y$ oscillations is proportional to $\| \vec{c}_{2z}^{MDM} + \xi_{EDM} \vec{e}_x \|$ and integral electric (or magnetic) field in the Wien filter.

The RF Wien filter is designed such that the E-field follows the B-field oscillations. It means that Lorentz force is zero only for one beam direction and the RF signal should be gated out when countercirculating beam comes. This can be achieved by installation of the RF Wien filters at two points where the countercirculating beams are opposite to each other (see Fig. H.1). Then gating out the RF signal of both Wien filters with beam revolution frequency allows to run CW and CCW beams simultaneously. The outcome is similar to the one discussed in section H.2: $\| \vec{c}_{2z}^{MDM} + \xi_{EDM} \vec{e}_x \|$ at two points of the ring is determined, and if direction $\vec{c}_{2z}^{MDM}$ can be predicted from the model assumptions, it allows to find $\xi_{EDM}$. However, direct extraction of EDM signal is also possible when both RF Wien filters are switched to make zero Lorentz force for opposite beam directions. In this case the directions of $\vec{c}_{2z}^{MDM}$ at the Wien filter locations change sign. Spin rotations produced by one RF Wien filter for CW and CCW beams are compared. Additionally, static solenoid is needed to suppress the $\vec{e}_z$ projection in $\vec{c}_{2z}^{MDM}$, otherwise $\| \vec{c}_{2z}^{MDM} + \xi_{EDM} \vec{e}_x \| \propto \xi_{EDM}^2$.

Another advantage of this option is that RF Wien filter is transparent for the off-momentum particles. The Wien filter RF phase and amplitude of the field can be adjusted such that only a slow build-up of vertical polarization is observed during the whole beam cycle. That allows to increase the statistical sensitivity of this method in 2.5 times in comparison to the method discussed in section H.4, assuming the same field integrals in the Wien filters. Disadvantage of the method is that direct extraction of EDM signal from the measured $P_y$ polarization build-ups produced with the same RF Wien filter for CW and CCW beams, depends on the equality of the orbits in the consecutive CW - CCW beam injections.

H.6.2 An option with static Wien filters in the prototype EDM ring

Instead of RF Wien filters described in section H.6.1, one or more Wien filters with static vertical electric and static horizontal magnetic fields can be used. The placement of Wien filters is not crucial. For a single Wien filter, all conclusions are the same as previously stated in section H.4. The only difference is that Eq. H.10 for $\nu_s^{cw} - \nu_s^{ccw}$ is calculated for CW and CCW beams that are running consecutively, and B-field of the Wien filter(s) is reversed between the injections in CW and CCW direction. This can lead to a systematic error if field reversal is not exact. Assuming that one can achieve two orders of magnitude higher field integrals for static fields, such method can have an advantage that it allows to reduce statistical error to EDM by two orders of magnitude compared to the one discussed in section H.4.
H.7 Summary and outlook

Here we propose a new method for measurement of charged particles EDM’s in electrostatic storage rings. One of the advantages of such rings is that CW and CCW bunches could be stored simultaneously which allows to cancel the systematic effects of ring lattice imperfections. The advantage of the method over the BNL proposal (see chapter 8) is that the ring operation mode is not fixed only to the energy of "frozen spin" which means it can be of much smaller size and different particle species could be studied. The disadvantage is that sensitivity to EDM signal is suppressed by 4 orders of magnitude compared to that at "frozen spin", assuming the electric field integral 37.5 KV in the Wien filter. Because of this the method seems as an intermediate step towards ultimate EDM precision searches, and it is applicable at the prototype (PTR) EDM ring. It serves as a complement for BNL proposal when applied at "frozen spin" for protons. It allows to control the systematic effects of unwanted MDM spin rotations produced by external magnetic fields when two Wien filters are used for spin tune mapping.

References

Appendix I

New ideas: Deuteron EDM Frequency Domain Determination

Abstract
This appendix describes suppressing the geometric phase and machine imperfection systematic errors, which are encountered in any frozen spin storage ring EDM measurement method based on observation of a slow, gradual change in the beam polarization vector. The geometric phase error is caused by non-commutating wobbling precessions of the polarization vector, which are significant only if the polarization vector precession rate is small. Geometric phase can be suppressed by dispensing with operating on the spin resonance (i.e., 3D frozen spin) state, in favour of operating on the 2D frozen spin state, represented by a rolling spin wheel. To eliminate the machine imperfection systematic error, the imperfection fields themselves are utilized as the drivers of the spin wheel.

The method is intended for a combined storage ring; the bend fields are magnetic and the frozen spin condition is met using multiple, uniformly-distributed, discrete Wien filters. Reversing the bending field (along with the beam direction) reverses the imperfection fields. The EDM measurement consists of measuring the difference of spin wheel roll rates, which is proportional to the EDM. Though motivated by the need to measure the deuteron EDM, the method can be applied also to the proton.

I.1 Motivation

Storage ring-based methods of search for the electric dipole moments (EDMs) of fundamental particles can be classified into two major categories, which we will call 1. Space Domain, and 2. Frequency Domain methods.

In the Space Domain paradigm, one measures a change in the spatial orientation of the beam polarization vector caused by the EDM.

The original storage ring, frozen spin-type method, proposed in [1], is a canonical example of a methodology in the space domain: an initially longitudinally-polarized beam is injected into the storage ring; the vertical component of its polarization vector is observed. Under ideal conditions, any tilting of the beam polarization vector from the horizontal plane is attributed to the action of the EDM.

Two technical difficulties are readily apparent with this approach:

1. it poses a challenging task for polarimetry [2];
2. it puts very stringent constraints on the precision of the accelerator optical element alignment.

The former is due to the requirement of detecting a change of about $5 \cdot 10^{-6}$ to the cross section asymmetry $\varepsilon_{LR}$ in order to get to the EDM sensitivity level of $10^{-29} e \cdot cm$. [1, p. 18]

The latter is to minimize the magnitude of the vertical plane magnetic dipole moment (MDM) precession frequency: [1, p. 11]

$$\omega_{\text{sys}} \approx \frac{\mu(E_0)}{\beta c \gamma^2}, \quad \text{(I.1)}$$

205
induced by machine imperfection fields. According to estimates done by Y. Senichev, if it is to be fulfilled, the geodetic installation precision of accelerator elements must reach $10^{-14}$ m. Today’s technology allows only for about $10^{-4}$ m.

At the practically-achievable level of element alignment uncertainty, $\omega_{syst} \gg \omega_{edm}$, and changes in the orientation of the polarisation vector are no longer EDM-driven.

Another crucial problem one faces in the space domain is geometric phase error. [3, p. 6] The problem here lies in the fact that, even if one can somehow make field imperfections (either due to optical element misalignment or spurious electro-magnetic fields) zero on average, since spin rotations are non-commutative, the polarisation rotation angle due to them will not be zero.

By contrast, the Frequency Domain methodology is founded on measuring the EDM contribution to the total (MDM and EDM together) spin precession angular velocity.

The polarisation vector is made to roll about a nearly-constant, definite direction vector $\bar{n}$, with an angular velocity that is high enough for its magnitude to be easily measurable at all times. Apart from easier polarimetry, the definiteness of the angular velocity vector is a safeguard against geometric phase error.

This “Spin Wheel” may be externally applied [4], or otherwise the machine imperfection fields may be utilized for the same purpose (wheel roll rate determined by equation (I.1)). The latter is made possible by the fact that $\omega_{syst}$ changes sign when the beam revolution direction is reversed. [1, p. 11]

### I.2 Universal SR EDM measurement problems

By way of introduction to the proposed measurement methodology, let us briefly summarize some measurement problems encountered by any EDM experiment performed in a storage ring; they can be grouped into two big categories:

- Problems solved by a Spin Wheel:
  - spurious electro-magnetic fields;
  - betatron motion.
- Problems having specific solutions:
  - spin decoherence;
  - machine imperfections.

#### I.2.1 Spin motion perturbation

Problems from the first category are ones introducing geometric phase error. Indeed, both the spurious and the focusing fields, when acting on a betatron-oscillating particle, perturb the direction and magnitude of its spin precession angular velocity vector. The effect is a spin kick in the direction defined by the perturbation.

Assume that the EDM provides a spin kick about the radial ($\hat{x}$-) axis. The magnitude of the angular velocity vector has a general form

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2},$$

where $\omega_y$ is minimized by fulfilling the frozen spin condition; $\omega_z$ (the constant part of which is due to machine imperfections) can be minimized via the installation of a longitudinal solenoid on the optic axis. In the space domain, one also tries to minimize the $\omega_{(E_v)}$ contribution to $\omega_x = \omega_{edm} + \omega_{(E_v)}$. Consequently, spin kicks must be minimized to (significantly) less than $\omega_{edm}$, so as to reduce geometric phase to less than the accumulated EDM phase.

\[1\] 1 m long, magnetic field approximately $10^{-6}$ T.
The benefit of having a Spin Wheel aligned with the EDM angular velocity is that orthogonal MDM contributions to the total angular velocity vector add up in squares, and hence their effect is greatly diminished:

\[
\omega = \sqrt{(\omega_{edm} + \omega_{SW})^2 + \omega_y^2 + \omega_z^2}
\]

\[
\approx (\omega_{edm} + \omega_{SW}) \cdot \left[ 1 + \frac{\omega_y^2 + \omega_z^2}{2\omega_{SW}^2} \right]^{1/2}
\]

\[
\approx (\omega_{edm} + \omega_{SW}) \cdot \left( 1 + \frac{\omega_y^2 + \omega_z^2}{2\omega_{SW}^2} \right)
\]

\[
\approx \omega_{SW} + \omega_{edm} + \frac{1}{2} \frac{\omega_y^2 + \omega_z^2}{\omega_{SW}}
\]

Since our goal is to observe the EDM-related value shift in \(\omega\), we need to minimize random variable \(\epsilon\):

\[
\frac{1}{2} \frac{\omega_y^2 + \omega_z^2}{\omega_{SW}} < \omega_{edm}
\]

Let’s make some preliminary estimates. Suppose \(\omega_{SW} \approx 50\) rad/sec (the reason for choosing this value will be explained shortly), \(\omega_{edm} \approx 10^{-9}\) rad/sec (corresponding to the EDM value \(10^{-29}\) e· cm). Then, \(\omega_y^2 + \omega_z^2\) must be reduced to less than \(10^{-7}\) rad/sec, or equivalently, either angular velocity to less than \(3 \cdot 10^{-4}\) rad/sec. This is several orders of magnitude greater than the expected standard error on the angular velocity estimate, [5] and hence should not be a problem to achieve.

One case left to be considered is MDM spin kicks about the \(\hat{x}\)-axis. These are not attenuated, and cause the most trouble. They come in three varieties: a) permanent, not caused by optical element misalignments; b) semi-permanent, caused by element tilts about the optic axis; c) spurious.

Semi-permanent radial spin kicks (be they caused by magnetic or electric fields) change sign when the beam revolution direction is reversed from clockwise (CW) to counter-clockwise (CCW). Spurious kicks can be dealt with by statistical averaging. Permanent, insensitive to either the guide field or the beam circulation direction, cannot be controlled. On the bright side, their sources should not be present under normal circumstances.

For more details on spin motion perturbation effects on the measurement of the EDM in frequency domain, please refer to [6].

I.2.2 Expected machine imperfection SW roll rate
In the estimates above, we used a roll rate \(\omega_{SW} \approx 50\) rad/sec for the spin wheel. This is our expected \(\omega_{syst}\) caused by machine imperfections.

Denote the standard deviation of the imperfection radial magnetic field distribution \(\sigma[B_x]\). For the whole ring, MDM precession will be distributed with a standard deviation [7]

\[
\sigma[\omega_x^{MDM}] = \frac{e}{m\gamma} G + 1 \frac{\sigma[B_x]}{\gamma \sqrt{n}}
\]

where \(n\) is the number of misaligned elements, \(G = (g - 2)/2\) is the anomalous magnetic dipole moment.

For deuterons in lattices [8] of \(n\) on the order of 100 elements, rotated about the optic axis by angles \(\theta_{tilt} \sim N(0, 10^{-4})\) rad, Y. Senichev estimates [7] \(\omega_x^{MDM}\) between 50 and 100 rad/sec.

Our simulations done in COSY INFINITY seem to confirm this result. In Figure I.1 you see the results of the simulation in which we rotated the 32 E+B spin rotator elements used in the frozen spin
(codename BNL) lattice [8] by angles randomly picked from the distribution \(N(\mu_0 \cdot (i - 5), \sigma_0)\), where \(\mu_0 = 10 \cdot \sigma_0 = 10^{-4}\) rad, \(i \in \{0, \ldots, 10\}\).

At \(\langle \Theta_{\text{tilt}} \rangle = 10^{-4}\) we observe a roll rate of 500 rad/sec. We should keep in mind, however, that Senichev assumes \(\sigma_{\Theta_{\text{tilt}}} = 10^{-4}\) rad, which means, for a lattice with \(n = 100\) tilted elements, a standard deviation of the mean \(\sigma_{\langle \Theta_{\text{tilt}} \rangle} = \sigma_{\Theta_{\text{tilt}}}/\sqrt{100} = 10^{-5}\). The dependence of \(\omega_{MDM}^x\) on \(\langle \Theta_{\text{tilt}} \rangle\) is linear, which means in an actual lattice we would observe an \(\omega_{\text{syst}} \leq 50\) rad/sec with 68% probability, and \(\omega_{\text{syst}} \leq 100\) rad/sec with 95% probability, and with 27% probability \(50 \leq \omega_{\text{syst}} \leq 100\).

![Graph](image.png)

**Fig. I.1:** Spin precession frequency (radial and vertical components) versus the mean E+B element tilt angle

### I.2.3 Spin decoherence
Spin coherence is a measure of preservation of polarisation in an initially fully-polarized beam. [9] Spin decoherence refers to the depolarisation caused by the difference in the beam particles’ spin precession frequencies.

The difference in spin tunes is due to the difference of the particles’ orbit lengths, and hence their equilibrium energy levels, on which spin tune depends. One way spin decoherence can be suppressed is by utilization of sextupole fields. We consider how this can be accomplished in [10].

### I.2.4 Machine imperfections
As we have seen, the problem with machine imperfections is twofold: a) they are practically impossible to remove at the present level of technology; but what’s even worse, b) their removal leaves one in the space domain, and opens the measurement up to geometric phase error.
Fortunately for us, the imperfection spin kicks they induce change sign when the beam circulation direction is reversed. Their magnitude is also sufficient for use as a Koop Wheel. The one remaining difficulty is the accuracy of the Koop wheel roll direction flipping. Hopefully, we can make a persuasive enough argument as to how this can accomplished.

I.3 Main methodology features

The method we propose is characterized by two main features:

1. It is a frequency domain method;
2. The fields induced by machine imperfections, instead of being suppressed, are used as a Koop Wheel.
   - The Koop Wheel roll direction is reversed by flipping the direction of the guide field;
   - its roll rate is controlled through observation of spin precession in the horizontal plane.

The advantages of the frequency domain, such as a) ease of polarimetry, and b) immunity to geometric phase error, have been discussed in previous sections. Now we will turn to the description of how machine imperfection fields can be used as a Koop Wheel.

I.4 EDM estimator statistic

Since the angular velocity measured in the frequency domain methodology includes contributions due to both the magnetic and electric dipole moments, the EDM estimator statistic requires two cycles to compose: one in which the Koop Wheel rolls forward, the other backward.

The change in the Koop Wheel roll direction is affected by flipping the direction of the guide field. When this is done: \( \vec{B} \mapsto -\vec{B} \), the beam circulation direction changes from clockwise (CW) to counter-clockwise (CCW): \( \vec{\beta} \mapsto -\vec{\beta} \), while the electrostatic field remains constant: \( \vec{E} \mapsto \vec{E} \). According to the T-BMT equation, spin precession frequency components change like:

\[
\begin{align*}
\omega_{x,CW} & = \omega_{x,MDM,CW} + \omega_{x,EDM} , \\
\omega_{x,CCW} & = \omega_{x,MDM,CCW} + \omega_{x,EDM} , \\
\omega_{x,MDM,CW} & = -\omega_{x,MDM,CCW} ,
\end{align*}
\]

(I.2a)

and the EDM estimator

\[
\omega_{x,EDM} := \frac{1}{2} (\omega_{x,CW} + \omega_{x,CCW} ) ,
\]

(I.2b)

\[
\omega_{x,EDM} = \omega_{x,EDM} + \frac{1}{2} \left( \omega_{x,MDM,CW} + \omega_{x,MDM,CCW} \right) ,
\]

(I.2c)

To keep the systematic error term \( \varepsilon \) below required precision, i.e. ensure that equation (I.2a) holds with sufficient accuracy, Y. Senichev devised [7] a guide field flipping procedure based on observation of the beam polarisation precession frequency in the horizontal plane.

To explain how it works, we need to introduce the concept of the effective Lorentz factor.
I.5 Effective Lorentz factor

Spin dynamics is described by the concepts of spin tune $\nu_s$ and invariant spin axis $\vec{n}$. Spin tune depends on the particle’s equilibrium-level energy, expressed by the Lorentz factor:

$$\nu_s^B = \gamma G,$$

$$\nu_s^E = \beta^2 \gamma \left( \frac{1}{\gamma^2 - 1} - G \right)$$

(1.3)

Unfortunately, not all beam particles share the same Lorentz factor. A particle involved in betatron motion will have a longer orbit, and as a direct consequence of the phase stability principle, in an accelerating structure utilizing an RF cavity, its equilibrium energy level must increase. Otherwise it cannot remain the bunch. In this section we analyze how the particle Lorentz factor should be modified when betatron motion, as well as non-linearities in the momentum compaction factor are accounted for.

The longitudinal dynamics of a particle on the reference orbit of a storage ring is described by the system of equations:

$$\frac{d}{dt} \Delta \phi = -\omega_{RF} \eta \delta,$$

$$\frac{d}{dt} \delta = \frac{q V_{RF} \omega_{RF}}{2 \pi h \beta^2 \gamma} (\sin \phi - \sin \phi_0).$$

(I.4)

In the equations above, $\Delta \phi = \phi - \phi_0$ and $\delta = (p - p_0) / p_0$ are the deviations of the particle’s phase and normalized momentum from those of the reference particle; all other symbols have their usual meanings.

The solutions of this system form a family of ellipses in the $(\phi, \delta)$-plane, all centered at the point $(\phi_0, \delta_0)$. However, if one considers a particle involved in betatron oscillations, and uses a higher-order Taylor expansion of the momentum compaction factor $\alpha = \alpha_0 + \alpha_1 \delta$, the first equation of the system transforms into: [11, p. 2579]

$$\frac{d \Delta \phi}{dt} = -\omega_{RF} \left[ \left( \frac{\Delta L}{L} \right)_\beta \left( \alpha_0 + \gamma^{-2} \right) \delta + \left( \alpha_1 - \alpha_0 \gamma^{-2} + \gamma^{-4} \right) \delta^2 \right],$$

where $\left( \frac{\Delta L}{L} \right)_\beta = \frac{\pi}{2 \epsilon_r} [\epsilon_x Q_x + \epsilon_y Q_y]$, is the betatron motion-related orbit lengthening; $\epsilon_x$ and $\epsilon_y$ are the horizontal and vertical beam emittances, and $Q_x, Q_y$ are the horizontal and vertical tunes.

The solutions of the transformed system are no longer centered at the same single point. Orbit lengthening and momentum deviation cause an equilibrium-level momentum shift [11, p. 2581]

$$\Delta \delta_{eq} = \frac{\gamma_0^2 \gamma_0^2}{\gamma_0^2 \gamma_0^2 - 1} \left[ \frac{\delta_m^2}{2} \left( \alpha_1 - \alpha_0 \gamma^{-2} + \gamma^{-4} \right) + \left( \frac{\Delta L}{L} \right)_\beta \right],$$

(1.5)

where $\delta_m$ is the amplitude of synchrotron oscillations.

We call the equilibrium energy level associated with the momentum shift (1.5), the effective Lorentz factor:

$$\gamma_{eff} = \gamma_0 + \beta_0^2 \gamma_0 \cdot \Delta \delta_{eq},$$

(1.6)

where $\gamma_0, \beta_0$ are the Lorentz factor and relative velocity factor of the reference particle.

Observe, that the effective Lorentz factor enables us to account for variation in the value of spin tune due to variation in the particle orbit length. It is crucial in the analysis of spin decoherence [10] and its suppression by means of sextupole fields.

It plays a big role, as well, in the successful reproduction of the MDM component to the total spin precession angular velocity.
I.6 Guide field flipping

Two aspects of the problem need to be paid attention to:

1. What needs to be kept constant from one measurement cycle to the next;
2. How it can be observed.

The goal of flipping the direction of the guide field is to accurately reproduce the radial component of the MDM spin precession frequency induced by machine imperfection fields. This point should not be overlooked: a mere reproduction of the magnetic field strength would not suffice, since the injection point of the beam’s centroid, and hence its orbit length — and, via equations (I.6) and (I.3), spin tune, — is subject to variation. (Apart from that, the accelerating structure might not be symmetrical, in terms of spin dynamics, with regard to reversal of the beam circulation direction.)

What needs to be reproduced, therefore, is not the field strength, but the effective Lorentz factor of the centroid.

Regarding the second question, we mentioned earlier that the Koop Wheel roll rate is controlled through measurement of the horizontal plane spin precession frequency. This plane was chosen because the EDM angular velocity vector points (mainly) in the radial direction; its vertical component is due to machine imperfection fields, and is small compared to the measured EDM effect. Therefore, in first approximation, when we manipulate the vertical component of the combined spin precession angular velocity, we manipulate the vertical component of the MDM angular velocity vector.

Moving on to the effective Lorentz factor calibration procedure. Let $\mathcal{T}$ denote the set of all trajectories that a particle might follow in the accelerator. $\mathcal{T} = \mathcal{S} \cup \mathcal{F}$, where $\mathcal{S}$ is the set of all stable trajectories, $\mathcal{F}$ are all trajectories such that if a particle gets on one, it will be lost from the bunch. Calibration is done in two phases:

1. In the first phase, the guide field value is set so that the beam particles are injected onto trajectories $t \in \mathcal{S}$.
2. In the second phase, it is fine-tuned further, so as to fulfill the FS condition in the horizontal plane. By doing this, we physically move the beam trajectories into the subset $\mathcal{S}|_{\omega_y = 0} \subset \mathcal{S}$ of trajectories for which $\omega_y = 0$.

Spin tune (and hence precession frequency) is an injective function of the effective Lorentz-factor $\gamma_{\text{eff}}$, which means $\omega_y(\gamma_{\text{eff}}^1) = \omega_y(\gamma_{\text{eff}}^2) \rightarrow \gamma_{\text{eff}}^1 = \gamma_{\text{eff}}^2$. The trajectory space $\mathcal{T}$ is partitioned into equivalence classes according to the value of $\gamma_{\text{eff}}$: trajectories characterized by the same $\gamma_{\text{eff}}$ are equivalent in terms of their spin dynamics (possess the same spin tune and invariant spin axis direction), and hence belong to the same equivalence class. Since $\omega_y(\gamma_{\text{eff}})$ is injective, there exists a unique $\gamma_{\text{eff}}^0$ at which $\omega_y(\gamma_{\text{eff}}^0) = 0$:

$$[\omega_y = 0] = [\gamma_{\text{eff}}^0] \equiv \mathcal{S}|_{\omega_y = 0}.$$

If the lattice didn’t use sextupole fields for the suppression of decoherence, $\mathcal{S}|_{\omega_y = 0}$ would be a singleton set. We have shown in [10] that if sextupoles are utilized, then $\exists \mathcal{D} \subset \mathcal{S}$ such that $\forall t_1, t_2 \in \mathcal{D}$: $\nu_s(t_1) = \nu_s(t_2)$, $\bar{n}(t_1) = \bar{n}(t_2)$. By adjusting the guide field strength we equate $\mathcal{D} = \mathcal{S}|_{\omega_y = 0}$, and hence $\mathcal{S}|_{\omega_y = 0}$ contains multiple trajectories. 

Therefore, once we ensured that the beam polarisation does not precess in the horizontal plane, all of the beam particles have $\gamma_{\text{eff}}^0$, equal for the CW and CCW beams.

Guide field flipping procedure simulation results can be found in [12].

2 Strictly speaking, even if sextupoles are used there remains some negligible dependence of spin tune on the particle orbit length (linear decoherence effects, cf. [10]). Because of that, the equalities for $\nu_s$ and $\bar{n}$ are approximate, and the set $\mathcal{S}|_{\omega_y = 0}$ should be viewed as fuzzy: we will consider trajectories for which $|\omega_y| < \delta$ for some small $\delta$ as belonging to $[\omega_y = 0]$.
I.7 Statistical precision

Members of the JEDI Collaboration have studied the statistical precision of spin precession angular velocity estimation from sparse (one detector event per 100 spin revolutions) [13] and dense [5] polarization data.

According to [13], the maximum likelihood estimator for the spin precession frequency estimate has a standard error

$$\sigma_{\hat{\omega}} = \frac{1}{PT} \sqrt{\frac{24}{N}},$$

where $N$ is the total number of recorded detector events, $P$ is the beam polarization, $T$ is the measurement time.

Assuming $N = 7.5 \times 10^8$ events, polarization $P = 0.4$, and cycle duration $T = 1,000$ seconds (same parameters as in the simulation done in [5]), we have $\sigma_{\hat{\omega}} \approx 4.5 \times 10^{-7}$ rad/sec at the cycle level. Estimates made in [5] agree with this result.

This precision is sufficient to obtain a mean estimate with statistical uncertainty $\sigma_{\langle \hat{\omega} \rangle} \approx 3 \times 10^{-9}$ rad/sec in one year of measurement, with the accelerator operational 70% of the time. An EDM of $10^{-29}$ e·cm should induce an $\omega_{edm}$ on the level of $10^{-9}$ rad/sec in storage rings proposed in [8]. Thus, we expect to be able to measure the deuteron EDM at the $10^{-29}$ e·cm level in one year of measurement time.

References


Appendix J

New ideas: Distinguishing the effects of EDM and magnet misalignment by Fourier analysis

Abstract
This appendix shows that, by measuring vertical polarisation in two properly separated positions in the storage ring, it is possible to estimate the magnitude of the major systematic uncertainty induced by ring imperfections. The imprecise positioning of the magnets causes the creation of a radial field, and the interaction of the magnetic dipole moment with this field induces the effect mimicking the EDM signal. The ring imperfections are distributed rather randomly along the ring, while the dipole magnets form a very regular pattern. Therefore the changes of the vertical polarisation induced by magnets misalignment have a non-harmonic pattern. On the other hand, the EDM-induced vertical polarisation has an almost harmonic pattern, since it results from the vertical field of ring dipoles, which is not strongly affected by their misalignments. Within a simple model the vertical polarisation induced by EDM and ring imperfections is calculated as a function of time. Then it is shown that Fourier analysis of obtained signals sampled twice per beam revolution allows to distinguish these two effects. It is done by comparison of the Fourier amplitudes for revolution frequency and for difference of this frequency and spin precession frequency. Even for unknown misalignments it is possible to predict, with the given likelihood, the magnitude of the systematic uncertainty induced by ring imperfections.

A reliable limit on the value of EDM in any experiment can be given only when systematic uncertainty is under control. In an experiment measuring EDM with a storage ring the most important systematic effect comes from the radial field arising due to magnets misalignment. For all proposed scenarios for EDM measurements based on the detection of induced vertical polarisation \( s_y(t) \), this systematic effect mimics the expected EDM signal. One might rely on the simulations of misalignment effect on \( s_y(t) \), but magnets rotations and displacements are in fact unknown. Therefore a direct experimental estimation of systematic uncertainty of misalignment effect by means of Fourier analysis of \( s_y(t) \) seems a much better solution.

The presented method of misalignment effect calculations is an extension of the formalism presented in [1]. The model is limited to particles moving on the central trajectory, but offers an analytic solution with a detailed insight into the general features characterising the time dependence of polarisation \( s_y(t) \). In the following example, for simplicity only, misalignment of COSY dipole magnets due to rotation around beam axis are considered. With a known placement of magnets and their individual misalignments the distributions of all fields are represented by Fourier series with \( V_0, V_j^V, V_j^R \) representing the Fourier coefficients for vertical field \( B_V(t) \) and \( R_0, R_j^V, R_j^R \) for radial field \( B_R(t) \). Then the solution of the BMT equation for longitudinal spin component \( s_z(t) \) is expressed by those coefficients and two frequencies: orbital \( \omega_o \) and that of spin precession \( \omega_s \). Finally, \( s_y(t) \) is obtained as the integral over time of the product \( s_z(t)B_V(t) \) for the EDM effect and of \( s_z(t)B_R(t) \) for the misalignment effect. For more details of derivation see [1].

To illustrate the time pattern for \( s_y(t) \) the first leading terms are presented:
Fig. J.1: Time dependence of vertical component of polarisation $s_y(t)$ induced by EDM (red line) and by magnets misalignment (blue line) shown for two periods of spin precession.

$$s_y(t) = \frac{\omega X}{2} \left[ X_0 \frac{\sin(\omega_s t)}{\omega_s} + \sum_{j=1}^{\infty} X_j^c \left[ \frac{\sin(j\omega_o t - \omega_s t)}{j\omega_o - \omega_s} + \frac{\sin(j\omega_o t + \omega_s t)}{j\omega_o + \omega_s} \right] + \right. \sum_{j=1}^{\infty} X_j^s \left[ \frac{\cos(j\omega_o t - \omega_s t)}{j\omega_o - \omega_s} + \frac{\cos(j\omega_o t + \omega_s t)}{j\omega_o + \omega_s} \right] + \ldots \right], \quad \text{(J.1)}$$

where for the misalignment effect $\omega_X = \omega_s$, $X_0 = R_0$, $X_j^c = R_j^c$, $X_j^s = R_j^s$ and for the EDM effect $\omega_X = D\beta c B_0 / \hbar$, $X_0 = V_0$, $X_j^c = V_j^c$, $X_j^s = V_j^s$ with $D$ being the EDM value and $B_0$ dipole magnet field.

Even though the functional time dependence of both effects is the same, different values of Fourier coefficients lead to different time histories for the two effects. In COSY as in any storage ring the magnets form a regular pattern and the vertical field disorders due to magnets misalignment are small since they scale with cosine of the misalignment angle. Therefore $V_j^c$ for odd $j$ and all $V_j^s$ coefficients are small. On the other hand radial field scales with sine of the misalignment angle, than its distribution is quite random and all radial field Fourier coefficients have arbitrary values. This causes some differences in time dependence of $s_y(t)$ for the EDM and the misalignment effects seen in Fig. J.1. The numerical results presented in this figure and hereafter are obtained for $D = 4.7 \cdot 10^{-21} \text{ e} \cdot \text{cm}$ and the measured COSY dipoles misalignment angles.

The differences in the time dependence of $s_y(t)$ can be quantified via Fourier analysis of the observed signals. From Eq. J.1 it is seen that Fourier amplitudes for $s_y(t)$ should peak at frequencies $\omega_s$ and $\omega_o \pm \omega_s$ (in general $j\omega_o \pm \omega_s$). These maxima can be determined by sampling (measuring) vertical polarisation with a proper frequency. In Fig. J.2 the Fourier amplitudes for $s_y(t)$ sampled with frequency $\omega_o$ and $2\omega_o$ are shown. The first one corresponds to polarisation measurement at one place on the orbit, while for the second the polarisation needs to be measured in two, reasonably separated places. It is seen that sampling with $\omega_o$ is not sufficient to distinguish between the EDM and the misalignment effects. For the parameters chosen for numerical calculations the Fourier amplitudes $F(\omega_s)$ at $\omega_s$ for both effects
are almost the same. Sampling $s_y(t)$ with $2\omega_0$ frequency, however, allows to observe a peak in Fourier amplitude $F(\omega_0 - \omega_s)$ at $\omega_0 - \omega_s$ frequency. In this case the amplitude for the EDM effect is by two orders of magnitude smaller than the amplitude for the misalignment effect. Hence, determination of the $F(\omega_0 - \omega_s)$ amplitude for misalignment effect allows to determine also the magnitude of the amplitude $F(\omega_s)$ for this effect. Since for the EDM measurement at COSY two polarimeters will be available, the presented method will allow to experimentally determine the misalignment-related systematic uncertainty for the measured limit of the EDM value.

The values of real misalignments of all magnets at COSY are known with a rather poor accuracy. In such case the presented method allows to calculate the probability of occurrence of a certain ratio of Fourier amplitudes $F(\omega_s)/F(\omega_0 - \omega_s)$. Then, setting a confidence level it is possible to determine an upper limit for the systematic effect contributing to the measured $F(\omega_s)$ amplitude. Since the magnitude of the Fourier amplitudes for the EDM effect depends very weakly on magnets misalignments, it is possible to determine the limit for the EDM value. An example of such analysis is shown in Fig. J.3. The probability distribution of the ratio $F(\omega_s)/F(\omega_0 - \omega_s)$ was obtained assuming that the rotation angles of COSY dipoles have Gaussian distribution with a standard deviation of $0.01^\circ$.

References

Fig. J.3: The likelihood of the Fourier amplitude ratio $F(\omega_s)/F(\omega_o - \omega_s)$ determined for the misalignment-induced $s_y(t)$. 
Appendix K

New ideas: External Polarimetry

Abstract
This appendix describes a pellet extraction scheme for extracting beam samples from the beam core, rather than from the beam tails (as had been assumed up till now). Though not a new idea, itself, pellet beam extraction has been, until now, very erratic, largely because of the poorly controlled “spray” of pellet directions. Recent pellet gun developments have made this approach much more promising. The appendix is largely didactic, collecting formulas needed for the design of the pellet beam extraction. For EDM, the merit of pellet beam sampling is the elimination of the need for beam heating to produce the beam tails (with their dubious lattice function dependence and questionable systematic validity) which enables internal target polarimetry, but cancels stochastic cooling possibilities. Because the pellets pass approximately through the beam bunch centres, pellet-produced beam samples will be very representative of the true particle distributions (that can be further monitored by optical tracking of the pellets).

K.1 Pellet-extracted beam sampling

K.1.1 Pellet-extracted beam sampling; qualitative

K.1.1.1 Successful pellet injector implementation
Sun et al [1] have demonstrated a lithium pellet injector that can be copied more or less unchanged for the beam sampling requirements of the EDM experiment. The upper part of Figure K.1 (copied directly from their figure) shows the Sun et al pellet injector. The lower part of the same figure shows the extra focusing (and isolation) stage needed to send pellets, one-by-one through our polarised proton (or other baryon) beam. The Sun application requires, fast lithium pellet microspheres, for the application of triggering an EAST Tokomak). Available pellet speeds range from 30 to 110 m/s, ideal for our pellet beam-sample extraction. Our application requires pellet material having highest possible charge number $Z$, for which pellet behaviour is expected to be closely similar.

K.1.1.2 General description of pellet-induced beam sample extraction
The ideal polarimetry for an EDM measurement experiment would be non-destructive and continuous for hour-long runs, with no beam extraction sampling required. However, at present, the only practical form of polarimetry—left-right asymmetry proton-carbon scattering—consumes stored particles. One can imagine such scattering polarimetry from an internal carbon target—for example from carbon pellets. It is easy to show that this cannot be practical. A pellet big enough to have satisfactory polarimetry scattering efficiency will kill the entire beam within seconds. Beam sample extraction onto a “thick” carbon target is therefore required—so that the particle can scatter within a thick external polarimetry target.

As it happens, the ability just mentioned, of a single pellet to destroy an entire beam can actually be exploited to produce very clean and efficient extraction of controllable samples from the core of a stored proton (or other baryon) beam. Basically one person’s “suddenly destroyed beam” can be another person’s “efficient slow-extracted beam sampling”. This is illustrated in Figure K.2. (Objection to this configuration for polarimetry, based on the obvious left-right asymmetry of the extraction apparatus, is
to 45 MeV, polarized proton beam
(maybe 3 m below)
1 cm
D (maybe 1 m)
impeller
rotating paddle
vacuum
everywhere
focusing reflector
circulating particle beam

Fig. K.1: The Sun et al lithium pellet launcher adopted for use as the pellet beam sampler of the EDM prototype storage ring. The ability to switch among four pellet types would be unnecessary but, otherwise, the design can just be copied. But the pellet sizes needed for the EDM application will be some five times smaller than for the Tokomak triggering application. Their apparatus fed more than one at a time too-small pellets (far smaller than they needed) but their paper explains how a single gap height could be reduced to repair this behaviour.

to be be addressed later.) When a particle in the circulating beam, by chance, passes through a transitory passing pellet in one straight section, the particle loses enough momentum that, when it gets to the next straight section, it has become physically separated from the main beam—i.e. it has been “extracted”.

The most important parameter, for the performance of the sampled beam extraction, is \( \Delta K_p \approx -100 \text{ KeV} \), the kinetic energy change of a particle (for example, proton) in its centred passage through a pellet. Roughly half of the protons hitting the pellet will suffer very nearly this same energy loss; the rest, because of their more glancing incidence, will suffer reduced energy loss, from this value all the way down to zero.

For slow protons—for example 45 MeV kinetic energy—the \( dE/dx \) stopping power of protons is large—about 7 times minimum-ionising. See Figure K.6. In virtually all cases the energy loss suffered by a beam particle passing through any single pellet is far larger than the maximum energy that can be
recovered in a single passage through an RF cavity (should one be encountered along the path). All such protons will therefore have been ejected from their stable RF buckets, but their radial positions have not instantaneously been altered in the process. The extracted “beam bunch” duration will be, for example, about 0.2 ms, which is the transit time for velocity \( v_P \) pellets from entry to exit of the beam bunch. Meanwhile, because of their far greater velocity \( v_p \), the beam bunches will have made perhaps 100 circulations of the storage ring, the extracted “bunch” will therefor be made up of 100 “sub-bunches”, each of the same length as the stored bunches, but staggered in time by time intervals equal to the ring circulation time \( T_0 \approx 1 \mu s \).

Apart from this spreading in time, the beam being extracted is still a pencil beam emerging from a point source. But most of these protons have off-momentum values near \( \Delta p/p = -0.001 \). At a point in the ring with dispersion \( D = 10 \) m, these about-to-be extracted protons are initially displaced from their nominal off-momentum closed orbits by about 1 cm. Interpreted as a betatron amplitude, this is almost twice the nominal beam bunch radius. After a horizontal betatron phase advance of \( \pi \) their radial betatron displacements will be reversed to -1 cm, relative to a nominal orbit that is, itself, also displaced by -1 cm. As a result, the transverse separation of extracted bunch relative to stored bunch is about 2 cm.

The extracted beam particles, though all starting from the same point source, also “remember” their initial betatron slope amplitudes. Downstream, the extracted sub-bunch transverse particle displacements (from their appropriately-reduced off-momentum closed orbit) will be approximately the

---

**Fig. K.2:** Top view of the left-right asymmetry of protons scattering through angles \( \theta \) from the seven graphene foils of the polarimeter target. (Some polarimeter components are traced from Figure 1(a) of reference [2].) Short hash marks along the polarimeter centreline actually represent, first, an entry scintillation counter, followed by seven carbon foil polarimeter graphene foil targets and, finally two exit scintillation counters. The figure shows how one quarter of a ring with horizontal betatron tune \( Q_x \approx 2 \) can act as a 180 degree spectrometer (even though it looks like 90 degrees), with point source at the pellet and the polarimeter at the “focus”. The regular beam focusing serve to focus the extracted beam as well.
same as those of the co-travelling bunch from which they were extracted. The separation of stored beam and extracted beam bunches may be about 4 times the nominal bunch radius. This is what can pass as “clean” slow beam sample extraction. (It is not unlike ion-stripping injection in which Liouville’s law is foiled by a sudden change of particle rigidity.)

There will also be multiple scattering suffered by each extracted proton in its passage through the pellet—for example $\theta_{\text{rms}} = \pm 2\,\text{mm}$ for this angular deflection. Though not a small angle, at least the core of the extracted bunch remains within the radial acceptance of the ring, both horizontally and vertically. The extracted beam will be broadened somewhat, and acquire transverse tails from this source. As it happens, though, the same horizontal phase advance that doubles the extracted beam separation also refocuses multiple-scattered protons back to a point focus at the polarimeter scattering target.

In short, when observed at the polarimeter in the next straight section, most of the protons that have touched the carbon pellet will have been slow-extracted into a bunch of much the same dimensions as the original bunch, somewhat broadened, but mainly displaced by 2 cm from the circulating beam. A noticeable exception to this analysis concerns protons that have barely grazed the pellet. Though almost certainly extracted from their stable buckets, these protons can decohere and form a more-or-less stable coasting beam of reduced radius, but surely at the percent level, at most. Though not welcome, such protons should have acceptably small effect on the EDM measurement—to be worried about later.

Suggested starting parameters for an EDM experiment are then: that the pellet material should have the highest atomic number $Z$ available, with radius $20\,\mu\text{m}$; the number of stored protons, $10^{10}$; the number of protons extracted by the first pellet, 25 million; and the total number of pellets, 400 (irrespective of the run length). However all parameters mentioned so far apply only to the starting beam conditions. As the beam intensity falls, say by a factor of two, to maintain the extracted beam flux will require the pellet rate to double. So the total number of pellets will be larger than has been stated so far. By controlling the rate at which pellets are launched the beam attenuation pattern can be made linear, or whatever is most favourable.

Making, for example, the assumption that the very first pellet is launched into a beam of $10^{10}$ protons, and the (unduly optimistic assumption of 100 percent extraction efficiency) the number of extracted beam protons through the polarimeter from just one pellet will be 25 million. Using detailed cross section values copied unchanged from the (invaluable) paper of M. Ieiri [2], the polarimeter efficiency is calculated to be 0.00034, with analysing power $A_{\text{pol}} = 0.78$. From the first pellet we therefore anticipate 5500 total polarimeter counts, with $4800 \pm 70$ scattering to the right (predictably, since we assume the proton beam is 100 percent vertical polarised) and $700 \pm 25$ scattering to the left. This would produce (statistically) a better than 2 percent r.m.s. beam polarisation measurement.

K.1.2 Experimental confirmation of wire and pellet beam extraction at COSY

K.1.2.1 Previous moving wire investigations

Though the pellet extraction of small beam samples from the centre of a beam bunch has not yet been demonstrated, nor the high quality of the extracted beam quantitatively confirmed, the concept has, itself, been confirmed experimentally, as show in Figure K.3. In this test by Keshelashvili and others, a stretched $10\,\mu\text{m}$ carbon fibre was passed suddenly and repeatedly, 10 times through a stored COSY beam in order to show that the basic considerations given here are correct. The upper oscilloscope picture indicates the resulting synchronous counting rate bursts in counters of the EDDA polarimeter. The bottom figure shows the beam intensity being reduced in a staircase-like fashion.

By reducing the target dimensionality from 2D to 1D, the concept of beam sampling has been confirmed. But, while a $10\,\mu\text{m}$ carbon fibre may seem hardly intrusive, the beam attenuation per wire transit is still three orders of magnitude too great for the intended application. The need for further dimensionality reduction from 1D to 0D—wire to point—seems inescapable. The proposed pellets, with radii three orders of magnitude less than the circulating beam transverse area will provide this needed
Furthermore, the possible performance degradation by electrostatic charging of an insulator in a beam has been shown to be unimportant, at least for a wire.

Fig. K.3: Results of an experimental investigation, by I. Keshelashvili et al. [3] of the interaction of a 10 $\mu$m radius carbon wire with the COSY beam for two consecutive cycles. The top graph shows the rate in a detector; the bottom part shows the stepwise reduction of beam intensity for each beam crossing of the wire.

### K.1.2.2 Pellet formulation applied to moving wire investigation

Later in this section, Eq. (K.4) is derived, giving the opacity $O_{BP}$ of a moving pellet. Here “opacity” is the fraction of the circulating beam particles that touch a single pellet (typically over many beam turns) during a single pellet transit. Here we simply copy this formula, with minor modification, to give $O_{BP}^W$, which is a crude approximation to the opacity of a single transit of a moving wire. The result is

$$O_{BP}^W \approx \left(\frac{rW}{r_B}\right) \frac{2r_B/\nu W}{C/\nu_p} = \frac{r_B}{rW} O_{BP}$$  \hspace{1cm} (K.1)

By the replacement $P \rightarrow W$, pellet radius $rP$ becomes wire radius $rW$, pellet transit time $tP$ becomes wire transit time $tW$, pellet material density $\rho P$ becomes wire material density $\rho W$, and pellet velocity $vP$ becomes wire velocity $vW$; (of these, only $rW$ and $vW$ appear in Eq. (K.1)). Apart from these, purely symbolic, changes, the only change has been to multiply the pellet opacity by a (large) multiplicative factor $r_B/rW$. Inclusion of this factor amounts to visualising the moving wire as being made up of a (large) number $r_B/rW$, of length $rW$, radius $rW$ cylindrical pellets stacked end to end. For $rW = rP = 10\mu$m and $r_B = 1$ cm, the wire opacity is one thousand times greater than the pellet opacity.

As an aside, it can be commented that it is the large factor $r_B/rW$ that makes pellets so much more satisfactory than wires for bunch sample extraction. But this factor does not impede our purpose here, of experimentally confirming the moving pellet formalism using moving wire experimentation.
K.1.3 Re-interpretation and revision of COSY moving wire beam experiments

The COSY experience with beam sampling by moving an obstacle rapidly through a circulating beam is summarised in Figure K.3, and can be characterised by two qualitative features: the staircase-like reduction of beam current in equal steps, synchronous with transits of a moving wire, and the further detection of similarly synchronous bursts of radiation in nearby counters of the EDDA polarimeter. The constant downward beam current steps prove that beam particles are hitting the moving carbon wire; the local EDDA counter radiation bursts suggest that the extracted beam energy is dissipated locally.

The former conclusion is incontrovertible, but the latter is not. It is our understanding that the EDDA counters are not sensitive to small angle particles less than ten degrees or so. Yet the dominant contribution to the total cross section for high energy charged particles incident on very thin targets is multiple scattering at angles much less than ten degrees. The present note therefore assumes that scattered beam particles are not contributing significantly to the EDDA signals. This and other contentions of the present note, can be tested experimentally using existing COSY moving wire apparatus, either with or without new instrumentation.

K.1.3.0.1 Moving wire investigation without new instrumentation—ready immediately.

The simplest suggested experiment is to replace the 10 µm carbon wire by a 10 µm tungsten wire. According to Eq. (K.1), the moving wire opacity $O_{W}^{B_P}$ is independent of the wire medium density $\rho_{W}$. In our model, every particle that touches a pellet is extracted, irrespective of the wire medium. The switch from carbon to tungsten wires should therefore not significantly affect the step-wise reduction of beam current shown in the bottom oscilloscope trace in Figure K.3. On the other hand the local, large angle radiation should be roughly proportional to the wire medium density. The effect of switching from carbon to tungsten should therefore increase the ratio of EDDA counts/pellet to beam current loss/pellet by an order of magnitude.

K.1.3.0.2 Moving wire investigation with new instrumentation—ready in a few months.

The proposed test without new instrumentation is a significant consistency test, but it does not confirm our contention that the extracted beam particles can be conveyed with significantly large efficiency onto a carbon polarimeter scattering target. What is needed, for example, is a downstream phosphor screen, or other radiation sensitive imaging device. Judicially-placed in the lattice, such an imaging device can determine, at least roughly, the angular distribution of beam particles scattered (at small angles) from the moving wire.

The choice of a high Z such as tungsten for moving wire medium helps any such investigation significantly. The sudden betatron amplitude discontinuity, $\Delta x_\beta$, derived later in these notes, is given by Eq. (K.16), which needs only the symbol conversion $\rho_{P} \rightarrow \rho_{W}$.

The switch from carbon to tungsten increases $\Delta x_\beta$ by an order of magnitude. Though the dispersion function at the moving wire is, presumably, more or less fixed, the displacement of the extracted beam is also proportional to $D_p$ at the screen location. In the COSY lattice there are natural high dispersion points (of order 10 m in the straight sections at arc centres). It seems natural to consider putting the extracted beam screen at one or the other of these points. This is still not enough though. It is also most favourable for the horizontal phase advance to be an odd multiple of $\pi$. To complete even a preliminary design the true COSY lattice functions have to be known, and preferably be tuneable, to optimise the extracted beam separation.

K.1.3.0.3 Full demonstration and calibration of pellet extraction—2+1 years.

A cartoon for a pellet extraction test set-up is shown in Figure K.4. Using a COSY lab cyclotron (or equivalent spectrometer at any lab) a 45 MeV proton beam can be used to confirm, optimise, and calibrate pellet beam sample extraction. The magnetic spectrometer mimics one quarter of the EDM proto-
type ring. Inset phosphor screen images show anticipated charge distributions, with and without pellet contribution. Charge densities are crudely represented by gray-scale shading. The dark elliptical region is the image of the main beam. The broken-line rectangle indicates a satisfactory placement region for an external polarimeter target. Because the less-strongly-deflected intensity overlaps the main beam, the on-target extraction efficiency has to be at least somewhat less than 100 percent.

**K.1.4 Quantitative formulation of pellet beam sampling**

It is necessary to establish many parameters for pellet beam sample extraction. Symbol definitions for the various parameters and kinematic quantities are given in Table K.1. Fortunately pellets are “everywhere” these days, and there is a large choice of materials from which accurate microspheres can be acquired. Parameters for materials that seem to be especially promising are given in Table K.2. It is not our purpose to determine the parameters with high accuracy. Rather, the initial purpose is to acquire a sufficiently quantitative understanding of the relative advantages of low-Z versus high-Z materials. (Surprisingly, it seems that high-Z pellets are more favourable for our application.)

In spite of the ubiquitous availability of high quality plastic pellets we have ruled out all organic materials, because their hydrogen content has the potential to harm the vacuum. This mainly leaves pure elements, metals and ceramics. To simplify the analysis we pretend that ceramics can be approximated as pure single-element metals, quartz as silicon, sapphire as aluminium, etc. Table K.2 contains physical properties of an incomplete list of satisfactory and available pellet materials limited in this way. There are many possibilities. The main deficiency in the list is the absence of a really high-Z pellet material, as indicated by question marks in the table. If no such pellets exist it can only be that there has, as yet, been no commercial application requiring high-Z pellets.

As well as being needed for analysing the kinematics of pellet acceleration, which is entirely describable by classical and statistical mechanics, physical properties are also needed in the table to calculate the slowing down by ionisation loss as well as the multiple Coulomb scattering of any beam particle that happens to find itself within the material of a pellet.

We picture our pellet bulk material as being in the condensed liquid state of particles that would “evapourate” to form an ideal gas if only they could be heated to a sufficiently high temperature without
burning or melting—which is not even close to possible. The requirement to extract one pellet at a time from a fluid of pellets is the main technical challenge in shooting pellets, one-by-one, through our particle beam. Fortunately the Su et al apparatus shown in Figure K.1 shows that it is possible to produce a reasonably well controllable pellet gun source with the parameters we need.

Ideally we could dial up our pellet gun, on demand, to deliver exactly one pellet with an exact speed and direction. In practice this is unrealistic since, once the pellet fluid medium has been shaken enough to make ejecting pellets one at a time possible, their momentum vectors will have much the same distributions and uncertainties as given by the Rayleigh-Maxwell distribution of ideal gas molecules.

Fortunately, for our application, the pellet beam requirements are not strict. The required average pellet rate will be of order 1 Hz, but the arrival times can be Poisson distributed in time. Also the pellet beam width need only be comparable with particle beam transverse dimensions of order one centimetre.

### K.1.5 Derivations of required formulas

#### K.1.5.1 Popcorn analogy

When cooking popcorn on a stove top, the kernels, when they pop, supply enough energy to stir things up enough to require the sauce pan lid to be kept on. But this also prevents steam from escaping, which can make the popcorn soggy. As a compromise one can leave the top slightly ajar. As a result, every once in a while, an unpopped kernel comes flying out through the opening between pan and lid. Voila! a source of fast corn pellets.

In our application we do not have popping kernels, and it is not even thinkable to supply enough heat to stir up the pellets thermally—they are far too heavy. We need a moving “impeller” to “evaporate” the pellets into a “vapor”. This necessarily makes the momentum of each particle uncertain, with a

<table>
<thead>
<tr>
<th>symbol</th>
<th>definition (MKS units in formulas, but MeV energies)</th>
<th>units in tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>pellet beam particle (proton, deuteron or helion, not electron)</td>
<td></td>
</tr>
<tr>
<td>$P_B$</td>
<td>beam of particles</td>
<td></td>
</tr>
<tr>
<td>$ZP/AP$</td>
<td>pellet material charge/mass number</td>
<td>gm/cm$^3$</td>
</tr>
<tr>
<td>$\rho_P$</td>
<td>mass density of pellet material</td>
<td>1/cm$^3$</td>
</tr>
<tr>
<td>$n_P$</td>
<td>number density of atoms in pellet material</td>
<td>1/cm$^3$</td>
</tr>
<tr>
<td>$n_e$</td>
<td>electron number density in pellet material</td>
<td>1/cm$^3$</td>
</tr>
<tr>
<td>$XP$</td>
<td>pellet material radiation “length” (i.e. times density)</td>
<td>gm/cm$^2$</td>
</tr>
<tr>
<td>$N_P$</td>
<td>number of atoms in a pellet</td>
<td></td>
</tr>
<tr>
<td>$M_P \approx N_P A_P m_p$</td>
<td>pellet mass</td>
<td>gm</td>
</tr>
<tr>
<td>$r_P$</td>
<td>radius of pellet microsphere</td>
<td>µm</td>
</tr>
<tr>
<td>$v_P$</td>
<td>speed of pellet</td>
<td>$&lt; \sim 100$ m/s</td>
</tr>
<tr>
<td>$t_P = 2 r_P \rho_P$</td>
<td>target “thickness” of pellet microsphere</td>
<td>gm/cm$^2$</td>
</tr>
<tr>
<td>$N_p$</td>
<td>total number of stored beam particles</td>
<td>$&lt; \sim 10^{10}$</td>
</tr>
<tr>
<td>$N_{extr.}$</td>
<td>number of beam particles extracted by a single pellet</td>
<td>$\sim 10^6$</td>
</tr>
<tr>
<td>$r_{\perp}$</td>
<td>transverse radius of (circular) particle beam</td>
<td>cm</td>
</tr>
<tr>
<td>$C$</td>
<td>circumference of storage ring</td>
<td>m</td>
</tr>
<tr>
<td>$v_p$</td>
<td>velocity of beam particle</td>
<td>m/s</td>
</tr>
<tr>
<td>$\eta_{sl}(v_p)$</td>
<td>slowing-down enhancement factor (relative to minimum ionising)</td>
<td>$\sim 7$</td>
</tr>
<tr>
<td>$K_p$</td>
<td>kinetic energy of beam particle</td>
<td>m/s</td>
</tr>
<tr>
<td>$p_p$</td>
<td>beam particle momentum (expressed in energy units)</td>
<td>MeV</td>
</tr>
<tr>
<td>$T_0 = C/v_p$</td>
<td>beam revolution period</td>
<td>s</td>
</tr>
<tr>
<td>$TP = 2 r_{\perp}/v_P$</td>
<td>pellet transit time through beam</td>
<td>s</td>
</tr>
<tr>
<td>$O_{bp}$</td>
<td>“Opacity” of one pellet transit to beam particles</td>
<td></td>
</tr>
<tr>
<td>$O_{bp}$</td>
<td>“Opacity” of one wire transit to beam particles</td>
<td></td>
</tr>
<tr>
<td>$\Delta K_p$</td>
<td>ionisation energy loss, particle through pellet centre</td>
<td></td>
</tr>
<tr>
<td>$\delta_p = \Delta p_p/p_p$</td>
<td>corresponding fractional momentum loss of particle</td>
<td>MeV</td>
</tr>
</tbody>
</table>

Table K.1: Definition of symbols for the various parameters and kinematic quantities. $m_p$ is the proton mass (which is approximately equal to the a.m.u.).
Table K.2: Material properties of high-quality, available microsphere pellets. They should be hydrogen-free, which rules out plastic. All materials are crudely treated as single element metals; quartz treated as silicon, sapphire (aluminium oxide) as aluminium, stainless steel as iron. Plausible coefficient of restitution \( C_r \) values are given in the final column. It has to be realised, though, that even if treating, for example, sapphire as aluminium may be crudely valid for calculating slowing down and multiple scattering of relativistic particles, it is not at all a sensible approximation for determining coefficients of restitution [6]. The value given for tungsten, though the result of an actual experiment [7], applies to bouncing for which the pellet velocity is much less than we require.

Maxwell-like distribution of velocities. In a laboratory scale enclosed vessel with transverse dimensions of order \( \ell \), there are enough micropellets to run all EDM experiments for centuries, (if none are wasted).

Say, therefore, that the volume of pellet material is less than the vessel volume by a factor of one thousand, with pellets all sitting, condensed, at the bottom of the vessel. Some sort of agitator can, however, stir up the pellets enough that any individual pellet of mass \( m \), with gravitational acceleration \( mg \), can have acquired a kinetic energy \( mv^2/2 \) of order \( mg\ell \), enough to have a significant probability of being, for example, in the top half of the vessel. This establishes, a velocity \( v \sim \sqrt{g\ell} \), independent of \( m \), which the agitator has to apply randomly to the pellets, in order for at least some of the pellets to behave like a gas.

This has set a lower limit requirement for the impeller velocity. But this limit is far lower than the pellet velocity we require. We could, as a response, use a much faster impeller. But this would be a mistake, since this would introduce large and unmanageable transverse velocities. (As always in accelerators) we should start with a low energy injector, before applying exclusively longitudinal acceleration. We therefore need two impellers, one to jiggle pellets free, and another to accelerate individual pellets to “high” speed \( v_P \sim 100 \text{ m/s} \).

In the apparatus of Sun et al. shown in Figure K.1, the initial agitation is supplied by the oscillating PZO piezo-electric element, coloured purple in the figure, and the secondary acceleration is provided by the rotating “paddle impeller”, coloured orange in the figure. (As commented earlier, the capability to switch pellet sizes—indicated by large red open arrow—is superfluous for our application.)

**K.1.5.2 Pellet acceleration by rotating paddle impeller**

When a micropellet approaches at right angles a (not necessarily made of the pellet material) flat surface at rest, with momentum \( p_{\text{inc.}} \), the pellet bounces with momentum \( p_{\text{refl.}} \). The coefficient of restitution [4] is defined as the ratio of these momenta;

\[
C_r(v_{\text{paddle}}) = \frac{p_{\text{refl.}}}{p_{\text{inc.}}}.
\]

which is a number in the range from 0 to 1, that depends on the pellet velocity, and on the pellet and surface materials. (The notation here is a bit garbled; \( C_r(v_{\text{paddle}}) \) depends on the paddle speed from which \( p_{\text{inc.}} \) acquires its value in the paddle rest frame, and \( p_{\text{refl.}} \) inherits the same velocity in its transformation to the laboratory.) In our case the flat surface is a paddle, far more massive than the pellet, and moving in the laboratory with velocity \( v_{\text{paddle}} \). In this case the pellet recoils with velocity

\[
v^P = v_{\text{paddle}}(1 + C_r(v_{\text{paddle}})),
\]
which can be as large as $2v_{\text{paddle}}$. The pellet will lose some of its speed in the reflection from the spherical focusing “mirror.” It will also acquire angular velocity (that will have no significant effect on the subsequent circulating beam sampling).

Figure K.5 shows the velocity dependence of sapphire pellets incident on aluminium. Coefficient of restitution values for a few possible pellet materials are also given in Table K.2.

![Figure K.5: Dependence of coefficient of restitution for aluminium oxide (sapphire) normally incident on aluminium [5]. In these notes paddle and pellet media are taken to be identical. (This is not really legitimate for sapphire on aluminium, since aluminium is less rigid than sapphire.)](image)

K.1.5.3 “Opacity” $O_{BP}$ of a pellet to beam particles

Our storage ring of circumference $C$ has some $N_p \approx 10^{10}$ particles circulating with period $T_0$ at speed $v_p$, with very small fractional momentum spread $\delta_B \sim 10^{-4}$, in a beam with circular cross section of radius $r_B$. Because a pellet is quite small, and is moving quickly, it is unlikely for any particular beam particle to come close enough to a pellet to be affected. In fact, this is a very sharp distinction, a particle either hits a pellet or it does not. Assuming the beam is distributed uniformly in a circle of radius $r_B$, in a single passage the probability is $(r_P/r_B)^2$. However, because the beam particles are relativistic and a pellet speed is much less, each beam particle has multiple opportunities, given by the pellet transit time multiplied by the beam circulation frequency, each time with the same probability. As a result the opacity, which is the probability that a proton will encounter a single pellet, is given by

$$O_{BP} \approx \left( \frac{r_P}{r_B} \right)^2 \frac{2r_B^2/v_P}{C/v_p} = 2 \frac{r_P}{r_B} \frac{v_P}{v_P T_0},$$

(K.4)

where $T_0$ is the beam revolution period. Later we will introduce $\rho P$ as the pellet material density, and $tP = 2 \frac{r_P}{r_B} \rho P$ as the “target thickness” of the pellet, expressed in gm/cm$^2$. Here we are anticipating the approximation that the particle path lengths though the pellet of a substantial fraction of the pellets differ little from the pellet diameter. In practice a pellet will be struck by many beam particles, but only a very small number of beam particles will be aware of the passage of the pellet. On the other hand, because the pellet is so massive, its passage will be unaffected, even though it is hit by many beam
particles. Furthermore, a single beam particle passing through the pellet will, at first, scarcely notice the interaction. But, because the binding of a particle in a stable RF bucket is so weak, such a particle is almost certainly doomed or, in less gloomy terms, “extracted” from its RF bucket. The number of beam particles extracted by a single pellet is then given by

\[ N_{\text{extr}} = O_B p N_p. \] (K.5)

Of course the circulating beam particles will be reduced by exactly this number, but the circulating beam will be otherwise unaffected. This has reduced our task to finding the fate of the \( N_{\text{extr}} \) “extracted” particles. (The quotation marks on “extracted” serve as a reminder that, though the particles are no longer captured in stable buckets, they have not necessarily been extracted from the storage ring and delivered to a polarimeter.

**K.1.5.4 Expressing pellet mass in terms of pellet “target thickness”**

The role of a pellet is to slow down the beam particles that happen to pass through it. This slowing down is caused almost entirely by collisions of the beam particle with electrons in the pellet. And yet the electrons make only a negligible contribution to the pellet mass \( MP \).

The pellet dynamics depends on pellet mass \( MP \) and the beam particle slowing down depends on the pellet “target thickness” \( t_P = 2r_P \rho_P \). The number of free parameters can be reduced by relating these two quantities.

\[ MP = \rho_P \frac{4\pi}{3} r_P^3 = \frac{2\pi}{3} r_P^2 t_P. \] (K.6)

**K.1.5.5 Slowing down of beam particle passing through pellet**

The slowing down of a weakly relativistic elementary particle passing through a medium falls inversely with its squared-velocity \( v_p^2 \), “bottoming out” at a “minimum ionization” value \( dE/dx \) as the speed approaches \( c \). This is illustrated graphically in Figure K.6. Minimum ionising values for our promising pellet media are given in Table K.2. One sees that these minimum ionisation values are approximately independent of the medium, with approximate value 1.6 MeV/(gm/cm\(^2\)). It was commented earlier that, since our beam particle velocities are significantly less than the speed of light, our slowing down is enhanced by some voltage-dependent factor \( \eta_p(v_p) \approx 7 \), where the value “7” is specific to our 45 MeV proton beam energy. This value can be regarded as constant for present purposes, since we are concentrating only on the determination of pellet parameters. With longitudinal position variable \( z \), we can therefore use

\[ \frac{dE_p}{dz} = -\eta_p(v_p) \frac{dE_p}{dx} \bigg|_{\text{min}} \approx -7 \times 1.6 \text{[MeV/(gm/cm-sq)]}. \] (K.7)

**K.1.5.6 Longitudinal momentum reduction of “extracted” particles**

To track extracted beam particles out of the ring it is particle momentum (in the form \( \delta = \delta_p/p \), rather than particle energy, that is needed. It would not be flagrantly wrong, and consistent with other relations used in these notes, to simply use the non-relativistic relation \( K = (1/2)p^2/m \) for this purpose. But, for greater generality, let us use a formula that is more nearly correct relativistically, starting with the mass-energy-momentum relationship;

\[ \gamma_p^2 m_p c^4 = c^2 = (m_pc^2 + K_p)^2 = m_p^2 c^4 + p_p^2 c^2. \] (K.8)

Solving for \( p_p \)

\[ p_p^2 c^2 = 2m_pc^2 K_p \left( 1 + \frac{K_p}{2m_pc^2} \right). \] (K.9)
Differentiating this equation, and keeping only the leading term in $K_p/(m_pc^2)$, yields

$$
\delta \equiv \frac{\Delta p_p}{p_p} \approx \frac{1}{2} \left( 1 + \frac{K_p}{2m_pc^2} \right) \frac{dK_p}{K_p}.
$$  (K.10)

Substituting from Eq. (K.7) produces

$$
\delta \approx \frac{1}{2} \left( 1 + \frac{K_p}{2m_pc^2} \right) \eta_p(v_p) \left. \frac{dE_p}{dx} \right|_{\min} tP,
$$  (K.11)

(which is negative).

**K1.5.7 Transverse displacement of “extracted” beam particles**

(Neglecting any pre-existing betatron or synchrotron amplitude of a beam particle passing through a pellet) let us assume the beam particle is on the design orbit as it enters the pellet. At the location in the ring of the pellet injector let the particle, horizontal dispersion function value be $D_p$, and the dispersion function slope be zero, meaning that the transverse position of a particle with fractional momentum offset $\delta$ is given by

$$
x_p = D_p \delta.
$$  (K.12)

On entry we have assumed $x_p = x_p' = 0$. Because the pellet is so “short”, the particle will still be on the design orbit (with any non-zero slope having been caused by multiple scattering which we are temporarily neglecting) as it exits the pellet. But, on exit, the pellets fractional momentum offset is given by Eq. (K.11); this means the pellet is not on its off-momentum closed orbit—the particle has acquired a (positive) horizontal betatron displacement given by

$$
x_{\beta}^{\text{out}} = -D_p \delta^{\text{out}},
$$  (K.13)

just right to cancel its sudden, newly-established (negative) off-momentum closed orbit displacement $D_p \delta^{\text{out}}$. In the absence of any further disturbance, the particle will continue to oscillate with betatron amplitude given by Eq. (K.12) about this newly-displaced closed orbit. For example, when the betatron
phase has increased by $\pi$, with $D_p$ assumed constant, the particle will be displaced from the true, on-momentum design orbit $2D_p \delta$, with $\delta$ given by Eq. (K.11).

In general, the betatron perturbation just calculated will simply be superimposed on any previously-neglected betatron and synchrotron amplitudes.

### K.1.5.8 Sudden particle translation expressed in terms of pellet opacity

The sudden transverse displacement $D_p, \delta$ (relative to its off-momentum closed orbit) of a beam particle that has passed through a pellet causes the particle to be extracted. For clean sample extraction we want to maximise this displacement (by increasing pellet size or atomic number). But, at the same time, we want to minimise the pellet opacity $O_{BP}$, in order to minimise beam particle consumption per pellet—for a bigger sample one need only send more pellets.

To analyse this compromise it is useful to express the sudden displacement in terms of the opacity—that is, to express $D_p \delta$ in terms of $O_{BP}$. Toward this end, we re-arrange Eq. (K.4) into

$$\frac{tP}{\rho P} = \sqrt{2O_{BP}r_B^2v_PT_0},$$

(K.14)

which, conveniently, depends on pellet parameters only through the opacity. We also combine Eq. (K.11) and Eq. (K.13);

$$\Delta x_\beta = -\frac{D_p}{2} \left( 1 + \frac{K_p}{2m_pc^2} \right) \varpi_p(v_p) \frac{dE_p}{dx} \bigg|_{\min} \left( \frac{tP}{\rho P} \right) \rho P,$$

(K.15)

and substitute from Eq. (K.14),

$$\Delta x_\beta = -\frac{D_p}{2} \left( 1 + \frac{K_p}{2m_pc^2} \right) \varpi_p(v_p) \frac{dE_p}{dx} \bigg|_{\min} \sqrt{2O_{BP}r_B^2v_PT_0 \rho P},$$

(K.16)

which is boxed to emphasise its importance. A striking implication of this equation is that, at fixed opacity, $\Delta x_\beta$ is proportional to the density $\rho P$ of the pellet material. The importance of this dependence can be assessed from the density column of Table K.2.

### K.1.5.9 Angular spread caused by multiple scattering

As well as the loss of momentum just calculated, each extracted beam particle acquires a multiple scattering angular distribution. The r.m.s. angular spread can be expressed in terms of the particle momentum $p_p$, in conjunction with the radiation length $XP$ and target thickness $tP$ of the pellet material. The radiation length, expressed in units of gm/cm-sq, is defined [8] by

$$XP = \frac{716.4}{ZP(ZP + 1) \ln \frac{288}{ZP}}.$$  

(K.17)

Values of $XP$ for promising pellet media are given in Table K.2. The r.m.s. angular spread caused by passage through the pellet with target thickness $tP$ and momentum $p_p$ is given by

$$\theta_{\text{r.m.s.}} = \frac{21 \text{ MeV}}{p_p c \beta_p} \sqrt{\frac{tP}{XP}} = \frac{21 \text{ MeV}}{\sqrt{2m_pc^2K_p (1 + K_p/(4m_pc^2))} \beta_p} \sqrt{\frac{tP}{XP}},$$

(K.18)

where $p_pc$ has been substituted from Eq. (K.9).
K.1.5.10 Pellet radius required for efficient bunch sampling extraction

As explained earlier, with dispersion function $D_p$ assumed constant, when the betatron phase has increased by $\pi$ (or any odd multiple of $\pi$), a particle passing through a pellet centre will be displaced from its previous off-momentum closed orbit by an amount $2D_p\delta$, with $\delta$ given by Eq. (K.11). Any polarimeter in the ring is assumed to be located at such a position.

Even particles touching a pellet will not, in general, pass through the pellet centre. About 1/2 of the pellets will be sufficiently off-centre for their path length through the pellet to be at least 30 percent less than the pellet diameter. These pellets we ignore, under the assumption that their energy loss has been insufficient for them to be differentiated from the surviving main beam, and therefore unlikely to register in the polarimetry. The path lengths of the remaining particles will all be approximately the same. They will be treated as if centred on the pellet.

References

[1] Z. Sun et al., First results of ELM triggering with a multichamber lithium granule injector into EAST discharges, IEEE Transactions on Plasma Science, 46, No. 5, 2018
[3] I. Keshelashvili, Juelich ballistic diamond pellet target for storage ring EDM measurement, ERC Consolidation Grant research proposal, 2016