An Analysis of a Potential Compact Positron Beam Source

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Abstract

For positron studies in plasma wakefield accelerators such as AWAKE, the development of new, cheaper, and compact positron beam sources is necessary. Using the GBAR experiment’s positron trap as an example source, this paper explores converting that trapped positron plasma into a usable beam. Bunching is initially accomplished by an electrostatic buncher and the beam is accelerated to 148 keV by pulsed electrostatic accelerators. This is necessary for injection into the $\beta$-matched rf cavities operating at 600 MHz, which bring the positron beam to a transverse emittance of 1.3 $\pi$ mrad mm, a longitudinal emittance of 93.3 $\pi$ keV mm, $\sigma_z$ of 1.85 mm and an energy of 22 MeV. The beamline used here is far simpler and less expensive than those at many facilities, such as SLAC, allowing for a cheap source of positron beams, potentially opening up positron beam studies to many facilities that could not previously afford such a source.

1 Introduction

1.1 Background

Plasma wakefield acceleration, or PWFA, is currently a promising way of accelerating charged particles that is far more efficient and compact than traditional rf accelerators. One of the greatest challenges of PWFA at the moment is that accelerating positron beams seems to be very difficult with this method. In rf cavities the phase can simply be changed to account for the different sign of the charge, while no such symmetry is easily exploitable in the case of PWFA. This topic has not been studied in much depth, although the hollow channel plasma wakefield acceleration technique does attempt to solve this issue [1]. One of the greatest barriers to furthering research on this topic is the lack of experimental facilities that can generate the positron beams necessary for this. As it stands, the only functional source of positron beams for PWFA experiments is at FACET-II at SLAC. This facility is approximately 1 km long and extremely expensive, making the construction of such a facility wildly out of reach for virtually all physics laboratories [2]. However, at GBAR at CERN, a method of generating a trapped, low-energy positron plasma with approximately $10^8$ particles has been developed. Potential exists for this beam to reach $10^{10}$ particles within a few years [3]. In this paper, the possibility of using this trap as a source for a positron beam is addressed, as well as the small and comparatively inexpensive linac needed to mold this plasma into a usable beam that can be fed into the AWAKE plasma cell, just as electron beams are currently fed in. While PWFA application is the primary interest of this paper, such a compact positron source would be of great interest to any facility interested in studying positron physics.

1.2 GBAR Positron Beam Source

1.2.1 Slow Positron Generation

At the moment, GBAR is able to produce positron beams with $10^8$ particles using a fairly simple and inexpensive method. The system impacts a 10 MeV electron beam on a sample of $^{22}$Na, which, through $\beta^+$ decay, generates positrons with an average energy of 1 MeV and maximum energy of 3 MeV. This process has an efficiency of about $5.5 \times 10^{-4}$ positrons per incoming electron, being quite inefficient. The wide spread here is undesirable, so a moderator is used to convert this into a beam with a much lower energy and energy spread. Tungsten, which has a negative work function,
as required for moderators, is used at GBAR. The moderator works by slowing down fast positrons through thermalization, with only very slow ones surviving to the surface of the material without annihilating. The surviving positrons are emitted with an energy corresponding to that of the moderator’s work function, which is, in this case, on the order of 1 eV. This is a highly inefficient process, with a moderation efficiency of $5 \times 10^{-4}$. Combining this with the efficiency of initial positron generation, this whole process has an efficiency of about $3 \times 10^{-7}$ slow positrons generated per input electron [3]. This is a value GBAR hopes to improve upon in the near future by using a Neon moderator that promises to be orders of magnitude more efficient. Such improvements would allow for higher charge positron beams, which would be extremely useful as a source.

1.2.2 Positron Trapping

The slow positron beam is then brought inside the 5 T trap, which requires the beam to be accelerated to 1 keV to overcome the magnetic mirror formed by transferring the beam from a region of virtually no magnetic field outside the trap to a 5 T field inside the trap. For the simulation studied here, a 1 T field was used, as that minimized emittance while keeping the beam compact. At 1 keV, there are no significant losses of positrons to the magnetic mirror. The positron beam then goes through an electron plasma of density $\sim 10^{11} \text{cm}^{-3}$ to slow it down. The beam then enters a 1000 V trap, where the well depth is increased as more positrons from many beams are accumulated. Once 1000 V is hit, the electrons are forced out of the trap so the pure positron plasma sits there with a low energy [3]. Currently, this allows generation of plasmas containing $10^8$ particles with the near goal of $10^{10}$ particles. In the simulation performed here, a trap of 1500 V was used, as that allowed for better bunching.

The trap can be used to eject the positrons by dropping the potential at the front, creating a parabolic potential and a correspondingly linear electric field used to bunch the beam. The start of the buncher is taken as $z = 0$ and the time when the buncher is turned on is taken as $t = 0$ for this simulation.

1.3 Current Positron Beam Sources

The positron beam generation at FACET-II is essentially similar to that of GBAR, with an electron beam impacting a target and generating positrons as such. However, instead of bringing the beam to rest and trapping them, the positrons are inserted into a damping ring, where they are collimated and manipulated. This damping ring is technically very complex, including dipoles, quadrupoles, sextupoles, and an accelerating component to bring the beam to 355 MeV and minimize transverse emittance. This produces a very high quality beam, but requires 54 large magnets around the damping ring to steer and modify it, which is unreasonably complex for most large facilities to build from nothing. FACET-II has inherited the damping ring from a previous experiment, so the modifications needed were a reasonable price for them, which is a uniquely fortunate situation [2].

2 Outline of Simulation

2.1 Generation of Initial Plasma Distribution

The results of the simulation of the beamline are highly dependent on the initial plasma distribution inside the trap, so it is therefore very critical that the initial distribution fed into ASTRA is accurate and represents what would be inside a plasma trap [4]. To ensure this, equation 1 was used to find the charge density function to check against the initial beam distribution used in the simulation [5].

$$\frac{\delta^2 \phi}{\delta r^2} + \frac{1}{r} \frac{\delta \phi}{\delta r} + \frac{\delta^2 \phi}{\delta z^2} = \frac{-n(r,z)}{\epsilon_0} \tag{1}$$

In equation 1, $\phi$ represents the potential inside the trap and $n(r,z)$ is a charge density. $n(r,z)$ is related to the potential by equation 2, making all terms in equation 1 actually dependent on $\phi$.

$$n(r,z) = Ce^{-\phi \epsilon_{eff}(r,z)} \tag{2}$$
C is a constant of integration found by ensuring the integral of $n(r,z)$ over the trap’s volume is equal to the total plasma charge and $T$ is the temperature of the plasma. $\phi_{eff}$ is dependent on $\phi$ and is given by equation 3.

$$\phi_{eff}(r,z) = \frac{1}{q} \left( \frac{1}{2} m \omega_r (\Omega_C - \omega_r) r^2 + q \phi(r,z) \right)$$  \hspace{1cm} (3)$$

$\Omega_C$ is the cyclotron frequency, given by $\frac{q B}{m}$, where $B$ is the strength of the magnetic field used to confine the plasma. $\omega_r$ is defined as $\frac{q n_0}{2 e B}$, where $n_0$ is the maximum density of the plasma. $n_0$ was found by considering the plasma to be of uniform density, an initial assumption explained below.

The Debye length of the plasma was calculated to be 60.6$\mu$m. It has been shown that if the Debye length is much less than the plasma radius, the density is highly uniform and only significantly drops off from the maximum at 1 Debye length before the edge of the plasma [6]. For this reason, it was decided that the plasma being of uniform density was an acceptable approximation for determining this constant. Our simulation would later verify this assumption.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trap radius</td>
<td>0.004 m</td>
</tr>
<tr>
<td>Trap height</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Potential at trap ends</td>
<td>1000 V</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>1 T</td>
</tr>
<tr>
<td>Plasma radius</td>
<td>0.001 m</td>
</tr>
<tr>
<td>Plasma height</td>
<td>0.09 m</td>
</tr>
<tr>
<td>Temperature</td>
<td>273 K</td>
</tr>
<tr>
<td>Number of positrons</td>
<td>$10^8$ particles</td>
</tr>
</tbody>
</table>

Table 1: Shows the parameters used to generate the initial plasma distribution as it is inside the trap before the electrostatic buncher is turned on.

A script in MATLAB was written to take an initial assumption about $n(r,z)$, in this case that it is constant over a cylindrical volume and zero elsewhere, and use that to calculate $\phi$, which in turn can be used to find $n(r,z)$ by equations 2 and 3. This process was repeated iteratively until it converged on a solution [7].

The full phase space distribution of the initial beam was necessary for the simulation, so the velocity distribution described in equation 4 was used to find the initial velocities of each particle after their initial positions were found. With this and the initial positions of all the particles, the initial state of the positron beam was fully defined. Note that the $v_z$ and $v_r$ velocity distributions will be centered on zero, as the mean term, $\omega_r r \hat{\theta}$ is only nonzero for $v_\theta$.

$$f_{eq}(r,z,v) = \frac{n(r,z)}{(2\pi k_B T/m)^{3/2}} e^{-\frac{1}{2} \frac{m}{k_B T} (v + \omega_r r \hat{\theta})^2}$$ \hspace{1cm} (4)$$

Figure 1: Phase space plot in $x$ for initial beam distribution

Figure 2: Phase space plot in $y$ for initial beam distribution

The $z$ phase space distribution reflects how the beam, in total, is at rest so $z$ momentum values are randomly distributed independent of $z$ and centered on zero. Typically, for a collection of rotating particles, a figure with amplitudes close to zero at large radii and large amplitudes at
small radii would be expected for the x and y phase spaces. However, since the rotation is largely of comparable magnitude to \( \dot{\gamma} \), by equations 5 and 6, the two both have a significant impact on the phase space distribution, so the characteristic form cannot be observed. Cooling the beam down would reveal this characteristic form more and reduce the thermal noise, but since the high temperature of the beam did not have a negative impact on the beam later in the simulation, such cooling was determined to be unnecessary.

\[
\begin{align*}
\dot{x} &= \dot{r} \cos \theta - r \dot{\theta} \sin \theta \\
\dot{y} &= \dot{r} \sin \theta + r \dot{\theta} \cos \theta
\end{align*}
\]

2.2 Beamline Design and Simulation

Since the beam starts already generated and essentially sitting with a very low initial velocity, the common method of using a 2.5 cell rf gun to quickly bring the beam to relativistic speeds was not an option. Additionally, the beam’s initial profile was extremely long with \( \sigma_z = 26 \mu m \) when inside the trap. Therefore, the two top priorities at the beginning of the beamline were to compress the bunch in z and accelerate it to \( \beta \sim 1 \) so that the beam can enter an rf cavity, be much shorter than the associated wavelength, and be fast enough that it does not quickly fall out of phase with the electromagnetic wave.

This was accomplished using three electrostatic devices: A low-field electrostatic buncher inside the trap and two higher field, pulsed electrostatic accelerators powered by 100kV power supplies. The electrostatic accelerators here are pulsed such that they turn on once the beam is fully contained inside them; that way, when each one turns on, it uniformly accelerates the beam. However, it does not need to be turned off before the beam leaves it, as the extra energy given to the back particles can be used to properly chirp the beam and compress it further, as it was calculated that such a device should bring the beam to \( \beta \approx 0.65 \), a point where velocity-based bunching is still possible.

The electrostatic buncher inside the trap that initially launches the beam consists of many rings concentric with the beam, where each ring can have its potential set. GBAR currently only uses potentials up to 1000 V but can go to 2000 V without causing any sparking. To kick the beam out of the buncher and get it to focus, a potential corresponding to the electric field given by equation 7 is created. The buncher runs from 0 cm to 10 cm. Note that the simulation in ASTRA begins at 0 cm, corresponding to the beginning of the buncher.

\[
E(x) = 2 \times 10^4 (1 - 5x)
\]

Note that E is in units of \( \frac{V}{m} \). The buncher is nonzero at the end so that particles at the front of the beam also get a kick and come to a focus outside the buncher. In this case, the beam comes to a focus at 14 cm.

\[
V(x) = -2 \times 10^4 (x - \frac{5x^2}{2})
\]

Note that the maximum of the potential, given by equation 8 in the domain of the buncher is 1500 V, greater than what GBAR typically does but far from impossible.

Since it would be undesirable for the front of the beam to get a kick the back doesn’t, the electrostatic buncher, which is placed to start at \( z = 0.127 \) m, is pulsed and turns on when the
beam is at the 14 cm focus point. This corresponds to a time of 9.2 ns. It is placed at 12.7 cm because the furthest back particles of the bunch are at 12.8 cm. This way, there is no charge loss from the device and all particles are accelerated. The electrostatic accelerator is left on once turned on, however, as it can then give a kick to the particles at the back of the bunch, bringing the beam to another focus. After this device, the beam comes to a focus at $z = 0.25$ m with $\sigma_z = 2$ mm.

Initially, only one electrostatic accelerator was used in the simulation, but the beam fell out of phase with the rf cavity too quickly to be of any use. Therefore, two identical electrostatic accelerators were used, with the second being places slightly behind the focus of the first, so the bunch would be uniformly accelerated to $\gamma = 1.29$, corresponding to $\beta = 0.63$. This was a sufficient speed for the beam to keep up with the rf cavity for at least three cells, meaning that a 600 MHz rf cavity could then be used to bring the beam to ultra-relativistic speeds and $\gamma \cong 6$. After three 600 MHz cells, the beam leaves and then reenters another 600 MHz accelerating cavity, although this one has 9 cells. By changing the rf cavity, the phase that the beam experiences is essentially reset, removing any error that would have been introduced by the beam falling out of phase when it is initially non-relativistic.

![Phase space plot in x for the beam briefly before being injected into the rf cavities](image1)

![Phase space plot in y for the beam briefly before being injected into the rf cavities](image2)

![Phase space plot in z for the beam briefly before being injected into the rf cavities](image3)

At the end of the second rf cavity, the beam has an energy of 22.07 MeV and a length of $\sigma_z = 1.85$ mm. This energy is comparable to the electron beam fed into AWAKE [8]. The x and y phase space distributions are quite standard, as shown in figures 7 and 8, but the z in figure 9 appears unusual due to the electrostatic acceleration and compression from earlier in the beamline. The electrostatic buncher made it so the beam doubles over on itself and here it seems to slightly overshoot so that the beam did not perfectly match with itself. Further work on this could optimize the electrostatic buncher so no trace of the bunching can be seen in the terminal z phase space plot, giving the lowest longitudinal emittance possible.

Tables 2, 4 and 3 describe the beamline used and relevant beamline parameters.

<table>
<thead>
<tr>
<th>Beamline element</th>
<th>Starting point (m)</th>
<th>Trigger time (ns)</th>
<th>Maximum Field (MV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrostatic buncher</td>
<td>0</td>
<td>0.0</td>
<td>0.02</td>
</tr>
<tr>
<td>Electrostatic accelerator 1</td>
<td>0.127</td>
<td>9.2</td>
<td>2</td>
</tr>
<tr>
<td>Electrostatic accelerator 2</td>
<td>0.243</td>
<td>10.17</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: Gives the information necessary to define the electrostatic fields used in the simulation.
3 Analysis

3.1 Proof of Correctness

3.1.1 Initial Distribution

To ensure the distribution was sampled from correctly, the number of particles selected for each x and y value was examined through a histogram.

Note that the number of particles per bin is essentially the same across the cross-section, except for at the very edge, where the probability quickly drops off. Therefore, the common mistake of not accounting for the change in coordinates when transforming a cylindrical probability to a Cartesian one was avoided. The variation in height for different bins is primarily due to the fact that only 2000 macroparticles were used, which is rather small for this histogram. However, it is demonstrated below that the result from a simulation with 2000 macroparticles varies negligibly from one with 60000 macroparticles, so this small number is fine in this case.
3.1.2 Macroparticle Number

Using a test beam that was virtually identical to the one used for the bulk of the simulations, except with a forward $z$ energy of 200 eV, tests were conducted to see if the number of macroparticles used, 2000, would be acceptable and not introduce significant error to the simulation. To do this, the beam was passed through a focus, where space charge would be at a maximum, and let to briefly drift. If the number of macroparticles needs to be greater than 2000, the phase space plots should vary with the number of macroparticles used, with the larger ones being more accurate. The focus was found to occur briefly before the beam was at 17 cm, so $z$ phase plots of the beam with different numbers of macroparticles were made there and are shown in figures 11 to 14.

![Figure 11: Phase space plot in $z$ for 2000 macroparticles](image1)

![Figure 12: Phase space plot in $z$ for 5000 macroparticles](image2)

![Figure 13: Phase space plot in $z$ for 10000 macroparticles](image3)

![Figure 14: Phase space plot in $z$ for 60000 macroparticles](image4)

It can be seen that the different plots strongly converge to the exact same figure, so the number of macroparticles used must be acceptable. This is not surprising; since the beam is quite large,
space charge should not have much of an effect at this point. When the beam is compressed, it is also accelerated, suppressing space charge.

### 3.2 Analytic Equation for Transverse Emittance

From the standard equation for transverse emittance, given by equation 9, an analytic equation for the transverse emittance of a plasma in a Penning trap was derived, using some assumptions that generally hold well in a Penning trap.

\[ \epsilon_n = \frac{1}{mc^2} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle xp_x \rangle^2} \]  

(9)

No correlation between \( x \) and \( p_x \) is anticipated, so the \( \langle xp_x \rangle \) term can be ignored. This lack of correlation can be seen in figure 1, where there is no relationship between \( p_x \) and \( x \) values.

It was assumed that \( k_BT \gg \omega_r r_p \). This implies that the thermal energy is much larger than the rotational energy of the plasma. This was shown to hold extremely well in a trap like the one used by GBAR down to temperatures on the order of 1 K, making this a reasonable assumption. This allows for use of equation 10, which, when combined with the standard definition of momentum, can be used to derive 11.

\[ k_BT = \frac{1}{2}mv^2 \]  

(10)

\[ p_x^2 = mk_BT \]  

(11)

To find \( \langle x^2 \rangle \), equation 12 was used under the assumption of constant density.

\[ \langle x^2 \rangle = \frac{\int x^2 n(x, y)dx dy}{\int n(x, y)dx dy} \]  

(12)

By combining 12 and 11, equation 9 can be rewritten in terms of the parameters of the trap, as in equation 13.

\[ \epsilon_n = \frac{1}{mc^2} \sqrt{\frac{qNmk_BT}{8\pi\epsilon_0 B\omega_r L_p}} \]  

(13)

This equation was shown to match the transverse emittance found by simulation to within 7%, indicating accuracy. This equation can then be used to find the emittance of a beam as a function of the plasma and trap parameters, as shown in figure 16 and 15. Note that for low positron number and low temperature, emittance values become extremely small, potentially making this technique particularly useful for facilities that can operate with low particle numbers or low temperatures. In these two figures, all the other parameters, such as magnetic field strength and \( \omega_r \), were held at the values given or implicitly defined by table 2.1.

Figure 15: Depicts emittance as a function of particle number with temperature held at 273 K

Figure 16: Depicts emittance as a function of temperature with particle number held at \( 10^8 \) positrons
### 3.3 Comparison with SLAC Source

The compact positron source described in this paper can be compared against the much larger source used at FACET-II as a benchmark. It is expected that the FACET-II beam will be superior in a number of parameters, most notably charge and energy, as the beamline there includes significantly more accelerating structures and is able to generate an initial beam with higher charge.

<table>
<thead>
<tr>
<th>Beam parameter</th>
<th>FACET-II Value</th>
<th>Value from Simulation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>335</td>
<td>22</td>
<td>MeV</td>
</tr>
<tr>
<td>Bunch length (rms)</td>
<td>0.35-0.41</td>
<td>1.85</td>
<td>mm</td>
</tr>
<tr>
<td>Energy spread (rms)</td>
<td>0.5-1.1</td>
<td>0.38</td>
<td>%</td>
</tr>
<tr>
<td>Transverse emittance</td>
<td>7.6</td>
<td>0.408</td>
<td>µm-rad</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>1</td>
<td>0.016</td>
<td>nC</td>
</tr>
</tbody>
</table>

Table 5: Compares the values of the beam produced by the FACET-II positron source and the positron source proposed by this paper.

As shown in table 5, the energy of the beam is significantly lower because the goal of our source is to simply produce a positron beam at fairly low energy that can then be injected into an accelerator, not to accelerate the beam. The bunch length from this method is significantly longer, but further optimization with the electrostatic buncher may be able to reduce this gap somewhat. Such optimization could be the focus of a future study. The other parameters are largely comparable, except for the bunch charge, which is about two orders of magnitude less. However, GBAR’s goal of being able to produce positron beams with $10^{10}$ particles would close this gap, allowing the method proposed here to use beams with a bunch charge of 1.6 nC.

### 3.4 Further Development

As it stands, the beam produced by this source lacks proper chirping, as seen in figure 9, so adding an additional rf cavity with a higher frequency at the end of the simulation, perhaps S-band, could be used to properly introduce chirping, allowing for magnetic chicanes to be used to further reduce the bunch length, if necessary.

Additionally, as also seen in figure 9, the beam essentially has two "heads" in phase space, separated by a fraction of a millimeter. If the S-band structure were added and significantly increased the energy of the beam, this odd shape may become trivial. Since the difference between the top and bottom of the two heads is on the order of a few MeV, if the beam is accelerated to much greater energies, the difference there would be trivial, the two heads would be crushed, and the only remnant of the odd shape would be a small contribution to the energy spread of the beam and a small increase to $\sigma_z$. However, if the S-band is only used to introduce enough energy to properly chirp the beam, the head would most likely remain. The two heads here are mostly likely caused by the electrostatic buncher, which causes the beam to fold over on itself. Here, it would seem that is done imperfectly, with the back not exactly matching the front, causing the two beam heads. By optimizing the forms and triggering times of the electrostatic beamline elements, particularly the electrostatic buncher, it should be possible to collapse the two heads into one, reducing $\sigma_z$ slightly.

Since the possibility of reaching $10^{10}$ would bring the charge of the beam to be roughly the same as larger than that of FACET-II, redoing these simulations for a higher charge beam would also be of great interest.

### 4 Conclusion

By using a positron source inspired by the GBAR experiment, a new kind of positron source which brings the beam to rest instead of placing it in a long damping ring was simulated and studied. Many of the parameters, such as transverse emittance, energy spread, and, to a lesser degree, bunch length, are competitive with those from major, complex positron sources, such as the one employed by FACET-II. The fact that the beam was initially at rest proved to be a major challenge, so using pulsed electrostatic beamline elements allowed for the beam to both be bunched and accelerated to a point where it can be injected into $\beta$-matched rf cavities. This electrostatic portion of the beamline did leave some artifacts in the z phase space of the final beam, such as the "double..."
headed" look, seen in figure 9, but with further optimization and improvement of this section, these artifacts may be removed. In any case, the effects do not seem to be detrimental to the beam and will most likely be washed away when it is accelerated further. Additionally, the design of the positron source proposed here can easily incorporate improvements to the beam source done by GBAR and similar experiments, meaning that as GBAR gets closer to reaching beams of $10^{10}$ positrons, this beam source gets closer to reaching beams of 1.6 nC, making it a strong candidate for a positron source for facilities wishing to study positron dynamics in detail, particularly for PWFA facilities, that is far less complex and expensive than current positron sources that require large damping rings.

References


