Measurement of the W boson rapidity, helicity, and differential cross sections in pp collisions at $\sqrt{s} = 13$ TeV

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Abstract

The differential cross section and charge asymmetry for inclusive W boson production at $\sqrt{s} = 13$ TeV is measured for the two transverse polarization states as a function of the W boson absolute rapidity. The measurement uses events in which a W boson decays to either an electron or a muon and a neutrino. The data sample of proton-proton collisions recorded with the CMS detector at the LHC in 2016 corresponds to an integrated luminosity of 35.9 fb$^{-1}$. The absolute differential cross section, and its value normalized to the total inclusive W boson production cross section, are measured over the rapidity range $|Y_{W}| < 2.5$. In addition, the W boson double-differential cross section, $d^2\sigma/dp_T^2d|\eta|$, and the charge asymmetry, are measured as a function of the charged lepton transverse momentum and pseudorapidity. The precision of these measurements is used to constrain the parton density functions (PDF) of the proton using the next-to-leading-order PDF set NNPDF3.0.

This document has been revised with respect to the earlier version from the same date.
1 Introduction

The standard model (SM) of particle physics provides a description of matter in terms of fundamental particles and their interactions mediated by vector bosons. The electromagnetic and weak interactions are described by a unified gauge theory based on the $SU(2)_L \times U(1)_Y$ symmetry group, where the photon, the W boson, and the Z boson act as mediators of the unified electroweak interaction [1–3].

The measurements of kinematic properties of W bosons produced at hadron colliders allow stringent tests of perturbative quantum chromodynamics (QCD) calculations and probe the nature of the electroweak interaction. In particular, measuring the polarization of the W boson is fundamental in determining its production mechanism.

At leading order in QCD, W bosons are produced at a hadron collider with small transverse momentum ($p_T$) through the annihilation of a quark and an antiquark: $u \bar{d}$ for the $W^+$ and $\bar{u}d$ for the $W^-$. At the LHC, W bosons with large rapidity ($Y_W$) are produced in the same direction as the quark that participates in the hard scattering. This is because one of the partons has to carry a large fraction ($x$) of the proton momentum and the parton distribution functions (PDFs) favor this being the quark [4]. Given the V-A coupling of the W to fermions in the standard model, the spin of the W boson is aligned with the flight direction of the antiquark, i.e., it is purely left-handed. At smaller $Y_W$, in the case the W boson is produced with significant $p_T$, the helicity of the W bosons become a mixture of left- and right-handed states, with the fractions depending on the relative size of the amplitudes of the three main processes: $ug \rightarrow W^+d$, $ud \rightarrow W^+g$ and $g\bar{d} \rightarrow W^+\bar{u}$. The PDFs at large values of $x$ determine the relative contributions of these amplitudes, favoring the production of left-handed W bosons at LHC over right-handed ones. Moreover, for higher values of the W boson $p_T$ ($p_T^{W}$), where the boson recoils against a jet, other processes beyond the leading order ones contribute to the production, and longitudinal polarization arises. The fraction of longitudinal polarization increases with $p_T^{W}$ in the kinematic region that contains the bulk of the W events, $p_T < 50$ GeV.

At the LHC W bosons are produced in large quantities and it is easy to trigger on their leptonic decays ($W \rightarrow \ell \nu$) with comparatively large purity. However, the escaping neutrino does not allow for a direct measurement of fully differential cross sections for W boson production. In particular, the polarization and rapidity distributions of the boson must be inferred by using the PDFs. Uncertainties stemming from the imperfect knowledge of these PDFs contribute a large fraction of the overall uncertainties in recent measurements of the mass of the W boson, and in other high precision measurements at the LHC [5, 6].

Constraints on the PDFs and their uncertainties are possible through many different measurements. Recently, the CMS and ATLAS collaborations published PDF constraints from double differential measurements of Z boson production and the accurate measurement of $\sin^2 \theta_W$ [7–9]. Studies of W bosons have been used by the ATLAS and CMS collaborations to set constraints on PDFs through the measurement of charge asymmetries, in particular as a function of the charged lepton pseudorapidity $\eta = -\ln \tan \frac{\theta}{2}$ [10–18].

Recently, a method has been proposed to directly measure the rapidity spectrum for W bosons at the LHC, differentially in three helicity states. It exploits the fact that the three helicity states of the leptonically decaying W boson behave differently in the two dimensional plane of observable lepton $p_T$ ($p_T^{\ell}$) and $\eta$ ($\eta^{\ell}$) [19].

This note describes an experimental implementation of this novel method of measuring the W boson production differentially in its helicity states, rapidity, and electric charge. In addition,
a measurement of the charge asymmetry as a function of $Y_W$ is presented. Furthermore, cross sections for $W$ boson production are provided as a function of the charged lepton kinematics in the two-dimensional plane of $p_T^{\ell}$ and $\eta^{\ell}$, unfolded to particle level.

This note is organized as follows. Section 2 gives a brief description of the CMS detector, followed by Section 3 detailing the data sample and the simulated samples used for this analysis. Section 4 summarizes the physics object and event selection. Section 5 describes the relevant backgrounds and the methods to estimate them. Section 6 explains the procedure to define the simulated 2D templates for $p_T^{\ell}$ and $\eta^{\ell}$, and the fitting strategy to perform the statistical analysis. The treatment of the systematic uncertainties is documented in Section 7. The results are presented in Section 8 and, finally, conclusions are drawn in Section 9.

### 2 The CMS detector

The central feature of the CMS apparatus is a superconducting solenoid of 6m internal diameter, providing a magnetic field of 3.8 T. A silicon pixel and strip tracker, a lead tungstate crystal electromagnetic calorimeter (ECAL), and a brass and scintillator hadron calorimeter (HCAL), each composed of a barrel and two end sections reside within the solenoid volume. Muons are measured in gas-ionization detectors embedded in the steel flux-return yoke outside the solenoid. Extensive forward calorimetry complements the coverage provided by the barrel and end detectors. A more detailed description of the CMS detector can be found in Ref. [20].

Events of interest are selected using a two-tiered trigger system [21]. The first level (L1), composed of custom hardware processors, uses information from the calorimeters and muon detectors to select events at a rate of around 100 kHz within a time interval of less than 4 $\mu$s. The second level, known as the high-level trigger (HLT), consists of a farm of processors running a version of the full event reconstruction software optimized for fast processing, and reduces the event rate to around 1 kHz before data storage. In this note the definition “online” refers to quantities computed either in the L1 or in the HLT processing, while “offline” refers to the ones evaluated later on the recorded events.

### 3 Data sample and simulated samples

The measurement is based on a data sample corresponding to an integrated luminosity of 35.9 fb$^{-1}$ of proton-proton collisions at a center-of-mass energy of 13 TeV recorded by the CMS experiment at the LHC during 2016.

Candidate events are selected with single-lepton triggers, with online $p_T$ thresholds of 24 GeV for muons and 27 GeV for electrons. For electrons, a higher threshold (up to about 40 GeV) for the level-1 hardware trigger was in place during the second half of the 2016 data taking period. These higher thresholds were present in the periods of highest instantaneous luminosity at the beginning of the LHC fills. Due to the higher thresholds for electrons the data sample for electrons is considerably smaller than that for muons and require a careful modelling of the trigger efficiencies as a function of electron $p_T$. Identification and isolation criteria are applied for these triggers, in order to suppress backgrounds before full event reconstruction.

Multiple Monte Carlo (MC) event generators are used to simulate the signal and background processes. The signal sample of $W$+jets is simulated at next-to-leading order (NLO) in perturbative QCD with the MADGRAPH5_aMC@NLO event generator [22]. Relevant background processes are simulated with MADGRAPH5_aMC@NLO ($Z \to \ell\ell$ and $W \to \tau\nu$) MADGRAPH5 (diboson and $t\bar{t}$ processes), as well as POWHEG 2.0 [23] (single-top processes). All simulated
4. Reconstruction and event selection

The analysis is performed by selecting $W \rightarrow \ell \nu$ candidate events characterized by a single prompt, energetic, and isolated lepton. A particle-flow algorithm [29] that aims to reconstruct all observable particles in the event is used. This algorithm classifies particles into either muons, electrons, photons, charged hadrons, or neutral hadrons. It optimally combines information from the central tracking system, energy deposits in the electromagnetic and hadronic calorimeters, and tracks in the muon detectors to reconstruct these individual particles and to determine quality criteria which are used to select the particles used in the distributions of the final state observables.

Muon candidates are required to have a transverse momentum $p^\mu_T > 26$ GeV and be within the geometrical acceptance, defined by $|\eta^\mu| < 2.4$. These values are chosen so that the inefficiency due to the trigger is minimal once the full selection is applied.

Quality requirements on the reconstructed muons are applied to ensure high purity of the selected events. These include requirements on the matching of the tracker information to the information from the muon system, as well as quality requirements on the combined track itself. In addition, a requirement on the relative isolation of the reconstructed muon is applied in order to suppress muons from background processes, such as leptonic heavy flavor decays. This isolation variable is defined as the pileup corrected ratio of the sum of the $p_T$ of all charged hadrons, neutral hadrons, and photons, divided by the $p_T$ of the muon itself. It is calculated for a cone around the muon of $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} < 0.4$ and it is required to be smaller than 15%.

Electron candidates are formed from energy clusters in the ECAL (called superclusters), with hits mostly spread along the $\phi$ direction, which are matched to tracks in the silicon tracker.

Electron identification is based on observables sensitive to the bremsstrahlung along the electron trajectory, and the geometrical and momentum-energy matching between the electron trajectory and the associated supercluster, as well as ECAL shower-shape observables and variables that allow for the rejection of the background arising from random associations of a track.
and a supercluster in the ECAL. Energetic photons produced in proton-proton collision may interact with the detector material and convert into electron-positron pairs. The electrons or positrons originating from such photon conversions are suppressed by requiring that there is no more than one missing tracker hit between the primary vertex and the first hit on the reconstructed track matched to the electron; candidates are also rejected if they form a pair with a nearby track that is consistent with a conversion. Additional details on electron reconstruction and identification can be found in [30, 31].

A relative isolation variable similar to that for muons is constructed for electrons, in a cone of $\Delta R < 0.3$ around the electrons. This variable is required to be less than a value that varies from around 20% in the barrel part of the detector to 8% in the endcap part. The values used are driven by similar requirements in the HLT reconstruction.

Offline selection criteria are generally equal to or tighter than the ones applied in the HLT. Despite this, differences in the definition of the identification variables defined in the online system and offline selection create differences between data and simulation that need dedicated corrections.

The analysis is carried out separately for $W^+$ and $W^-$, and aims to measure the charge asymmetry in $W$ production, so the charge misidentification has to be suppressed as much as possible. Thus the offline electron selection also employs a tight requirement for the charge definition, which reduces the charge misidentification to 0.02% (0.2%) in the barrel (endcap) regions in the $p_T$ range of interest [32].

Events coming from $W \rightarrow \ell\nu$ decays are expected to contain one charged lepton (electron or muon) and significant missing transverse energy resulting from the neutrino. The quantity $E_T^{\text{miss}}$ is defined as the magnitude of the vector sum of all reconstructed particle transverse momenta in the event. No direct requirement on $E_T^{\text{miss}}$ is applied but a requirement is placed on the transverse mass, defined as $m_T = \sqrt{2p_T^\ell E_T^{\text{miss}}(1 - \cos\Delta\phi)}$, where $\Delta\phi$ is the angle in the transverse plane between the directions of the lepton $p_T$ and the $E_T^{\text{miss}}$. Events are selected with $m_T > 40$ GeV. This requirement rejects a large fraction of QCD multijet backgrounds.

Events from background processes that are expected to produce multiple leptons, mainly $Z \rightarrow \ell\ell$, $t\bar{t}$, and diboson production, are suppressed by a veto on the presence of additional electrons or muons in the event. In order to maximize the rejection efficiency for these events they are rejected if additional leptons, identified with looser criteria than the selected lepton, have $p_T > 10$ GeV.

4.1 Efficiency corrections

The measurement of differential cross sections relies crucially on the estimation of the lepton efficiencies both in the collision data as well as in the MC simulation as this is among the dominant contributions to the uncertainty. For the total absolute cross sections the uncertainties are dominated by the luminosity uncertainty. In the case of normalized differential cross sections the correlation of the luminosity uncertainty between the inclusive and differential measurements is such that it mostly cancels out in their ratio, meaning that the dominant uncertainties are the ones related to the lepton efficiency that are not fully correlated through the lepton kinematics phase space.

The lepton efficiency is determined separately for three different steps in the event selection:

- trigger (L1+HLT);
- offline reconstruction;
The lepton efficiency for each step is determined with respect to the previous step.

A technique called tag-and-probe is used in which the efficiency for each step is measured for MC simulation and collision data using samples of $Z \to \ell\ell$ events with very high purity [33]. The sample is defined by selecting events with exactly two leptons. One lepton candidate, denoted as the *tag*, satisfies tight identification and isolation requirements. The other lepton candidate, denoted as the *probe*, is selected with the selection criteria that depend on the efficiency of the above steps being measured. The number of probes passing and failing the selection is determined from fits to the invariant mass distribution, with $Z \to \ell\ell$ signal and background components. The backgrounds in these fits stem largely from QCD multijet events and are at the percent level. In certain regions of phase-space, especially in the sample of failing probes, these backgrounds contribute significantly, requiring an accurate modeling of the background components. The nominal efficiency in collision data is estimated by fitting the Z signal using a binned template derived from simulation, convolved with a Gaussian function with a floating scale and width to describe the effect of detector resolution. An exponential function is used for the background. The nominal efficiency in MC simulation is derived from a simple ratio of passing probes over all probes.

Systematic uncertainties in efficiencies are derived by varying the signal and background models. The alternative signal shape is a Breit-Wigner with nominal Z boson mass and width convolved with an asymmetric resolution function (Crystal-Ball function [34]) with floating parameters. The alternative background is modeled by a function which models the phase space in the invariant mass for two leptons satisfying the minimum $p_T$ criteria applied.

For each step, the tag-and-probe method is applied to data and to simulated samples, and the efficiency is computed as a function of lepton $p_T$ and $\eta$. The ratio of efficiencies in data and simulation is computed together with the associated statistical and systematic uncertainties and is used to weight the simulated W boson events. The uncertainties in the efficiencies are propagated as a systematic uncertainty in the cross section measurements. The analysis strategy demands a very high granularity in the lepton kinematics for which the efficiencies are computed in slices of $\Delta \eta = 0.1$ and steps of $p_T$ ranging from 1.5 GeV to 5 GeV. A smoothing is applied as a function of lepton $p_T$ for each slice in $\eta$, modeled by an error function. Systematic uncertainties associated with this method are propagated to the measurement and are discussed in Sec. 7.1.3. These include a correlated component across $\eta$ and an uncorrelated component related to the statistical uncertainty in each of the slices in $\eta$.

### 5 Background estimation

The selection requirements described in Sec. 4 result in a data sample of $(114 \times 10^6) \ W^+ \ W^- \ W^+ \ W^-$ candidate events in data in the muon (electron) final state, with very little background. A summary of the inclusive background-to-signal ratios is shown in Table 1. The most significant residual background is QCD multijet production, where the selected non-prompt leptons stem from either semileptonic decays of heavy-flavor hadrons or are the product of misidentified jets (usually from light quarks). The former is the principal source of QCD background in the muon channel; the latter dominates the background in the electron channel, along with the production of electron-positron pairs from photon conversions.

The non-prompt lepton background is estimated directly from data. A control sample (the *application* sample) is defined by one lepton candidate that fails the standard lepton selection
criteria, but passes a looser selection. The efficiency, $\epsilon_{\text{pass}}$, for such a loose lepton object to pass the standard selection is determined using another independent sample (the QCD enriched sample) dominated by events with non-prompt leptons from QCD multijet processes. This QCD enriched sample, that is disjoint to the signal sample by means of the requirement $m_T < 40 \text{ GeV}$, is defined by one loosely identified lepton and a jet with $p_T > 45 \text{ GeV}$ recoiling against it. The measured efficiency for the leptons in this sample, parameterized as a function of $p_T$ and $\eta$ of the lepton, is then used to weight the events in the application sample by $\epsilon_{\text{pass}}/(1 - \epsilon_{\text{pass}})$, to obtain the estimated contribution from the non-prompt lepton background in the signal region. The $\epsilon_{\text{pass}}$ is computed with granularity of $\Delta \eta = 0.1$, and in each $\eta$ bin it is parameterized as a linear function of $p_T$.

The systematic uncertainties from the determination of $\epsilon_{\text{pass}}$ dominate the overall uncertainty of this method. The systematic uncertainty has two sources: the dependence of $\epsilon_{\text{pass}}$ on the sample composition, and the method itself. The first source is estimated by modifying the jet $p_T$ threshold in the QCD enriched sample, which modifies the sample composition by changing the fractions of gluon- or quark-induced jet. In addition, the parameters of the linear fit as a function of $p_T$ are varied within their measured uncertainties to get an alternative weighting. The uncertainty in the method itself is estimated from a closure test on a background dominated region, obtained by inverting the $M_T$ requirement. The differential agreement in the two-dimensional ($p_{T\ell}, \eta_{\ell}$) plane is rather good in both electrons and muons, and varies with lepton $\eta$ and $p_T$, being within 20% for both the electron and muon channel in the whole kinematic range and better than 10% for lepton $p_T > 30 \text{ GeV}$. The level of agreement in the background dominated region is used as an estimate of the normalization systematic of this process in the signal extraction described in Sec. 7. In the case of electrons, where this background is larger than in the muon case, the central value of the QCD background is re-scaled by the values derived in this closure test.

A small fraction of the events passing the selection criteria are due to other electroweak processes and this contribution is estimated from simulation. Drell-Yan events in which the $Z$ boson decays to a pair of muons or electrons and one of the two leptons falls outside the detector acceptance mimic the signature of $W$ events rather closely. A smaller effect from Drell-Yan production stems from $Z \rightarrow \tau \tau$ decays where one $\tau$ lepton decays leptonically and the other hadronically. Additionally, events from $W \rightarrow \tau v$ decays are treated as background in this analysis. The light leptons from the $\tau$ decays exhibit in general lower $p_T$ lepton than signal events and are strongly suppressed by the minimum $p_T$ requirements. Other backgrounds arise from $t\bar{t}$ and single-top production, with one of the top quarks producing a $W$ that subsequently decays leptonically. There are small contributions to the background from diboson ($WW, WZ, ZZ$) production. Finally, for the electron channel only, the background from $W \rightarrow e\nu$ where the lepton is reconstructed with the wrong charge is considered. This background is completely negligible for the muon final state.

## 6 Template construction and fitting procedure

The measurement strategy is to fit 2D templates in the charged lepton kinematic observables of $p_{T\ell}$ versus $\eta_{\ell}$ to the observed 2D distribution in data. While each of the background processes will result in a single such template, the simulated $W$ boson signal is divided into its three helicity states as well as into slices of the underlying $W$ boson rapidity, $|Y_W|$. The procedure of constructing these helicity-rapidity templates is described below.
Table 1: Estimated background-to-signal ratios in the $W \rightarrow \mu \nu$ and $W \rightarrow e\nu$ channels. The DY simulation includes $\ell = e, \mu, \tau$.

<table>
<thead>
<tr>
<th>Processes</th>
<th>Bkg. to sig. ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow \ell\ell$ (DY)</td>
<td>5.2% 3.9%</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>3.2% 1.3%</td>
</tr>
<tr>
<td>$WW+WZ+ZZ$</td>
<td>0.1% 0.1%</td>
</tr>
<tr>
<td>Top</td>
<td>0.5% 0.5%</td>
</tr>
<tr>
<td>charge flips</td>
<td>- 0.02%</td>
</tr>
<tr>
<td>QCD</td>
<td>5.5% 8.2%</td>
</tr>
</tbody>
</table>

6.1 Construction of helicity and rapidity signal templates

The inclusive W boson production cross section at a hadron collider, with its subsequent leptonic decay, neglecting the small terms which are exclusively NLO, is given by [35]:

$$\frac{dN}{d\cos\theta^* d\phi^*} \propto \left(1 + \cos^2\theta^*\right) + \frac{1}{2}A_0(1 - 3 \cos^2\theta^*) + A_4 \sin 2\theta^* \cos \phi^* + \frac{1}{2}A_2 \sin^2\theta^* \cos 2\phi^* + A_3 \sin \theta^* \cos \phi^* + A_4 \cos \theta^*. \quad (1)$$

In the Collins-Soper frame of reference, the angles $\theta^*$ and $\phi^*$ are the angles between the lepton and W boson directions of motion, where the lepton refers to the charged lepton in the case of $W^-$, and the neutrino in the case of $W^+$ [36]. The angular coefficients $A_0$ to $A_4$ in Eq. 1 depend on the W boson charge, $p_W^T$ and $Y_W$, and receive contributions from QCD at leading and higher orders. When integrating Eq. 1 over $\phi^*$, the cross section is written as:

$$\frac{dN}{d\cos\theta^*} \propto \left(1 + \cos^2\theta^*\right) + \frac{1}{2}A_0(1 - 3 \cos^2\theta^*) + A_4 \cos \theta^*. \quad (2)$$

This expression can equivalently be written as a function of the helicity amplitudes [37], given by:

$$\frac{1}{N} \frac{dN}{d\cos\theta^* dp^W_T dY_W} = \frac{3}{8}(1 \pm \cos \theta^*)^2 \cdot f_L(p_T^W, Y_W) + \frac{3}{8}(1 \pm \cos \theta^*)^2 \cdot f_R(p_T^W, Y_W) + \frac{3}{4} \sin^2 \theta^* \cdot f_0(p_T^W, Y_W), \quad (3)$$

where the coefficients $f_\mu$ and the term $\cos \theta^*$ correspond to the helicity fractions and decay angle, and the upper (lower) sign corresponds to $W^+$ ($W^-$) bosons, respectively. Thus, the fractions of left-handed, right-handed, and longitudinal W bosons ($f_L, f_R$ and $f_0$, respectively) are related to the coefficients $A_i$ of Eq. 2 by $A_0 \propto f_0$ and $A_4 \propto (f_L - f_R)$ depending on the W boson charge, where by definition $f_1 > 0$ and $f_L + f_R + f_0 = 1$. The generated leptons are considered before any final state radiation (“pre-FSR leptons”).

As there is no helicity information in the simulated MC signal sample, a reweighting procedure is implemented based on the production kinematics of the W boson, and the kinematics of the leptonic decay of the W boson.
The coefficients $f_i$ depend strongly on the production kinematics of the $W$ boson, namely its $p_T^W$, its rapidity, $|Y_W|$, and its charge. Therefore, a reweighting procedure is devised in which the $\cos \theta^*$ distribution is fitted in bins of $p_T^W$ and $|Y_W|$ separately for each charge to extract the predicted $f_i$. Each simulated event is reweighted three separate times to obtain pure samples of left-handed, right-handed, and longitudinally polarized $W$ bosons. The results of this procedure are illustrated in Fig. 1, where the simulated signal is split into the three helicity states by reweighting of the extracted helicity fractions $f_i$. Distributions for $p_T^W$ and $|Y_W|$ are shown for both charges of $W$ bosons along with the resulting distribution of the charged lepton $\eta$.

The distributions of $p_T^W$ and $|Y_W|$ are vastly different for the three helicity components. While the $W_L$ and $W_R$ components behave the same as a function of $p_T$, their behavior in $|Y_W|$ is very different. Their production cross section is equal at $|Y_W| = 0$, but the $W_L$ component increases up to a maximum at $|Y_W|$ between 3 and 3.5, whereas the $W_R$ component decreases monotonically with higher $|Y_W|$. The $W_0$ component has an overall much lower production cross section, which is relatively flat in $|Y_W|$ and increases as a function of $p_T$, as expected in the Collins-Soper reference frame. The very different distributions in $|Y_W|$ of the $W_R$ and $W_L$ components paired with the preferential decay direction of the charged lepton for these two helicity states results in distinctly different $\eta^\ell$ distributions. For positively charged $W$ bosons at a given $|Y_W|$, the $W_L$ component causes the charged lepton to have values of $\eta^\ell$ closer to zero. In contrast, the positively charged $W_R$ component tends to have larger values of $|\eta^\ell|$. For negatively charged $W$ bosons the opposite is true, i.e., the charged lepton $|\eta^\ell|$ will tend to be large for left-handed $W$ bosons, whereas right-handed $W^-$ bosons lead to leptons observed mostly at small values of $|\eta^\ell|$.

Figure 1: Distributions of the $W$ boson $p_T^W$ (left), $|Y_W|$ (center), and the resulting $\eta$ distribution of the charged lepton (right) after reweighting each of the helicity components for positive (top row) and negative charge (bottom row) $W$ bosons.
6.2 Fitting strategy for the rapidity-helicity measurement

The characteristic behavior of the lepton kinematics for different polarizations of the W boson can be exploited to measure the cross section for W boson production differentially in $|Y_W|$ and the three helicity components. This is done by splitting each of the three helicity states into bins of $|Y_W|$ and constructing the charged lepton $p_T^\ell$ versus $\eta^\ell$ templates for each of the helicity and charge components. An example of a 2D templates is shown in Fig. 2, where three different templates are shown for $W^+$ bosons. The blue template is obtained from $W^+_R$ produced from 0 to 0.25 in $|Y_W|$, the red template from $W^+_R$ produced between 0.50 and 0.75 in $|Y_W|$, and the green template from $W^+_L$ produced between 2.00 and 2.25 in $|Y_W|$. The behavior described above can clearly be seen. Another important aspect of the underlying physics can also be understood from Fig. 2: while the W bosons are produced in orthogonal regions of phase space, the resulting templates for the observable leptons overlap considerably for the different helicity and rapidity bins. This overlap is most striking for adjacent bins in $|Y_W|$ in a given helicity state. In Fig. 2, the two right-handed W boson and the left-handed W boson distributions show sizeable overlap, albeit with contrasting shapes as a function of the observable lepton kinematics. A consequence of the large overlaps in general, and in neighboring bins in rapidity in particular, are large (anti-)correlations in the fitted differential cross sections in helicity and rapidity.

As indicated by Fig. 2, the 2D templates in the observable lepton kinematics extend from the minimum $p_T^\ell$ requirement of 26 (30) GeV for muons (electrons) to a maximum value of 45 GeV in bins with width of 1 GeV. In the observable $\eta^\ell$, the width of the bins is 0.1, extending from $-2.4$ ($-2.5$) to 2.4 (2.5) for muons (electrons).

In order to extract the differential cross sections in W rapidity for the three helicity states, the full sample of simulated W boson events is divided using the method described earlier into the three helicity components and 10 bins of $|Y_W|$ of width 0.25 up to $|Y_W| = 2.5$. These separate signal processes are left freely floating in a maximum likelihood (ML) fit to the observed 2D distribution for $p_T^\ell$ versus $\eta^\ell$. All events above the threshold $|Y_W| = 2.5$ are fixed to the prediction from MC and are treated as background due to the rapid loss in acceptance for certain
charge and helicity combinations. Additionally, the longitudinal polarization states are fixed to the MC prediction. This results in 40 freely floating cross sections in the fit, corresponding to the 10 bins in W boson rapidity for each charge and left and right polarizations.

6.3 Fitting strategy for the double-differential W boson cross section

The double differential W boson cross sections, as a function of $p_T^\ell$ and $|\eta^\ell|$, are measured with an analogous technique. The double differential cross section for each charge of the W boson is denoted by

$$\sigma^\pm = \frac{d\sigma}{d|\eta|^d p_T^\ell} (pp \rightarrow W^\pm + X \rightarrow \ell^\pm + X),$$

and can be measured in very fine bins of $|\eta^\ell|$ and $p_T^\ell$. Current theoretical calculations predict these cross sections with next-to-next-to-leading-order (NNLO) accuracy in perturbative QCD, and such a measurement is a more rigorous test of these calculations than the previous studies performed by the CDF and D0 collaborations at the Tevatron pp collider [10, 11], or by the ATLAS, CMS and LHCb collaborations at the LHC [12–18], which all measured the cross section as a function of reconstructed $|\eta^\ell|$ only. The CDF collaboration has also inferred the charge asymmetry as a function of $|Y_W|$ in [10]. When integrating either in the $|\eta^\ell|$ or in the $p_T^\ell$ dimension, the classical differential cross section measurement can be recovered.

This measurement is performed by fitting the same 2D distributions of $p_T^\ell$ versus lepton $\eta^\ell$, with different signal processes. Instead of constructing each signal template from an underlying $|Y_W|$ and helicity state, each signal process in this measurement corresponds to the underlying generated lepton $p_T$ and lepton $|\eta|$ bin. The generated leptons in this measurement are obtained by a so-called “dressing” procedure, where electroweak radiation is added back to the charged lepton momentum within a cone of $\Delta R < 0.1$. The unfolding takes bin-by-bin differences in generated versus reconstructed $p_T^\ell$ and $|\eta^\ell|$ into account. The resulting number of underlying signal processes increases from the 40 processes in the helicity/rapidity fit to a total of 324, corresponding to 18 bins in the $p_T^\ell$ times 18 bins in $|\eta^\ell|$. The generated $p_T^\ell$ ranges from 26 GeV to 56 GeV in bins of width between 1.5 GeV and 2 GeV, where the bins at low and high values of $p_T^\ell$ are wider than around the Jacobian peak. The bin width in $|\eta^\ell|$ is 0.1 up to $|\eta^\ell| = 1.3$, followed by 4 bins of width 0.2, and a final bin ranging from $|\eta^\ell| = 2.1$ to 2.4. Events in which the generated leptons are outside of the reconstructed acceptances are treated as a background component in this fit. The treatment of the backgrounds and the systematic uncertainties remains the same as for the rapidity/helicity fit.

6.4 Likelihood construction and fitting

A ML fit is performed to extract the parameters of interest. The software package in which this ML fit is implemented is based on the TENSORFLOW software package originally developed for machine learning applications [38]. The benefit of such an implementation is the native support of analytic gradients in TENSORFLOW, as well as the ability to parallelize the fits. This gives a major improvement over previous implementations of profiled ML fits in terms of speed, memory consumption, and numerical stability.

The negative log-likelihood function can be written as

$$L = -\ln(L(\text{data} | \vec{\mu}, \vec{\theta})) = \sum_i \left(-n_i^{\text{obs}} \ln n_i^{\exp}(\vec{\mu}, \vec{\theta}) + n_i^{\exp}(\vec{\mu}, \vec{\theta}) \right) + \frac{1}{2} \sum_k (\theta_k - \theta_0^k)^2$$

$$n_i^{\exp}(\vec{\mu}, \vec{\theta}) = \sum_p \mu_p n_{i,p}^{\exp} \prod_k \kappa_{i,p,k}$$
6. Template construction and fitting procedure

where: \( n_{i,p}^{\text{exp}} \) is the expected yield per bin per process; \( \mu_p \) is the freely floating signal strength multiplier per signal process, fixed to unity for background processes; \( \theta_k \) are the nuisance parameters associated with each systematic uncertainty; and \( \kappa_{i,p,k} \) is the size of the systematic effect per bin, per process, per nuisance. The systematic uncertainties are implemented with a unit Gaussian constraint on the nuisance parameter \( \theta_k \) such that the factor \( \kappa_{i,p,k} \) multiplying the yield corresponds to a log-normal distribution with mean-parameter 0 and width-parameter \( \ln \kappa_{i,p,k} \). This parameterization corresponds to the one used by the LHC combination working group for the Higgs boson search and measurements [39].

The signal strength modifiers are extracted directly from the ML likelihood fit. The uncertainties and the covariance matrices are obtained from the test statistic \( q(\bar{\mu}) \), defined as

\[
q(\bar{\mu}) = -2 \cdot \log \left( \frac{\mathcal{L}(\bar{\mu}|\hat{\theta})}{\mathcal{L}(\hat{\mu}|\hat{\theta})} \right),
\]

where \( \hat{\theta} \) represents the maximum likelihood estimators of a given set of parameters \( \bar{\alpha} \), with \( \alpha = \mu, \theta, \bar{\nu} \). The unfolded cross sections are extracted simultaneously in the ML fit by correlating the dependence of the true cross section, \( \bar{\sigma}(\bar{\theta}) \) and extracting the interpolated cross section at the values of the nuisance parameters \( \hat{\theta} \) corresponding to the minimum of the negative-log-likelihood.

While the cross section vectors \( \bar{\sigma} \) are left freely floating when fitting for the rapidity/helicity or the double-differential cross sections, it is also possible to fix these parameters to their expected values. Performing the fit in such a way allows for the direct measurement of the constraints that are being set by the data on every nuisance parameter. This is especially interesting for the case of the PDF uncertainties, as the large and quite pure selected sample of W bosons can place strong constraints on the PDF uncertainties by using the charged lepton kinematics.

6.5 Measurement of the charge asymmetry and unpolarized cross sections

The fit to the data is performed simultaneously for the two charge categories and to the three helicity states. Therefore, the minimization can yield combinations of the measured cross sections with the proper propagation of the uncertainties through the fit covariance matrix, differentially either in rapidity or double-differentially in \( p_T^\ell \) and \( |\eta^\ell| \).

One of the additional quantities considered is the polarized W boson charge asymmetry, defined as:

\[
A^{\text{pol}}(|\eta_W|) = \frac{d\sigma^{\text{pol}} / dY_W(W^+ \rightarrow \ell^+ \nu) - d\sigma^{\text{pol}} / dY_W(W^- \rightarrow \ell^- \bar{\nu})}{d\sigma^{\text{pol}} / dY_W(W^+ \rightarrow \ell^+ \nu) + d\sigma^{\text{pol}} / dY_W(W^- \rightarrow \ell^- \bar{\nu})},
\]

where \( \text{pol} \) represents the W polarization state. The charge asymmetry, unfolded into \( |\eta_W| \) through the ML fit, differentially in the three polarizations, provides a more direct constraint on the PDF than the previous measurements performed at LHC, which are performed differentially in the reconstructed lepton pseudorapidity [12, 16]. In the CDF collaboration measurement [10] the W boson charge asymmetry was unfolded in \( |\eta_W| \), but inclusively in the W boson helicity state.

The charge asymmetry of W bosons, which can also be determined from the double-differential cross section measurement, is written as:

\[
A(|\eta^\ell|, p_T^\ell) = \frac{d\sigma^+ / d|\eta^\ell| d p_T^\ell - d\sigma^- / d|\eta^\ell| d p_T^\ell}{d\sigma^+ / d|\eta^\ell| d p_T^\ell + d\sigma^- / d|\eta^\ell| d p_T^\ell}.
\]
If the distribution is integrated over $p_T^\ell$ the results can be compared directly with previous measurements of $A(|\eta|^\ell)$ at hardon colliders. Similarly, when integrating over $|\eta|^\ell$, the $A(p_T^\ell)$ can be found. These 1D distributions as functions of $p_T^\ell$ and $|\eta|^\ell$ are obtained by integrating over the other variable after performing the fully differential 2D fit. Associated uncertainties are taken into account properly from the full 2D covariance matrix of the fit.

7 Systematic uncertainties

This section describes the treatment of systematic uncertainties from experimental sources as well as from modeling and theory uncertainties. In general, systematic uncertainties are divided into two types: those affecting only the normalization of the templates and those affecting the shape of the templates.

Normalization uncertainties are treated as log-normal nuisance parameters acting on a given source of background or signal. They are allowed to change the overall normalization of the process by the quoted value, while retaining the relative contributions of the process in each of the lepton $p_T$ versus lepton $\eta$ bins.

Shape uncertainties do the exact opposite. While the integral of a background or signal component is kept constant at the central value, the relative shape of the 2D template is allowed to float. This necessitates both an up and down variation of each shape nuisance parameter. This is equivalent to a log-normal uncertainty on a bin-by-bin basis.

Uncertainties are also allowed to be a combination of the two, i.e. to change the normalization as well as the shape of the 2D templates simultaneously.

7.1 Experimental uncertainties

7.1.1 QCD multijet background

The QCD multijet background is estimated from data sidebands in the lepton identification and isolation variables, as described in Section 5. A normalization uncertainty is applied to this background estimation, in coarse bins of $|\eta|^\ell$. Its size is derived from a closure test in the background dominated region enriched in QCD multijets constructed by inverting the requirement on the transverse mass of the lepton and the missing transverse momentum ($m_T < 40/30$ GeV for the $\mu/e$ channel). It amounts to about 5% in the muon final state for all the $|\eta|^\ell$ bins, and 0.5% to 5% in the electron final state, with larger uncertainties at higher $|\eta|^\ell$. The smaller uncertainty for electrons is related to the increased size of the fake-lepton control sample. Each of these normalization uncertainties is treated as uncorrelated to the others.

A systematic uncertainty in the normalization of the QCD multijet background is also estimated by a closure test in the background dominated region in bins of $p_T^\ell 3$ GeV (5 GeV) wide. The uncertainties range from 30% to 15% (10% to 20%) depending on the $p_T^\ell$ region for the muon (electron) final state. These normalization uncertainties are also considered uncorrelated among each other.

The closure test is also evaluated for the two charges separately, weighting the events with the charge independent $\epsilon_{\text{pass}}$ misidentification probability. The two estimates are found to be consistent within the uncertainties, with a very similar dependency on the lepton $\eta$ and $p_T$. A further check has been carried out, by computing a charge-dependent $\epsilon_{\text{pass}}^\pm$. Based on these checks, we introduce in the muon case, an additional charge-dependent uncertainty of 2%, in the same coarse bins of $|\eta|^\ell$, and no additional uncertainty for electrons.
Another shape uncertainty is applied to cover for the uncertainty in the extraction of the QCD multijet efficiency $\epsilon_{\text{pass}}$. This lepton misidentification rate, $\epsilon_{\text{pass}}$, is extracted through a linear fit to the $p_T$ of the lepton, which has an uncertainty associated to it. While a variation of the offset parameter of this fit is absorbed by the normalization uncertainty, the linear parameter of the fit is varied, which therefore varies the QCD multijet background as a function of $p_T^{\ell}$. This uncertainty is applied in the same uncorrelated bins of $|\eta|$ as for the normalization uncertainty.

In total, 46 (55) nuisance parameters that affect the QCD multijet background estimation are considered for each charge of the muon (electron) final state.

### 7.1.2 Lepton momentum scale

The lepton momentum scales are calibrated and corrected using the Z boson standard candle. In the muon case, the non-closure of the momentum scale obtained with fits to the $Z \rightarrow \mu\mu$ invariant mass is found to be of the order of $10^{-4}$. For such a precision, a detailed nuisance model has been implemented to cover residual effects [40] that can remain after the calibration procedure has been applied.

Systematic uncertainties in the derivation of the muon momentum scale corrections are considered. These uncertainties are related to: the modeling of the Z transverse momentum, electroweak effects on the Z line shape and the effect of the acceptance on the dimuon invariant mass. These uncertainties are finely grained in muon $\eta$ and $p_T$. Furthermore, the uncertainty in the limited data and simulated Z sample is estimated from 100 statistical replicas of the two datasets. Every replica is constructed from a subset of the total event ensemble via a bootstrap procedure. Each of them is also finely binned in muon $\eta$ and $p_T$. The 99 independent statistical uncertainties are diagonalized with the procedure of Ref. [41] and their independent contributions are included as shape nuisance effects.

For electron candidates, the observed residual differences in the energy scales for the data and the simulated Z sample are of the order of $10^{-3}$. A similar procedure to that used for the muon momentum scale is adopted. Two systematic effects are considered in fine bins of $\eta^e$ and $p_T^e$. One is the difference in the Z mass value obtained by fitting the mass peak for $Z \rightarrow e^+e^-$ events in two different ways. The first fit is to a MC template convolved with a Gaussian resolution function and the second to a functional form made of a Breit-Wigner lineshape for a Z boson convolved with a Crystal-Ball function, with floating mean and width parameters. The effect is the main contribution to the systematic uncertainty, and ranges from 0.1% to 0.2% for $p_T^e < 45$ GeV and 0.2–0.3% at higher values. The second, smaller systematic effect comes from the modeling of the Z transverse momentum. As for the muon case, the limited statistics of the samples used to derive the energy scale corrections is accounted for by 100 replicas of the data and MC, diagonalized to get 99 independent nuisance parameters.

For both lepton flavors the precision in the estimate of the momentum scale decreases when increasing $\eta^\ell$. The W boson sample with a lepton in the more forward regions of the detector still has sufficient statistical power to allow the fit to constrain the momentum scale nuisance parameters. If the systematic effect related to the momentum scale is fully correlated across the full $\eta^\ell$ acceptance, then an eventual constraint of it in the profiling procedure driven by the large effect on the templates at high $\eta^\ell$ may result in an unphysical constraint in the central region. This is avoided by de-correlating the nuisance parameters related to the various momentum scale systematics in wide bins of $\eta^\ell$, for both muons and electrons. In contrast, the parameters relating to the statistical part of this uncertainty are kept fully correlated across $\eta^\ell$.

Given that the systematic uncertainty in the momentum scale of the leptons allows for the
\( p_T \) of a lepton to be changed and therefore for bin-to-bin migration, it is applied as a shape uncertainty.

### 7.1.3 Lepton efficiency scale factors

Data-to-MC efficiency scale factors are derived through a tag-and-probe method, also using \( Z \rightarrow \ell\ell \) events. Two systematic uncertainties are considered. The first uncertainty comes from the scale factor and depends on the functional forms used to describe the background and signal components when fitting the efficiencies in each bin of \( \eta^\ell \) as a function of \( p_T^\ell \) of the probe lepton. Alternative fits are performed by using different models for the dilepton invariant mass lineshape for either the \( Z \) events or for the combinatorial background events, resulting in different efficiencies. This systematic change is assumed to be correlated among all bins in \( \eta^\ell \). The size of this uncertainty ranges from a few per mille at low values of \( \eta^\ell \), to around 1-2% in the very forward region. Another uncertainty in the lepton efficiency scale factors that is uncorrelated between \( \eta^\ell \) bins arises from the statistical uncertainties in the size of the Z sample in each \( \eta^\ell \) bin. These uncertainties are derived by varying the parameters of the error function, used to interpolate between the measured efficiency values as a function of \( \eta^\ell \), described in Section 4.1, by their uncertainties. This introduces three nuisance parameters for each bin in \( \eta^\ell \), resulting in a total of 144 (150) nuisance parameters per charge in the muon (electron) final state. These systematics are considered uncorrelated in each charge, since they are measured independently for each charge. An additional uncertainty in the trigger efficiency is considered for events with electrons in the endcap region of the detector. This uncertainty is due to a radiation-induced shift in the ECAL timing in the 2016 data taking period, which led to early event readout (referred to as pre-firing) in the L1 trigger and a resulting reduction in efficiency for events with significant energy deposits in the electromagnetic forward ECAL. The correction is estimated using a set of \( Z \rightarrow e^+e^- \) from unbiased triggers, which is statistically limited. The uncertainty ranges from 0.5% for \(|\eta| \approx 1.5\) to 10% at \(|\eta| \approx 2.5\) for electrons from W decays.

### 7.1.4 Extra lepton veto

The efficiency for the requirement that only one lepton be present in the event, which is especially effective in reducing the \( Z \rightarrow \ell\ell \) background, is also affected by differences in the efficiencies in data and in MC. As more background survives the selection at higher \(|\eta^\ell|\), where the uncertainty in the efficiency is larger, a conservative normalization uncertainty is applied, equal to 2% (3%) for the muon (electron) channel. In the electron channel, an additional uncertainty is included to account for the L1 trigger pre-firing effect, described previously, in \( Z \rightarrow e^+e^- \) events in which one electron is in the ECAL endcap. This uncertainty ranges from 2% at low electron \( p_T \), and is up to 10% in the highest \(|\eta^\ell|\) and \( p_T^\ell \) bins.

### 7.1.5 Charge misidentification

The probability of a charge flip for a muon in the \( p_T^\ell \) range considered is found to be negligible \((10^{-5})[42]\), thus no uncertainty is introduced to describe this effect. For the electrons, the statistical uncertainty in the estimate of the charge flips in \( Z \rightarrow e^+e^- \) events reconstructed with same-sign or opposite-sign events is used. It is dominated by the limited sample of same-sign events in the 2016 dataset [32]. The uncertainty assigned to this small background component in the electron channel only is 30%.
7. Systematic uncertainties

7.1.6 Integrated luminosity

A normalization uncertainty is assigned to the imperfect knowledge of the integrated luminosity. This is applied as an overall normalization uncertainty in all processes estimated from MC simulation and is set at a value of 2.5% [43].

7.2 Modeling and theory uncertainties

7.2.1 Missing higher orders in QCD

Theory uncertainties resulting from missing higher orders in the QCD calculations are implemented in the following way. Renormalization and factorization scales, \( \mu_R \) and \( \mu_F \), respectively, are changed to half and twice their original value. This change is propagated to the resulting weight for each simulated event in three variations: the uncorrelated ones in which either \( \mu_R \) or \( \mu_F \) is varied, and the correlated one in which both are varied simultaneously, but in the same direction, i.e. both up or down by a factor of two. This uncertainty is applied to all signal processes as well as the \( Z \to \ell\ell \) background. For the signal processes, these variations lead to a normalization shift that is largely independent of \( \eta^Z \). The impact on the shape of the \( p_T \) distribution is minimal up to \( p_T < 35 \text{ GeV} \), however, for \( p_T > 35 \text{ GeV} \) a significant modification of the predicted \( p_T \) distribution is seen. These uncertainties are allowed to change both the normalization and the shape of the overall 2D templates. In the case of the signal, they are split into several components, as described below, to account for the uncertainty in the \( p_T \) distribution.

7.2.2 \( p_T^W \) modeling

Improper knowledge of the \( p_T^W \) spectrum results in an uncertainty that affects the \( p_T^{\ell} \) spectrum. It is most important in the region of low \( p_T^W \), a region in which fixed-order perturbative calculations lead to divergent cross sections as \( p_T^W \) approaches zero, which requires resummation. The nominal templates are evaluated from the \textsc{MadGraph5}_\textsc{aMC@NLO} simulated sample, with the \( p_T^W \) spectrum reweighted to the \( p_T^Z \) distribution measured in data as described in Section 3. Uncertainties are associated to the \( p_T^W \) modeling in such a way as to reduce the sensitivity to the theoretical prediction, at the cost of increasing the statistical uncertainty of the results. Namely, the uncertainties in \( \mu_R \) and \( \mu_F \), described above, are binned in ten bins in \( W p_T: [0.0, 2.9, 4.7, 6.7, 9.0, 11.8, 15.3, 20.1, 27.2, 40.2, 13000] \) GeV. These nuisance parameters are uncorrelated for each charge. In the case of the polarized cross section measurement an uncorrelated uncertainty is used for each helicity state to account for the different production mechanisms of the longitudinal, left- and right-handed W bosons. The \( \mu_R \) and \( \mu_F \) uncertainties in the \( W \to \tau\nu \) process are binned in the same \( p_T^W \) bins, albeit integrated in polarization, and so are uncorrelated with the signal processes.

7.2.3 Parton distribution functions

Event weights in the MC simulations that account for 100 variations of the NNPDF3.0 PDF set, referred to as replica sets, are used to evaluate the PDF uncertainty in the predictions. These 100 replicas are transformed to a Hessian representation, in order to facilitate the treatment of PDF uncertainties in the analysis via the procedure described in Ref. [41], with 60 eigenvectors and a starting scale of 1 GeV. Because the PDFs determine the kinematics and the differential polarization of the W boson, variations of the PDFs alter the relative contribution of the W boson helicity states in \( p_T^W \) and \( |Y_W| \). Thus, the reweighting of the signal templates described in Section 6.1 is repeated independently for each of the 60 Hessian variations. Each signal
process is reweighted once for each of the 60 independent variations as the up variation. The corresponding down variation is obtained by mirroring the up variation with respect to the nominal template. Given that the underlying PDF uncertainties also affect the DY and $W \rightarrow \tau \nu$ backgrounds, the same procedure is applied to the simulated events for these backgrounds, and the uncertainties are treated as fully correlated between the signal and these two background processes. Both a change of the overall normalization of the templates as well as a change in shape results from this procedure. The magnitude of the Hessian variations are 1% or lower for the normalization, but show very different behavior in the $p_T$ versus $\eta$ plane, from which a constraint on these PDF uncertainties is expected.

### 7.2.4 Choice of $\alpha_S$ value

The 100 PDF replicas of the NNPDF3.0 set are accompanied by two variations of the strong coupling constant. The central value of $\alpha_S = 0.1180$ is varied from 0.1195 to 0.1165. Both normalization and shape are affected by this variation.

### 7.2.5 Simulated background cross sections

The backgrounds derived from simulation, namely Drell-Yan production, diboson production, $W \rightarrow \tau \nu$, and all top quark backgrounds are subject to an overall normalization-only uncertainty. The main contributions to the theory uncertainty in the Z and W production cross section arises from PDF uncertainties, $\alpha_S$ and $\mu_R$ and $\mu_F$. These are accounted for as shape nuisance parameters affecting the templates of such processes, and they are fully correlated with the same parameters affecting the signal. For the $W \rightarrow \tau \nu$ process a further 4% normalization uncertainty is assigned, to address the residual uncertainty due to the much lower $p_T$ of the decay lepton, and different phase space of the rest of the event.

For the top quark and diboson backgrounds, the kinematic distributions are well modeled by the higher-order MC generators. The uncertainties assigned to the normalization are 6% and 16% respectively, motivated by the large theoretical cross section uncertainty for each of the contributing processes. Because these processes make a small contribution to the selected sample of events, the effect of these relatively large uncertainties is small.

### 7.2.6 Choice of the $m_W$ value

The central value of the W boson mass $m_W = 80.419$ GeV is used as the nominal value in the simulation. Events are reweighted to two alternative values of $m_W$, with values $\pm 50$ MeV with respect the nominal, using a Breit-Wigner assumption for the invariant mass at generator level.

### 7.2.7 Modeling of QED radiation

The simulation of the signal processes models the lepton FSR through the QED showering in PYTHIA 8 within the MADGRAPH5_aMC@NLO MC generator. An uncertainty in this modeling is assessed by considering an alternative showering program, called PHOTOS [44]. A large sample of $W \rightarrow \ell \nu$ ($\ell = e^+, e^-, \mu^+, \mu^-$ separately) has been produced at generator level only, interfacing a QCD NLO MC program to either PYTHIA 8 or PHOTOS. The variable sensitive to FSR, which accounts for the different radiation rate and, in case of radiation, for the harder FSR photon spectrum produced by PHOTOS with respect to PYTHIA 8, is the ratio $r_{FSR} = \frac{p_T^{\text{PHOTOS}}}{p_T^{\text{PYTHIA}}}$ between the dressed lepton $p_T$ and the bare lepton $p_T$ (after radiation). Alternative templates are built by reweighting the nominal MADGRAPH5_aMC@NLO events by the ratio between PHOTOS and PYTHIA 8 as a function of $r_{FSR}$. 
The effect of QED FSR is largely different for the two lepton flavors, because of the differences in the lepton masses and the estimate of the lepton momentum. For the electrons it is derived from a combination of the measurements using the track and the ECAL supercluster. The latter dominates the estimate for the energy range exploited in this analysis, and its reconstruction algorithm, optimized to gather the bremsstrahlung photons, also efficiently collects the FSR photons. For the muons only the track is used, and there is no explicit recovery of the FSR. For these reasons the nuisance parameters related to this effect are kept uncorrelated between the two lepton flavors.

7.2.8 Statistical uncertainty of the W simulation

An uncertainty is assigned to the limited size of the MC sample used to build the signal templates. The sample size, when considering the negative weights of the NLO corrections, corresponds to approximatively one fifth of the data sample. This is included in the likelihood with the Beeston-Barlow approach [45] and represents one of the dominant contributions to the systematic uncertainty.

A summary of the systematic uncertainties applied is shown in Table 2. They amount to 1176 nuisance parameters for the helicity fit.

7.3 Impact of uncertainties in the measured quantities

The breakdown of the effects of the systematic uncertainties on the measured quantities (signal strength modifiers for one process, $\mu_p$ in Eq. 5, absolute cross sections $\sigma_p$, or normalized cross sections $\sigma_p/\sigma_{tot}$) is presented as the impact of an uncertainty in the parameter of interest. It is defined as the shift that is induced as one nuisance parameter, $\theta_k$ in Eq. 5, is fixed and brought to its $+1\sigma$ or $-1\sigma$ post-fit values, with all other parameters profiled as normal. The procedure is applied to groups of uncertainties, gathered such that each group includes strongly correlated sources. Groups are defined for:

- **luminosity**: uncertainty in integrated luminosity,
- **efficiency stat.**: uncorrelated part (in $\eta^\ell$) of the lepton efficiency systematics,
- **efficiency syst.**: correlated part (in $\eta^\ell$) of the lepton efficiency systematics (coming from the tag-and-probe method), level-1 prefire uncertainty for the signal electron or the second electron from $Z \rightarrow e^+e^-$ events,
- **fakes**: includes both the normalization and shape uncertainties related to the background from QCD multijet events,
- **lepton scale**: uncertainty in the lepton momentum scale,
- **other experimental**: systematic uncertainties estimated from simulation and the extra-lepton veto,
- **other bkg**: normalization uncertainties for all backgrounds, except for the nonprompt background,
- **PDFs $\oplus \alpha_S$**: 60 Hessian variations of the NNPDF3.0 PDF set and $\alpha_S$,
- $\mu_F,\mu_R,\mu_F,\mu_R$: separate $\mu_R$ and $\mu_F$ variations, plus the correlated variation of both $\mu_R - \mu_F$,
- **FSR**: modeling of final state radiation,
- **MC statistics**: statistical uncertainty per bin of the template for all the samples,
- **statistical**: the statistical uncertainty in the data sample.
Table 2: Systematic uncertainties for each source and process. Quoted numbers correspond to the size of log-normal nuisance parameters applied in the fit, while a “yes” in a given cell corresponds to the given systematic uncertainty being applied as a shape variation over the full 2D template space.

<table>
<thead>
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<th>source/process</th>
<th>signal</th>
<th>Drell-Yan W → τν</th>
<th>QCD</th>
<th>Top</th>
<th>dibosons</th>
<th>charge flips</th>
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<tr>
<td><strong>Normalization uncertainty for W → ℓν (ℓ = μ, e)</strong></td>
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<td>-</td>
<td>-</td>
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The impact of each group is the effect of the simultaneous variation of all the parameters included in it, by using the covariance matrix of the fit. These groups cover all the nuisance parameters included in the likelihood and are mutually exclusive. Figure 3 summarizes the relative impact of groups of systematic uncertainties for two illustrative measurements: the normalized cross sections and the charge asymmetry for $W_L$, for the combination of the muon and electron final states. The impact of uncertainties that are completely correlated among all the rapidity bins mostly cancel when considering either the cross section normalized to the total cross section or in the charge asymmetry.

In a similar manner, the effect of the statistical and systematic uncertainties is shown for the normalized double-differential cross section, and for its charge asymmetry. For simplicity the distribution is integrated over $p_T^l$ and it is shown as a function of $|\eta^l|$ in Fig. 4.

Figure 3: Top: relative impact of groups of uncertainties (as defined in the text) on the signal normalized cross sections for the $W^-$ case. Bottom: absolute impact of uncertainties on the charge asymmetry of $W_L$. All impacts are shown for the combination of the muon and electron channels.
Figure 4: Top: relative impact of groups of systematic uncertainties (as defined in the text) on the signal normalized cross sections for the $W^+$ case. Bottom: relative impact of uncertainties on charge asymmetry (bottom). All impacts are shown for the combination of the muon and electron channels.
8. Results and interpretations

The template fit to the \( p_{T}^{\ell}, \eta^{\ell} \) distribution is performed on the four independent channels: \( W^{+} \rightarrow \mu^{+}\nu, W^{-} \rightarrow \mu^{-}\bar{\nu}, W^{+} \rightarrow e^{+}\nu, W^{-} \rightarrow e^{-}\bar{\nu} \). The observed events as a function of lepton \( \eta \) and \( p_{T}^{\ell} \) are shown in Figs. 5 (6) for the muon final states and Figs. 7 (8) for the electron final state, for the positive (negative) charges. These distributions represent the two-dimensional templates, unrolled into one dimension, such that \( \text{bin}_{\text{unrolled}} = 1 + \text{bin}_{\eta} + 48(50) \cdot \text{bin}_{p_{T}^{\ell}} \), with \( \text{bin}_{\eta} \in [0, 48(50)] \) and \( \text{bin}_{p_{T}^{\ell}} \in [0, 18(14)] \) for the muon (electron) channel, along with the one-dimensional projections in each of the two variables. In the projections, the sum in quadrature of the uncertainties in the two-dimensional distribution is shown, neglecting any correlations. These uncertainties are therefore for illustration purpose only.

![Figure 5: Distributions of \( \eta^{\mu} \) (a), \( p_{T}^{\ell} \) (b) and bin\(_{\text{unrolled}} \) (c) for \( W^{+} \rightarrow \mu^{+}\nu \) events for observed data superimposed on signal plus background events. The signal and background processes are normalized to the result of the template fit. The cyan band over the data-to-prediction ratio represents the uncertainty in the yield in each bin after the profiling process.](image)

8.1 Cross section measurements

The \( W^{\pm} \rightarrow \ell\nu \) cross section measurements are performed in both the electron and muon channels, by using the negative log likelihood minimization in Eq. 5. This provides a cross-check of experimental consistency of the two decay modes, and provides a method of reducing the impact of the statistical and systematic uncertainties when combining the measurements in the two channels and accounting for correlated and uncorrelated uncertainties.
8.1.1 Combination procedure

Measurements in different channels are combined by simultaneously minimizing the likelihood across channels, with common signal strengths and nuisance parameters as appropriate. Uncertainties that are correlated among channels are those corresponding to the integrated luminosity, the knowledge of specific process cross section in the background normalizations when the process is estimated from simulation, and effects which are common to multiple processes. Uncertainties related to the estimate of the QCD background are considered uncorrelated between electron and muon channels, since they originate from the closure test of the estimate in the background dominated enriched regions, which are independent of each other. The estimate of the lepton misidentification probability $\epsilon_{\text{pass}}$ is also done independently. The tests done on the application of a common $\epsilon_{\text{pass}}$ to positive and negative charges separately show that this systematic uncertainty can be considered 100% correlated between the two charges for each lepton flavor.

The statistical uncertainties in the efficiency correction factors are implemented uncorrelated among positive and negative charges, and uncorrelated among the channels, since they are derived from independent samples. The fully correlated part of the systematic uncertainty in the efficiency within a channel is assumed uncorrelated between electrons and muons, since
8. Results and interpretations

![Graphs showing distributions of lepton $\eta$, $p_T$, and $\eta$ unrolled lepton](image)

Figure 7: Distributions of $\eta$ (a), $p_T$ (b) and bin unrolled (c) for $W^+ \rightarrow e^+ \nu$ events for observed data superimposed on signal plus background events. The signal and background processes are normalized to the result of the template fit. The cyan band over the data-to-prediction ratio represents the uncertainty in the yield in each bin after the profiling process.

the dominant effects from the $Z \rightarrow \ell\ell$ lineshape and the background sources are very different.

Most of the theoretical uncertainties are assumed 100% correlated between the four channels. They are uncertainties on the boson transverse momentum spectrum modeling due to $\mu_F$ and $\mu_R$ uncertainties and the uncertainty in the knowledge of $\alpha_s$. Another large group of nuisance parameters that are correlated among all the channels represent the effects of the PDF variations within the NNPDF3.0 set used on both the shape of the templates used and their normalization. The 60 nuisance parameters associated with the Hessian representation of the 100 PDF replicas as well as the uncertainty in $\alpha_s$ are 100% correlated among all the four lepton flavor and charge channels. These 60+1 systematic uncertainties are also fully correlated with the respective uncertainties considered for the $Z$ and $W \rightarrow \tau\nu$ processes.

8.1.2 Differential cross sections in $|Y_W|$  

The measured $|Y_W|$-dependent cross section, for the left- and right-handed polarizations, is extracted from the fit in 9 bins of $|Y_W|$, with a constant width of $\Delta Y_W = 0.25$ in a range $|Y_W| < 2.5$. Two additional bins, $2.5 < |Y_W| < 2.75$ and $2.75 < |Y_W| < 10$ that integrate over the kinematic region in which the detector acceptance is small, are fixed to the expectation from MadGRAPH5_aMC@NLO with a large 30% normalization uncertainty. In order to achieve a
partial cancellation of uncertainties which are largely correlated among all $|Y_W|$ bins, the cross sections are normalized to the fitted total W cross section integrating over all the rapidity bins within the acceptance. As stated before, the longitudinally polarized component is fixed to the MadGraph5_aMC@NLO prediction with a 30% normalization uncertainty, and therefore not a freely floating parameter in the fit, and hence only the $W_L$ and $W_R$ components are shown in the following.

The measured W boson production cross sections, split into the left and right helicity states, for the combination of electron and muon channels, are presented in Fig. 9, normalized to the total cross section in the whole rapidity range. The experimental distributions are compared to the theoretical prediction from MadGraph5_aMC@NLO. The uncertainty shown in the theory prediction includes the contribution from the PDFs (NNPDF3.0 set), the envelope of the $\mu_F$ and $\mu_R$ variations, and $\alpha_S$. The line shown within the band represents the prediction from MadGraph5_aMC@NLO after the reweighting of the sample using the $p_T^Z$ weights. It is evident that the reweighting procedure, while it modulates the boson $p_T$ distribution and therefore the $p_T^Z$ template, has only a mild effect on the shape of $|Y_W|$. The main systematic uncertainty in the signal cross section, the 2.5% uncertainty in the integrated luminosity [43], is fully correlated across all the rapidity bins, thus it cancels out when taking the ratio to the total W cross section.

Figure 8: Distributions of $\eta'$ (a), $p_T$ (b) and bin$_{unrolled}$ (c) for $W^+ \to e^-\bar{\nu}$ events for observed data superimposed on signal plus background events. The signal and background processes are normalized to the result of the template fit. The cyan band over the data-to-prediction ratio represents the uncertainty in the yield in each bin after the profiling process.
In addition to these normalized and unpolarized cross sections, the results of the fits can also be filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and $\alpha_S$. Also shown is the ratio of the prediction from MADGRAPH5_aMC@NLO* to the data. The light-filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and $\alpha_S$.

The ratio of the expected normalized cross section using the nominal MADGRAPH5_aMC@NLO simulation to the measured one in data is also presented. As described in Sec. 6.5, the fitted $|Y_W|$-dependent cross sections can be used to simultaneously derive the differential charge asymmetry. This is presented, differentially in $|Y_W|$ and polarization, in Fig. 9 (c).

There are significant correlated uncertainties between neighboring $W$ rapidity bins. The cross section results differential in $W$ rapidity were tested for statistical compatibility with a smooth functional shape, taking these correlations into account. Toy MC experiments have shown that the results are quantitatively consistent with smooth third-order polynomial functions of $|Y_W|$. This test was performed simultaneously in both helicity states, both charges, and all $|Y_W|$ bins, taking into account the full covariance matrix of the fit.

Results are also shown as an unpolarized cross section, i.e., by summing over all helicity states as a function of $|Y_W|$. Both the unpolarized normalized cross section, and the unpolarized charge asymmetry as a function of $|Y_W|$ are shown in Fig. 10.

In addition to these normalized and unpolarized cross sections, the results of the fits can also be
presented as absolute cross sections. This is shown in Fig. 11, where the absolute, unpolarized cross sections are shown for the combined flavor fit. Generally good agreement is observed in the shape of the measured distribution with respect to the expectation, albeit with an offset of the order of a few percent.

![Figure 11: Measured absolute W → ℓν (left) or W → ℓ¯ν (right) cross section as a function of |Y_W| from the combined flavor fit. Also shown is the ratio of the prediction from MADGRAPH5_aMC@NLO to the data. The light-filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and αS.](image)

### 8.1.3 Double-differential cross sections in \( p_T^\ell \) and \( |\eta^\ell| \)

Double-differential cross sections in \( p_T^\ell \) and \( |\eta^\ell| \) are measured from a fit to the observed data in the \( (p_T^\ell, |\eta^\ell|) \) plane. The underlying generated templates are unfolded to the dressed lepton definition in 18 bins of \( p_T^\ell \) and 18 bins in \( |\eta^\ell| \), as described in Section 6.3. These cross sections are shown in Fig. 12, normalized to the total cross section. These results come from the combination of the muon and electron final states, divided into two categories of the lepton charge. From the measured cross sections, the double-differential charge asymmetry is computed, where the uncertainty is computed from the full covariance matrix from the fit.

While these normalized cross sections of the combined flavor fit represent the result with the smallest total uncertainty due to the cancellation of the fully correlated components, the absolute cross sections are also of interest. In particular, the agreement of the absolute cross sections between the flavor channels highlights the experimental agreement between the flavors after consideration of all systematic uncertainties. These plots are displayed in Fig. 13, where the measured absolute cross sections are shown separately for the electron, muon, and combined flavor fit. Good agreement is found within the uncertainties in the regions with sufficient statistics. Uncertainties become very large in the high \( |\eta^\ell| \) region for the electron-only fit, rendering a precise comparison difficult.

From the results of this fit, the single-differential cross section is measured by integrating in one of the two dimensions, as a function of the other variable. Along with these cross sections, the charge asymmetry differential in one dimension is extracted. This approach has the added value, with respect to a single-differential measurement, that it is independent of the modeling of the lepton kinematics in the variable which is integrated away. The resulting absolute cross sections, as a function of the lepton pseudorapidity, and the relative charge asymmetry, from the combination of the two channels, are shown Fig. 14. This result can be directly compared with previous measurements performed at 7 and 8 TeV by the CMS and ATLAS collaborations [12, 42].
Figure 12: Normalized double-differential cross section and charge asymmetry as a function of the $p_T^W$ and $|\eta^W|$, unrolled in a 1D histogram along $|\eta^W|$ for the positive (negative) charge on top (middle). The bottom plot shows the resulting charge asymmetry. The lower panel in each plot shows the ratio (difference) of the observed and expected cross section (asymmetry).
Figure 13: Absolute double-differential cross section as a function of $p_T^c$ and $|\eta^f|$, unrolled in a 1D histogram along $p_T^c$ in bins of $|\eta^f|$ for the positive (negative) charge on top (bottom). The combined muon+electron fit is shown in green markers, the muon-only fit in blue markers, and the electron-only fit is shown in red markers.

Figure 14: Absolute differential cross section as a function of $|\eta^f|$ for the $W^+ \rightarrow \ell^+ \nu$ channel (left) and $W^- \rightarrow \ell^- \bar{\nu}$ channel (middle). The corresponding charge asymmetry is shown on the right. The measurement is the result of the combination of the electron and muon channels. The lower panel in each plot shows the ratio (difference) of observation and expectation for the cross section (asymmetry) and the relative uncertainty.
8. Results and interpretations

8.2 Constraint of the PDF nuisances through likelihood profiling

When the cross section parameters in the likelihood function of Eq. 5 are fixed to their expected values ($\mu_p = 1$) within their uncertainties, the fit has the statistical power to constrain the PDF nuisance parameters. This procedure corresponds to the PDF profiling method described in [12], with associated caveats about the interpretation of constraints far from the initial predictions. The constraints in this case are derived directly from the detector-level measurements rather than passing through an intermediate step of unfolded cross sections. The input PDF and Monte Carlo prediction are both accurate to NLO in QCD, with the Monte Carlo prediction implicitly including resummation corrections through the matching to the parton shower. The theoretical uncertainties included in this procedure for missing higher orders in QCD correspond to the full model used for the measurement as described in Section 7.2. This is in contrast to typical global PDF fits or QCD analyses which are performed at NNLO accuracy, though at fixed order without resummation, and with the inclusion of missing higher order uncertainties only in dedicated studies at NLO so far [46, 47]. For each variation the fit input value (pre-fit) is trivially represented by a parameter with mean zero and width one. The expected post-fit values of these parameters all have mean zero, but a constrained uncertainty after the likelihood profiling procedure, i.e., width smaller than unity. Finally, the points representing the observed post-fit values of the parameters may have mean different from zero, signifying a pull of the associated systematic uncertainty, and width smaller than one. Figure 15 shows these constraints of the PDF nuisance parameters, where the 60 variations correspond to the orthogonalized Hessian PDF variations corresponding to the NNPDF3.0 replicas, and $\alpha_S$. All of the variants, i.e., pre-fit, post-fit expected, and post-fit observed are shown. Post-fit constraints of down to $\simeq 70\%$ of the pre-fit values are observed in some Hessian variations, while the mean constraint is closer to $\simeq 90\%$.

Figure 15: Pulls and constraints of the 60 Hessian variations of the NNPDF3.0 PDF set, and of the $\alpha_S$ parameter, from the combined fit of muon and electron channels. The underlying fit is performed by fixing the W cross sections to their expectation in all helicity and charge processes. The cyan band represents the input values (which all have zero mean and width one), the orange bands show the post-fit expected values, and black points represent the observed pulls and constraint values. The post-fit nuisance parameter values with respect to the prefit values and uncertainties give a $\chi^2/dof$ of 117/61.
9 Conclusions

A differential measurement of the W boson cross sections, measured as a function of the W boson rapidity, $|Y_W|$, and for the two charges separately, $W^+ \rightarrow \ell^+ \nu$ and $W^- \rightarrow \ell^- \bar{\nu}$, is presented by the CMS Collaboration. Double-differential cross section measurements of the W boson are presented as a function of the charged lepton transverse momentum, $p_T^\ell$, and lepton pseudorapidity, $|\eta^\ell|$. For both measurements, the differential charge asymmetry is also presented. The measurement is based on data taken in pp collisions at the LHC at a center-of-mass energy of $\sqrt{s} = 13$ TeV with an integrated luminosity of 35.9 fb$^{-1}$. Differential cross sections are presented both absolute and normalized to the total production cross section within a given acceptance. For the helicity measurement, the range $|Y_W| < 2.5$ is considered, while for the double-differential cross section the range $|\eta^\ell| < 2.4$ and $26 < p_T^\ell < 55$ GeV is used. The measurement is performed using both the muon and the electron channels, combined together considering all sources of correlated and uncorrelated uncertainties.

The precision in the measurement as a function of $|Y_W|$, using a combination of the two channels, is about 2% in central $|Y_W|$ bins, and 5% to 20%, depending on the charge-polarization combination, in the outermost acceptance bins. The precision of the double-differential cross section, relative to the total, is about 1% in each of the 324 bins relative to the central part of the detector, $|\eta^\ell| < 1$, and better than 2.5% up to $|\eta^\ell| < 2$ for each of the two W charges. Charge asymmetries are also measured, differentially in $|Y_W|$ and polarization, as well as in $p_T^\ell$ and $|\eta^\ell|$. The uncertainties in these asymmetries range from 0.1% in high-acceptance regions to roughly 2.5% in regions of phase space with lower detector acceptance.

This measurement is used to constrain PDF parameters in a simultaneous fit of the measurements from the two channels and the two W charges. The constraints are derived at detector level on 60 uncorrelated eigenvalues of the NNPDF3.0 set of PDFs within the MADGRAPH5_aMC@NLO generator code, and show a total constraint down to $\simeq 70\%$ of the pre-fit uncertainties for certain variations of the PDF nuisance parameters.
References


A.1 Helicity and rapidity analysis

Figure 16 shows the absolute polarized cross sections as a function of $|Y_W|$ for the left-handed and right-handed helicity states from the combination of muon and electron channels. Also shown is the ratio of the prediction from MADGRAPH5_aMC@NLO to the data. The light-filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and $\alpha_S$.

Figure 17 shows again the absolute unpolarized cross sections as a function of the W boson rapidity. This figure, however, also shows the comparison between the two lepton flavors. i.e., performing the fits separately once in the muon-only, once in the electron-only, and once in the flavor combination shows the experimental agreement of the different flavor channels. Such fits are shown in Fig. 11, where these results are shown for both charges of the W boson. It is
shown that the single flavor fits agree within their uncertainty with each other as well as with the combined flavor fit. It is worthwhile to note, however, that the “true” correlation structure of the three different fits cannot be trivially displayed in the ratios of the flavors shown in the lower panels of Fig. 11.

Figure 17: Measured absolute $W \rightarrow \ell^+\nu$ (left) or $W \rightarrow \ell^-\bar{\nu}$ (right) cross section as a function of $|Y_W|$ from three distinct fits: the combination of muon and electron channels (green), the muon-only fit (blue), and the electron-only fit (red) in the top panel. Also shown is the ratio of the prediction from MADGRAPH5_aMC@NLO to the data in the middle panel, as well as the ratio between the muon-only fit and the electron-only fit in the bottom panel. The light-filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and $\alpha_S$.

Figure 18 shows the distribution of the $A_4$ coefficient extracted as a function of $|Y_W|$ from the combined fit to the electron and muon channel for the two charges of the $W$ boson.

Figure 19 shows the correlation coefficients between the different signal processes split into their helicity components from the combined electron+muon fit for the two charges of the $W$ boson. The numbering corresponds to the bins in $|Y_W|$ of width 0.25 starting at zero. It is worthwhile to note here that the correlations of neighboring bins in rapidity are large, especially for each helicity. However, there are also non-trivial correlations across the helicity states.

Figure 20 shows the correlation coefficients between the different PDF nuisance parameters in the combined electron+muon fit. The numbering of the PDF nuisances derives from the conversion of the NNPDF3.0 replicas to 60 orthogonal Hessian nuisance parameters and carries no physical meaning.

Figure 21 shows the post-fit pulls and their post-fit constraints of the nuisance parameters associated with the QCD scale systematics. The numbering corresponds to the bins in the $p_T$ spectrum in increasing order. The numbers result from the combined fit to the electron and
Figure 18: Measured $A_4$ coefficient for $W \rightarrow \ell^+\nu$ (left) and $W \rightarrow \ell^-\bar{\nu}$ (right) extracted from the fit of the polarized cross sections to the combined electron+muon fit. Also shown is the difference between the prediction from MADGRAPH5_aMC@NLO and the measured values. The light-filled band corresponds to the expected uncertainty from the PDF variations, QCD scales and $\alpha_s$.

Figure 19: Correlation coefficients between the helicity-dependent signal cross sections for $W \rightarrow \ell^+\nu$ (left) and $W \rightarrow \ell^-\bar{\nu}$ (right) extracted from the fit to the combined electron+muon fit.

### A.1 Impacts for the helicity analysis

### A.2 2D differential cross section

#### A.2.1 Impacts for the 2D differential cross section analysis
Figure 20: Correlation coefficients between the 60 PDF nuisance parameters extracted from the fit to the combined electron+muon fit.

Figure 21: Post-fit pulls and constraints of the nuisance parameters associated with the QCD scale systematics. The numbering refers to bins in the $p_T$ spectrum in increasing order. The nuisance parameters applied to the “left” polarization are shown on the left while the ones associated with the “right” polarization are shown on the right.
Figure 22: Remaining impacts on the normalized polarized cross sections as a function of the rapidity. Shown are the impacts of the nuisance groups for $W^+_R$ (top), $W^+_L$ (middle), and $W^-_R$ (bottom).
Figure 23: Impacts on the absolute polarized cross sections as a function of the rapidity. Shown are the impacts of the nuisance groups for $W^+_L$ (top) and $W^+R$ (bottom).
Figure 24: Impacts on the absolute polarized cross sections as a function of the rapidity. Shown are the impacts of the nuisance groups for $W_\nu^-$ (top) and $W_\nu^+$ (bottom).

Figure 25: Impacts on the charge asymmetry for $W_\nu^+$. 

CMS Preliminary

$35.9 \text{ fb}^{-1} (13 \text{ TeV})$

Uncertainties on cross section for $W_\nu^+ \rightarrow l^+ \nu \ p_T^l \in [26, 45] \text{ GeV}$

- lepton scale
- efficiency stat.
- luminosity
- efficiency syst.
- $|p_T^l| \geq 26 \text{ GeV}$
- $|p_T^l| \geq 26 \text{ GeV}$
- other experimental
- statistical
- MC statistics

Total uncertainty

CMS Preliminary

$35.9 \text{ fb}^{-1} (13 \text{ TeV})$

Uncertainties on charge asymmetry for $W_\nu^+ \rightarrow l^+ \nu \ p_T^l \in [26, 45] \text{ GeV}$

- lepton scale
- efficiency stat.
- luminosity
- efficiency syst.
- $|p_T^l| \geq 26 \text{ GeV}$
- $|p_T^l| \geq 26 \text{ GeV}$
- other experimental
- statistical
- MC statistics

Total uncertainty
Figure 26: Impacts on the unpolarized absolute cross sections for $W^+$ (top), $W^-$ (middle), and the unpolarized charge asymmetry (bottom).
Figure 27: Impacts on the unpolarized normalized cross sections for $W^+$ (top) and $W^-$ (bottom).
Figure 28: Impacts on the $A_4$ coefficient as a function of $|Y_W|$ for $W^+$ (top) and $W^-$ (bottom).

Figure 29: Unrolled cross section for the combined electron+muon fit unrolled along $p_T^f$ in bins of $|p_T^f|$ for $W^+$ (top) and $W^-$ (bottom). Shown in the colored bands are the prediction from MADGRAPH5-AMC@NLO with the PDF+$\alpha_S$ variations in blue, and the total theoretical uncertainty in bordeaux.
Figure 30: Normalized cross sections as a function of $|\eta^l|$, integrated over $p_T^l$ for $W^+$ (left) and $W^−$ (right). Shown in the colored bands are the prediction from MADGRAPH5_aMC@NLO with the PDF+\(\alpha_S\) variations in blue, and the total theoretical uncertainty in bordeaux.

Figure 31: Absolute cross sections as a function of $p_T^l$, integrated over $|\eta^l|$ for $W^+$ (left) and $W^−$ (right). Shown in the colored bands are the prediction from MADGRAPH5_aMC@NLO with the PDF+\(\alpha_S\) variations in blue, and the total theoretical uncertainty in bordeaux.

Figure 32: Normalized cross sections as a function of $p_T^l$, integrated over $|\eta^l|$ for $W^+$ (left) and $W^−$ (middle), and the resulting charge asymmetry (right). Shown in the colored bands are the prediction from MADGRAPH5_aMC@NLO with the PDF+\(\alpha_S\) variations in blue, and the total theoretical uncertainty in bordeaux.
Figure 33: Remaining impacts of the nuisance groups on the normalized cross sections as a function of $|\eta^{\ell}|$, integrated in $p_T^{\ell}$, for $W^−$.

Figure 34: Remaining impacts of the nuisance groups on the absolute cross sections as a function of $|\eta^{\ell}|$, integrated in $p_T^{\ell}$, for $W^+$ (top) and $W^−$ (bottom).
Figure 35: Remaining impacts of the nuisance groups on the normalized cross sections as a function of $p_T^\ell$, integrated over $|\eta^\ell|$, for $W^+$ (top), $W^-$ (middle), and the resulting charge asymmetry (bottom).
Figure 36: Remaining impacts of the nuisance groups on the absolute cross sections as a function of $p_T^\ell$, integrated over $|\eta^\ell|$, for $W^+$ (top) and $W^-$ (bottom).