SYMMETRY BREAKING INDUCED BY TOP LOOPS

J.P. Fatelo†, J.-M. Gérard, T. Hambye†, J. Weyers

Institut de Physique Théorique
UCL
B-1348 Louvain-la-Neuve, Belgium.

Abstract

It is argued that top quark loops trigger symmetry breaking in the Standard Electroweak Model. The Higgs boson is then expected to be lighter than 400 GeV. Further speculations on this dynamical mechanism even suggest a Higgs boson observable at LEP 200.

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† Chercheur IISN, Belgium.
Recent experimental evidence [1] for a top quark with a mass around 174 GeV implies that the strongest force in the electroweak sector of the Standard Model is due to the Yukawa coupling of the Higgs scalar to the top quark. Indeed, with \( m_t = \frac{g v}{\sqrt{2}} \) and the vacuum expectation value \( v \approx 247 \text{ GeV} \), the coupling constant \( g \) is close to 1.

In this note we argue that symmetry breaking (SB) of the gauge group \( SU(2)_L \times U(1) \) is a dynamical effect driven by this Yukawa force. More precisely, in the framework of an effective electroweak theory, our suggestion is that SB does not have to be put in by hand at tree level but is induced by top quark loops. One immediate consequence is a range of allowed values for the Higgs boson mass \( 30 \text{ GeV} \lesssim m_H \lesssim 400 \text{ GeV} \) depending on the values of the physical cut-off \( \Lambda \). Furthermore, if we try to implement the idea that top one-loop effects exhaust the physics of SB by making the effective scalar interactions as small as possible, we find \( \Lambda \approx 1 \text{ TeV} \) and \( m_H \approx 80 \text{ GeV} \).

In a completely different context, namely that of a fundamental renormalizable electroweak theory, a similar SB mechanism can apply and appears to be an attractive alternative to the usual Higgs mechanism. Here again since top quark loops trigger SB it is tempting to neglect scalar loops or, more technically, to assume that \( \lambda_R \) - the renormalized quartic scalar coupling - is quite small if not zero. Should one take \( \lambda_R = 0 \) at the scale \( v \) one would find the same rather small value for the Higgs boson mass, namely around 80 GeV.
I. Effective Electroweak Theory

Let us first consider the Lagrangian

\[ \mathcal{L} = \bar{\Psi} i \partial \Psi + \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\mu^2}{2} \phi^2 + \frac{g}{\sqrt{2}} \bar{\Psi} \Psi \phi \]  

(1)

where \( \Psi \) is the top quark field and \( \phi \) the neutral component of the standard Higgs doublet.

Eq(1) is nothing but the Lagrangian of a massless fermion interacting with a massive scalar \( (\mu^2 > 0) \). It is the relevant part of the Standard Model Lagrangian for symmetry breaking, as we shall see. We explicitly assume no quartic scalar self interactions. This is perfectly acceptable in the context of an effective theory, i.e. a theory cut off at some physical scale \( \Lambda \).

The one-loop quantum corrections to the tree-level potential \( V^{(0)}(\phi) = \frac{1}{2} \mu^2 \phi^2 \) are obtained from the infinite series of Feynman diagrams given in Fig.1.

The resulting effective potential reads [2,3]

\[ V^{(1)} = \frac{\mu^2 \phi^2}{2} - \frac{N_c}{8\pi^2} \int_0^\Lambda q^2 dq^2 \ln \left( 1 + \frac{g^2 \phi^2}{2q^2} \right) \]

\[ = \frac{\mu^2 \phi^2}{2} - \frac{N_c}{16\pi^2} \left\{ \Lambda^4 \ln \left( 1 + \frac{g^2 \phi^2}{2\Lambda^2} \right) + \frac{g^2 \phi^2 \Lambda^2}{2} \right\} - \frac{g^4 \phi^4}{4} \ln \left( 1 + \frac{2\Lambda^2}{g^2 \phi^2} \right) \]

(2)

with \( N_c \) the number of colors.

From Eq.(2) it is clear that the classical minimum \( \langle \phi \rangle = 0 \) of the tree level potential can be turned into a maximum by the one loop corrections. A new minimum
then appears at $\langle \phi \rangle = v \neq 0$, while the potential of Eq.(2) remains bounded from below. Indeed, with $m_t = g t / \sqrt{2}$, the extremum condition on $V^{(1)}$ admits the unique non trivial solution
\[
\frac{m_t^2}{\Lambda^2} \ln \left( 1 + \frac{\Lambda^2}{m_t^2} \right) = 1 - \frac{8\pi^2 \mu^2}{N_c g^2 \Lambda^2}
\]  
if $0 < \mu^2 < \frac{N_c g^2 \Lambda^2}{8\pi^2}$. In words, for quantum SB to occur, the scalar mass term at tree level must be genuine. The change of sign of the second derivative of the potential at the origin is entirely due to the one-loop corrections.

The Higgs mass is defined by
\[
\frac{\partial^2 V^{(1)}}{\partial \phi^2} |_{\phi = v} = m_H^2 = \frac{N_c g^2}{4\pi^2} \left\{ \ln \left( 1 + \frac{\Lambda^2}{m_t^2} \right) - \frac{\Lambda^2}{\Lambda^2 + m_t^2} \right\} m_t^2
\]  
and the fourth derivative of $V^{(1)}$ at the vacuum expectation value reads
\[
\frac{\partial^4 V^{(1)}}{\partial \phi^4} |_{\phi = v} = \frac{3N_c g^4}{8\pi^2} \left\{ \ln \left( 1 + \frac{\Lambda^2}{m_t^2} \right) + \frac{9m_t^2}{\Lambda^2 + m_t^2} - \frac{8m_t^4}{(\Lambda^2 + m_t^2)^2} + \frac{8}{3} \frac{m_t^6}{(\Lambda^2 + m_t^2)^3} - \frac{11}{3} \right\}
\]  

It is not difficult to include one-loop gauge boson contributions to Eq.(2). One then obtains
\[
V^{(1)} = \frac{1}{2} \mu^2 \phi^2 - \frac{1}{32\pi^2} \int_0^\Lambda dq^2 q^2 \left\{ 4N_c \ln \left[ 1 + \frac{g^2 \phi^2}{2q^2} \right] - 6 \ln \left[ 1 + \frac{g_2^2 \phi^2}{4q^2} \right] - 3 \ln \left[ 1 + \frac{(g_1^2 + g_2^2) \phi^2}{4q^2} \right] \right\}
\]  
with $g_1$ and $g_2$ the $U(1)$ and $SU(2)_L$ gauge couplings, respectively. In our normalization the $W$ and $Z$ masses are given by $M_W^2 = \frac{g_2^2 v^2}{4}$ and $M_Z^2 = \frac{(g_1^2 + g_2^2) v^2}{4}$. It
is then straightforward to compute the corresponding modifications to Eq.(3) and Eq.(4).

Notice that the gauge boson contributions alone would lead to \( \frac{\partial^2 V^{(1)}}{\partial \phi^2} |_{\phi=v} < 0 \). A real Higgs mass requires (see Fig.2)

\[
m_t \gtrsim \left( \frac{6M_W^4 + 3M_Z^4}{4N_c} \right)^{1/4} \approx 78 \text{ GeV}.
\]

(7)

A heavy top is therefore an essential ingredient for the quantum SB mechanism advocated in this letter. Furthermore with \( m_t \) around 174 GeV, the contributions of the gauge bosons to the effective potential are numerically quite small, at most of the order of a few percent. They are included in Fig. 2 where we plot \( m_H \) as a function of \( m_t \) for different values of the cut-off \( \Lambda \). If we take \( \Lambda \) as small as the vacuum expectation value \( v \), we find the lower limit \( m_H \approx 30 \text{ GeV} \). However, the experimental bound [4] for the Higgs mass \( m_H \gtrsim 60 \text{ GeV} \) together with the range allowed by LEP and CDF data [5] for the top mass require \( \Lambda \) to be greater than about 500 GeV. On the other hand taking \( \Lambda \) smaller than the GUT scale implies the following upper bound

\[
m_H \lesssim 400 \text{ GeV}.
\]

(8)

In fact for values of \( \Lambda \) of the order of or larger than the GUT scale, the one-loop approximation breaks down since one expects the ratio of the two-loop to the one-loop top contributions to be of the order of \( \frac{N_c g^2}{16\pi^2} \ln \frac{\Lambda^2}{m_t^2} \).
Below the GUT scale and for quite an extended range of the cut-off the one loop approximation should give a reasonable description of the physics involved provided higher order scalar contributions remain small. The one-loop scalar contribution vanishes by assumption since $V^{(0)}$ does not contain a self interacting $\phi^4$ term or, equivalently, since $\left. \frac{\partial^4 V^{(0)}}{\partial \phi^4} \right|_{\phi=0} = 0$. Should we impose that $\left. \frac{\partial^4 V^{(1)}}{\partial \phi^4} \right|_{\phi=v}$ remains zero - scalar self interactions vanish at the true vacumm - the cut-off would turn out to be around 1 TeV (see Eq.(5)) and the one-loop approximation looks trustworthy. Combining Eq(4) with Eq.(5), this further constraint on $V^{(1)}$ implies then

$$m_H = \frac{2 m_1^2}{\pi v} + O \left( \frac{m_1^2}{\Lambda^2} \right)$$

(9)

i.e. a Higgs mass of about 80 GeV.

To put it differently our proposal for SB in the Standard Model is that all the physics of the phenomenon is well described by single top quark loops. Pushing the idea to the limit namely making sure that all other contributions remain negligible leads to a rather low cut-off $\Lambda (\approx 1 \text{ TeV})$ and a light Higgs scalar.

II. Renormalizable Electroweak Theory

It is worthwhile to repeat the calculations of the previous section in the framework of a renormalizable theory. The effective potential at the one-loop level reads [2,3]

$$V_R = \frac{1}{2} \mu_R^2 \phi^2 + \frac{1}{4!} \lambda_R \phi^4 + \frac{1}{(4\pi)^2} \phi^4 \left[ \frac{\lambda_R^2}{12} - \frac{N_c}{4} g_R^4 \ln \left( \frac{\phi^2}{M^2} - \frac{25}{6} \right) \right]$$

(10)
In Eq.(10) $\mu_R^2$ and $\lambda_R$ are finite renormalized parameters defined by

$$\mu_R^2 = \left. \frac{\partial^2 V_R}{\partial \phi^2} \right|_{\phi=0}$$

and $\lambda_R = \left. \frac{\partial^4 V_R}{\partial \phi^4} \right|_{\phi=M}$ with $M$ an arbitrary scale.

We have included the would-be Goldstone boson contributions in Eq.(10); without them the term $\frac{\lambda_R^2}{12}$ would have been $\frac{\lambda_R^2}{16}$. Gauge boson contributions have been neglected because, once again, their effect is at most of a few percent.

To simplify the discussion let us choose the renormalization scale $M = v$. In that case a non trivial minimum of $V_R$ can occur only if $\mu_R^2 < 0$. Defining as usual $m_H^2 = \left. \frac{\partial^2 V_R}{\partial \phi^2} \right|_{\phi=v}$ we obtain

$$m_H^2 = \left( \frac{N_c g_R^4}{3\pi^2} + \frac{\lambda_R}{3} - \frac{\lambda_R^2}{9\pi^2} \right) v^2.$$  \hspace{1cm} (11)

For small $\lambda_R$ (at the scale $v$), the Higgs mass is dominated by the contribution of the Yukawa coupling and, again, one finds

$$m_H = \frac{2 m_t^2}{\pi} + O(\lambda_R) \approx 80 \text{ GeV}.$$ \hspace{1cm} (12)

In view of the present uncertainties on $m_t$, it is not worthwhile to include QCD corrections. Also we are well aware of triviality and stability issues [6] concerning the effective potential when $\lambda_R \simeq 0$. We simply recall that they are again related to the (non) appearance of new physics at some scale $\Lambda$, but we will not discuss these points here. Our main point, much like in the previous section, is that top loop effects can be responsible for a significant fraction if not all of the Higgs mass.

To summarize, our suggestion in this letter is that the large Yukawa coupling
responsible for a heavy top quark plays a most important dynamical role in the Standard Model: we have shown that single top quark loops alone are enough to trigger symmetry breaking at the quantum level. In the effective Lagrangian approach of Section I, this scenario requires \( m_H \lesssim 400 \text{ GeV} \). Pushing our suggestion to the extreme, namely assuming that all the relevant physics is given by the top one-loop effective potential seems to us an attractive and economical alternative to the usual Higgs mechanism. It then requires scalar self interactions to be as small as possible. In the effective as well as in the renormalizable case, this assumption leads to \( m_H = \frac{2 m_t^2}{\pi v} \approx 80 \text{ GeV} \), which is in a range accessible [7] at LEP 200.

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References


Fig. 1  Massless top quark contributions to the one-loop effective potential.
Directed lines represent the top, dashed lines the scalar.

Fig. 2  Higgs mass as a function of top mass for different values of the cut off $\Lambda$.
The horizontal line corresponds to the present experimental lower bound $m_H \gtrsim 60\ GeV$ and the vertical lines to a reasonable range of values for $m_t$. 