Update on the $b \to s$ anomalies

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ABSTRACT

We present a brief update of our model-independent analyses of the $b \to s$ data presented in the articles published in Phys. Rev. D96 (2017) 095034 and Phys. Rev. D98 (2018) 095027 based on new data on $R_K$ by LHCb, on $R_{K^*}$ by Belle, and on $B_{s,d} \to \mu^+\mu^-$ by ATLAS.

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$\ast$This is an addendum to Ref. [1], "Lepton Nonuniversality in Exclusive $b \to s\ell^+\ell^-$ Decays", and Ref. [2], "Hadronic and New Physics Contributions to $b \to s$ Transitions".

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**New data:** Using the theoretical framework introduced in Refs. [1,2] we update our results in view of the following new experimental measurements:

- The most awaited one is the LHCb measurement of the lepton-universality testing observable $R_K \equiv \text{BR}(B^+ \to K^+ \mu^+\mu^-)/\text{BR}(B^+ \to K^+e^+e^-)$. The LHCb measurement using 5 fb$^{-1}$ of data collected with the center of mass energies of 7, 8 and 13 TeV for $R_K$ in the low-dilepton mass ($q^2$) bin leads to

$$R_K([1.1, 6.0] \text{ GeV}^2) = 0.846^{+0.016}_{-0.014} ,$$

where the first and second uncertainties are the systematic and statistical errors, respectively. Compared to the previous LHCb measurement based on 3 fb$^{-1}$ of data [4], the central value is now closer to the SM prediction, but the significance of the tension is still 2.5σ due to the smaller uncertainty of the new measurement.

- Moreover, there has been new experimental results on another lepton-universality testing observable $R_{K^*} \equiv \text{BR}(B \to K^{*+}\mu^+\mu^-)/\text{BR}(B \to K^{*+}e^+e^-)$ by the Belle collaboration [5], both for the neutral and charged $B$ mesons. The results are given in three low-$q^2$ bins and one high-$q^2$ bin which for the combined charged and neutral channels are

$$R_{K^*}([0.045, 1.1] \text{ GeV}^2) = 0.52^{+0.36}_{-0.26} \pm 0.05, \quad R_{K^*}([1.1, 6.0] \text{ GeV}^2) = 0.96^{+0.45}_{-0.29} \pm 0.11,$$

$$R_{K^*}([0.1, 8] \text{ GeV}^2) = 0.90^{+0.27}_{-0.21} \pm 0.10, \quad R_{K^*}([15, 19] \text{ GeV}^2) = 1.18^{+0.52}_{-0.32} \pm 0.10.$$

For our analysis we consider the [0.1, 8] GeV$^2$ bin (together with the high-$q^2$ bin) and do not use the very low $q^2$ bin below 0.1 GeV$^2$ as advocated by Ref. [4] in order to avoid near-threshold uncertainties which would be present when the lower range of the bin is set to the the di-muon threshold.

We note that the Belle measurement for the low-$q^2$ bin, [0.045, 1.0], which we do not use, has a tension with the SM prediction which is slightly more than 1σ, while the other bins are all well in agreement with the SM at the 1σ-level. All the $R_{K^*}$ measurements of Belle are in agreement with the LHCb measurement [7] due to the large uncertainties of the Belle results.

- Our update also takes into account new experimental data on $B_{s,d} \to \mu^+\mu^-$ by ATLAS [8]. We have combined this new result with the previous results of CMS [9] and LHCb [10] building a joint 2D likelihood (see Fig.1) with common $f_d/f_s$ and $\text{BR}(B^+ \to J/\psi K^+) \times \text{BR}(J/\psi \to \mu^+\mu^-)$ which finally leads us to

$$\text{BR}(B_s \to \mu^+\mu^-) = 2.65^{+0.43}_{-0.39} \times 10^{-9}, \quad \text{BR}(B_d \to \mu^+\mu^-) = 1.09^{+0.74}_{-0.68} \times 10^{-10}.$$

![Figure 1: 2D likelihood plot where the contours are 1, 2 and 3σ (in terms of $\Delta\chi^2$). The numbers correspond to the absolute $\chi^2$ and the black box is the SM prediction.](image)

The calculation of the observables is performed with SuperIso v4.1 [11]. The statistical methods used for our study are described in Refs. [12,13]. In particular, we compute the theoretical covariance matrix for all the observables and consider the experimental correlations provided by the experiments. For the hadronic corrections, we do not consider hadronic parameters as in Refs. [2,14] but use 10% error assumption as explained in [13].
Comparison of $R_K$ and $R_{K^*}$ data with other $b \to s$ data: The hadronic contributions which are usually the main source of theoretical uncertainty cancel out in the case of the potentially lepton flavour violating ratios $R_K$ and $R_{K^*}$ and, thus, very precise predictions are possible in the SM \cite{15}. In contrast, the power corrections to the angular observables and other observables in the exclusive $b \to s$ sector are still not really under control and are usually guessed estimated to 10%, 20% or even higher percentages of the leading nonfactorisable contributions to those observables. However, there is a promising approach based on analyticity which may lead to a clear estimate of such effects and which may allow for a clear separation of hadronic and new physics (NP) effects in these observables \cite{16}.

As argued in Ref. \cite{3}, the present situation suggests separate analyses of the theoretically very clean ratios and the other $b \to s$ observables. In Table \cite{4} the one-operator fits to new physics has been compared when considering all the relevant data on $b \to s$ transitions except for $R_K$ and $R_{K^*}$ and when only considering the data on $R_K$ and $R_{K^*}$ \cite{4}. We note that the NP significance of the ratios is reduced compared to our previous analysis \cite{4}, mainly because of the new measurements of $R_{K^*}$ by Belle which are compatible with the SM predictions at the 1σ-level as stated above. But within the one-operator fits we find again that the NP analyses of the two sets of observables are less coherent than often stated, especially regarding the coefficients $C^\mu, e_{10}$.

\begin{table}[h]
\centering
\begin{tabular}{llll}
\hline
 & All observables except $R_K$, $R_{K^*}$ & & Only $R_K$, $R_{K^*}$  \\
 & $(\chi^2_{SM} = 100.2)$ & & $(\chi^2_{SM} = 16.9)$  \\
\hline
 & b.f. value & $\chi^2_{min}$ & PullSM  \\
$\delta C_9$ & $-1.00 \pm 0.20$ & 82.5 & 4.2σ  \\
$\delta C^\mu_9$ & $-1.03 \pm 0.20$ & 80.3 & 4.5σ  \\
$\delta C_9^e$ & $0.72 \pm 0.58$ & 98.9 & 1.1σ  \\
$\delta C_{10}$ & $0.25 \pm 0.23$ & 98.9 & 1.1σ  \\
$\delta C^\mu_9$ & $0.32 \pm 0.22$ & 98.0 & 1.5σ  \\
$\delta C_{10}$ & $-0.56 \pm 0.50$ & 99.1 & 1.0σ  \\
$\delta C^\mu_{10}$ & $-0.48 \pm 0.15$ & 89.1 & 3.3σ  \\
$\delta C^\mu_{10}$ & $0.33 \pm 0.29$ & 99.0 & 1.1σ  \\
\hline
\end{tabular}
\caption{Comparison of one-operator NP fits where the $\delta C^\mu_{LL}$ basis corresponds to $\delta C^\mu_9 = -\delta C^\mu_{10}$. On the left hand side all relevant data on $b \to s$ transitions except $R_K$ and $R_{K^*}$ (with 10% error assumption for the power corrections) is used and on the right hand side only the data on $R_K$, $R_{K^*}$ is considered.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{llll}
\hline
 & All observables except $R_K$, $R_{K^*}$, $B_{s,d} \to \mu^+\mu^-$ & & Only $R_K$, $R_{K^*}$, $B_{s,d} \to \mu^+\mu^-$  \\
 & $(\chi^2_{SM} = 90.7)$ & & $(\chi^2_{SM} = 19.0)$  \\
\hline
 & b.f. value & $\chi^2_{min}$ & PullSM  \\
$\delta C_9$ & $-1.03 \pm 0.20$ & 81.0 & 4.3σ  \\
$\delta C^\mu_9$ & $-1.05 \pm 0.19$ & 78.8 & 4.6σ  \\
$\delta C_9$ & $0.72 \pm 0.58$ & 98.5 & 1.1σ  \\
$\delta C_{10}$ & $0.27 \pm 0.28$ & 98.7 & 1.0σ  \\
$\delta C^\mu_{10}$ & $0.38 \pm 0.28$ & 97.7 & 1.4σ  \\
$\delta C_{10}$ & $-0.56 \pm 0.50$ & 98.7 & 1.0σ  \\
$\delta C^\mu_{11}$ & $-0.50 \pm 0.16$ & 88.8 & 3.3σ  \\
$\delta C^\mu_{11}$ & $0.33 \pm 0.29$ & 98.6 & 1.1σ  \\
\hline
\end{tabular}
\caption{Comparison of one operator NP fits where the $\delta C^\mu_{LL}$ basis corresponds to $\delta C^\mu_9 = -\delta C^\mu_{10}$. On the left hand side all relevant data on $b \to s$ transitions except $R_K$, $R_{K^*}$, $B_{s,d} \to \mu^+\mu^-$ (with 10% error assumption for the power corrections) is used and on the right hand side only the data on $R_K$, $R_{K^*}$, $B_{s,d} \to \mu^+\mu^-$ is considered.}
\end{table}

One may expect that the observables $B_{s,d} \to \mu^+\mu^-$ are responsible for the finding that NP in $C^\mu_{10}$ is favoured in the fit to the ratios $R_{K^{(*)}}$ but not in the fit to the rest of the $b \to s$ transitions. However, when besides $R_K$, $R_{K^*}$ also the $B_{s,d} \to \mu^+\mu^-$ observables are removed from the rest of the $b \to s$ observables and compared to the fit when considering the data on $R_K$, $R_{K^*}$, $B_{s,d} \to \mu^+\mu^-$ we find that at least within the one-operator fits the observables $B_{s,d} \to \mu^+\mu^-$ do not play a major role: The results in Table\cite{2} are very similar with the ones in Table\cite{1} This feature is consistent with our finding in Ref.\cite{1} that the observables $B_{s,d} \to \mu^+\mu^-$ will not play a primary role in the future differentiation between the

\footnote{The right (left) hand side results of Table\cite{1} in this paper give the updated results of Table 1 (2) in Ref.\cite{1} where here we have not normalised to the SM values.}
NP hypotheses for the ratios $R_{K^{(s)}}$. However, with the new average for $BR(B_s \to \mu^+\mu^-)$ which includes the ATLAS measurement, there is a tension of $1.5\sigma$ with the SM prediction which suggests the same direction for $C^u_{10}$, as it is preferred by the $R_{K^{(s)}}$ fit. This can also be seen by comparing the right hand sides of Tables 1 and 2 where there is a slight increase in the SM-Pull when the data on $B_s \to \mu^+\mu^-$ is added to the $R_{K^{(s)}}$ fit.

In the next step we compare the two sets of observables by two-operator fits again. In Fig. 2 the two operator fits for $\{C^e_9, C^u_9\}$, $\{C^\mu_{10}, C^\mu_{9}\}$ and for $\{C^\mu_{10}, C^e_{10}\}$ are shown, using only the data on $R_K, R_{K^*}$ or all observables except $R_K, R_{K^*}$ where the effect of moving the data on $B_s,d \to \mu^+\mu^-$ observables from one set to the other has been shown with the black and gray contours. The latter ones nicely show the influence of these observables when more than one operator is considered. Independent of these effects one finds that the two sets of observables are compatible at least at the 2$\sigma$-level.

**Figure 2:** Two operator fits to NP. The contours correspond to the 68 and 95% confidence level regions. On the left hand side we have considered all observables except $R_K$ and $R_{K^*}$ with the assumption of 10% power corrections. On the right hand side we have only used the data on $R_K, R_{K^*}$. Pull$_{SM}$ for the 1$^{st}$, 2$^{nd}$, 3$^{rd}$ rows are respectively, 4.1, 4.1, 1.1$\sigma$ (3.1, 3.2, 3.1$\sigma$), for the plots on the left (right). The black (gray) dashed and solid contours correspond to excluding (including) the data on $B_s,d \to \mu^+\mu^-$ from (to) the fits of the left (right) hand side.

**Global fit** In Table 3 the global one-operator fits to NP are given where all the relevant data on $b \to s$ transitions are considered. In Fig. 3 the two operator fits for $\{C^e_9, C^u_9\}$, $\{C^\mu_{10}, C^\mu_{9}\}$ and $\{C^\mu_{10}, C^e_{10}\}$ can be seen. These fits are always done under the assumption of 10% power corrections in the angular observables. Compared with our previous analysis in Ref. 2 the NP significance in the one- and also in the two-operator fits is reduced by at least 0.5$\sigma$. Only in cases of flavour-symmetric $C_9$ and $C_{10}$ which are independent from the changes in the ratios one finds the same NP significance as expected.

The observables $B_{s,d} \to \mu^+\mu^-$ are usually used to strongly constrain NP effects in scalar and pseudoscalar operators. As a consequence, a general usage is to consider the contributions from the
All observables ($\chi^2_{\text{SM}} = 117.03$)

<table>
<thead>
<tr>
<th>b.f. value</th>
<th>$\chi^2_{\text{min}}$</th>
<th>Pull_{SM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta C_9$</td>
<td>$-1.01 \pm 0.20$</td>
<td>99.2</td>
</tr>
<tr>
<td>$\delta C_9'$</td>
<td>$-0.93 \pm 0.17$</td>
<td>89.4</td>
</tr>
<tr>
<td>$\delta C_9''$</td>
<td>$0.78 \pm 0.26$</td>
<td>106.6</td>
</tr>
<tr>
<td>$\delta C_{10}$</td>
<td>$0.25 \pm 0.23$</td>
<td>115.7</td>
</tr>
<tr>
<td>$\delta C_{10}'$</td>
<td>$0.53 \pm 0.17$</td>
<td>105.8</td>
</tr>
<tr>
<td>$\delta C_{10}''$</td>
<td>$-0.73 \pm 0.23$</td>
<td>105.2</td>
</tr>
<tr>
<td>$\delta C_{10,\text{LL}}$</td>
<td>$-0.41 \pm 0.10$</td>
<td>96.6</td>
</tr>
<tr>
<td>$\delta C_{10,\text{LL}}'$</td>
<td>$0.40 \pm 0.13$</td>
<td>105.8</td>
</tr>
</tbody>
</table>

Table 3: Best fit values and errors in the one operator fits to all the relevant data on $b \rightarrow s$ transitions, assuming 10% error for the power corrections.

Figure 3: Two operator fits to NP, considering all observables (with the assumption of 10% power corrections). Pull_{SM} in the $\{C_9, C_9'\}, \{C_{10}, C_9'\}, \{C_{10}, C_{10}'\}$ fits are 4.9, 4.9, 3.2$\sigma$, respectively.

classical and pseudoscalar as vanishingly small. However, as mentioned in Ref. [2], this is only valid when the relation between the scalar and pseudoscalar operators ($C_{Q_1} = -C_{Q_2}$) is assumed, which breaks the possible degeneracy between $C_{Q_2}$ and $C_{10}$ and allows for strong constraints on $C_{Q_{1,2}}$. In general scenarios, $C_{Q_2}$ and $C_{10}$ can have simultaneously large values which compensates, while indeed $C_{Q_1}$ is rather constrained (for more details see Ref. [2]).

Finally, we note that there have been other model-independent analyses presented recently which update previous analyses [1, 2, 17–21] based on the new experimental data. We find small differences with these updated analyses [21, 24] only in the different choices of bins in the new Belle measurement and by slightly different treatments of power corrections and of form factors.

In summary, the overall picture of the $b \rightarrow s$ anomalies remains the same as before taking into account the new results from LHCb, Belle and ATLAS on $R_{K^*}, R_{K^*}$ and $B_s \rightarrow \mu^+\mu^-$. Although, the significance of the new physics description of the $R_{K^*}$ data is now reduced by more than half a $\sigma$. Nevertheless, the future measurements of these theoretically very clean ratios and similar observables which are sensitive to lepton flavour non-universality have a great potential to unambiguously establish lepton non-universal new physics.
References


