The determination of absolute cross-sections in the ISR requires a monitor. The monitoring procedure has to be precise and simple. In the following a possible monitor is proposed.

The number of events per unit of time of a process of cross-section \( \sigma \) for a beam of square cross-section and uniform density is given by:

\[
N_{\text{int}} = \sigma \times \frac{c \nu_1 \nu_2}{h \tan \frac{\alpha}{2} (2\pi R)^2}
\]  

(1)

where:

- \( h \) is the beam height
- \( \alpha \) is the crossing angle (\( \alpha = 15^\circ \))
- \( c \) is the light velocity
- \( R \) is the machine radius
- \( \nu_1, \nu_2 \) is the total number of protons in the two rings.

The beam width does not appear in Eq. (1) because particles in the same plane always meet somewhere.

If, as in practice, the beam profile is not a squared box but some kind of bell-shaped curve, the interaction rate is still given by a formula of type (1) in which an "effective" beam height is introduced, given by:

\[
\frac{1}{h_{\text{eff}}} = \frac{\int S_1(z) \cdot S_2(z) \, dz}{\int S_1(z) \, dz \cdot \int S_2(z) \, dz}
\]

(2)

where \( S_1(z) \) and \( S_2(z) \) are the (arbitrarily normalized) beam shapes.

An absolute cross-section determination requires measuring beam currents, shapes, and overlaps at the interaction region.
We propose to determine separately the three integrals

\[ I_1 = \int S_1(z) \, dz ; \quad I_2 = \int S_2(z) \, dz ; \]
\[ I_{12} = \int S_1(z) \, S_2(z) \, dz \]

by quickly sweeping a thin wire at a constant velocity \( v \) across the beam in the interaction region.

Forward emitted particles in the interactions of the two beams with the wire are detected by two separate counter telescopes. Let \( N_1 \) and \( N_2 \) be the number of counts recorded by the telescopes during the beam scan. Let us also record chance coincidences (resolving time \( \delta \)) between the two telescopes, \( N_{12} \). Then:

\[ \frac{1}{N_{\text{off}}} = \frac{I_1 I_2}{I_{12}} = \frac{N_1 N_2}{N_{12}} \frac{1}{v \delta} \quad (3) \]

The number of stored particles \( y_1 \) and \( y_2 \) can be determined either from the stored current or alternatively from the temperature rise of the wire during the scan. In this case, an additional scan of each individual beam would be required.

The effects of the scan across the stacks are now considered in more detail:

1) Effects of wire passage on the stacks. The main effect of the passage of the wire is a blow-up of beam because of multiple scattering. If we denote with \( D \) the wire diameter, and with \( n \) the number of revolutions of protons during the scan time, the fraction of the beam

\[ f = \frac{D}{h} n \quad (4) \]

passes across the wire. Typically for \( D = 10 \mu, h = 1 \text{ cm}, n = 30 \) revolutions \( (v = 10 \text{ cm/msec}) \), we have \( f \approx 0.03 \).

The average thickness of material traversed by the protons of the stack is:

\[ T = \frac{1}{\sqrt{2}} \frac{D^2}{h} n \quad (5) \]

*) Beam-beam counts and gas counts are probably negligible during the scan. Otherwise they have to be subtracted out.
For the numerical values indicated, \( T = 2.3 \times 10^{-5} \) cm, about 10\(^{-4}\) g/cm\(^2\) of matter in the case of a quartz wire. The amount of matter of the wire can be compared with the thickness of residual gas that the beam traverses per unit of time:

\[
T_{\text{Gas}} = 3 \times 10^{14} \frac{P}{L} \left( \text{g} \text{ cm}^{-2} \text{ sec}^{-1} \right)
\]  

(6)

where \( P \) is indicating the average pressure in the vacuum chamber in Torr. For \( P = 10^{-9} \) Torr we get \( T_{\text{Gas}} = 3 \times 10^{-5} \) g cm\(^{-2}\) sec\(^{-1}\); that is, the passage of the wire across the stack is equivalent to the residual gas effects over about 3 seconds. Therefore, repeating the beam scan, for instance every few minutes has negligible effects on the beam lifetimes and sizes.

The number of nuclear interactions during the scan for an interaction mean free path \( L = 50 \) cm is:

\[
N_{\text{int}} = \frac{\nu \frac{P}{L}}{\sqrt{2} \frac{P}{L} h} = 4 \times 10^{-7} \nu_p,
\]

(7)

that is \( N_{\text{int}} = 1.6 \times 10^8 \) for the nominal stack intensity \( \nu_p = 4 \times 10^{14} \) protons. This number, although negligible compared to the stack intensity, is large enough for very convenient detection of the forward-emitted particles.

ii) Effects of beams on the wire. Let us consider thermal stresses due to beam friction on the wire. Taking for the specific heat of quartz the figure:

\[
C_y = 3.5 \text{ joule cm}^{-3} \text{ deg}^{-1}
\]

we get a temperature increase:

\[
\Delta t = \frac{1.1 \times 10^{-13} N}{L} \frac{P}{h} \nu_p \nu^2 \text{ C}^0,
\]

(8)

where \( L \) is the beam width in the horizontal plane. For a full stack, \( N_p = 2 \times 4 \times 10^{14} = 8 \times 10^{14} \), \( h = 1 \) cm, \( L = 6 \) cm, and according to Eq. (8)

\[
\Delta t = 440 \text{ C}^0,
\]
which is a severe but still acceptable condition.

The temperature increase would, however, be a serious limitation if scanning time is appreciably longer.

Heat losses due to radiations and conduction have been calculated and found completely negligible even at the highest temperatures.

There are two possible ways to realize the beam scan:

i) The wire is moved rapidly across the beams. In order to avoid melting at the highest stored currents, wire speeds of the order of 100 m/sec are required. This is probably quite unrealistic, and a speed of the order of 10 m/sec is perhaps a more reasonable figure. In this case, the maximum admissible stack intensity should be limited to about \( v = 4 \times 10^{13} \) protons.

ii) Firstly, the wire is put close to the stacks, which are then deflected by equal amounts so as to be scanned by the wire. The required deflections can be easily reduced by placing two identical vertically-bending magnets at \( \frac{1}{2} \) betatron wavelength upstream and downstream from the interaction point. This requires the relatively modest bending power of approximately 1 kG metre at the maximum stack energy \( E = 28 \) GeV. The filling time of the magnets is of the order of 100 μsec. The magnetic field must rise with a very precise and constant rate.

In principle, the first method is probably cleaner. However, it might require major engineering development work. The second method has no fast-moving part, and it is much simpler to realize. It is essential that beam displacements are done without distortions of the beam shape.

We would like to remark that the technique of determining the beam profile with a thin wire scan can be tested at the CPS. It may turn out that it could be of interest as an instrument for the accelerator, no device for precise determination of the beam shape being available so far at the Argonne accelerator, the beam shape was determined by observing the beam interactions in the residual gas. However, this method conflicts with the extremely severe requirements on the residual gas pressure in the ISR interaction region.
Finally, such a facility would provide a useful way of calibrating and testing detectors at the ISR by simulating, at a high rate, actual beam-beam interactions.
Let $\frac{dN_1}{dt}$, $\frac{dN_2}{dt}$, and $\frac{dN_{12}}{dt}$ be the instantaneous rates and chance coincidences in the two telescopes, respectively. Then

$$\frac{dN_1}{dt} = K_1 S_1(z), \quad \frac{dN_2}{dt} = K_2 S_2(z), \quad (A1)$$

with $K_1$, $K_2$ arbitrary normalization factors, depending on the telescope geometries, cross-section, machine energy, etc. We do not require that two telescopes must be identical; that is we allow $K_1 \neq K_2$. However $K_1$ and $K_2$ are constant during the scan-time. If we indicate with $v = \frac{dz}{dt} =$ constant, the scan velocity, expression (A1) gives:

$$\frac{dN_1}{dt} \cdot v \cdot dt = K_1 S_1(z) \cdot dz, \quad \frac{dN_2}{dt} \cdot v \cdot dt = K_2 S_2(z) \quad (A2)$$

and making this product also:

$$\frac{dN_1}{dt} \cdot \frac{dN_2}{dt} \cdot v \cdot dt = K_1 K_2 S_1(z) S_2(z) \cdot dz \quad (A3)$$

Introducing in formula (A3) the formula for chance coincidences:

$$\frac{dN_{12}}{dt} = \frac{dN_1}{dt} \cdot \frac{dN_2}{dt} \cdot \delta, \quad (A4)$$

where $\delta$ is the resolving time of the coincidence circuit, we get:

$$\frac{dN_{12}}{dt} \cdot \frac{v}{\delta} \cdot dt = K_1 K_2 S_1(z) S_2(z) \cdot dz \quad (A5)$$

Integrating formulae (A2) and (A5) throughout the scan we get:
\[ I_1 = \int S_1(z) \, dz = \frac{V}{K_1} \int \frac{dN_1}{dt} \, dt = \frac{V}{K_1} N_1 \]

\[ I_2 = \int S_2(z) \, dz = \frac{V}{K_2} \int \frac{dN_2}{dt} \, dt = \frac{V}{K_2} N_2 \]

\[ I_{12} = \int S_1(z)S_2(z) \, dz = \frac{V}{\delta K_1 K_2} \int \frac{dN_{12}}{dt} \, dt = \frac{V}{\delta K_1 K_2} N_{12} \]

from which it follows immediately that:

\[ \frac{1}{h_{\text{eff}}} = \frac{I_{12}}{I_{112}} = \frac{N_{12}}{N_1 N_2} \cdot \frac{1}{V} \]

which is just formula (3) of page 2.
Effective Beam-Beam Interaction Height = \( v \delta \frac{N_1 N_2}{N_{12}} \)