Abstract
In view of the LHC Injectors Upgrade (LIU) and the challenging high brightness target beam parameters, a broad range of possible working points for the Proton Synchrotron (PS) is being investigated. High order resonances have been identified, both structural resonances driven by space charge due to the lattice harmonics of the PS, and resonances excited by multipolar components in the machine. This paper provides a summary of the performed tune scan studies, covering both experimental and simulation results. Furthermore, non-linear analysis techniques have been used to characterize the resonances and their effect on the beam in presence of space charge.

INTRODUCTION

The space charge potential of a bunched beam generates a highly non-linear force that can dominate low-energy, high-brightness synchrotrons as the PS. It introduces an incoherent tune shift that depends on the longitudinal beam profile, the transverse beam sizes and the form of the distributions. This tune spread can be analytically estimated for Gaussian beam distributions [1]. Figure 1 shows relevant resonances together with the expected space charge tune spread for the beam types of Table 1, used for the studies presented in this paper. For high-brightness beams, such as the LHC operational beam, the tune spread is about $\Delta Q_h \approx 0.2$ and $\Delta Q_v \approx 0.25$, covering a large part of the available tune space. The vertical integer resonance is crossed, possibly contributing to the observed emittance blow-up [2]. On the other hand, losses are observed for vertical tunes above $Q_v = 6.25$ which limit the accessible tune space. Previous studies have shown that at least part of the losses are caused by the 8th order space charge driven resonance at $Q_v = 6.25$ [3]. More recently, a series of Machine Development (MD) studies have been performed to better understand the non-linearities that govern the long injection plateau of 1.2 s and possibly identify additional resonances contributing to the observed losses [4]. In this paper, the experimental results of a vertical tune scan will be discussed and the observed data will be complemented with simulations.

EXPERIMENTAL SETUP

To understand the mechanism leading to the losses discussed above, the lattice had to be kept as linear as possible. On the injection plateau of the PS, the beam suffers from a head-tail instability that is usually suppressed through linear coupling [5]. For this reason, in normal operation, the (1,-1) resonance is further enhanced using skew quadrupoles. In the high vertical tune regime, which is of interest for our studies, this technique could not be applied as the coupling resonance would not be always overlapped. The beam had to be stabilized using the transverse feedback [6]. To eliminate the skew quadrupole components that result in the natural excitation of the (1,-1) in our experiment, the closest tune approach was used for empirical correction. In this technique, the horizontal and vertical tunes are being crossed and the minimum tune separation directly yields the coupling coefficient. The currents of the available skew quadrupoles were varied until a configuration that minimized the coupling was found.

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>LHC</th>
<th>MD-beam</th>
<th>low-br.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_h$ [mm rad]</td>
<td>3.5</td>
<td>2</td>
<td>4.5</td>
</tr>
<tr>
<td>$e_v$ [mm rad]</td>
<td>2.5</td>
<td>1.1</td>
<td>4</td>
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<tr>
<td>Intensity [$10^{10}$ ppb]</td>
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<td>41</td>
<td>44</td>
</tr>
<tr>
<td>$\Delta p/p_{rms}$ [$10^{-3}$]</td>
<td>1.02</td>
<td>0.54</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 1: Beam Parameters (Fig. 1)
In previous studies, it was observed that the (0,3) resonance is naturally excited in the PS [7]. Using the skew sextupole correctors of the PS, this resonance could be compensated [8]. Figure 2a shows the strength and the phase of the corresponding resonance driving term (RDT) for the four available skew sextupole magnets when powered individually with the same current as calculated in PTC [9]. The sextupoles selected to compensate the resonance are the “XSK14” and “XSK52”, since the vectors of their RDTs are almost perpendicular (the “XSK10” was not connected). Therefore, powering them with different current values, the RDT space can be scanned in order to compensate the unknown skew sextupole component in the lattice. The described technique was applied using the “low-brightness” beam so that any effects connected to space charge would be negligible. The beam is injected at \( Q_v = 6.37 \) and the (0,3) resonance is dynamically crossed from 800 ms to 900 ms, as the tune is changed to \( Q_v = 6.30 \). The losses as a function of the current configuration of the two skew sextupoles are shown in Fig. 2b. The effect of the sextupoles in the intensity evolution of the beam is shown in Fig. 2c. The change of the slope in the intensity curve from 800 ms to 900 ms indicates the excitation of the resonance. The configuration giving the minimum amount of losses is the one best compensating the resonance. The chromaticity was kept at the natural value, \( Q_h' \approx -5.7 \) and \( Q_v' \approx -7.6 \), avoiding the non-linear components needed for the correction.

**EXPERIMENTAL RESULTS**

A vertical tune scan was performed using the “MD-beam”. The horizontal tune is kept constant throughout the scan at 6.16. This choice allows the distinction of the (0,8) and (2,6) resonances, while avoiding crossing of the horizontal integer. The vertical tune is varied in steps of 0.05 from 6.24 to 6.37. The results of the scan are shown in Fig. 3. The bunch profile distributions used to calculate the Root Mean Square (RMS) sizes, were taken with wire scanners at 185 ms (i.e. 15 ms after injection [10]) and at the end of the plateau at 1285 ms. The intensity was monitored throughout the cycle and the values corresponding to the wire scanner measurements are the ones considered. The RMS size of the vertical plane is influenced by the (0,8) resonance, since a significant reduction is observed at \( Q_v = 6.27 \). For tunes \( Q_v > 6.30 \) the RMS sizes of both planes is increased suggesting blow-up of the profiles and the formation of tails in the transverse distributions, as they are affected by the coupled resonance (2,6). The intensity ratio shows two minima right after the crossing of the two structural resonances (0,8) and (2,6). The (0,3) seems to only slightly affect the observables suggesting that it’s corrected to a sufficient extent for this scan. The (4,4) resonance does not seem to have a strong effect on this beam. This patterns of losses and blow-up of the transverse distributions as a function of the working point seems to follow the known periodic resonance crossing mechanism for bunched beams under the space charge force [11], i.e. when the working point is lying right above the resonance, particles in the tails of the transverse distributions get lost, whereas the blow-up occurs for even higher working points when particles in the core of the bunch are trapped by the resonance. The excitation of the \( Q_v = 50 \) is a well known issue [3]. However, the measurements presented here indicate for the first time that also the \( 2Q_h + 6Q_v = 50 \) structural nonlinear coupling resonance is excited. This result suggests

![Figure 2](image_url)

**Figure 2:** (a) RDT amplitude and phase for the skew sextupoles installed in the PS. (b) Scan of the currents of the “XSK14” and “XSK52” sextupoles with losses indicated by the color bar. (c) Intensity along the injection plateau corresponding to the natural excitation (no sextupoles) and the extreme cases of (b) together with the tune variation as a function of time.

![Figure 3](image_url)

**Figure 3:** Ratios of final values (at 1285 ms) over initial values (at 185 ms) of the horizontal (blue) and vertical (green) RMS sizes of the bunch profiles and the intensity (red). Vertical lines indicate the relevant resonances (top).
that the space charge force can potentially excite all of the 8th order systematic resonances, since the 50th harmonic coincides with the main harmonic of the PS lattice [12].

**SIMULATION STUDIES**

The excitation of resonances is studied using the Frequency Map Analysis (FMA) technique [13] for tracking data including space charge. The PS lattice is matched using MAD-X [14] and the tracking is done with PTC in PyORB [15]. The space charge is included using the PyORBIT frozen model in which the kick is analytically calculated from the lattice functions, the intensity, the longitudinal and the transverse parameters of the beam using the Bassetti-Erskine formula [16]. The experimentally measured values are the ones used for the space charge calculation. The particles chosen for each study have the same longitudinal action. The tunes of the test particles are calculated for two consecutive synchrotron periods, applying the PyNAFF code [17] to the turn by turn data. The indicator for the resonance excitation is the tune diffusion coefficient [18]. For on-momentum particles, the resulting FMA is shown in Figs. 4a, 4b. The excitation of the structural resonances is shown in the tune and configuration spaces. The same technique is applied to off-momentum particles as shown in Figs. 4c, 4d. In this case the tunes are modulated through the synchrotron motion due to the varying space charge potential along the longitudinal line density profile of the beam and the machine chromaticity (which is again kept at the natural values). Therefore, the resonances appear broader compared to the on-momentum case, since a larger number of particles see them along their synchrotron motion. Additionally, the calculated diffusion for all particles crossing resonances appears larger. In fact, the losses observed in the experiment are most likely caused by the periodic crossing of these resonances [11].

**CONCLUSION**

In the PS, resonances, either lattice induced or space charge driven, have been identified giving a better understanding of the losses observed on the injection plateau. The measurements indicate the observation of multiple 8th order structural resonances, in accordance with FMA studies applied to particle tracking simulations.
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REFERENCES