Letter of Intent: The MUonE Project

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1 Executive summary

We propose an experiment at the CERN SPS to measure the hadronic component of the running electromagnetic coupling $\alpha(t)$, in a momentum transfer $(t)$ region relevant for the calculation of the muon g-2 anomaly. The hadronic part of the running can be determined from the precise measurement of the shape of the differential cross section for the $\mu e \rightarrow \mu e$ elastic process, using high energy muons impinging on electrons at rest in a low-Z target. The measurement does not rely on the absolute knowledge of the luminosity.

The muon g-2 anomalous magnetic moment $a_\mu$ is sensitive to physics beyond the standard model and the present discrepancy between measurements and prediction is tantalising. The knowledge of the electromagnetic coupling running, and in particular of its hadronic component, represents a major source of uncertainty for the g-2 prediction: we aim at measuring it directly for the first time, with a precision interesting for g-2 measurements.

The CERN M2 beam is a unique facility worldwide, providing high energy muon beams with the necessary high intensity needed to perform such a measurement.

We plan to build the MUonE detector using state-of-the-art silicon strip detectors. The development of the chosen tracking sensors and front-end electronics is well advanced, being carried out by the CMS collaboration for the HL-LHC upgrade of the experiment. The DAQ will be the one developed for the CMS sensors. In particular the online track reconstruction based on FPGAs could be important to reduce the computing task. We plan to validate in a pilot run how well the solution works in the case of the MUonE application. The detector will be equipped at the end of the tracker with a calorimeter covering the acceptance of the last stations, and a muon filter aimed at rejecting possible hadron contamination in the beam. The main challenge of the proposed experiment is the control of systematic effects at the same level of the statistical precision. In this respect the efficiency of the tracking system must be highly uniform in the whole acceptance and, together with other detector parameters (e.g. angular resolution), must be precisely determined from data.

The MUonE collaboration asks for three weeks of Pilot Run in 2021 to assess the detector performance in view of the measurement foreseen in 2022-24.
2 Introduction

For almost a century, the values of the magnetic moments of the elementary constituents of matter have been permanently under investigation, both experimentally and theoretically.

Starting from the sixties and seventies, when CERN and the European physics have played a leading role in the determination of the muon anomaly with a series of pioneering experiments and fundamental contributions, efforts to reach the ultimate precision have been constantly pursued until today.

In terms of the gyromagnetic proportionality factor $g$ the anomalous magnetic moment of the muon can be defined as $a_{\mu} = (g - 2)/2$. To date, the most accurate determination of $a_{\mu}$ has been obtained by the Brookhaven BNL E821 experiment resulting in $a_{\mu}^{\text{exp}} = 11659209.1(6.3) \times 10^{-10}$ [1, 2, 3, 4, 5], with the error, dominated by statistics, corresponding to 0.54 ppm.

The Standard Model value $a_{\mu}^{\text{SM}} = 11659182.0(3.6) \times 10^{-10}$ [6] shows an interesting difference $a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 27.1(7.3) \times 10^{-10}$ with the result of making $a_{\mu}$ the quantity whose value mostly differs from the value predicted by the Standard Model with a discrepancy in the $(3.5 - 4)\sigma$ range.

This remarkable difference is a long standing issue [7] shared by all the recent estimates of the hadronic contribution, all showing a similar discrepancy [8] that opens the way to the interpretation that it may be a signal of new physics giving room to potentially interesting scenarios [9].

On the experimental side the new E989/Muon g-2 experiment[10] is presently already running at Fermilab and is expected to improve the precision by a factor of four [11]. A first result, with an error comparable to the E821 one, is expected soon. In addition, a completely new low-energy approach to measuring the muon g-2 is being developed by the E34 collaboration at J-PARC [12]. An analogous improvement is therefore required in the precision of the theoretical prediction, as its uncertainty will become the main limitation of this formidable test of the Standard Model.

Hadronic loops contributions to $a_{\mu}^{\text{SM}}$ give rise to its main theoretical uncertainties. They mainly originate from the leading hadronic vacuum polarization term $a_{\mu}^{\text{HLO}}$ and from the hadronic light-by-light contribution $a_{\mu}^{\text{HLBL}}$ which has the second largest error in the theoretical evaluation. The leading hadronic vacuum polarization contributions to $a_{\mu}^{\text{HLO}}$, due to the non-perturbative nature of the strong interactions at low energy, is traditionally computed via a dispersion integral on the hadron production cross section in $e^+ e^-$ annihilation [13]. Furthermore, the functional form of the s-channel cross section $e^+ e^- \rightarrow \text{hadrons}$, being densely populated with resonances and modulated by threshold effects, makes the dispersive approach evaluation of $a_{\mu}^{\text{HLO}}$ highly challenging with an experimental error dominated by systematic uncertainties [5] and, even if future, forthcoming, data from $e^+ e^-$ colliders, as BES III and Belle II, will possibly reduce the uncertainties, the present data do not allow any further improvement on the accuracy.

Alternative evaluations of $a_{\mu}^{\text{HLO}}$ can be obtained via lattice QCD calculations [14, 15, 16]. However, in spite of their continuous progress, these lattice determinations are not yet competitive with the dispersive ones obtained via timelike data.

The aim of this proposal is to represent a new and alternative contribution in this quest.

We propose a new experiment, MUonE [17], based on an alternative method to determine $a_{\mu}^{\text{HLO}}$ by measuring the effective electromagnetic coupling in the spacelike region [18]. We aim at a new independent determination of the hadronic vacuum polarization contribution to the anomaly $a_{\mu}^{\text{HLO}}$ to be compared with timelike dispersive and lattice results toward a firmer assessment of $a_{\mu}$. 
3 The MUonE project

3.1 A new method to measure $a^{HLO}_\mu$

MUonE is a new experiment to measure the hadronic part of the running of the electromagnetic coupling constant in the spacelike region by the scattering of high-energy muons on atomic electrons on a low-Z target through the elastic process $\mu e \rightarrow \mu e$ [17].

The differential cross section of this process, measured as a function of the squared momentum transfer in the spacelike domain $t = q^2 < 0$, provides direct sensitivity to $a^{HLO}_\mu$ [18].

By using the muon beam of 150 GeV (or higher), with an average rate of $\sim 1.3 \times 10^7 \mu \text{sec}^{-1}$, currently available at the CERN North Area (the M2 beam), a statistical uncertainty of $\sim 0.3\%$ can be achieved on $a^{HLO}_\mu$ after $\sim 2$ years of data taking.

The sensitivity to $a^{HLO}_\mu$ is obtained by integrating the effective fine-structure constant $\Delta \alpha$ in the formula:

$$a^{HLO}_\mu = \frac{\alpha}{\pi} \int_0^1 dx \left(1 - x\right) \Delta \alpha_{\text{had}}[t(x)],$$

where $\Delta \alpha_{\text{had}}(t)$ is a smooth function and is the hadronic contribution to the running of $\alpha$, evaluated at

$$t(x) = \frac{x^2 m^2}{x - 1} < 0,$$

$\Delta \alpha$ is given by the sum of the hadronic, the leptonic, the top quark and Weak contributions:

$$\Delta \alpha = \Delta \alpha_{\text{had}} + \Delta \alpha_{\text{lep}} + \Delta \alpha_{\text{top}} + \Delta \alpha_{\text{Weak}}$$

The pure leptonic, the top quark and the Weak contributions can be calculated in perturbation theory. The leptonic one is known to three loops in QED [19] and up to four loops in specific $q^2$ limits [20, 21, 22]. The hadronic contribution, due to the non-perturbative nature of Quantum Chromodynamics at low energies cannot be directly evaluated and relies on an experimental determination. The timelike behaviour of the vacuum polarization diagram, due to the virtual quark contributions to the imaginary part of the $e^+ e^-$ annihilation at low energy, undergoes significant variations caused by the presence of resonances and threshold effects. In contrast to the dispersive integral, the integrand of Eq.(1) is a smooth function free of resonance poles. Figure 1-left shows $\Delta \alpha_{\text{had}}$ as a function of the variables $x$ and $t$. The range $x \in (0, 1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The expected integrand of Eq.(1), is plotted in Fig. 1-right. Note that $\Delta \alpha_{\text{had}}(t_{\text{peak}}) \simeq 7.86 \times 10^{-4}$.

This technique is similar to the one used for the measurement of the pion form factor described in [23].

3.2 Precision requested for the measurement

The big challenge of this experiment is that all the systematics effects should be known at 10 ppm. In order to correctly extract $\Delta \alpha$ a Monte Carlo code for the process, accurate to the Next to Next to Leading (NNLO) level of radiative contributions, must be available. Currently is available a fully differential Monte Carlo code containing the Next to Leading Order (NLO) radiative corrections (RC), including the NLO Electroweak corrections.

The effect of the contributions of the hadronic vacuum polarization to the $\mu e \rightarrow \mu e$ differential cross section is to increase it by a few per mille, mainly in the kinematical region where the outgoing electron angle is below 10 mrad. A precise determination of $a^{HLO}_\mu$, both theoretically and experimentally, requires not only high statistics, but also a high level of control of systematic uncertainties, as the final goal of the experiment consists of the determination of the shape of the differential cross section with a $\sim 10$ ppm systematic uncertainty at the peak of the integrand function (see Fig. 1).
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Fig. 1: Left: $\Delta \alpha_{\text{had}}[f(x)] \times 10^4$ (red) and, for comparison, $\Delta \alpha_{\text{lep}}[f(x)] \times 10^4$ (blue), as a function of $x$ and $t$ (upper scale). Right: the integrand $(1-x)\Delta \alpha_{\text{had}}[f(x)] \times 10^5$ as a function of $x$ and $t$.

Fig. 2: The relation between the muon and electron scattering angles for 150 GeV incident muon beam momentum. The blue triangles indicate the reference values of the variable $x$ and the electron energy.
Experimental systematic uncertainties Several crucial requirements on the experimental side are: i) to keep the efficiency highly uniform over the entire $q^2$ range; ii) to control the alignment of the tracker elements with very high precision; iii) to describe with an accuracy at the level of 1% the effects of the multiple scattering on the electrons of relatively low energy. Multiple scattering breaks the muon-electron two-body angular correlation, moving events out of the kinematic line in the 2D plot of Fig. 2. In addition, multiple scattering, in general, causes acoplanarity, while two-body events are planar within the resolution.

Data themselves will be used to measure directly some of the quantities which must be kept under control. For example, the high number of muons passing through the apparatus will be used to monitor periodically the alignment and the efficiency. The muon interactions like $\mu \rightarrow \mu \gamma$ and $\mu \rightarrow \mu e^+e^-$ must be recorded to extrapolate precisely the background subtraction and contamination to the final elastic sample. In this respect the proposed modularity of the apparatus will help.

Theoretical systematic uncertainties The differential cross section must be calculated by including all the radiative corrections contributing to the Next-to-Next-to Leading (NNLO) level of accuracy (see Section 12 for a detailed discussion). At the next-to-leading order (NLO), the class of QED corrections are known for $\mu e$ scattering. The typical theoretical uncertainty of a QED NLO calculation [24] is of the order of 1%. The techniques to match NLO fixed order calculation with all-orders resummed calculations of leading logarithmic contributions are well known and allow to reach accuracies at the 0.1% level. In order to go beyond this limit, the NNLO QED corrections and relevant weak contributions are required. First results towards the final goal have been recently obtained [25, 26, 27, 28].
4 The experimental apparatus

The goal of the experiment is to precisely measure the shape of the differential cross section of the $\mu - e$ elastic scattering. Muons with $\sim$150 GeV/c momentum, from the CERN M2 muon beamline, impinge on the atomic electrons of Beryllium targets. The measurement of the angles of the particles involved in the scattering is performed by tracking devices. Therefore, the multiple Coulomb scattering has to be minimized in order not to spoil the track measurements. The total target thickness of 60 cm, required to collect the necessary statistics in a reasonable running time, is divided into 40 slices, each instrumented with its tracking system. The apparatus is therefore a sequence of 40 identical stations, composed each of a 15 mm thick Be target and tracking sensors, with a lever arm of about 1 m.

The apparatus is equipped with an electromagnetic calorimeter (ECAL), placed downstream all stations, which provides particle identification and a measurement of the electron energy.

Should the contamination of pions in the muon beam be not low enough, a muon filter, instrumented with muon chambers, will be added to the apparatus, downstream ECAL.

A schematic view of the experimental apparatus is shown in Fig. 3.

![Fig. 3: Schematic view of the MUonE experimental apparatus (not to scale).](image)

4.1 The Tracking system

4.1.1 Overview and general concept

The tracking system represents the heart of the MUonE detector concept, which is based on the precise measurement of the scattering angles of the outgoing electron and muon, with respect to the direction of the incoming muon beam. The detector must be fully efficient over the $q^2$ range of interest, which in practice corresponds to an electron energy greater than 1 GeV. With a muon beam momentum around 150 GeV, this requirement corresponds to a maximum scattering angle of about 30 mrad, implying that a limited angular coverage is sufficient. An active area of $10 \times 10$ cm$^2$ for the tracking modules contains all the kinematics of the event. Another feature characterizing MUonE is the re-use of the muon beam in a modular system based on individual stations, which allows the statistics of $\mu e$ events to be increased, limiting at the same time the longitudinal length of the target, with the advantage of greatly reducing multiple scattering, at the same total event rate. Tracking between two targets (and beyond) is used both to measure the scattered electron and muon and to provide the direction of the beam for next target. See Fig. 6. In practice this modularity requirement implies a length of approximately one meter for each station, together with a target thickness of 15 mm of Be, in order to keep a reasonable total length of the apparatus.

Another important requirement is related to the detector angular resolution. Muons can be distinguished from electrons using solely the angular information, with the exception of a limited ambiguity region, which is determined by the angular resolution itself. As an example, in Fig. 4 the distribution of the two measured scattering angles, $\theta_{\text{left}}$ and $\theta_{\text{right}}$, is shown for events simulated with different angular resolutions. Here $\theta_{\text{left}}$ and $\theta_{\text{right}}$ are the scattering angles selected randomly, without particle identification. The increased range of the ambiguity region going from the ideal case (only multiple scattering from the target) to increasing detector resolution is clearly seen. An additional motivation for high angular resolution is an accurate definition of elastic events, i.e. an accurate definition of the signal region. Inelastic events with a photon in the final state ($\mu e \rightarrow \mu e \gamma$) do not follow the elastic curves shown in
Fig. 4 and need to be either identified with the calorimeter or rejected by means of angular cuts. The angular resolution is related to the spatial resolution of the tracker planes, orthogonal to the direction of the beam: an angular resolution of 0.02 mrad over one meter length corresponds to a spacial resolution of 20 µm, which can be be achieved with state-of-the-art silicon detectors.

![Fig. 4: Distribution of the two measured scattering angles, θ_{left} versus θ_{right} in mrad, for events simulated with ideal angular resolution (only multiple scattering from the target included, top left panel), angular resolution of 0.02 mrad (top right panel), 0.06 mrad (bottom left panel) and 0.1 mrad (bottom right panel), respectively. In blue are the points corresponding to the correct particle identification, in green the wrong identification.](image)

Very high and uniform detection efficiency in a high beam rate environment is another relevant requirement. Silicon detector with high signal-to-noise ratio (S/N ≈ 25) as the ones currently developed for use at colliders can achieve detection efficiency close to 100% [29] and sustain high rate (70 MHz). In addition the required coverage of 30 mrad can be achieved with a single silicon sensor over a distance of 1 m with state-of-the-art technology, ensuring an active area over the full detector, without cracks.

Two sensitive planes, each plane measuring the two coordinates orthogonal to the beam direction (x,y), is the minimum requirement for the determination of a track direction. For silicon strip detectors each plane must consist of two sensors to measure the x and y coordinates. In practice, as it is desirable to measure the tracking efficiency from data themselves, a third sensitive plane is added, making a total of six sensors for each station. One of the planes has to be rotated to resolve position ambiguities. Three sensitive planes can also be used to provide transverse (x,y) alignment with data-based methods.

As multiple scattering is an important source of systematic uncertainty in MUonE, it is also desirable to avoid additional detectors, solely dedicated to triggering. In this sense silicon sensor have the advantage, over other tracking detector, that can provide trigger capabilities, if equipped with proper electronics [29, 30, 31].
In conclusion, the main conceptual points related to the structure of a tracking station for MUonE are the following ones:

- tracking detectors between two targets (N and N+1) used both for measuring the linear trajectory of the two scattered particles from target N (electron and muon) and for determining the trajectory of the incoming beam for target N+1;
- complete angular coverage of the interesting region, possibly with a single sensor in the detection plane in order to provide uniformity;
- use of the muon beam itself for high-statistics measurement of the efficiency and high-precision transverse alignment, requiring three planes measuring x and y coordinates.

Considering the above conceptual considerations, a silicon-detector-based tracking system represents an ideal choice for MUonE, satisfying these relevant requirements:

- high spatial precision,
- compactness in order to allow modularity to be achieved
- high rate sustainability
- high efficiency and uniformity,
- easy integration in trigger system.

### 4.1.2 Silicon sensor choice: the CMS modules

The development of a new dedicated silicon detector, with the necessary front-end readout electronics, requires a few years of R&D, followed by procurement and production. We have made a survey of existing silicon sensors and electronics, already designed for current experiments (or their upgrades), in order to make an adequate choice for our application, based on already existing (or being produced) components.

The relevant figures of merit for our choice are the detector active area, the single hit resolution, the material amount and the front-end response time and maximum readout rate. In Appendix 17.1, details are given on the possible solutions which have been considered.

The silicon strip sensors being produced for the CMS Tracker upgrade feature a large active area (a single sensor is sufficient to cover the full MUonE required acceptance) and an adequate spatial resolution. They can also sustain the high readout rate required for MUonE (40 MHz) with their accompanying front-end electronics. In the so-called 2S configuration they are well-suited for track triggering. They also represent a good compromise for detector thickness. Their production schedule is also in line with the timescale of MUonE, as described in Section 14, and they have been chosen for the experiment.

The silicon sensors foreseen for the CMS HL-LHC Outer Tracker (OT) are 320 μm thick sensors with n-in-p polarity produced by Hamamatsu Photonics (HPK). In particular, the sensor considered for MUonE have been designed for the so-called 2S modules of the CMS OT, which are square sensors with an area of 10 cm × 10 cm. The strips are capacitively-coupled, with a pitch of 90 μm and are segmented in two approximately 5 cm long strips. At each side of the sensor 1016 strips are read out by eight ASIC chips (CMC Binary Chips, CBC), for a total of 2032 channels. In the CMS OT 2S modules two closely-spaced silicon sensors reading the same coordinate are mounted together and read out by common front-end CBC ASICs capable of correlating the hits from the two sensors for triggering purpose. For MUonE we
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plan to exploit this track triggering capability, as described in Sec. 6. The chosen distance between two sensors in a 2S module is 1.8 mm.

Data from the front-end CBC ASIC are aggregated by a concentrator ASIC (CIC) receiving from each CBC six parallel lines operating at 320 Mb/s each, corresponding to 48 bits at 40 MHz. The CIC sorts, buffers and format these data and sends it to the LpGBT serializer chip hosted in the service hybrid (SEH), together with the Versatile Link chips. The LpGBT can transmit data at 5.12 or 10.24 Gb/s. Correlated hits in the two sensors in a module forming a stub (trigger data) are transmitted by the front-end electronics at 40 MHz. Single hits (readout data) can be readout at 750 kHz, in case of a L1 trigger.

An exploded view of the 2S module with the various components and a picture of a 2S prototype module are shown in Fig. 5 (left and right panels, respectively). The power dissipated by the front-end electronics corresponds to about 5 W. In CMS the 2S modules are mounted on supporting structures with mounting points providing cooling to the entire module, including its readout electronics. For MUonE we plan to provide cooling by mean of the same mounting points, with a dedicated support and cooling system.

The final arrangement of the setup is shown in Fig. 6. as anticipated it consists of a repetition of 40 times the same station, with at the end a calorimeter and a muon filter.
Fig. 6: *(Not on scale)* A sketch of one station, which is repeated 40 times in the final apparatus. Lower figure represents the last station, which is followed by a calorimeter and the muon filter.
4.2 The Electromagnetic Calorimeter

A forward electromagnetic calorimeter (ECAL) covering part of the total acceptance for the elastic scattering is motivated by the following reasons:

- **Particle Identification (PID)**, namely, the unambiguous identification of the electron and muon tracks, especially in the kinematic region $\theta_e \approx \theta_\mu$, where the tracking system fails to recognize the two particles. This can be achieved with the ECAL either using the electron angle measurement, obtained through the impact point of the e.m. shower, or using the electron energy measurement, which constrains the kinematics of the elastic scattering.

- **Measurement of the electron energy** $E_e$. This measurement adds redundancy to the overall system and can help to countercheck the results obtained with the tracker, to assess systematic effects, and to control the background.

- **Event selection**, for example, tagging events where the energy deposited in the ECAL is above a given threshold, or where the presence of 2 showers (i.e., electron + $\gamma$-ray) has been detected.

To accomplish those goals, we propose a homogeneous ECAL placed downstream all the tracker stations, composed of lead tungstate (PbWO$_4$) crystals, similar to those used by the CMS electromagnetic calorimeter, whose performances are described in [32]. Crystals (section 2.5 × 2.5 cm$^2$, 23 cm long, 26 $X_0$) are readout by solid state sensors (SiPM or APD). PbWO$_4$ has a fast light scintillation emission time, a good light yield, and allows compact dimensions. The main properties of some scintillating materials are summarized in Table 5 in the appendix.

Since the tracker system is about 40 m long and includes passive material that spoils the detector performance, the ECAL transverse dimension will not cover the full acceptance to serve all the stations. The angular region where the identification is ambiguous, is for $\theta_e \leq 5$ mrad, and to cover this angular range, allowing some space to account for the lateral shower containment, the ECAL transverse dimension would be of the order of 1 × 1 m$^2$. With such area, full containment is achieved for electrons with energy $E \gtrsim 30$ GeV, while angular acceptance is for $E \gtrsim 10$ GeV.

We have simulated a larger active area and limited the apparatus to 20 target stations. In the full apparatus they correspond to the stations from 21 to 40. The final ECAL dimension will be optimized with a dedicated study. The mechanical structure which holds the crystals is not simulated.

The production of the scintillation photons in the crystals and its fluctuation, the transport (reflection and absorption) of the photons to the solid state sensors, the production (and fluctuation) of photoelectrons in the sensors, and the signal electronic readout with its noise are not included in the simulation. To obtain a realistic response of the detector, we account for the effects of such processes by re-processing the energy deposited in each crystal, obtained by the MC simulation, adding in quadrature the uncorrelated Gaussian fluctuations (after ref. [32]): $\sigma_{\text{electronics}} = 25$ MeV for each crystal, which includes all contributions to the electronics noise, and $\sigma_{\text{sys}} = 0.1 \% \cdot E_{\text{crystal}}$ ($E_{\text{crystal}}$ is the energy deposited in the single crystal), which accounts for all systematic effects.

The fluctuations of the light production, transport and conversion are negligible with respect to the fluctuations of the deposited energy (with $\sigma(E)/E \lesssim 5\%$ as discussed below), and therefore are not considered here.

We have simulated electrons emitted from all targets at energies : 1, 2, 5, 10, 20, 50, 75, 100, and 130 GeV, at proper emission angles.

We have reconstructed the electromagnetic shower in a 5 × 5 crystal matrix centered around the crystal with the maximal energy deposition. Crystals with an energy deposited below 10 MeV are discarded.
4.2.1 Position/angular measurement

To determine the point of impact of the scattered electrons on the ECAL we use the method described in [33] and presented in the appendix. The technique uses logarithmic weights for the energy fraction observed in the individual detector cells, without the need to model the shower transverse development neither to introduce additional corrections for the position or angle of incidence. The $z$ coordinate of the electron impact point is assumed to be at the surface of the calorimeter. The electron angle can be then estimated with simple trigonometry knowing the $\mu$-e vertex position.

![Graph](image_url)

**Fig. 7:** Difference (mm) between the projected impact position and the reconstructed beam centroid, along the $x$ direction, for a 50 GeV electron produced in the 1st (left, black), 10th (center, blue), and 20th (last) (right, maroon) target. In the final apparatus these stations correspond respectively to 21st, 30th, 40th. Note the different range of the axes.

As an example, Fig. 7 shows the distributions of the difference between the projected electron impact point and the reconstructed shower centroid along the $x$ direction for a 50 GeV electron produced in the 1st, the 10th and the last target. A clear dependence on the station position exists, arising from the different material budget crossed by the e.m. shower during its development before entering the ECAL. The point of impact from the farthest stations are worst measured, but the emission angle determination benefits from a longer level arm, which overcompensates that drawback. This can be clearly seen in Fig. 8 which reports the angular resolution as a function of the electron energy for different targets. For all targets, the angular resolution $\sigma_{\theta}$ at $E > 40$ GeV is slightly dependent on $E$ and in the $0.1 \div 0.3$ mrad range, apart from the last target, for which $\sigma_{\theta} \sim 0.4$ mrad. Those numbers are only a factor of 5 worse than the ones obtainable with the tracker system.

With reference to PID, such resolution allows that the two segment tracks can be extrapolated from the tracker to the ECAL to disentangle which track is associated to the e.m. shower, so to unambiguously identify the electron.

4.2.2 Energy resolution

To study the energy resolution in our configuration we select elastic scattering events produced in the target closest to the calorimeter, in order to minimize the amount of material encountered by the electron before hitting the ECAL.

We show in the appendix, see Fig. 37, the quantity $\delta E = 1 - E_{\text{cal}}/E_{\text{true}}$ in the nine energy bins. At this stage of the analysis we do not subtract the energy deposited by the muon (on average, 300 MeV): this
Fig. 8: Angular resolution (mrad) of the reconstructed electron direction, as a function of the electron energy, produced in the 1st (black), 5th (green), 10th (blue), 15th (red), and 20th (last) (maroon) target. In the final apparatus, these stations correspond to 21st, 25th, 30th, 35th and 40th.

explains the negative value of $\delta E$ at low $E_{e,\text{true}}$. At high energy we see that the collected energy is few percent smaller than the true. This effect is mostly due to the lateral loss of the shower (at high energy, as many as 60 crystals are interested by the shower) and can be corrected through calibration (see [32]).

The energy resolution of the calorimeter is affected by radiation in the silicon detectors and in the Be targets for electrons produced far away from the calorimeter. The Fig. 9 shows the $\delta E$ distribution for a 50 GeV electron produced in the 1st, 10th, and last target. The Fig. 10 shows the rms width of the $\delta E$ distributions as a function of the true energy for electrons produced in five different targets. A clear pattern emerges for electrons above 20 GeV. The energy reconstruction can in principle be improved with an algorithm which combines the energy measured in the calorimeter with the pattern of hits reconstructed along the electron track direction in the silicon detectors, considered as a preshower. This will be explored in more details in the future.

The measurement of the energy released in the ECAL allows an independent measurement of kinematic variable $t$, complementary to that based on the tracker system. It is less sensitive to hard photon radiation (which, being emitted collinearly to the electron, deposits its energy in the same calorimeter blocks as the electron) and might provide the most precise measurement at high energy.
Fig. 9: $\delta E$ (GeV) for a 50 GeV electron produced in the 1st (left, black), 10th (center, blue), and 20th (last, right, maroon) target. In the final apparatus, these stations correspond to 21st, 30th, and 40th. Note the different range of the axes.

Fig. 10: Rms width of the $\delta E$ distributions as a function of the electron energy, for electrons produced in the 1st (black), 5th (green), 10th (blue), 15th (red), and 20th (maroon) target. In the final apparatus, these stations correspond to 21st, 25th, 30th, 35th, and 40th.
4.3 Mechanics

As described in Sect. 4, the MUonE detector has a modular structure with 40 identical and independent stations. Each station is composed by a beryllium target and three tracking layers made of two silicon modules having strips oriented vertically and horizontally, and at a stereo angle (the middle layer). The longitudinal station length is \( \sim 1 \text{ m} \).

The transverse dimensions are defined by the event kinematics and an active area of \( 10 \times 10 \text{ cm}^2 \) for the tracking modules is sufficient to contain each event.

The mechanics to hold a station is relatively simple, but it must satisfy few stringent requirements. Since the event kinematics is defined solely by the reconstructed angular quantities, the hit position has to be known with very high accuracy. In particular, while the transverse \((x, y)\) module position can be accurately reconstructed offline by accumulating many straight tracks, the \(z\) position of each module has to be stable by construction within \( 10 \text{ \(\mu\)m} \). Note that this precision is required \textit{within} each station, and not between different stations.

The experimental area will be thermally controlled. However, the supporting material of the tracking stations must have a very small Coefficient of Thermal Expansion (CTE). For a 1 m long station, a CTE of order \( 10^{-6} \text{ K}^{-1} \) is required in order to keep the \(z\) position of a station stable at the \( 10 \text{ \(\mu\)m} \) level for temperature variations of \( \Delta T = \pm 5^\circ \text{C} \).

An additional requirement to the supporting structure is that it must be \textit{transparent} to the large flux of low energy \((E < 500 \text{ MeV})\) electrons emitted at large angle. This is required in order to limit the amount of particles backscattering into the active volume due to interactions with the surrounding material.

The most advanced studies concern the tracker mechanics, which will be based on a carbon fiber structure with inserts supporting the beryllium target and the silicon modules. Carbon fiber has a negligible CTE in the longitudinal direction, which can be theoretically reduced to zero in the region of interest \((T \sim 300 \text{ \(^\circ\)K})\). Moreover its density \(d=1.42 \text{ gr/cm}^3\) is very low, thus limiting the backscattering effect.

A simple scheme of the support structure is shown in Fig.11. The base is a U shaped carbon fiber structure; the vertical dimension - 20 mm in figure - guarantees stiffness to vertical sag.

Each silicon layer is held in position by a frame, here called \textit{insert}, not drawn in the picture. In the current project, this frame is limited to the region where the sensors are, but it can be extended rather easily to cover the full station in order to protect it and make it mechanically more stable. An analysis of the rotational mode of the carbon fiber structure will be performed in order to decide about the external frame.

This frame includes the cooling system, evidenced in the picture. The cooling tube has typical dimensions of 3 mm and it is in thermal contact with the edge of the sensors, through conductive glue, to remove heat. The cooling fluid has not yet been fully identified, but the constraints here are much more relaxed with respect to experiments in which the tracker is buried inside the detector. A stable temperature of the order of few degrees is sufficient to keep the dark current at an acceptable level.

A Coordinate Measurement Machine (CMM) is able to define the relative position of the inserts with a precision better than \( 5 \text{ \(\mu\)m} \) in a meter scale. Such a machine is routinely used in the laboratories of INFN-Pisa: it has a survey volume of \( 1500 \times 2500 \text{ mm}^2 \) and it is installed in a class 10,000 clean room, with temperature and humidity control. This machine can be used to align the tracker modules with the required precision.

Mechanical precision features can be glued which allow to define a repeatable local coordinate system, used to install the station in the experimental hall.

Standard survey reference holes \((\phi = 8 \text{ mm})\) and target supports will be inserted in the structure, in
agreement with the survey team, in order to have an absolute module survey in situ.

![Support structure in carbon fiber.](image)

**Fig. 11:** Support structure in carbon fiber.

Each module will be placed on a support structure that must:

- be rigid enough to withstand the weight of the module without flexing significantly;
- have damping capability to dissipate energy from induced vibrations;
- have the possibility of aligning the axis of the module with the beam at the millimeter level.

A simple and stable solution, optimal for the first Run which will have a limited number of modules, can be achieved by means of a standard honeycomb optical breadboard and an alignment platform. A more integrated solution will be designed for the full structure.

Since the supporting table and the module will have different CTEs, the relative coupling must be carried out by means of a suitable mechanism (e.g. kinematic coupling).

Finally, the system of roll-in and roll-out of the whole apparatus will be carefully thought and designed to meet the requests of precisely positioning of the tracker elements, and keeping in mind that alternation with other M2 users could be mandatory.

The mechanical frame for the calorimeter will be studied once the optimization is finalized (cf sect. 4.2).
5 The Beam

The experiment will employ the upgraded M2 muon beam at the CERN SPS as described in details in Ref [34]. The M2 beam line delivers high energy and high-intensity muon and hadron beams towards the experimental hall EHN2 as well as low intensity electron beams for calibration. The beams are derived from the SPS primary proton beam of 400 GeV/c with an intensity between $10^{12}$ and $10^{13}$ protons per SPS spill. The beam impinges on a primary beryllium production target, where mainly secondary protons, electrons, pions and kaons are produced. These secondary particles are then transported in a long, evacuated beam line [35], which allows the pions and kaons to decay into muons. Left-over hadrons are stopped with the help of a 9.9 m thick Beryllium absorber through which muons pass basically unharmed. The muons are then momentum-selected with the help of large magnetic collimators. The beam then has still a very large halo component that is gradually being cleaned by further magnetic collimation. Typically, muons with momenta in the range of 100 and 225 GeV/c can be selected. The typical maximal intensity for a beam energy of $\sim 160$ GeV is $\sim 5 \times 10^7 \mu^+$/s for a SPS spill with $10^{13}$ protons on target. The typical SPS cycle for fixed-target (FT) operation lasts at least 14.8 s, including 4.8 s spill duration, i.e. the time during which the beam is slowly extracted. The number of FT cycles is about 2-3 per minute depending on LHC fillings and constraints by other users.

The EHN2 hall is currently occupied by the COMPASS experiment. For MuonE installation a location in the upstream region of the hall has been deemed suitable, where currently two CEDAR detectors are housed (details are given in Appendix 17.3). This tentative location is shown in Fig. 12. The beam is relatively large in the present configuration at the location of the CEDARs. Therefore optics studies were performed within EN-EA-LE to estimate the beam size for a modified configuration along with integration studies to estimate required beam line modifications, costs etc. Both old and modified optics are shown in Appendix 17.3.

5.1 Beam Parameters

The optics has been studied for the upstream location to provide a parallel beam with a central momentum of 160 GeV/c with the help of TRANSPORT [36] and HALO [37], to fit the beam within the detector dimensions of 10 cm x 10 cm. Figure 13 shows the beam spot at the entrance of MuonE with $\sigma_x = 26$ mm and $\sigma_y = 27$ mm, while Fig. 14 shows the angular divergence with $\sigma_x = 0.27$ mrad and $\sigma_y = 0.20$ mrad. The beam momentum distribution is shown in Fig. 15-left for a 160 GeV/c beam with $\sigma_p = 6$ GeV/c. The halo of the muon beam for $|r| > 15$ cm and $|x|, |y| < 3$ m is shown in Fig. 15-right, which is...
\( \sim 20\% \) of the nominal beam flux. Further details about the study is presented in \cite{38}.

**Fig. 13:** Beam spot size at MuonE with \( \sigma_x = 26 \text{ mm} \) and \( \sigma_y = 27 \text{ mm} \)

**Fig. 14:** Beam Divergence at MuonE with \( \sigma_x' = 0.3 \text{ mrad} \) and \( \sigma_y' = 0.2 \text{ mrad} \).

**Fig. 15:** Left: momentum distributions at MuonE with \( \sigma_p = 5.9 \text{ GeV/c} \) for a 160 GeV/c incoming beam. Right: muon Halo distribution at the entrance to the upstream position for \(|r| > 15 \text{ cm}\) and \(|x|, |y| < 3 \text{ m}\). The halo flux is \( \sim 20\% \) of the nominal beam flux at this position.

### 5.2 Beam Momentum Measurement

The upstream location is also compatible with the request to measure the beam momentum, which can be done using Bend 6 as a spectrometer. Currently, the COMPASS collaboration uses their so-called Beam Momentum Station (BMS), which is located upstream of the CEDARs and consists of beam defining hodoscopes labelled BM01-06 as shown in Fig. 16. This detector set-up has been chosen for an estimation
of the achievable momentum resolution in case a similar detector or possibly even the same system would be allowed to be used in the future. For this, the entire beamline was simulated with HALO and particle hits were scored at the BMS hodoscope positions. The BMS hodoscopes have a detector resolution of 1.3 mm for BM01-4 and 0.7 mm and 0.4 mm for BM05 and BM06 respectively. The hodoscopes can define the beam only in the vertical (Y) co-ordinate. The system of bending magnets (Bend 6) consists of three 5 m vertical bends with 3.3 Tm each. The momentum reconstruction with the BMS was estimated and Fig. 17 (left) shows the momentum resolution $\sim 0.9\%$ for a 160 GeV/c incoming beam from simulation which matches the results from data of COMPASS. Fig. 17 (right) shows the estimated improvement that can be expected if better resolution tracking detectors would be used.

Fig. 16: Schematic of the beamline showing the BMS

Fig. 17: Left: Momentum resolution of 0.9% estimated for the current BMS setup with simulation. Right: Estimated momentum resolution $\frac{d\rho}{\rho}$ [%] as a function of detector resolution.
6 TRIGGER and DAQ

6.1 Introduction

The readout system for the experiment is proposed to be based on a variant of the system developed for the HL-LHC upgrade of the CMS tracker, which is described in [29]. The detector modules are the 2S-modules to be deployed in the outer radial region of the CMS tracker (Fig. 18). These modules have two silicon microstrip sensor planes separated by a few mm, with the strips in the sensors parallel to each other, read out by a 130 nm CMOS ASIC called the CBC (CMS Binary Chip)[39, 40]. The strips on both sensors are connected to the inputs of a single CBC with the upper and lower layer strips interleaved to adjacent channels of the CBC. If the sensors are correctly aligned, simple patterns can be identified which represent hits matching the criteria for high-$p_T$ “stubs” in CMS (Fig. 19). For this application a stub is simply a pair of matching hits in the two layers, representing a space point in one dimension. Up to 3 stubs per 40 MHz clock cycle can be read out by each CBC. The power consumption of a 2S-module is about 5.4 W, with the 16 CBCs contributing about 2.2 W.

The 2S-modules are in advanced state of development; the CBC is complete and has been available for module prototyping for some time, the high density hybrids which contain the bump-bonded CBCs and concentrator ASIC (CIC) have been challenging to manufacture but are now believed to be final, the final version of the CIC is due to be manufactured in mid-2019, and the service hybrid which contains DC-DC converters and optical links for data transfer and control is in an advanced state. Prototype (n-on-p) sensors have been available and the production order is currently being negotiated. The CMS schedule foresees module production starting in 2020, and final prototypes will be available before then.

6.2 Stub logic

A hit in either sensor layer is defined by a cluster, which may be one channel wide, or greater if adjacent strips on the same layer have also produced a hit. The first stage of the stub-finding logic examines the width of each cluster in each layer, and rejects unsuitable candidates by comparison with a programmable value. In addition, the logic generates “bend” information, which is relevant in CMS in view of the magnetic field. For MUonE, the bend can be considered to be a short vector representing the lateral difference in strip position between the clusters on the two sensor layers. When the data are output, 8 bits are allocated to the position of the stub, giving half-strip resolution, and 4 bits to the bend vector.

Stubs are read out at the full 40 MHz LHC clock rate, while in CMS the full hit data representing clusters
in the individual sensors are read out at a lower rate, up to 1 MHz, on receipt of a Level-1 trigger. For this project, it is probable that only the stub data will be needed since most of the spatial information will be preserved with good precision. The stub logic will suppress background hits from noise in a single layer, and any accidental tracks crossing the sensors at a large enough angle to create wider clusters. The width and relative lateral location of the clusters used to form stubs is programmable, so can be tuned for optimal performance, as can the response of the comparators on each channel.

An important difference between MUonE and CMS for the front-end electronics is that in the beam to be used for this experiment, muons will arrive asynchronously compared to the 40 MHz LHC clock and generate signals with random phases with respect to the clock. This means that muons, or secondary particles from an interaction, which arrive close in time to one another will produce multiple hits in the sensors, while a particle which arrives significantly away from the optimal sample time will not be registered (Fig. 20). The width of the time window depends on the comparator threshold, and the CBC
has several options for selecting the duration of the comparator output which can be studied for optimised performance (Fig. 21).

![Front-end circuit and different comparator operation modes.](image)

**Fig. 21:** Front-end circuit and different comparator operation modes.

### 6.3 Stub rates

The output rate of the MUonE readout system is determined by the intensity of the M2 muon beam and the occupancy in the detector modules. The intensity of the beam is rather high, of order 50 MHz. However, the module occupancy should be quite small, since most muons will cross the detector without interacting. Sampling of the MUonE detector must be done at 40 MHz as for CMS which reduces pileup to very few overlapping muons per clock cycle, easing pattern recognition and tracking. As a consequence of the low detector occupancy and the simplicity of the $\mu - e$ events the DAQ system does not need a custom hardware trigger and online event reconstruction using stub data should be possible in firmware on the front-end DAQ boards, reducing the computational resources required. This is discussed in the next section.

The expected multiplicity from beam particles can be estimated from

$$m = I_\mu \Delta t = 1.25$$

assuming an incident muon rate, $I_\mu$, of 50 MHz, and a sensitive time interval of 25 ns. $\Delta t$ may be adjusted to be more than one clock cycle by choosing one of the operation modes referred to in Fig. 21.

To estimate possible inefficiencies in the signal detection related to the bandwidth limitations, we evaluated the occupancy of the different ASICs, using a GEANT4 based simulation code. We assumed the intensity of the muon beam to be 50 MHz, with a beam profile of $\sigma_x = \sigma_y = 1.5$ cm. Figure 22 shows the transverse profile of the muon beam projected on the sensor surface, superimposed with the mapping of the corresponding ASICs. The simulation allowed to define the distribution of the number of stubs per ASIC, and to estimate the overall data throughput (see next section).

Figure 23 shows the distribution of the number of stubs recorded by the CBC in the hottest region of the sensor. The probability of detecting more than three stubs is of order 0.1%. Saturation due to pileup is a genuine random effect, not biasing the signal detection efficiency. Note that by setting the voltage threshold required to detect a hit it is possible to tune the readout data rate.

### 6.4 DAQ

Communication between a single module and the back-end system is established over a pair of optical fibres running the lpGBT protocol [41]. The down-link runs at 2.56 Gb/s and is used to provide clock,
fast commands and 'slow control' configuration data to the module, while the up-link provides 5.12 Gb/s of bandwidth to read out all data to the DAQ. As described, it is expected that MUonE will not implement a trigger to read out the front-end devices, but instead make use of the 40 MHz stub data streams from each module directly. This implies that the ultimate constraint in terms of readout bandwidth from the module is defined by the CIC, where no more than 16 stubs can be transferred to the DAQ in 8BX per CIC (32 stubs in 8BX per module). With an expected average muon multiplicity of 1.25 muons per BX per module, there should be sufficient margin to handle fluctuations in the stub rate.

Reflecting the modularity of the tracking system, and use of detectors developed for the CMS tracker upgrade, it is proposed to implement a scalable DAQ solution using readout hardware designed for a number of different applications in CMS. The Serenity development platform [42], itself a prototype of the CMS Tracker DAQ hardware required for Phase 2, is shown in Fig. 24, and consists of four elements.
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– Carrier Card - the carrier card is compatible with the Advanced Telecommunications Computing Architecture (ATCA) standard [43] and hosts the common board services including power, clocking, optical interfaces, the intelligent platform management controller (IPMC) functionality, and a pluggable x86 CPU Computer-on-Module to control and manage the board.

– Daughter Cards - the daughter cards host the data-processing FPGAs and utilise interposer arrays from Samtec (Z-RAY) [44] to interface with the carrier card. Two interposer sites are available on the carrier card.

– Optics - the carrier card provides connectivity for up to 12 Tx and 12 Rx Samtec Firefly [45] pluggable optical modules, with each module supporting up to 12 channels at either 1-16 Gb/s or 25-28 Gb/s.

– Framework - a framework of generic, flexible firmware and software is provided as part of the platform, factorising board-specific functionality and operation from user defined firmware or software, reducing barriers to entry while improving maintainability.

To meet the needs of the various sub-systems making use of the hardware in CMS, the flexibility of the platform is afforded via the use of interchangeable FPGA daughter cards and pluggable optics in combination. One possible configuration of the Serenity platform, based on the proposal for the CMS Tracker, is presented in Fig. 24. Two daughter cards hosting state-of-the-art Xilinx Kintex Ultrascale+ KU15P FPGAs [46] each provides 36 bidirectional links for communication with detector electronics over the lpGBT protocol. Each KU15P daughter card also provides 8 bidirectional links (and a maximum of 24 if desired) at 25 Gb/s for communicating with other processing or DAQ hardware. Applied to MUonE, each FPGA could therefore service six tracking stations (36 modules), meaning a single Serenity card could handle 12 tracking stations in total.

The expected data throughput from each module in the MUonE detector can be estimated by assuming: a stub can be encoded in 16 bits (11 bit position in half-strip resolution, 1 bit module side, 4 bit stub vector information), a header of 32 bits per module is required (containing a module identifier, an event identifier, and possibly some module status information), and a conservative rate of 3 stubs per BX per module is expected (the muon multiplicity is estimated at 1.25 muons per BX per module). Simulations indicate that each station can be considered independent from the others; it is rare that downstream stations are significantly affected by secondary particles generated upstream. Therefore it is reasonable to assume that the data rate scales proportionally with the number of detector stations. At 40 MHz this implies a conservative average data rate of ~20 Gb/s per station, meaning each Serenity card would output on average around 240 Gb/s of data. This could be easily accommodated by the 400 Gb/s optical output provided in the Serenity configuration above.

Four Serenity cards, handling the first layer of processing, would be able to read out a detector consisting of up to 48 tracking stations. This would include lpGBT decoding, packet capture and alignment, and CIC data unpacking to extract the stubs. In addition, the cards would implement the logic required to control the modules and configure the front end ASICs. One important aspect is the need to ensure the full detector is time-aligned and so this processing layer will also include the handling of the relative timing between modules in the system in order to account for fibre length differences and particle time of flight throughout the apparatus. The firmware and control software required to implement the DAQ logic is a simplified version of the DAQ package developed for the CMS Tracker and largely exists.

The data from the first layer could then be delivered to a second layer of processing electronics, consisting of a single Serenity. The Serenity in this layer would be required to read in 64 links at 25 Gb/s, which is easily achievable with a slightly modified configuration of the card illustrated in Fig. 24. Similarly the FPGAs on this layer can be selected according to the density of processing logic required. The expected input data rate would be <1 Tb/s. This second layer would be responsible for both event building and
event selection, described in the next section. A relatively simple event filter implemented in the FPGAs of the second layer could reduce the demands on the computing system downstream by suppressing the event rate to $<400$ kHz from 40 MHz, resulting in a conservative uncompressed output data rate of $<10$ Gb/s. A number of options exist to allow the filtered events to be transferred from the Serenity to a storage server (Ethernet 1G/10G, x4 PCIe Gen3 etc.) and can be optimised once the data throughput has been better estimated.
7 Simulation

7.1 Detector description in GEANT4

The experiment simulation is based on the Geant4 [47] toolkit version 10.4.1 and it is under active development. The code is written in such a way to exploit the Geant4 multi-threading capabilities (event based parallelism).

The code allows the simulation of different detector geometries by simply changing flags or geometrical parameters at run-time.

The tracking part is subdivided in stations, optionally it is possible to add an electromagnetic calorimeter at the end of the stations sequence. There are some predefined geometrical descriptions; the basic station is made of a target followed by a number of silicon tracker planes. The target is made of beryllium, and both the size and the material can be changed at run-time. The currently chosen geometry setup of a station is composed by one target and by twelve silicon tracker planes 320 µm thick. The tracker planes are grouped in pairs, with a gap of 1.8 mm.

The distance between two subsequent pairs is 1 cm and there are 3 of such structures in a station. In each structure the first pair will measure the x coordinate, the second one will measure the y coordinate.

The current calorimeter description is made of equal input cross section crystals; the crystals can be slightly tilted, pointing to the muon beam, or parallel to the beam line. It is possible to choose, at the moment, between two materials: PbWO4 and CsI(Tl), but other materials can be easily added if needed.

The simulation can run with different generator setups:

- **basic process**: the program simulates the interaction of a single muon with the detector.
- **forced interaction**: the mu-e scattering process is produced utilizing the Von Neumann algorithm for the kinematics, while the interaction vertex is placed inside a randomly chosen target.
- **NLO process**: The Geant4 generator has not the ability to simulate Next to Leading Order events; in order to study the NLO events the current version of the simulation program allow to read NLO events generated by a different program, put the generated event in a randomly chosen target and make Geant4 follow the particles making them interact with the detectors.

The output of the simulation is a set of ROOT [48] TTree that store information from the interaction of the particles with the detectors. The program does not simulate the electronic response, it is foreseen that the simulation of these effects will be done taking the output of the Geant4 based program and running on it specific simulations, giving a more flexible software structure and reducing the need of computing resources.

The code source is maintained in a git repository available to all collaborators.

7.2 Generators

A Monte Carlo event generator (EG) including QED NLO radiative corrections has been developed and finalized for preliminary simulations and feasibility studies. Other theoretical effects beyond NLO (NNLO corrections, resummation, etc.) are not available into an event generator yet but currently they are under consideration and we definitely plan to make them available into a generator in the near future.

The theoretical formulation of the NLO Monte Carlo EG is based upon the features sketched in Sec. 12.1 and discussed at length in [49]. The code is not publicly available at the moment, but it will in due time: nevertheless a few Monte Carlo samples containing \( \mathcal{O}(10^{11}) \) events have been generated and can be used by the MUonE collaboration.
The code is a derivation of the widely used BabaYaga@NLO EG [50]. Its main features are sketched in the Appendix, Sec. 17.4.

An example of an analysis performed with the NLO EG is shown in Fig. 25. In the figures, for the \( q_e \) and \( q_{\mu} \) observables the \( R \) quantity is defined as the ratio

\[
R_i = \frac{d\sigma(\Delta\alpha_{\text{had}}(q^2) \neq 0)}{d\sigma(\Delta\alpha_{\text{had}}(q^2) = 0)}
\]

In words, \( R \) shows the effect on the given observable of the hadronic part of the QED running coupling constant, which is the quantity that MUonE aims to extract from data. The left-side plots show that on \( q_e \) the LO “signal” (black histograms) is highly reduced at NLO if no cuts are applied (red dots) and recovered when they are applied (blue dots for \( \Delta\phi_{\mu < 3.5 \text{ mrad}} \) and violet dots for \( \theta_{\mu > 0.2 \text{ mrad}} \)). On the other hand, in the right-side plots, the \( q_{\mu} \) variable looks much more insensitive to both inclusion of NLO effects and application of experimental cuts.
8 The Tracking

8.1 Tracking Algorithms

The events of the scattering of high-energy muons on atomic electrons of a low-Z target through the elastic process, $\mu e \rightarrow \mu e$, are characterized by relatively simple topology. This means three tracks to be reconstructed in the detector, i.e. the direction and momentum of the incident muon and directions of the outgoing electron and muon. The proposed detector setup (see Sec. 4) assumes relatively clean physics environment, i.e. low detector occupancy and no hardware-based trigger. The boosted kinematics of the collision allows the detector to cover almost 100% of the acceptance, while the time structure of the beam allows to keep the background related to incoming particles at low level.

In practice, as there is no CPU time limit, the track reconstruction is meant to be offline. In this way, the quality of the track reconstruction can be maximized. This may boost much the reconstruction efficiency and precision, keeping the acceptance corrections at low level.

The detector layout (see Sec. 4) assumes 40 identical stations, each consisting of a 1.5 cm thick layer of Be coupled to 3 Si modules covering a relative distance of one meter from one target to the next, and spaced by intermediate air gaps.

The expected relatively low detector occupancy and the assumption of no limits on the CPU time brings the pattern recognition on the achievement of the maximum possible track reconstruction efficiency. In the first stage it will be performed in the $x$-$z$ and $y$-$z$ projections, by constructing pairs from all the combinations of hits in $x$ and $y$ layers separately. For each pair of hits a two-dimensional (2-d) line in $x$-$z$ or $y$-$z$ projections will be determined, and all the hits within a certain window will be then collected. Only the lines with at least three hits will be accepted. No unique combinations of hits forming 2-d lines are imposed at this stage.

The output set of the combinations of hits constituting two-dimensional lines will be sorted according to the number of collected hits and the pseudo-$\chi^2$, defined as the $\chi^2$ of hits with respect to the initial 2-d line defined by the hit pair. The best two-dimensional line candidates will be selected by choosing the ones with the maximum number of collected hits and (in case of the same number of hits) the lowest pseudo-$\chi^2$.

The final combinations of the hits constituting 2-d lines in $x$-$z$ or $y$-$z$ projections will be fitted and the hits from stereo layers will be employed for proper assignment of the $x$-$z$ and $y$-$z$ lines.

Two-dimensional fit. The robust fit to the selected combinations of the hits constituting 2-d lines in $x$-$z$ or $y$-$z$ projections will be performed to reconstruct 2-d tracks. Such a fast fitting procedure is performed with the removal of outlier hits, and determined with the uncertainties of the $x$ and $y$ hit positions as referred in Sec. 4. All the fitted $x$-$z$ and $y$-$z$ lines will be then paired to create three-dimensional (3-d) lines requiring a maximum number of hits shared in stereo planes. All 3-d lines will be finally fitted using MINUIT minimization package [51], with all the hits collected within a certain window with respect to the initial 3-d line (see next section).

Reconstruction in three dimensions. After the stage of the two-dimensional reconstruction all combinations of line segments in $x$-$z$ and $y$-$z$ projections will be combined into 3-d track candidates. Solving the ambiguities of assignment of 2-d lines to the 3-d lines will take great advantage from the stereo ($u$, $v$) middle layers. No prior requirements on quality of such combination will be imposed to maximize the reconstruction efficiency. For each 3-d track candidate the initial parameters of 3-d line will be determined from the corresponding 2-d lines. Such 3-d line will be treated as a seed for the track fit. All hits along initial trajectory will be collected within a certain window. The iterative fitting procedure will be performed using a least square method. After each iteration the outlier hits will be removed and the fit will be repeated until no outlier is found. As no unique combinations of hits was imposed up to this
point, the collections of tracks may contain clones or duplicate candidates. A clone removal procedure will be applied. First the identical solution will be reduced to single candidate. Next the tracks will be sorted according to the number of hits and $\chi^2$ per degree of freedom of the least square fit. The tracks with the largest number of hits will be accepted first. To accept the track at least three hits in $x$-$z$ and three hits in $y$-$z$ projections are required. For the same number of hits the candidate with lower $\chi^2/ndf$ will be taken. After accepting a track, the hits used by the track will be marked as used. Then the next track is searched for containing minimum required number of hits not marked as used. The final collection of unique tracks are passed to the last stage of event reconstruction.

### 8.2 Event reconstruction

The set of reconstructed tracks are used to search for elastic $\mu - e$ topology. In the first step all combinations of track pairs are checked to intersect in one of the Be targets within a certain window. Then the third track incoming to the target close to the intersection point is searched for. For the three tracks initially compatible with muon-electron scattering a global fit will be performed to reach the best possible sensitivity for the scattering angles of outgoing muon and electron. The least square fit will include the $\chi^2$ contribution from tracks and the $\chi^2$ contribution from the single optimal point of intersection in the target. For the reconstructed muon-electron scattering candidate, the total $\chi^2$ and other variables like acoplanarity will be used to filter good candidates. In the final step the tracks will be extrapolated to calorimeter and muon stations, if in their acceptance, to identify outgoing muon and electron. The measured angles of outgoing muon and electron with respect to the incoming muon are correlated to the momenta of outgoing particles (kinematics is smeared by the detector resolution and radiative effects). The knowledge of the momentum will enable to perform a Kalman fit including contribution from multiple scattering. Optimal parameters at the target will be determined and the final scattering angles will be calculated.

The performance of event reconstruction will be checked using simulation based on Geant4 package [47]. Some of the results quoted are obtained from Fast Simulation, which will be used extensively also in the future to get high statistics samples for detailed study. As an example Fig. 26 shows the performance of the tracking algorithm, implemented as a Kalman filter, obtained assuming two different target thickness of 1 cm or 2 cm, and when tracking through a single station or extending the length of the tracking level arm to the closest downstream station. One can observe in particular that for particles momenta above 120 GeV/c by adding hits of the downstream station helps improving the angular resolution.

![Fig. 26: Left: the comparison of the tracking algorithm performance assuming two different target thickness of 1 cm (black circles) and 2 cm (red squares), in logarithmic scale. Right: the algorithm performance varying the length of the tracking level arm, by extending the tracking to the next downstream station.](image)

In the case the first tracking reconstruction will be applied in the FPGA, the optimization of the algorithms will be made.
9 Elastic events: the Analysis

9.1 Selection criteria

A muon that undergoes elastic scattering can be identified by the reconstruction of two charged particles in a tracking station.

The following criteria will be applied within the event selection algorithm, and are being studied with the simulation of the detector to estimate efficiency and purity.

1. a cut on the track candidate quality ($\chi^2$) helps to eliminate combinatorial background and fakes.

2. multiplicity of reconstructed track candidates. There must be at least two. Higher multiplicity can identify control channels (like $\mu \rightarrow \mu e^+ e^-$ or reject large part of nuclear interactions background).

3. there must be a common vertex reconstructed in the target from at least two track candidates.

4. there must be a single candidate as incoming track in the 3 tracking stations immediately upstream, consistent with the vertex position in the following target.

5. the elastic events being planar, a cut in acoplanarity will help in rejecting background (residuals from nuclear interactions, pair production, etc.)

Further filtering criteria are under investigations with the fast and full simulations, in order to optimize the efficiencies and the background rejection.

It is also envisaged to collect events where a single $\mu$ passes straight through the apparatus, to provide the alignment of the detector.

9.2 Background

We used GEANT4 for studying the background, and this work is in progress. Interactions of high-energy muons have been simulated considering the relevant processes, as indicated in [52]. For high energy muons ($\geq 100$ GeV), the dominant processes are ionization, bremsstrahlung, electron-positron pair production, and nuclear interactions.

By analyzing the available statistics of fully simulated events (order of several $10^7$ events) we established that the main source of background is due to electron-positron pairs, where one of the $e^+$ or the $e^-$ is out of the detector acceptance or has been reabsorbed in the material. We estimated the contribution of the pair background to be of the order of $10^{-4}$. The production of higher statistics data samples by means of a computing cluster is ongoing.

The contribution to the background due to muon nuclear interactions is expected to have in many cases a recognizable signature (track multiplicity). The muon nuclear cross section in beryllium at 150 GeV is predicted to be 0.49 mb, and we are studying with a full simulation the fraction of nuclear interactions which have two particles traced in the station and which are coplanar.

In order to evaluate the level of the background with data, we plan to record events with three tracks in the acceptance, and use the Monte Carlo to estimate the effective contribution of the two tracks to the selected events.

9.3 Use of the inverse kinematic method

The kinematics of the elastic scattering of a muon on an electron at rest is completely described by three quantities: the incident muon energy $E_{\mu}$, and the two scattering angles of the muon and the electron with respect to the incident beam direction, $\theta_{\mu}$ and $\theta_e$ respectively. The knowledge of two of these three
quantities allows the determination of the missing one. In our case the two known quantities are the scattering angles. We apply this method to the particle identification, as discussed below, and to the determination of the average beam energy, as presented in 10.2.3.

The analytic treatment of the elastic scattering is a textbook case, for example ref. [53], to which we refer. We obtain $E_\mu$ by numerically solving the equation system in terms of the only variables $q_\mu$ and $q_e$.

We developed a toy Monte Carlo simulation to generate elastic interactions and to compute the effect of the target material and of the tracker intrinsic resolution on the measurement of the scattering angles. Then we compute back the value of $E_\mu$. We used the nominal beam momentum $p_{beam} = 150$ GeV/c, with a Gaussian momentum spread $\sigma(p_{beam})/p_{beam} = 3.75\%$. Events are generated randomly inside the target, and the two angles $\theta_\mu$ and $\theta_e$ are then smeared according to a Gaussian distribution with a standard deviation $\sigma = \sqrt{\sigma_{MCS}^2 + \sigma_{TRK}^2}$, where $\sigma_{MCS}$ is the usual effect of the multiple coulombian scattering, function of the particle momentum, and $\sigma_{TRK}$ is the tracker intrinsic resolution. Finally, the value of the beam energy $E_\mu$ is computed.

In Fig.27–left we show the calculated beam energy as a function of the electron scattering angle, for a 1 cm thick Be target and a tracker resolution $\sigma_{TRK} = 20$ $\mu$rad. The $E_\mu$ distributions have different shape and width as a function of the particle energies. At low $\theta_e$, that is close to the electron maximum energy and to the muon minimum energy, the determination of $E_\mu$ suffers from the uncertainties caused by the muon multiple scattering. On the opposite side, at large $\theta_e$, multiple scattering of the electron dominates.

![Graph](image)

Fig. 27: Left: distribution of $E_\mu$ (GeV) vs $\theta_e$ (mrad). The red dots show the distribution when the particle identity has been swapped. The blue lines delimit the 1.5 ÷ 5.0 mrad angular range used for the beam energy determination. Right: distribution of $E_\mu$ (GeV) for the electron angular range $\theta_e = 1.5 ÷ 5.0$ mrad. The red histogram shows the distribution obtained when the particle identity has been swapped.

9.3.1 Particle ID

The inverse kinematic method can be exploited to identify each particle. In fact, if the scattering angle is assigned to the wrong particle (for instance, the particle with scattering angle $\theta_\mu$ is classified as an electron), the algorithm cannot find a solution, or converges to a beam energy value outside the nominal beam range. On the opposite, in the angular range where $\theta_\mu \approx \theta_e$, the two particles are hardly distinguishable (their mass being negligible with respect to their energy), and the algorithm succeeds to determine $E_\mu$.

This has been verified, in the whole kinematics domain, by swapping the identity of the two particles in the flow of the aforementioned simulation, namely, associating the angle $\theta_\mu$ to the particle with mass $m$
and the angle $\theta_e$ to the particle with mass $M$.

The result is shown in Fig. 27–left (red dots). Events with the wrong particle ID appear only for $\theta_e \lesssim 5$ mrad.

In Fig. 27–right the red histogram shows the $E_\mu$ distribution in the $1.5 \div 5.0$ mrad $\theta_e$-range obtained when the particle identity is swapped, overlapped to the correct ID case. The fraction of wrong ID events within $\pm 3$ standard deviations from the mean value $\langle E_\mu \rangle$ is about 40%. This illustrates that the tracker system alone is not enough for the particle ID in the whole kinematic range of the elastic scattering. The presence of an electromagnetic calorimeter is crucial for this task.
10 Strategy to fit the hadronic contribution

10.1 Extraction of the hadronic contribution

The hadronic contribution to the running of $\alpha$ is most easily displayed by considering the ratio $R_{\text{had}}$ of the observed angular distributions and the theoretical predictions evaluated for $\alpha(t)$ corresponding to only the leptonic running. In the simplest case, for the electron angle at LO, this ratio is simply proportional to:

$$R_{\text{LO}}^{\text{had}}(t) = \left(1 - \frac{\Delta\alpha_{\text{had}}(t)}{1 - \Delta\alpha_{\text{lep}}(t)}\right)^{-2} \simeq 1 + 2 \frac{\Delta\alpha_{\text{had}}(t)}{1 - \Delta\alpha_{\text{lep}}(t)},$$

(6)

where $\Delta\alpha_{\text{lep}} \lesssim 1\%$ and $\Delta\alpha_{\text{had}} \lesssim 0.1\%$ in the MUonE kinematical range. At NLO the ratio becomes a complex expression and is evaluated by Monte Carlo simulation. The used observables are the inclusive distributions of the muon and of the electron scattering angle, as well as the two-dimensional distribution of these two. The extraction of the hadronic contribution is carried out by a template fit method, as described below. All the results described here have been obtained with the NLO MC generator with no approximation.

The most important experimental features modifying the ideal theoretical prediction have been included in a fast Monte Carlo simulation. The M2 beam energy spread has been accounted for by a 3.75% Gaussian distribution, around the nominal value of 150 GeV. Detector resolution and material effects have been taken into account by a parameterization tuned on a dedicated full simulation based on GEANT4.

The measured electron and muon angles have been obtained by smearing the generator-level quantities according to the expected effects of the intrinsic detector resolution and the multiple scattering on one beryllium target and on the detector itself, comprising 6 silicon-strip CMS modules for one tracking station (3 measured points on both the $x$ and $y$ transverse coordinates). As an alternative a somewhat ideal detector model has been assumed, by completely neglecting the multiple scattering on the detector material and assuming an intrinsic angular resolution of 0.02 mrad.

The expected distribution of the electron scattering angle is shown in Fig. 28-left without any selection. Detector effects produce large distortions, mostly due to multiple scattering of electrons with energies as low as 1 GeV. These low energy electrons populate not only the large $\theta_e$ region but also the low $\theta_e$ through radiative processes with (at least) one real photon emitted. Given the expected high background rate at low $\theta_e$, as well as the strong effects of multiple scattering on low energy signal electrons, a simple selection has also been studied, removing events with $\theta_e < 0.2$ mrad, as shown in Fig. 28-right. This simple cut reduces significantly the detector effects and helps recovering most of the elastic signal at low $\theta_e$. The flattening of the distribution observed for the MUonE detector at large $\theta_e$ results from the cut on $\theta_\mu$, given the worse intrinsic resolution in comparison with the other best detector option.

The expected distribution of the muon scattering angle is shown in Fig. 29-left. Relatively large detector effects are visible only at the boundaries of the kinematical range. At small $\theta_\mu$, the intrinsic resolution is dominant, since the muon keeps almost the full beam energy, so the material effects are small. At the maximum angle $\approx 4.8$ mrad the visible effect is the resolution at the edge of the phase space. This is mostly due to multiple scattering of the outgoing muon, whose momentum is about 20 GeV in that region. The ratio $R_{\text{had}}$ as a function of the muon scattering angle is shown in Fig. 29-right, for a pseudo-experiment with the nominal MUonE integrated luminosity of $1.5 \times 10^7$ nb$^{-1}$. The maximum observable effect of the hadronic running is about 0.2% as expected.

To extract the hadronic contribution from the measured angular distributions, the $\Delta\alpha_{\text{had}}(t)$ is modelled by a two-parameter analytical function, which approximates very well the Jegerlehner’s numerical parameterization [54, 55] used in the NLO Monte Carlo generator (based on time-like hadroproduction data and perturbative QCD). The chosen function is physically inspired, with logarithmic dependency at large $|t|$ and linear behaviour at small $|t|$, as expected from general principles.
The signal extraction is carried out by a template fit method. Template distributions for the scattering angles $\theta_\mu$ and $\theta_e$, both 1D and 2D, have been calculated from NLO Monte Carlo events on a grid of points in the parameter space sampling the region around the expected reference values. To this purpose a reweighting technique has been used to reduce the needed computing time without any loss of accuracy. The template fit is then carried out by a $\chi^2$ minimization, comparing the angular distribution of pseudodata with the predictions obtained for the scanned grid points. Figure 29-right shows the resulting fit obtained for the muon angular distribution in the given pseudo-experiment. The fit quality is quite good.
The statistical uncertainty of the fit has been tested by repeating many pseudoexperiments, each one with statistics equivalent to the MUonE expected luminosity. The resulting $\alpha_{\text{LO}}^{\mu}$ is finally obtained by calculating the master integral of Eq. 1 using the fitted parameters to describe the $\Delta\alpha_{\text{had}}(t)$. From 3,000 pseudoexperiments we get $\alpha_{\mu}^{\text{HLO}} = (689.8 \pm 2.3) \times 10^{-10}$, where the errors are only the statistical ones. It must be noted, as detailed in Sec. 10.2.2, that the fit function is extrapolated to the full $x$ range of Eq. 1 to obtain this value. More details on the fit and the used parameterization are given in the Appendix 17.5.

The reference value of $\alpha_{\mu}^{\text{HLO}}$, corresponding to the used Jegerlehner’s parameterization, is $688.6 \times 10^{-10}$, which is about half the statistical error from the expected fit result. This small difference is understood as a technical error in the fitting procedure.

10.2 Strategy for the systematic uncertainties

The reference figure is the expected statistical uncertainty of 0.3% on $\alpha_{\mu}^{\text{HLO}}$, which is estimated for the nominal MUonE integrated luminosity of $1.5 \times 10^7$ nb$^{-1}$. Any systematic effect should be kept within a comparable value. The most harmful effects are those variable across the kinematical range (i.e. versus $t$), as they affect the shape of the differential cross section, hence the running of $\alpha(t)$. Many effects are playing at this challenging precision. The most important experimental systematic uncertainties are related to:

- Normalization (flat, correlated systematic error);
- Fit model and extrapolation;
- Average beam energy scale;
- Beam energy spread;
- Multiple scattering;
- Tracker efficiency and reconstruction uniformity;
- Tracker alignment (in particular, longitudinal positioning);
- $e - \mu$ identification;

In addition, theoretical uncertainties need to be controlled at a similar precision, as mentioned in Sec. 3.2. Other possible experimental systematics are expected to have a lesser impact. In particular: the determination of the position of the interaction vertex; the event selection; the residual background. Here the proposed methods to address the major systematics are discussed. Tests of the fit stability in the presence of the various systematic effects and of their impact on the measurement is still an on-going work.

10.2.1 Normalization uncertainty

This can be thought as a flat, correlated systematic error, due to the experimental normalization or the theory prediction. The MUonE experiment does not rely on a precise knowledge of the luminosity. The hadronic running of $\alpha(t)$ is extracted from the precise measurement of the shape of the differential cross section. The measurement has to be constrained to the prediction at the boundary of the MUonE acceptance, in the so-called normalization region. In that region the value of $\Delta\alpha_{\text{had}}(t)$ is hundred times smaller than at the peak of the integrand in the master Eq. (1). A preliminary study constraining $\Delta\alpha_{\text{had}}$ in the normalization region, at $x \simeq 0.3$, to be consistent with the timelike estimate within a safe uncertainty, showed that it is possible to absorb the normalization uncertainty without any loss of accuracy in the MUonE measurement.
10.2.2 Fit model

The parameterization of $\Delta \alpha_{\text{had}}(t)$ (given in appendix at Eq. 14) is used both to measure the hadronic running within the MUonE acceptance, and to extrapolate to the full $x$ range. At the maximum momentum transfer, the experiment gets to $x_{\text{max}} = 0.932$ for the nominal beam energy of 150 GeV. The extrapolation from $x_{\text{max}}$ to $x = 1$ accounts for about 13% of the $a_H^{\text{LO}}$ integral. Uncertainty in the extrapolation can be tested by varying the fit parameters within their uncertainties, or using alternative functional forms, as Padé approximants. Second (or third) order polynomials in $t$ can be used for the fit within the acceptance but cannot be extrapolated to the full $x$ range, due to their diverging limit for $x \to 1$ (that is for $t \to -\infty$). A small systematic error, below 0.2%, has been estimated by comparing the extrapolated result to the original reference. The origin is understood to come from the technical difficulty of the template fits with the correlated parameters. An improved fit method has already been developed, so this error could be even reduced.

10.2.3 Average beam energy

The average beam energy scale of the M2 beam is known at a level of about 1%. In the experiment the incoming muon energy can be measured event-by-event by the BMS spectrometer to $\sim 0.9\%$. Given the high muon beam intensity this can provide an extremely precise statistical determination of the average beam energy, allowing to closely monitor the time variations. However it cannot assess the systematic uncertainty of the average beam energy scale, which needs to be controlled by a physics process. The requested precision is challenging, since a systematic miscalibration of the average beam energy of only 5 MeV is seen to distort the LO $\mu - e$ cross section by almost a factor $10^{-5}$ in the signal region.

Two methods have been tested to calibrate the beam energy scale, both relying on the kinematics of the elastic $\mu - e$ scattering.

The first method uses the inverse kinematic method, described in Sec. 9.3. It is thought to work at best when the particle identification is achieved, possibly requiring the ECAL information. The best determination of the average beam energy $\langle E_\mu \rangle$ is obtained selecting events in the kinematic region where the uncertainties induced by the multiple Coulomb scattering are lower. Such condition occurs when the angle $\theta_e$ is $1.5 \leq \theta_e \leq 5.0$ mrad, where the electron and muon energies are similar. This can be clearly seen in Fig. 27-left, where the selected region is shown. The distribution of $E_\mu$ in such angular range is reported in Fig. 27-right, together with the fit to a gaussian curve. The distribution width $\sigma_{1.5-5.0} \approx 6$ GeV is mainly due to the original beam momentum spread ($\approx 5.6$ GeV), while the combined effect of the tracker resolution and multiple scattering, which adds in quadrature, amounts to about 2.5 GeV.

The second method, previously used by the NA7 experiment [23], does not require particle identification, provided that clean elastic events are selected. In this case the relevant observable is the average angle of the two tracks for events where the angular difference is below an appropriate cut. In fact the sensitivity is maximum around the equal-angle condition, where both $e$ and $\mu$ are scattered at $\theta \approx \sqrt{2m_e/P_{\text{beam}}}$. This method is robust against transverse misalignments, which give a null effect at first order. Similarly event-by-event variations of the incoming muon direction do not affect the average angle. Instead the method is crucially dependent on the precise longitudinal positioning of the detector layers within each tracking station, as any systematic shift in the average angle translates into a shift of the beam energy determination. Many experimental effects tend to smear the average angle distribution and have been considered to assess the performance of the method. The beam energy spread significantly widens the observed distribution and is one of the most serious effects to be taken into account. The intrinsic detector resolution depends both on the Si sensor features and on the lever arm of the tracking station. The multiple scattering on the target and detector material is less relevant as in the selected region the two tracks have energies around $E_{\text{beam}}/2 = 75$ GeV.

Both methods allow to reach the statistical accuracy of $1 - 2$ MeV, which requires to collect about $10^7$
selected events. Thanks to the high beam intensity such number can be obtained in a few days of data taking for each single station.

An important validation could be obtained by applying these methods to several consecutive stations. In this way one could probe the expected ionization energy loss of the beam muons as they cross the detector targets. The expectation is of about 5 MeV for one target, i.e. the right scale we need to be sensitive to. Each detector station will provide a statistically independent calibration, hence the individual results could be averaged, further reducing the collection time to a few hours.

Uncorrelated systematics would also be reduced, most notably those related to the longitudinal positioning of individual planes within each station, and to other metrological parameters, such as, for example, the target thickness.

### 10.2.4 Beam energy spread

The M2 beam energy spread directly affects the differential $\mu e$ cross section up to a relative change exceeding $10^{-4}$ in the signal region, therefore it needs to be accurately measured. By integrating enough data the limit precision on the measured beam energy profile will be determined by the BMS resolution, discussed in 5.2. Assuming to know the spectrometer resolution to 20 – 25% the impact on the final $\alpha^{HLO}_\mu$ is estimated to be negligible.

### 10.2.5 Multiple scattering

As already mentioned, multiple scattering is one of the most important effects to be controlled, in particular for electrons at large $\theta_e$ (corresponding to low energy). Section 11.1 describes the results of test-beam measurements compared to the expectations of GEANT4 Monte Carlo simulation. Although the agreement with GEANT4 is quite good in describing the core of the distributions, the requested precision is higher. This leaves the only possibility to use the MUonE data itself to precisely tune the MC prediction in the normalization region, and Monte Carlo or physics-inspired models to determine the energy dependence of the effect, extending to the signal region. The expected multiple scattering for high energies could be tested by in-situ measurements of the deflection of beam muons in small dedicated runs spanning the range of accessible energies at the M2 beam line.

If measurements with electron beams in the low energy region turn out to be necessary, we envisage to take data at the East Hall beams at CERN, in parallel with MUonE taking data, using one or two stations of the MUonE apparatus.
11 Testbeams

11.1 2017 testbeam

A precise knowledge of the Coulomb Multiple Scattering (MSC in the following) is required to account for the effects of the targets and to describe the response of the detector. The acceptable experimental uncertainties due to MSC effects, must be known to the percent level. We decided therefore to measure and test the precision achievable with the MSC models used by GEANT4 describing the experimental data. To this purpose a test beam has been performed in 2017 at the beam line H8 of CERN using 12 and 20 GeV electrons, colliding on carbon targets of different thickness. The detector has been set up by the UA9 collaboration [56]. It allows precise tracking thanks to the high-resolution of their Si micro-strip detectors.

The apparatus is described in detail in [56, 57]. A tracking module measures the hit position with a resolution of 7 µm. The corresponding intrinsic angular resolution is therefore of the order of 0.01 mrad. Targets were made of Isostatic Graphite, with density 1.83 g/cm$^3$.

Running the detector for a week, large data samples were collected, each of about $1.5 \times 10^7$, with electrons of 12 GeV and 20 GeV through targets of 8 mm and 20 mm. To align the tracking planes, runs with high energetic pions without target were taken (alignment runs).

For studying the MSC we selected events with only one incoming electron ($q_{in}$) and only one outgoing electron ($q_{out}$). The distribution of the angular deflections ($\Delta \theta = q_{out} - q_{in}$) can be represented as the convolution of the detector resolution function $f_d(\theta)$ with the target scattering angles distribution function $f_t(\theta)$:

$$f(\theta) = (f_d \ast f_t)(\theta) \quad (7)$$

$f_d(\theta)$ was measured with the alignment runs. $f_t(\theta)$ had been determined with GEANT4 simulations. Corrections have been applied to account for effects of energy loss and multiple scattering in the detector. The electron energy loss of about 5%, occurring while crossing the upstream detector, before hitting the target, has been taken into account.

Figures 30 show an example of the measured scattering angles compared to the results of the simulations (x and y projections). The agreement of the distributions in the core (the central region accounting for 90% of the integral), looks satisfactory. In the tails the agreement cannot be assessed precisely enough, because of the limited statistics, two to three orders magnitude less than that at the peak values.

The experimental data were also fitted with analytic models of the angular distributions. Examples are shown in Fig. 31. The function consists of a Gaussian and a Students $t$-distributions. Its mathematical expression can be found in [57]. The successful result in fitting the data with the mathematical model is remarkable.

We measured the detector response resolution function, as a function of the beam energy in the range from 12 to 180 GeV. The multiple scattering resolution function (scaling as $1/E$) added in quadrature to the intrinsic resolution results in about 0.018 mrad at the highest energy.
Fig. 30: Multiple scattering of 12 GeV electrons on 20 mm C target. First raw: comparisons of measurements and GEANT4 (10.4.4 opt 4) results. Second raw: fractional differences.

Fig. 31: Fit of the multiple scattering analytic angular distribution to data. In the first raw: 12 GeV electrons colliding on 8 mm C target. In the second raw: 20 GeV electrons colliding on 8 mm C target.
11.2 2018 testbeam

The 2018 test run was performed in the EHN2 downstream experimental area, exploiting the high-intensity, high-emittance and high-momentum muons that result either from the decays of the damped pion beam used by COMPASS or directly from the M2 muon beam, which is occasionally used for the COMPASS calibrations.

The apparatus  The experimental setup is shown in Fig. 32 left. Its core was the tracking system, which consisted of sixteen $9.5 \times 9.5 \times 0.041 \text{ cm}^3$ single-sided silicon microstrip sensors manufactured by Hamamatsu on high resistivity substrates. Each sensor features 384 channels, with $121 \mu \text{m}$ readout pitch and the floating strip scheme; the analog readout allows an intrinsic spatial resolution of $\sim 35 \mu \text{m}$ [58]. A homogeneous ECAL with $\sim 8 \times 8 \text{ cm}^2$ front section and PMT-based readout allowed the measurement of the output electron energy. Its calibration was performed at the CERN East Hall T9 beamline.

Several configurations, which differ one from the other by the number of 8 mm thick graphite targets, by the tracking system lever arms, by the number of $\pi/4$ stereo layers and by the calorimeter type, were tested. In the final setup, 6 (10) silicon planes were placed at the input (output) stage of a single target and a BGO calorimeter was used (Fig. 32 right). The output tracking stage was $\sim 1.3 \text{ m}$ long, which resulted in a $\sim 35 \text{ mrad}$ angular acceptance upper limit. Further details can be found in [59].

The analysis  During the $\sim 7.5$ month long run, $\sim 1.4 \times 10^9$ events were collected, with several geometrical configurations. The final analysis was performed with $\sim 35 \times 10^6$ (i.e. $\sim 24\%$) events with single or double input particles, and corresponding to the final detector configuration with the BGO calorimeter.

The main goal of the testbeam was to observe the $\mu - e$ elastic scattering, achieving a maximum possible track reconstruction efficiency and precision, and to determine the correlation between the scattered angles of the outcomeing muons and electrons.

The track reconstruction procedure used in the testbeam data analysis is very similar to the one described in Sec. 8.1. The first stage consists of constructing the pairs from all the combinations of hits in $x$, $y$ and stereo layers, separately in the $x$-$z$ and $y$-$z$ projections. For each pair of hits two-dimensional (2-d) lines in $x$-$z$ or $y$-$z$ have been constructed, collecting all the hits within a certain window in the $x$-$y$ plane, and accepting only the 2-d line candidates with at least three hits, as shown in Fig. 33.

A simple clusterization of adjacent hits has been applied to reduce the number of cloned tracks and to improve the reconstruction of two close tracks. No unique combinations of hits forming 2-d lines have been imposed at this stage. The robust fit to the combinations of the hits constituting 2-d lines in $x$-$z$ or $y$-$z$ projections has been used assuming $30 \mu \text{m}$ uncertainty for the $x$ and $y$ hit position, with removal of outlier hits. At the second stage, all the fitted $x$-$z$ and $y$-$z$ lines have been paired to create three-dimensinal...
(3-d) lines requiring a maximum number of hits shared in stereo planes. All 3-d lines have been finally fitted using a least square method.

The track based alignment has been performed aligning first the stereo layers and then re-aligning all the x and y layers. The alignment procedure was based on collecting good quality tracks with at least 10 hits, and minimizing the residuals of every station one-by-one using the iterative least square method based procedure.

The final tracking procedure was based on constructing the 3-d track candidates from all combinations of line segments in x-z and y-z projections, and then fitting such 3-d track candidates using a least square method. All hits along initial trajectory have been collected within a certain window and the outlier hits have been removed within 5 $\sigma$. The convergence was achieved if there were no change in fitted parameters. Such reconstructed 3-d tracks have been sorted according to the number of hits and minimum $\chi^2$ per degree of freedom. To finally accept the track at least three hits in x-z and three hits in y-z projections have been required. From multiple track candidates with shared hits the candidate with lower $\chi^2$/ndf have been taken.

The reconstructed tracks have been used to search for events of $\mu - e$ topology. About $7.1 \times 10^3$ candidates have been selected from the data sample considered, imposing a loose requirements on the difference in x and y positions of the combinations of two tracks at the target z position. Then the third track incoming to the target close to the intersection point has been found. For these three tracks initially compatible with $\mu - e$ scattering, a global fit has been performed, based on the least square method including the $\chi^2$ contribution from tracks and the $\chi^2$ contribution from the single optimal point of intersection in the target. Finally, the scattering angles of the outgoing muon and electron have been determined, to perform the study of the correlation between these angles, with a loose cut on the acoplanarity of the event. In Fig. 34 the angular correlation is shown (left), together with the (energy,angle) correlation for the electron (right). The Monte Carlo is reproducing quite well the observed features, but the analysis is still under way and what is shown here is very preliminary.
Fig. 33: Reconstruction of two dimensional tracks of the elastic events candidates. The empty circles represent hits not included in the reconstruction.

Fig. 34: Left: the $\theta_e$ vs $\theta_\mu$ correlation and right: The $\theta_e$ vs the $E_e$ as measured in the calorimeter.
12 Theory

In this Section we will discuss the present status and the future prospects of the theoretical prediction for the $\mu e \rightarrow \mu e$ scattering cross section. The radiative corrections (RC) to this process will be required at the highest levels of theoretical precision in order to match the foreseen experimental accuracy. They represent a primary potential source of systematic uncertainty which must be carefully investigated.

The Feynman diagram contributing to the leading order (LO) QED amplitude of the $\mu e \rightarrow \mu e$ scattering process is represented by the $t$-channel exchange of a photon between the electron and the muon. The corresponding differential cross-section results to be:

$$\frac{d\sigma_0}{dt} = 4\pi\alpha^2 \frac{(M^2 + m^2)^2 - s u + t^2/2}{t^2 \lambda(s, M^2, m^2)}, \tag{8}$$

where $m$ and $M$ are the electron and muon mass respectively, $s, t$ and $u$ are the usual Mandelstam variables, $\alpha$ is the fine-structure constant, and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen function. Equation (8) is valid both for positive and negative muons, both options being possible for the muon beams available at CERN’s North Area.

Next-to-Leading order (NLO) QED corrections to the differential cross section were computed long time ago by applying some approximations in [60, 61, 62, 63, 64, 65, 66] and more recently revisited [67]. The complete calculation of the full set of NLO QED corrections and of NLO electroweak corrections with the development of a fully differential Monte Carlo code was completed [49]. The NLO corrections to $\mu e$ scattering are discussed in Section (12.1).

The knowledge of QED corrections at next-to-next-to-leading order (NNLO), will be crucial to adequately interfacing the high-precision data of MUonE. These are not known. Due to the extensive and systematic study carried on for the Bhabha scattering at two loops, some of the results already obtained [68, 69, 70] and results for $t\bar{t}$ production in QCD [71, 72], can be applied to $\mu e$ scattering as well. Even if the NNLO QED corrections have not been systematically evaluated, the hierarchical structure of the logarithmic contributions is well known [73]. In a general process, as is natural for a perturbative expansion, one has that the QED contributions can be organized in a series containing increasing powers of the coupling constant $\alpha$:

$$\sigma = \sigma^{(0)} + \frac{\alpha}{\pi} \sigma^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 \sigma^{(3)} + \cdots + O\left(\frac{\alpha}{\pi}\right)^n \sigma^{(n)} + \ldots \tag{9}$$

Each of the $\sigma^{(n)}$ contains the contributions of the corresponding order in $\alpha$, is accompanied by a coefficient and is multiplied either by a constant or by a number of logarithms of collinear origin, denoted by $L$ as well as of logarithms of infrared (IR) origin, denoted by $\ell$. The general term is of the form $c_{n,\eta} \left(\frac{\alpha}{\pi}\right)^n L^n \ell^\varepsilon$ with $0 \leq \eta, \varepsilon \leq n$ (The maximal power of $\eta$ and $\varepsilon$ being 1 for each power of $\alpha$). If the required accuracy of the cross section is of the $\mathcal{O}(10^{-5})$, the evaluation of the perturbative series has to be extended to take into account all the terms contributing to the aimed accuracy. Within the range of the accessible energies for the MUonE experiment the value of $\alpha/\pi$ can approximately estimated to be $\frac{\alpha}{\pi} \approx \frac{1}{337} \approx 2.5 \cdot 10^{-3}$ correspondingly $\left(\frac{\alpha}{\pi}\right)^2 \approx 5.0 \cdot 10^{-6}$ and $\left(\frac{\alpha}{\pi}\right)^3 \approx 1.5 \cdot 10^{-8}$. By taking the coefficients $c_{n,\eta}$ to be of the $\mathcal{O}(10)$ and by assuming the value of $t$ at the peak in the Fig. 1, $t \approx -0.108 \text{ GeV}^2$, being the large logarithm $L = \ln \frac{\Lambda^2}{m^2} \approx 10$ it follows that:

$$c_{1,1} \left(\frac{\alpha}{\pi}\right) L \approx 0.2, \quad c_{2,2} \left(\frac{\alpha}{\pi}\right)^2 L^2 \approx 5 \cdot 10^{-3}, \quad c_{2,1} \left(\frac{\alpha}{\pi}\right)^2 L \approx 5 \cdot 10^{-4}, \quad c_{2,0} \left(\frac{\alpha}{\pi}\right)^2 \approx 5 \cdot 10^{-5}. \tag{10}$$

Furthermore it turns out that:

$$c_{3,3} \left(\frac{\alpha}{\pi}\right)^3 L^3 \approx 1.5 \cdot 10^{-4}, \quad c_{3,2} \left(\frac{\alpha}{\pi}\right)^3 L^2 \approx 1.5 \cdot 10^{-5}, \quad c_{3,1} \left(\frac{\alpha}{\pi}\right)^3 L \approx 1.5 \cdot 10^{-6}. \tag{11}$$
Since each term of the perturbative series contains a coefficient $c_{n,\eta}$ that we assume to be of the $\mathcal{O}(10)$, to reach a precision of the $\mathcal{O}(10^{-5})$, all the terms to the second order in $\alpha$ have to be taken into account together with the leading and next-to-leading logarithmic terms to the third order. Therefore also constant terms should be evaluated to the second order and by taking the third order leading and next-to-leading logarithmic ones we estimate that an accuracy of $1 \cdot 10^{-5}$ can be obtained.

A first step towards the calculation of the full NNLO QED corrections to $\mu e$ scattering was taken in [74, 75, 76], where the master integrals for the two-loop planar and non-planar four-point Feynman diagrams were computed. These integrals were calculated setting the electron mass to zero, while retaining full dependence on the muon one. The extraction of the leading electron mass effects from the massless $\mu e$ scattering amplitudes has been recently addressed in [77] (see also [78, 79, 80]). The NNLO QED corrections to $\mu e$ scattering are discussed in Section (12.2). The hadronic corrections to $\mu e$ scattering have been computed very recently [28]. They are discussed in Section (12.3). Future prospects for the $\mu e$ scattering theoretical prediction are discussed in the final Section (12.4).

12.1 NLO corrections

In this subsection we briefly report of the calculation of the NLO RC in the Standard Model, which are available in a fully exclusive Monte Carlo event generator. The Monte Carlo implementation of NLO RC is further discussed in 7.2 and extensively detailed in Ref. [49], to which we refer for all the details of the calculation, for their implementation and for references to the relevant literature.

At NLO, all the relevant Feynman diagrams for one-loop virtual and one-photon emission corrections at $\mathcal{O}(\alpha)$ must be accounted for. Here we only mention a few general results and features: i) at LO, $Z$ exchange diagrams must be included because they affect the distributions at the level of $\mathcal{O}(10^{-5})$; ii) at NLO, EW corrections are, as expected, dominated, by far, by QED, and the purely weak part, although recently computed in [49], can be safely neglected for the sake of speed in numerical calculations; iii) up to NLO, the muon and electron masses must be kept everywhere, i.e. neglecting finite mass effects could result in a large theoretical error; iv) the impact of NLO RC is non trivial and depends on the observable under consideration; v) experimental selection criteria change the size of QED NLO corrections and the interplay between experimental cuts and RC must be studied in detail to reduce systematic uncertainties of both experimental and theoretical origin. In Fig. 35 (from [49], where also the setups in the labels are described), we show the impact of QED NLO RC for the process $\mu^+ e^- \rightarrow \mu^+ e^-$ by considering four different experimental setups. In the upper plots of Fig. 35 the $t$ Mandelstam variables as calculated on the electron (left, $t_{ee}$) or muon (right, $t_{\mu\mu}$) current are considered. We notice that NLO RCs lie in the 10-40% range, increasing towards the edge of the distributions where the $|t|_{ee,\mu\mu}$ are larger. In the lower plots, the scattered electron ($\theta_e$) and muon ($\theta_\mu$) angles are shown. We notice that the QED NLO RC on $\theta_e$ can become quite sizable, but an elasticity cut reduces their impact: the large effects in the absence of the cut can be ascribed to radiative events $\mu^+ e^- \rightarrow \mu^+ e^- \gamma$, when large-angle (soft) electrons are scattered in the low $q_e$ region due to the emission of an hard photon.

12.2 NNLO QED corrections

The determination of the QED cross section at NNLO accuracy demands for three types of contributions: i) double-virtual terms, due to virtual two-loop four-point diagrams, for the process $\mu e \rightarrow \mu e$; ii) real-virtual terms, due to one-loop five-point diagrams, for the process $\mu e \rightarrow \mu e \gamma$; iii) double-real terms, due to tree-level six-point diagrams, for the process $\mu e \rightarrow \mu e \gamma \gamma$. These contributions must be combined into the squared amplitudes and integrated over the different phase-space, respectively for two-, three-, and four-body final states. The final integration is carried out numerically by Monte Carlo methods, and since the individual contributions, upon the renormalization of ultraviolet (UV) divergencies, may still

---

1We remark that $t_{ee}$ and $t_{\mu\mu}$ do not coincide beyond LO.
suffer from infrared (IR) singularities, the integration procedure must be supplied with subtraction terms to regularise them.

Until recently, none of the above ingredients was available in the literature. Concerning the double-virtual terms, in three steps [74, 76, 75, 81], all functions needed for the determination of the two-loop scattering amplitude for $\mu e \rightarrow \mu e$, coming from planar and non-planar two-loop diagrams, were computed. In particular, given the hierarchy between the electron mass $m$ and the muon mass $M$, $m/M \approx 5 \cdot 10^{-3}$, these functions were computed in the approximation $m = 0$. A set of about 120 basic integrals was identified and evaluated analytically, through the differential equations method [82, 83, 84, 85, 86] by means of the Magnus exponential matrix [87]. They admit a series representation around $d = 4$ dimensions, whose coefficients are combinations of generalized polylogarithms (GPLs) [88, 89, 90, 91]. The extraction of the leading electron mass effects from the massless $\mu e$ scattering amplitudes has been recently addressed in [77] (see also [78, 79, 80]). The impact of the electron mass on the two-loop amplitudes can be investigated by means of a very efficient decomposition in terms of basic functions based on rational reconstruction over finite fields [92] and their purely numerically integration, by using methods which have been developed recently [76, 81, 93, 94].

To NNLO accuracy electron pair production effects should be also considered and taken into account. One and two-loop diagrams with vacuum polarization insertions in the photon propagator were considered some time ago for the case of the Bhabha scattering in the massless limit with the 0.1% accuracy [95, 96]. Pair production amplitudes show, in fact, potentially dangerous unrenormalizable singularities of the form $(\alpha/\pi)^2 L^3$ which can be only cancelled by real pair production diagrams at the same order. The resummation of pair production contributions can be also shown to take place to all orders of the pertur-

**Fig. 35:** Impact of QED NLO radiative corrections on the $t_{ee}$ and $t_{\mu\mu}$ Mandelstam variables and $\theta_\mu$ and $\theta_\mu$ angles. The upper panels of the plots show the absolute differential cross-sections and the lower panels represent the ratio of the QED NLO over the LO distribution. See text for details.
bative expansion [97].

The next steps will be devoted to the actual determination of the UV renormalised two-loop amplitude. Also the evaluation of the real-virtual contributions are under investigation and will be tackled by means of the adaptive integrand decomposition technique [98, 99] developed for the one-loop amplitudes. For the study of the IR singularities, needed for the identification of a proper subtraction scheme, concepts and ideas developed within the Soft Collinear Effective Theory (SCET) approach to QCD, along the line of [100] will be employed.

Further radiative terms that should be also evaluated on the benchmark of the 10 ppm are those of heavy pair production $\mu\mu$, $pp$, and $tt$.

12.3 NNLO hadronic corrections

The hadronic corrections to $\mu e$ scattering have been computed very recently [28]. While at NLO these corrections are simply proportional to the product of the LO QED predictions and the hadronic part of the vacuum polarization, at NNLO their evaluation is complicated by the presence of non-factorizable two-loop diagrams. In [28] their calculation was presented by using the dispersive approach on the hadronic $e^+e^-$ annihilation, timelike, data $^2$. Recently, by taking advantage of the hyperspherical integration method, it was shown that these non-factorizable diagrams can also be calculated by employing the hadronic vacuum polarization in the spacelike region without using timelike data [27].

The results of [28] show that corrections to the differential scattering cross sections with respect to $t\varepsilon$, the square of the difference of the initial and final electron momenta, are of the order $10^{-4}$–$10^{-5}$ for most of the kinematic region spanned by $t\varepsilon$. These corrections will therefore play a crucial role in the analysis of MUonE’s data. The relative theoretical uncertainty of these predictions, induced by the experimental error of the hadronic $e^+e^-$ annihilation data, is estimated to be about 1% or less. It is therefore well below the precision expected at MUonE.

12.4 Future developments

MUonE’s extremely-high accuracy demands for the resummation of classes of radiative corrections which are potentially enhanced by large logarithms. As discussed before radiative corrections can be organized in a power series of $\frac{\alpha}{\pi}$ times powers of $L = \log \left(-t/m^2\right)$ and $\ell = -2\log(2\Delta\alpha/\sqrt{s})$, where $L$ and $\ell$ are the collinear and IR (or soft) logarithms respectively. We refer here only to the case of radiation from the electron leg, which is numerically the most relevant one. In the definition of $\ell$, $\Delta\alpha$ is related to the maximum energy allowed for the radiation, which is in general a function of the applied cuts and of the particular observable under consideration. Thanks to the factorization theorems of the soft and collinear radiation, by the resummation techniques one can exponentiate the leading logarithmic corrections to all orders in $\alpha$ with terms of the form $\alpha^n(L-1)^n\ell^m$. A general framework for implementing numerically the leading logarithmic resummation is provided either by the Parton Shower (PS) approach or by the YFS formalism. These methods can be improved to include consistently NLO RC as well since the IR logarithm $\ell$ is correctly resummed to all orders in $\alpha$ by construction, the theoretical error formally starts with contributions $\alpha^2L$ not enhanced by any IR logarithm $\ell$ [106]. Going one step further and by following the same line of reasoning, when the complete NNLO RC will be available and a NNLO matched PS (or $\mathcal{O}(\alpha^2)$ YFS) will be implemented, according to past experience in Bhabha scattering at $\mathcal{O}(\alpha^2L)$, we expect that the error due to missing corrections will start at order $\alpha^3L^2$, not enhanced by any IR logarithm $\ell$; i.e., we expect that the theoretical error on any distribution will be of the order of:

$$\left(\frac{\alpha}{\pi}\right)^3(L)^2 \times k' \simeq 10^{-6} \times k', \quad (12)$$

$^2$This approach, originally based on [13], has also been employed to calculate the hadronic corrections to muon decay [101, 102] and Bhabha scattering [103, 104, 105].
with $k'$ being of $\mathcal{O}(1)$.

We conclude that the resummation of the leading logarithmic QED corrections, combined with the exact NNLO QED and the hadronic corrections, is expected to describe any relevant $\mu e$ scattering distribution with an ultimate theoretical precision of $\mathcal{O}(10^{-6})$.

Finally, a potential source of theoretical uncertainty may arise also from the fact that the initial-state electrons are bound rather than free. Besides the off-shell effects due to the finite binding energy, also the electron momentum distribution must be considered. As our target material, Beryllium, is a metal, the electrons are mainly delocalized and these effects are expected to be almost negligible. However, the highly energetic incident muons can also scatter off core valence electrons. For these bound state effects can be more relevant. Work is in progress to quantitatively estimate and assess these effects.
13 Pilot Run in 2021

The MUonE collaboration requests 3 weeks of the M2 beam, at the end of the running period of 2021. The plan is to operate the MUonE detector once the running of COMPASS will be over. The pilot run aims to achieve several goals, as explained in the following.

13.1 Apparatus, location and beam

The apparatus will consist of 2 stations and is sketched in Fig. 36

Each station consists of a thin target and six CMS tracking modules described in Sect. 4.1.2. Six other tracking modules will be installed upstream the detector for tracking the incoming muons. In total the apparatus will consist of 18 tracking modules, and can be considered as a prototype of the final setup, which in principle will be an extension to 40 stations, of the two operating in 2021. The apparatus will be installed in the space available upstream COMPASS. The characteristics of the M2 beam, in terms of the required intensity and focusing, are described in Section 4.

13.2 Pilot run motivations

Aim of the Pilot Run is to demonstrate the validity of the design and the operation of the MUonE project. The main objectives are:

- Confirm the system engineering, i.e. assembly, mounting and cooling;
- Assessing the detector counting rate capability;
- Checking the signal integrity in the process of data transfer for the DAQ;
- Proving the validity of the trigger-less operation mode;
- Evaluate the FPGA real-time processing, to distinguish muons passing stations without interacting, to demonstrate the ability to identify and reconstruct $\mu - e$ events in real time.
- Testing the procedure for the alignment of the sensors: tools and methodology;
- Monitoring mechanical and thermal stability;
- estimating the systematic error budget in the operating conditions

13.3 Activities in 2020-2021

In order to prepare the pilot run, many activities are planned from now to the beginning of 2021.

- The sensors and front-end electronics will be tested by the CMS tracker group, in the frame of the activity for their Upgrade-II. The 36 Si strip sensors (24 to equip the two stations and the 12 for the upstream tracking elements) will be delivered to MUonE by the beginning of 2021.
– The DAQ system will be prepared in advance to be ready to take data with good efficiency.

– The mechanical support must guarantee stability and precise knowledge of the trackers positions within the stations. The high precision assembly of the tracking elements with the support will be tested in the laboratories. The method to monitor the distances between the Si trackers elements, based on laser interferometry, will be studied and tested as well.

– Assuming we can run for three weeks, the elastic events produced in the detector acceptance, in the energy range of $E_e > 1$ GeV, is expected to be of the order of few $10^8$, depending on the duty cycle of the beam and the reconstruction and selection efficiencies, for which study a full simulation is being prepared.

With the above statistics, a preliminary measurement of the differential elastic cross section should be possible. The Pilot Run will be fundamental to evaluate other aspects of the analysis, which would be an important step towards the demonstration of the measurement feasibility.
14 Tentative Schedule

Assuming that the Pilot Run will validate the detector and its operation with the 2 stations, we will add stations in steps.

The plan, in agreement with the CMS tracker collaboration, is that they could provide the tracking stations to MUonE with the following time profile:

- 50% of stations delivered by spring 2022 (20 stations)
- 50% by end of 2022 (20 stations)

Within the assumption above, the MUonE collaboration will request therefore M2 beam time to start data collection with half the apparatus in 2022. Then, with the complete apparatus in 2023 and 2024, we plan to request the necessary beam time to accumulate a statistics as large as possible.

Assuming the present maximum intensity of the M2 beam at 150 GeV, MUonE would need $\sim 4 \times 10^7$ s total running time to achieve a statistical error of $\sim 0.3\%$.

15 Collaboration and Cost

This section is work-in-progress.

The Collaboration  The MUonE collaboration is still under construction. It has two components, because theory and experiment need both a large effort to get the necessary precision in the calculations as well as in the measurement.

The current status is summarized in Table 1 and Table 2, showing also the field of activity and interest in which groups are involved. The current manpower amounts to: 35 senior, 7 post-doc, 4 PhD, 2 graduate students on the experimental side, and 11 senior, 8 post-doc and 1 PhD on the theory side. We are in contact with several groups at CERN, especially for the beam optimization, for survey, alignment, thermalization studies.

<table>
<thead>
<tr>
<th>Group</th>
<th>Researchers</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bologna</td>
<td>6</td>
<td>Calorimeter, simulation, systematics</td>
</tr>
<tr>
<td>Firenze</td>
<td>1</td>
<td>Tracker</td>
</tr>
<tr>
<td>Imperial College</td>
<td>5</td>
<td>Electronics, DAQ</td>
</tr>
<tr>
<td>Krakow</td>
<td>5</td>
<td>DAQ, tracking algorithms, analysis</td>
</tr>
<tr>
<td>Milano-Bicocca</td>
<td>3</td>
<td>Mechanics, simulation, analysis</td>
</tr>
<tr>
<td>Budker Institute</td>
<td>3</td>
<td>Simulation, systematics, analysis</td>
</tr>
<tr>
<td>Padova</td>
<td>7</td>
<td>Calorimeter, simulation, analysis</td>
</tr>
<tr>
<td>Pisa</td>
<td>9</td>
<td>Tracker, mechanics, analysis</td>
</tr>
<tr>
<td>Trieste</td>
<td>2</td>
<td>Mechanics</td>
</tr>
<tr>
<td>Shangai Jiao Teng U.</td>
<td>2</td>
<td>Simulation, analysis</td>
</tr>
<tr>
<td>University of Liverpool</td>
<td>2</td>
<td>Mechanics, alignment, simulation</td>
</tr>
<tr>
<td>University of Virginia</td>
<td>3</td>
<td>Calorimeter, simulation, systematics</td>
</tr>
<tr>
<td>CERN</td>
<td>see text</td>
<td>Survey, alignment, thermalization, beam</td>
</tr>
</tbody>
</table>

Cost estimate  At the moment only a very rough cost estimate is possible, because it depends largely from which solutions we will adopt for the different aspects (e.g. mechanics and calorimetry). In particular, the estimate for the calorimeter cost is not available at this time, considering that we are exploring the possibility of using spare crystals from CMS, and the associated electronics. We are evaluating if an analogous strategy as adopted for the tracker, would be applicable for the calorimeter.
Table 2: Current manpower for the theory

<table>
<thead>
<tr>
<th>Group</th>
<th>Researchers</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Padova</td>
<td>11</td>
<td>NNLO QED, hadronic calculations</td>
</tr>
<tr>
<td>Parma</td>
<td>1</td>
<td>NNLO calculations, resummation</td>
</tr>
<tr>
<td>Pavia</td>
<td>5</td>
<td>Higher order calculations, theory MC development</td>
</tr>
<tr>
<td>PSI</td>
<td>2</td>
<td>NNLO calculations, resummation</td>
</tr>
<tr>
<td>Dublin</td>
<td>1</td>
<td>Lattice QCD</td>
</tr>
</tbody>
</table>

A very tentative estimate is summarized in in Table 3. The final estimate will be made on the basis of the final choices and drawings.

Table 3: Preliminary cost estimates

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost estimate (kEuro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si sensors and electronics</td>
<td>750 (+ 15 pilot run)</td>
</tr>
<tr>
<td>DAQ</td>
<td>140</td>
</tr>
<tr>
<td>Mechanics (without survey system)</td>
<td>120</td>
</tr>
<tr>
<td>Calorimeter (PbWO₄)</td>
<td>See text</td>
</tr>
<tr>
<td>Muon-filter and active chambers</td>
<td>Recycling existent material</td>
</tr>
<tr>
<td>Computing</td>
<td>150 – 500</td>
</tr>
<tr>
<td>Targets</td>
<td>40</td>
</tr>
<tr>
<td>Service and infrastructure</td>
<td>350 – 450</td>
</tr>
</tbody>
</table>

16 Summary and overall timeline

The design and the aim of the MUonE detector have been described. It includes the silicon trackers as developed for the CMS upgrade-II, a calorimeter (still under optimization with simulation), and possibly a muon filter. The MUonE Collaboration requests three weeks for a Pilot run at the end of 2021 to validate these ideas, and to try to do a measurement provided the statistics will allow it. If successful, half of the detector will be assembled at the beginning of 2022 and completed at the end of the 2022. The full detector could therefore run in the Run 3 period, to collect the maximum of the statistics, provided all the milestones from the Pilot Run will have been satisfied.
17 APPENDIX

17.1 The Tracker

Table 4 summarizes technology, geometrical, resolution and material information for the detectors that have been considered. Monolithic Active Pixels (MAPS) detector are attractive for the small thickness, however they generally present slow timing ($\approx 1\mu s$) and low readout rate (an exception is the MuPix detector being produced for the Mu3e experiment).

Table 4: Characteristics of silicon detector developed for some high energy physics experiments (current or with upgrade in preparation). The ALICE upgrade quoted figures are for the inner tracker. Belle 2 strips features strips on both sides of the detector, with orthogonal directions. CMS upgrade 2S modules figures are given for two modules with orthogonal readout. ADD REFERENCES FOR EACH EXPERIMENT.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>technology</th>
<th>length x [cm]</th>
<th>length y [cm]</th>
<th>pitch/pixel [µm]</th>
<th>resolution [µm]</th>
<th>thickness [x/X₀]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALICE upg</td>
<td>MAPS</td>
<td>27</td>
<td>1.5</td>
<td>30</td>
<td>2</td>
<td>0.4 %</td>
</tr>
<tr>
<td>Belle 2 strip</td>
<td>Hybrid/strip</td>
<td>12.5</td>
<td>4</td>
<td>50-160</td>
<td>10-24</td>
<td>0.7 %</td>
</tr>
<tr>
<td>CMS upg 2S</td>
<td>Hybrid/strip</td>
<td>10</td>
<td>10</td>
<td>90</td>
<td>18</td>
<td>0.6 %</td>
</tr>
<tr>
<td>CMS upg pix</td>
<td>Hybrid/pixels</td>
<td>3.7</td>
<td>4.4</td>
<td>50</td>
<td>7</td>
<td>2.0 %</td>
</tr>
<tr>
<td>Mimosa 26</td>
<td>MAPS</td>
<td>1.1</td>
<td>2.1</td>
<td>18.4</td>
<td>3.2</td>
<td>0.1 %</td>
</tr>
<tr>
<td>Mu3e pix</td>
<td>MAPS</td>
<td>2</td>
<td>2</td>
<td>80</td>
<td>38</td>
<td>0.1 %</td>
</tr>
<tr>
<td>LHCb pix</td>
<td>Hybrid/pixels</td>
<td>4.2</td>
<td>1.4</td>
<td>55</td>
<td>12</td>
<td>0.9 %</td>
</tr>
</tbody>
</table>

17.2 The ECAL

Table 5: Properties of some scintillating crystals. \( \tau_{fast} \) and \( \tau_{slow} \) are the lifetimes of the fast and slow component, respectively, of the emitted scintillation light. \( \lambda_{MAX} \) is the wavelength of the maximum of the emission spectrum.

<table>
<thead>
<tr>
<th>Material</th>
<th>density (g/cm(^3))</th>
<th>( X₀ ) (cm)</th>
<th>Light yield (phot/MeV)</th>
<th>( \tau_{fast} ) (ns)</th>
<th>( \tau_{slow} ) (ns)</th>
<th>( \lambda_{MAX} ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PbWO(_4)</td>
<td>8.28</td>
<td>0.89</td>
<td>( 2 \cdot 10^2 )</td>
<td>3.5</td>
<td>30</td>
<td>440</td>
</tr>
<tr>
<td>CsI (pure)</td>
<td>4.51</td>
<td>1.86</td>
<td>( 2 \cdot 10^3 )</td>
<td>6</td>
<td>35</td>
<td>315</td>
</tr>
<tr>
<td>BaF(_2)</td>
<td>4.88</td>
<td>2.03</td>
<td>( 1.2 \cdot 10^4 )</td>
<td>0.9</td>
<td>630</td>
<td>190/220 - 310</td>
</tr>
<tr>
<td>CsI(Tl)</td>
<td>4.51</td>
<td>1.86</td>
<td>( 5.4 \cdot 10^4 )</td>
<td>1000</td>
<td>-</td>
<td>550</td>
</tr>
<tr>
<td>BGO</td>
<td>7.13</td>
<td>1.12</td>
<td>( 2.8 \cdot 10^3 )</td>
<td>300</td>
<td>-</td>
<td>480</td>
</tr>
<tr>
<td>NaI(Tl)</td>
<td>3.67</td>
<td>2.59</td>
<td>( 4 \cdot 10^4 )</td>
<td>230</td>
<td>-</td>
<td>410</td>
</tr>
</tbody>
</table>

17.2.1 Electromagnetic shower centroid

The electromagnetic shower centroid is used as the best estimate of the point of impact (PoI) of the electron. Following the treatment in [33], the position is measured with the following equation:

\[
\bar{x}_{PoI} = \frac{\sum_i w_i \bar{x}_i}{\sum_i w_i}, \quad w_i = \max \left\{ 0, \left[ W_0 + \ln \left( \frac{E_i}{E_T} \right) \right] \right\},
\]

where

- \( i \) is the cell index that runs over a 9 \times 9 crystal area centred in the crystal with the highest energy deposit (assumed as the approximate electron PoI)
– $\vec{x}_i$ is the coordinate vector of the i-th cell centre
– $E_i$ is the energy deposited in the i-th cell
– $E_T = \sum E_i$ is the total energy deposited in the ECAL by the shower
– $W_0 = 4.0$ is a dimensionless parameter set at the value that maximise the spatial resolution.

### 17.2.2 Energy measurements

Fig. 37: $\delta_e = 1 - E_{cal}/E_{true}$ in the energy bins 1, 2, 5, 10, 20, 50, 75, 100, and 130 GeV, for electrons generated in the last target (the closest to the ECAL).

### 17.3 The Beam

The present muon beam optics is shown in Fig 38.
As can be seen from the optics, the beam is quite large for the present configuration at the location of the CEDAR. Therefore optics studies were performed within EN-EA-LE to estimate the beam size for a modified configuration along with integration studies to estimate required beam line modifications, cost etc.

Figure 39 shows the modified optics.

17.4 NLO Monte Carlo details

Here we summarize the main feature of the developed NLO Monte Carlo event generator:

1. the generated events are fully exclusive, \textit{i.e.} all the momenta of the event particles can be stored in such a way that any observable can be studied or any further effect can be applied (\textit{e.g.} detector simulation, experimental cuts, etc.);

2. events can be generated both at LO and NLO;

3. both \textit{weighted} and \textit{unweighted (constant weight)} events can be generated: the use of weighted events speeds up event generation and, generally, reduces the Monte Carlo statistical error;

4. the incoming muon energy (beam momentum) can be spread by a gaussian distribution around its nominal value;

5. since the main goal of MUonE is the direct measurement of $\Delta\alpha_{\text{had}}(q^2)$ at space-like momenta ($q^2 < 0$), the hadronic correction to the vacuum polarization can be switched on and off, all the rest
of the inputs remaining unchanged. This gives the possibility of studying the effect of $\Delta \alpha_{\text{had}}(q^2)$ on any observable at NLO theoretical accuracy, also including experimental effects\textsuperscript{3};

6. the events can be stored into \texttt{Root} n-tuples for further analysis. The storage format has been agreed among us and includes all the relevant information for each run input and for each generated event. Particular attention has been paid to make the format flexible for future developments (e.g. inclusion of multiple-photon emission) and to the easiness of concatenating independent sub-samples;

7. a \texttt{Root} interface has been developed for reading, analysing and manipulating generated samples. The interface is available upon request and will be soon made available on a public software repository.

\textsuperscript{3}In the EG, Fred Jegerlehner’s routine \texttt{hadr5n12.f} is used to account for the hadronic correction to the vacuum polarization, in order to simulate the expected effect to be measured. The routine is available at \url{http://www-com.physik.hu-berlin.de/~fjeger/software.html}. Of course, any other parameterization can be interfaced and used.
17.5 Analysis strategy

The $\Delta \alpha_{\text{had}}(t)$ parameterization used in the template fits is:

$$
\Delta \alpha_{\text{had}}(t) = k \left\{ -\frac{5}{9} \frac{4M}{3t} + \left( \frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M^2}{t^2}}} \log \frac{1 - \sqrt{1 - \frac{4M^2}{t^2}}}{1 + \sqrt{1 - \frac{4M^2}{t^2}}} \right\} \quad (14)
$$

This ansatz adopts the functional form of the leading-order expression for the photon vacuum polarization induced by a lepton pair in the space-like region (where the $M$ parameter would be substituted by the lepton $m^2$ and $k$ would be just $\alpha/\pi$). The same form is also valid for the contribution of $t\bar{t}$ pairs (with $M = m_{\text{top}}^2$ and $k = \frac{\alpha}{\pi} Q^2 N_c$, with $Q = 2/3$ the top electric charge and $N_c = 3$ the number of colours). Since the hadronic contribution to the running $\alpha$ is not calculable in perturbation theory, being dominated by light quark/hadron exchanges, the parameters $k$ and $M$ do not have a precise physics interpretation. At large $|t|$ the dependency is logarithmic, proportional to $\log \frac{M^2}{\xi}$, as expected from general principles. In the limit of very small $t$ Eq. 14 goes like:

$$
\Delta \alpha_{\text{had}}(t) \simeq -\frac{1}{15} \frac{k}{M}
$$

(15)

This corresponds to the dominant behaviour in the MUonE kinematical region. Correcting terms, corresponding to quadratic and higher orders in $t$ are incorporated in the form of Eq. 14. Actually, due to the dominant low-$t$ dependence, the $k$ and $M$ parameters are highly correlated and make Eq. 14 unpractical for the experimental extraction of the hadronic running. Thus it is convenient to use $K = k/M$ as fit parameter, substituting $k = KM$ in Eq. 14.

Results obtained by fitting the one-dimensional $\theta_\mu$ distribution for 3,000 toy experiments, each one with statistics equivalent to the MUonE expected luminosity are shown in Fig. 40. The fit quality is quite good, with average $\chi^2 = 24.7 \pm 7.9$ for 28 degrees of freedom. The fit values are $K = (0.13659 \pm 0.00072)$ GeV$^{-2}$, $M = (0.0562 \pm 0.0022)$ GeV$^2$. The expected final result is $\alpha_{\mu}^{\text{LO}} = (689.8 \pm 2.3) \times 10^{-10}$, where the errors are only the statistical ones.

The result has been verified by fitting independently the electron angular distribution, and the joint $\theta_e - \theta_\mu$ distribution with the same technique. The fitted values are almost identical, as is the fit quality.
Fig. 40: Results of the template fit of the muon angular distribution, obtained in 3,000 toy experiments, each one with statistics corresponding to the nominal MUonE integrated luminosity of $1.5 \times 10^7$ nb$^{-1}$. (Top row) The fitted parameters describing $\Delta a^\text{had}(t)$. With respect to Eq. 14 the fit parameter $K = k/M$ replaces $k$ to reduce the correlation with $M$; (Bottom left) the fit $\chi^2$ for 28 degrees of freedom; (Bottom right) the resulting $a^\text{HLO}_\mu$. 
References


[81] S. Di Vita et al. “Master integrals for the NNLO virtual corrections to $q\bar{q} \to t\bar{t}$ scattering in QCD: the non-planar graphs”. In: (2019). arXiv: 1904.10964 [hep-ph].


