The CKM angle $\gamma / \phi_3$

Frédéric Machefert, LAL(Orsay)
On behalf of the LHCb collaboration
The angle $\gamma / \phi_3$ and the methods to measure it.
The $\gamma / \phi_3$ angle

$$\frac{\gamma}{\phi_3} \equiv \arg \left( -\frac{V_{ud} \cdot V_{ub}^*}{V_{cd} \cdot V_{cb}^*} \right) \simeq \arg \left( -\frac{V_{ub}^*}{V_{cb}^*} \right)$$

- The angle $\gamma$ can be measured when the $b \rightarrow u$ and $b \rightarrow c$ type decays interfere to exhibit CP violation
- $\gamma$ can be measured in CPV tree decays with tiny theory uncertainties
  - No $V_{tx}$ term → the only angle that can be measured without penguin pollution (indirect measurements contain loops) \( \frac{\delta \gamma}{\gamma} \approx 10^{-7} \)
- $B \rightarrow D^{(*)} K^{(*)}$ decays satisfy those criteria and some potentially give large CP violation

\[ \frac{\gamma}{\phi_3} = \left( 72.1^{+5.4}_{-5.8} \right)^\circ \quad \frac{\gamma}{\phi_3} = \left( 65.3^{+1.0}_{-2.5} \right)^\circ \]

Babar, Belle, LHCb

Aim is to measure it at the degree-level precision

[PRD 92(3) 033002]

Standard Candle
The methods to measure $\gamma$ : text book example

$\gamma \equiv \arg \left( -\frac{V_{ud} \cdot V_{ub}^*}{V_{cd} \cdot V_{cb}^*} \right) \approx \arg \left( -\frac{V_{ub}^*}{V_{cb}^*} \right)$

$A \left( B^- \rightarrow D^0 K^- \right) = r_B e^{i(\delta_B - \gamma)}$

The typical CP violation observables which are measured are

- Charge asymmetries
  $$A = \frac{\Gamma \left( B^- \rightarrow f_D K^- \right) - \Gamma \left( B^+ \rightarrow \bar{f}_D K^+ \right)}{\Gamma \left( B^- \rightarrow f_D K^- \right) + \Gamma \left( B^+ \rightarrow \bar{f}_D K^+ \right)}$$

- Partial width ratios
  $$R = \frac{\Gamma \left( B^- \rightarrow f_D K^- \right) + \Gamma \left( B^+ \rightarrow \bar{f}_D K^+ \right)}{\Gamma \left( B^- \rightarrow f_{D^{\text{fav}}} K^- \right) + \Gamma \left( B^+ \rightarrow \bar{f}_{D^{\text{fav}}} K^+ \right)}$$

The hadronic parameters (measured) are

- Amplitude ratio $r_B$
- Strong phase difference $\delta_B$

Many methods using different D decays (D decay parameters needed)

Improves the systematics : no CPV in favoured decays
GLW versus ADS methods

GLW: CP eigenstates
Gronau, London, Wyler
[PLB 352 (1991), 483]
[PLB 265 (1991), 172]

ADS: large interference
(=large asymmetries)
[PRL 78 (1997) 3257]

GLW:
\[ \Gamma (B^+ \rightarrow f D K^-) \propto 1 + r_B^2 + 2 r_B \cos (\delta_B + \gamma) \]

ADS:
\[ \Gamma (B^+ \rightarrow f D K^-) \propto (r_D^f)^2 + r_B^2 + 2 r_B r_D^f \cos (\delta_B + \delta_D^f + \gamma) \]
**GLW versus ADS methods**

**GLW: CP eigenstates**
Gronau, London, Wyler
[PLB 352 (1991), 483]
[PLB 265 (1991), 172]

\[ r_{B}e^{i(\delta_{B}-\gamma)} \]

\[ D^{0} K^{-} \]

\[ B^{-} \]

\[ (\pi^{+} \pi^{-} \frac{K+K^{-}}{2}) K^{-} \]

\[ D^{0} K^{-} \]

\[ r_{D}=1, \delta_{D}=0 \]

**ADS: large interference**
(= large asymmetries)
[PRL 78 (1997) 3257]

\[ r_{B}e^{i(\delta_{B}-\gamma)} \]

\[ D^{0} K^{-} \]

\[ B^{-} \]

\[ (K^{+} \pi^{-}) K^{-} \]

\[ D^{0} K^{-} \]

\[ r_{D}e^{i\delta_{D}} \]

external measurements

Favoured and suppressed decays in both
Interfering decay amplitudes

**GLW:**
\[ R_{CP\pm} = 1 + r_{B}^{2} \pm 2 r_{B} \cos \delta_{B} \cos \gamma \]

**ADS:**
\[ R_{ADS} = r_{B}^{2} + r_{D}^{2} + 2 r_{B} r_{D} \cos \gamma \cos(\delta_{B} + \delta_{D}) \]

\[ A_{CP\pm} = \frac{\pm 2 r_{B} + \sin \delta_{B} \sin \gamma}{R_{CP\pm}} \]

\[ A_{ADS} = \frac{2 r_{B} r_{D} \sin \gamma \sin(\delta_{B} + \delta_{D})}{R_{ADS}} \]
GLW versus ADS methods: D four body decays

GLW and ADS methods can be extended to more complex final states

**Quasi - GLW**

\[ \frac{r_B e^{i(\delta_B - \gamma)}}{r_B e^{i(\delta_B - \gamma)}} \]

\[ B^- (\pi^+ \pi^- \pi^+ \pi^-) \]

\[ D^0 K^- \]

Fractional CP-even content

\[ F_+ = 0.737 \pm 0.028 \]

\[ 2F_+ - 1 \sim 0.5 \]

[Malde et al. PLB 747 (2015) 9]

[Atwood and Soni, PRD 68 (2003) 033003]

[arXiv:1602.07430]

**Quasi - ADS**

\[ \frac{r_B e^{i(\delta_B - \gamma)}}{D^0 K^-} \]

\[ B^- (K^+ \pi^- \pi^+ \pi^-) K^- \]

Coherence factor

\[ \kappa_{3\pi_D} = 0.43^{+0.17}_{-0.13} \]

GLW:  \[ \Gamma (B^+ \to f_D K^+) \propto 1 + r_B + (2F_+ - 1)2 r_B \cos (\delta_B + \gamma) \]

ADS:  \[ \Gamma (B^+ \to f_D K^+) \propto (r_D^f)^2 r_B^2 + 2r_B r_D^f \kappa_D^f \cos (\delta_B + \delta_D^f + \gamma) \]
GGSZ method : 3-body final state

- The idea is to perform the GLW/ADS type analysis across the D decay phase space
- For example:
  - $D^0 \to K_s^0 \pi^+ \pi^-$ has contributions from
    - Singly-Cabibbo-suppressed decay $D^0 \to K_s^0 \rho^0$
    - Double-Cabibbo-suppressed decay $D^0 \to K^{*+} \pi^-$
  - Interference between the contributions enhances the sensitivity and reduces the ambiguities in $\gamma$ determination
    
    $$ d \Gamma_B \pm (x) = A_{\pm, \mp}^2 + r_B^2 A_{-\pm, \pm}^2 + 2 A_{\pm, \mp} A_{-\mp, \pm} [r_B \cos (\delta_B \pm \gamma) \cos (\delta_D \pm \gamma) + r_B \sin (\delta_B \pm \gamma) \sin (\delta_D \pm \gamma)] $$

- 2 situations:
  - Model-dependent : fit with full amplitude model for $(x_\pm, y_\pm)$
  - Model-independent : choose binning scheme in Dalitz plane to minimize $d\Gamma$ variations across bin and fit simultaneously in each bin for $(x_\pm, y_\pm)$
Example of the $D^0 \to K_s^0 \pi^+ \pi^-$

- Sensitivity to $\gamma$ is given by a comparison of the $D$ Dalitz distributions for the $B^+$ and $B^-$ mesons
- CP asymmetry is looked at in bins of the Dalitz space

[PRD 82 (2010) 112006]
The time-dependent method with $B_s^0 \to D_s^{\pm}K_{s}^{+/-}$

- $B_s^0$ and anti-$B_s^0$ can both decay in the same $D_s^{\pm}K_{s}^{+/-}$ final state
  - Either via $b \to uW$ or $b \to cW$
- The interference is given by the neutral $B_s^0$ mixing process
  - Requires the knowledge of $-2\beta_s \equiv \phi_s$
  - The weak phase is $(\gamma - 2\beta_s)$

![Diagram of the decay process](image)

- The initial $B_s^0$ flavour must be tagged
- The meson oscillation has to be observed $\to$ time dependent analysis
  - Fit of the decay-time decay rate
  - Also requires the knowledge of $\Gamma_s, \Delta\Gamma_s, \Delta m_s$
The time-dependent method with $B_s^0 \rightarrow D_s^{\pm} K^{\mp}$

- Fit the decay time dependent asymmetry

\[
A_{CP}(t) = \frac{\Gamma_{B_s^0 \rightarrow f}(t) - \Gamma_{B_s^0 \rightarrow \bar{f}}(t)}{\Gamma_{B_s^0 \rightarrow f}(t) + \Gamma_{B_s^0 \rightarrow \bar{f}}(t)} = \frac{S_f \sin(\Delta m_s t) - C_f \cos(\Delta m_s t)}{\cosh(\frac{\Delta \Gamma_s t}{2}) + A_f^{\Delta \Gamma_s} \sinh(\frac{\Delta \Gamma_s t}{2})}
\]

\[
C_f = -C_{\bar{f}} = \frac{1-r_B^2}{1+r_B^2}
\]

\[
A_{f(\bar{f})}^{\Delta \Gamma} = \frac{-2r_B \cos(\gamma - 2\beta \mp \delta_B)}{1+r_B^2}
\]

\[
S_{f(\bar{f})} = \pm 2r_B \sin(\gamma - 2\beta \mp \delta_B)
\]

\[\text{Im}[2\lambda_f/(1+|\lambda_f|^2)]\]

\[\text{Re}[2\lambda_f/(1+|\lambda_f|^2)]\]
B Dalitz method

- Analysis the Dalitz of the 3 body decay of the B in $B^0 \to DK^+\pi^-$
  - Excellent sensitivity to $\gamma$
  - GW method (Gershon-Williams ArXiv:0909.1495)

- Multiple interfering resonances $\rightarrow$ increased sensitivity to $\gamma$
  - $D^{*0}(2400)^-, D^*_2(2460)^-, K^*(892)^0, K^*(1410)^0, K^*_2(1430)^0$

- Fit B decay Dalitz plot for cartesian parameters
  - Similar to the GGSZ method but with the B, not the D
    - $D\to KK, \pi\pi$ (GLW-Dalitz)
    - $D\to K^+/\pi^-/\pi^+$ (ADS-Dalitz)
    - $D\to K_s^0 \pi^+\pi^-$ (GGSZ-Dalitz : double Dalitz !)

![LHCb](image_url)
e+e- colliders
The pioneers of the $\gamma$ measurement

- BaBar and Belle pioneered the measurement of $\gamma$ using all the methods at hand
  - GLW, ADS, GGSZ and time-dependent methods

BaBar @ PEP-II
- 9 GeV $e^-$ + 3.1 GeV $e^+$

Belle @ KEKB
- 8 GeV $e^-$ + 3.5 GeV $e^+$
e+e- colliders – GLW method

- $\Delta E$ projections of the fits to the $B^{+/-} \rightarrow D_{CP^{+}} K^{+/-}$ candidates

[BaBar]
Full data sample

[Belle]
Full data sample

[J. C74 (2014) 3026]
e+e- colliders – ADS method

Belle

\[ B^- \rightarrow D(K^+\pi^-)K^- \]

- \( \Delta E \) and NN distributions in a signal enriched region

Belle

\[ B^- \rightarrow D^*K^- \]

\[ D^* \rightarrow D\gamma \]

\[ D \rightarrow K^+\pi^- \]

[J. C74 (2014) 3026]
**e+e- colliders – GGSZ method – Selection of the events**

**Belle**
Full data sample

**BaBar**
Full data sample

<table>
<thead>
<tr>
<th>Mode</th>
<th>Belle, $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ (Poluektov, 2010)</th>
<th>$B_{\perp}B$,$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ (del Amo Sanchez, 2010b)</th>
<th>$B_{\perp}B$, $D^0 \rightarrow K_S^0 K^+ K^-$ (del Amo Sanchez, 2010b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\perp} \rightarrow D K_{\perp}$</td>
<td>756 (71%)</td>
<td>896 ± 35 (68%)</td>
<td>154 ± 14 (82%)</td>
</tr>
<tr>
<td>$B_{\perp} \rightarrow D^* K_{\perp}$, $D^* \rightarrow D \pi^0$</td>
<td>149 (78%)</td>
<td>255 ± 21 (81%)</td>
<td>56 ± 11 (87%)</td>
</tr>
<tr>
<td>$B_{\perp} \rightarrow D^* K_{\perp}$, $D^* \rightarrow D \gamma$</td>
<td>141 (42%)</td>
<td>193 ± 19 (55%)</td>
<td>30 ± 7 (78%)</td>
</tr>
<tr>
<td>$B_{\perp} \rightarrow D K_{\perp}^*$</td>
<td>54 ± 8 (65%) (Poluektov, 2006)</td>
<td>163 ± 18 (58%)</td>
<td>28 ± 6 (81%)</td>
</tr>
</tbody>
</table>

**yield(purity)**

*References*

[J. C74 (2014) 3026]
BaBar model dependent analysis

BaBar used both the $D^0 \rightarrow K_s \pi \pi$ (left) and $D^0 \rightarrow K_s KK$ (right) modes.

Optimal binning of the $D^0 \rightarrow K_s \pi \pi$ decay produced by Cleo-c and used by Belle to measure $\gamma$ without any model assumption.

The parameters $c_i$ and $s_i$ include information about the cosine and sine of the phase difference $\frac{1}{2} \delta_D (m_+^2, m_-^2)$ averaged over a bin.

\[ s_\pm = m_\pm^2 \]
\[ s_0 = m_{h^+ h^-}^2 \]

[J. C74 (2014) 3026]
Time dependent measurements involve \((2\beta + \gamma)\):

- The \(B^0\)-anti\(B^0\) mixing, \(B \rightarrow D^{(*)} h, h = \pi, \rho\) decay
- The \(B^0_s\)-anti\(B^0_s\) mixing, \(B \rightarrow D^*_S^{(*)} h, h = \pi, K\) decay

Belle measurement of the distance between the tagged and reconstructed B vertices along \(z\) and where the lepton (tag) has either the same or opposite charge as the low-momentum pion of the decay.

A non-zero amplitude of the sinusoidal shape is the sign of CP violation.

[J. C74 (2014) 3026]
Combination of the BaBar and belle measurements using a frequentist procedure and the GLW, ADS and GGSZ methods

<table>
<thead>
<tr>
<th></th>
<th>$\phi_3$ ($^\circ$)</th>
<th>$r_B(DK)$</th>
<th>$\delta_B(DK)$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>69 ± 17</td>
<td>0.090$^{+0.016}_{-0.017}$</td>
<td>105 ± 19</td>
</tr>
<tr>
<td>Belle</td>
<td>68 ± 14</td>
<td>0.112 ± 0.015</td>
<td>116$^{+19}_{-21}$</td>
</tr>
<tr>
<td>$B$ Factors</td>
<td>67 ± 11</td>
<td>0.102 ± 0.011</td>
<td>111$^{+13}_{-14}$</td>
</tr>
</tbody>
</table>

[J. C74 (2014) 3026]
Hadron collider - LHCb
# y angle measurement with LHCb

<table>
<thead>
<tr>
<th>B decay</th>
<th>D decay</th>
<th>Method</th>
<th>Ref.</th>
<th>Dataset</th>
<th>Status since last combination</th>
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</thead>
<tbody>
<tr>
<td>B⁺ → DK⁺</td>
<td>D → h⁺h⁻</td>
<td>GLW</td>
<td>14</td>
<td>Run 1 &amp; 2</td>
<td>Minor update</td>
</tr>
<tr>
<td>B⁺ → DK⁺</td>
<td>D → h⁺h⁻</td>
<td>ADS</td>
<td>15</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>B⁺ → DK⁺</td>
<td>D → h⁺π⁻π⁺π⁻</td>
<td>GLW/ADS</td>
<td>15</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>B⁺ → DK⁺</td>
<td>D → h⁺h⁻π⁰</td>
<td>GLW/ADS</td>
<td>16</td>
<td>Run 1</td>
<td>As before</td>
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<tr>
<td>B⁺ → DK⁺</td>
<td>D → K⁺h⁻h⁻</td>
<td>GGSZ</td>
<td>17</td>
<td>Run 1</td>
<td>As before</td>
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<tr>
<td>B⁺ → DK⁺</td>
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<td>GGSZ</td>
<td>18</td>
<td>Run 2</td>
<td>New</td>
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<tr>
<td>B⁺ → DK⁺</td>
<td>D → K⁺h⁻h⁻</td>
<td>GLS</td>
<td>19</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>B⁺ → DK⁺</td>
<td>D → h⁺h⁻</td>
<td>GLW</td>
<td>14</td>
<td>Run 1 &amp; 2</td>
<td>Minor update</td>
</tr>
<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → h⁺h⁻</td>
<td>GLW/ADS</td>
<td>20</td>
<td>Run 1 &amp; 2</td>
<td>Updated results</td>
</tr>
<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → h⁺π⁻π⁺π⁻</td>
<td>GLW/ADS</td>
<td>20</td>
<td>Run 1 &amp; 2</td>
<td>New</td>
</tr>
<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → h⁺π⁻π⁺π⁻</td>
<td>GLW/ADS</td>
<td>21</td>
<td>Run 1</td>
<td>As before</td>
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<td>B⁺ → DK⁺⁺</td>
<td>D → h⁺h⁻</td>
<td>GLW/ADS</td>
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<td>As before</td>
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<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → K⁺h⁻</td>
<td>ADS</td>
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<td>Run 1</td>
<td>As before</td>
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<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → h⁺h⁻</td>
<td>GLW-Dalitz</td>
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<td>As before</td>
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<td>B⁺ → DK⁺⁺</td>
<td>D → K⁺h⁻</td>
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<td>As before</td>
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<td>B⁺ → DK⁺⁺</td>
<td>D → D⁺⁺K⁺</td>
<td>TD</td>
<td>25</td>
<td>Run 1</td>
<td>Updated results</td>
</tr>
<tr>
<td>B⁺ → DK⁺⁺</td>
<td>D → D⁺⁺K⁺</td>
<td>TD</td>
<td>26</td>
<td>Run 1</td>
<td>New</td>
</tr>
</tbody>
</table>

[LHCb-CONF-2018]

Results based on
1, 2 fb⁻¹ @ 7, 8 TeV (Run I)
2 fb⁻¹ @ 13 TeV (Run II)

NB: Hadron collider measurement at Fermilab of R_{CP} and A_{CP} by CDF with a limited precision [arXiv:0809.4809v2]
ADS/GLW: from $B^{+/-}\rightarrow D K^{*+/-}$ decays

- Uses both two- and four-body D decays

- 12 CP observables from $B^{-}\rightarrow D(-f)K^{*}(892)^{-}$, with $K^{*}(892)^{-}\rightarrow K_{s}^{0}\pi^{-}$
  - $f = K^{-}K^{+}, \pi^{-}\pi^{+}, K^{-/+}\pi^{+/−}, \pi^{+}\pi^{-}\pi^{+}\pi^{-}, K^{-/+}\pi^{+/−}\pi^{+}\pi^{-}$

$$A_{CP}^{f} = \frac{\Gamma(B^{-}\rightarrow f_{D} K^{*-}) - \Gamma(B^{+}\rightarrow \bar{f}_{D} K^{*+})}{\Gamma(B^{-}\rightarrow f_{D} K^{*-}) + \Gamma(B^{+}\rightarrow \bar{f}_{D} K^{*+})}$$

$$R_{f} = \frac{\Gamma(B^{-}\rightarrow f_{D} K^{*-}) + \Gamma(B^{+}\rightarrow \bar{f}_{D} K^{*+})}{\Gamma(B^{-}\rightarrow f_{D}^{fav} K^{*-}) + \Gamma(B^{+}\rightarrow \bar{f}_{D}^{fav} K^{*+})} \times \frac{BR(f^{fav}_{D})}{BR(f_{D})}$$

- CP violation neglected in the D decays:
  - $A_{CP} = A_{raw} - A_{production} - A_{det}$
  - $A_{CP}^{KK} = A_{CP}^{\pi\pi} = A_{CP}^{+}$
  - $R_{KK} = R_{\pi\pi} = R_{CP}^{+}$

- No CP violation expected in favoured decays
  - $B^{-}\rightarrow [K^{-}\pi^{+}]_{D} K^{-}, B^{-}\rightarrow [K^{-}\pi^{+}\pi^{+}\pi^{-}]_{D} K^{-}$
ADS/GLW: from $B^{+/-}\rightarrow DK^{+/-}$ decays

- **GLW modes**

  \[ R_{CP\pm} = 1 + r_B^2 \pm 2 \kappa r_B \cos \delta_B \cos \gamma \quad A_{CP\pm} = \pm \frac{2 \kappa r_B + \sin \delta_B \sin \gamma}{R_{CP\pm}} \]

  - $\kappa$ accounts for $K^0_s\pi$ not from $K^*$ (pure $K^*$: $\kappa=1$)
  
  - Simulations give $\kappa=0.95\pm0.06$

  \[ R_{\pi\pi\pi\pi} = 1 + r_B^2 \pm 2 \kappa \left(2F_{4\pi} - 1\right) r_B \cos \delta_B \cos \gamma \quad A_{CP\pm} = \pm \frac{2 \kappa \left(2F_{4\pi} - 1\right) r_B + \sin \delta_B \sin \gamma}{R_{CP\pm}} \]

- **ADS decays require more inputs**

  \[ R_{K^\pi}^\pm = \frac{r_B^2 + (r_{K^\pi}^{D})^2 + 2 \kappa r_B r_{K^\pi}^{D} \cos \left(\delta_B + \delta_{K^\pi}^D \pm \gamma\right)}{1 + (r_B r_{K^\pi}^{D})^2 + 2 \kappa r_B r_{K^\pi}^{D} \cos \left(\delta_B - \delta_{K^\pi}^D \pm \gamma\right)} \]

  \[ R_{K3\pi}^\pm = \frac{r_B^2 + (r_{K3\pi}^{D})^2 + 2 \kappa r_B \kappa_{K3\pi}^D r_{K3\pi}^{D} \cos \left(\delta_B + \delta_{K3\pi}^D \pm \gamma\right)}{1 + (r_B r_{K3\pi}^{D})^2 + 2 \kappa r_B \kappa_{K3\pi}^D r_{K3\pi}^{D} \cos \left(\delta_B - \delta_{K3\pi}^D \pm \gamma\right)} \]

  \[ \kappa_{K3\pi}^D \neq 1 \]

  \[ \pi^+\pi^-\pi^+\pi^- \text{ is not a pure CP eigenstate} \]

  - $F_{4\pi} \sim 0.75$

  - $r_{K^\pi}^D, \delta_{K^\pi}^D$

  - $r_{K3\pi}^D, \delta_{K3\pi}^D, \kappa_{K3\pi}^D$

  [PLB 747 (2015) 9]

  [arXiv:1612.07233]

  [PRL 116 (2016) 241801]

  [PLB 757 (2016) 520]
ADS/GLW: from $B^{+/-} \rightarrow DK^{**+/-}$ decays

- Fit to the favoured decays in an extended range determines signal and background models

Simultaneous fit to 56 subsamples

[JHEP 11 (2017) 156]
ADS/GLW: from $B^{+/-} \rightarrow DK^{*+/-}$ decays

$A_{CP^+} = 0.08 \pm 0.06 \pm 0.01$

$R_{CP^+} = 1.18 \pm 0.08 \pm 0.01$

$A_{ADS}^{K \pi} = 0.011 \pm 0.004 \pm 0.001$

$R_{ADS}^{K \pi} = -0.81 \pm 0.17 \pm 0.04$

$A_{ADS}^{K^3 \pi} = -0.45 \pm 0.21 \pm 0.14$

$R_{ADS}^{K^3 \pi} = 0.011 \pm 0.005 \pm 0.003$

[JHEP 11 (2017) 156]
LHCb published recently an update on $\gamma$ from fully reconstructed $B^- \rightarrow (h^+ h^-)_D h^-$

- $h^+ h^- = K^+ K^-, \pi^+ \pi^-, K^- \pi^+$

Increased sensitivity to $\gamma$ by adding new $B \rightarrow D^* K$ modes

- Same selection for $B^{-/+} \rightarrow D^* h^{-/+}$ and $B^{-/+} \rightarrow D h^{-/+}$
- Fully and partially reconstructed decays in the same spectrum

Small $D^*-D$ mass difference and conservation of parity and angular momentum

- Distinctive signatures of $D^* \rightarrow D \pi^0$ and $D^* \rightarrow D \gamma$

19 CP observables: asymmetries and ratios of partial widths

[PLB 777 (2018) 16]
Values of CP observables determined by a simultaneous fit to 12 independent samples: $B^{+/—} \rightarrow D^{(*)}K^{+/—}$, $D \rightarrow hh$ and $D^{*—} (hh) \gamma/\pi^0$

- $K^{+/—}$ and $\pi^{+/—}$ detection asymmetry from $D^+ \rightarrow K^- \pi^+ \pi^+$ and $D^+ \rightarrow K^0_s \pi^+$
- Production asymmetry from high yield $B^— \rightarrow [K^-\pi^+]_D \pi^-$ decay

[PLB 777 (2018) 16]
A positive CP asymmetry observed in $B^{+/−} \rightarrow D_{CP}K^{+/−}$

$B^{-} \rightarrow D^{*}K^{-}, D^{*} \rightarrow D\pi^{0}$
- $D = D^{0} + r_{B} e^{i(\delta_{B} − \gamma)} D^{0}$

$B^{-} \rightarrow D^{*}K^{-}, D^{*} \rightarrow D\gamma$
- $D = D^{0} + r_{B} e^{i(\delta_{B} + \pi − \gamma)} D^{0}$

CP asymmetries from $D^{*} \rightarrow D\pi^{0}$ and $D^{*} \rightarrow D\gamma$ have opposite signs

$$R_{CP^{+}} = 0.989 \pm 0.013 \pm 0.010$$
$$A_{CP^{+}} = 0.124 \pm 0.012 \pm 0.002$$

[PLB 777 (2018) 16]
GGSZ mode: $B^{+/0} \rightarrow DK^{+/0}$, $D \rightarrow K^0_s \pi^+ \pi^- / D \rightarrow K^0_s K^+ K^-$

$D^0 \rightarrow K^0_S \pi^+ \pi^-$ [JHEP 08 (2018) 176]

- $B\rightarrow D\pi^-$ used to estimate contamination in $B\rightarrow DK^-$ sample due to $\pi$-$K$ misID
- Fit to the 8 $B^{+/0} \rightarrow Dh^{+/0}$ subsamples, integrated over the DP, fixes the signal/bkg shapes
- Yield/bin of $B^{+/0} \rightarrow D\pi^{+/0}$: direct mass fit
- Yield/bin of $B^{+/0} \rightarrow DK^{+/0}$:

$$S_i^\pm = \frac{N_{tot}^i (DK)}{\sqrt{\sum_{-n}^{+n} N_i^\pm}}$$

Population/bin normalisation

$N_i^\pm = h_B^i \left[ F_i^\pm + (x^2 + y^2) F_i^\perp + 2 \sqrt{F_i F_i^\perp (x_c^\pm y_s^\pm)} \right]$  

With $x^\pm \equiv r_B \cos(\delta_B \pm \gamma)$, $y^\pm \equiv r_B \sin(\delta_B \pm \gamma)$

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Frédéric Machefert - February 2019  
KEK Flavour Factory workshop 2019
GGSZ mode: $B^{+/−} \to DK^{+/−}$, $D \to K^0_s \pi^+ \pi^− / D \to K^0_s K^+ K^−$

- First observation ($6.4\sigma$) of CPV in $B^{+/−} \to DK^{+/−}$ with $D0 \to K^0_s h^+ h^−$

Cleo-c
[PRD 82 (2010) 112006]
- LHCb published (LHCb-CONF-2018-002) a new combination
- 98 observables and 40 free parameters
LHCb combination

- **Breakdown of the measurement by methods**
- **Contributions from B\(^+\), variation in precision and # of solutions**

![Graphs showing measurement breakdown and contributions](image)

- **Breakdown according to the B meson type**
- **Expected uncertainty from full Run I + II dataset: 3-4°**
World average and prospects
World Average

- Present world average

\[ \gamma = 73.5^{+4.2}_{-5.1} \]°

No sector real tension on the global fit
External inputs

- Multi-body decays of the D provide key methods to extract $\gamma$ and can be analysed in 2 ways:
  - Model-dependent: irreducible systematics
  - Model-independent: requires external inputs for the hadronic param. of the D decay (strong phase, coherence factors, ...)
  - Need quantum correlated charm threshold data:
    - Cleo-c (0.8/fb) and BESIII (taking data)
Prospect for the measurement of $\gamma$

- LHCb and Belle have started ambitious upgrade programs

Each experiment aims at reaching $\sim 1^\circ$
Backup
The CKM matrix

- In the SM, quarks can change flavour by emission of a $W$ boson
- Quark mixing in the SM is described by the 3x3 unitarity matrix

\[
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix}
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}
\]

- The matrix elements determine the transition amplitude

- Parametrized by three mixing angles and a CP violating phase
The CKM matrix

- The CKM matrix exhibits a clear hierarchy
  \[ \sin(\theta_{13}) \ll \sin(\theta_{23}) \ll \sin(\theta_{12}) \ll 1 \]

- So, often expressed in term of the Wolfenstein parameterisation

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4)
\]

- Constrained behaviour of the flavour sector and strong constraints on new physics

- CKM is the only source of CP violation in the SM

\[
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0
\]
\[
\Rightarrow \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + 1 + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0
\]

- The CKM angle \( \gamma / \phi_3 \) is among the least well known constraints