HOW TO DESIGN A COMB LINE BAND-PASS FILTER

D. Boussard

1. INTRODUCTION

Comb filters are very attractive in the several hundred Megahertz range, because they are relatively small (λ/8 or less) and they give good electrical performance. This type of filter consists of an array of resonating lines coupled together. These lines are short-circuited on one end, the other end being loaded by a capacitor. The theory and the design procedure for comb filters are given in Ref. 1. However, the design is valid for lines having rectangular cross-sections, and in addition, the coupling elements to the resonators should also be rectangular bars. (There are n + 2 bars for a n pole filter.)

This design is not very attractive from a mechanical point of view, because rectangular bars are more difficult to machine and to position correctly than round rods. We present here a design which makes use of only n round rods for a n pole filter, the coupling elements being simply tapping points on the outer lines.

2. CHARACTERISTIC IMPEDANCES AND COUPLING CAPACITANCES OF TWO ROUND RODS BETWEEN PARALLEL PLATES

The odd and even mode impedances of two round rods between parallel plates (Fig. 1) are given in Ref. 1 (p. 177)

\[ Z_e - Z_0 = \frac{120}{\sqrt{\varepsilon_r}} \log \coth \frac{\pi s}{2b} \]  \hspace{1cm} (1)

\[ Z_e + Z_0 = \frac{120}{\sqrt{\varepsilon_r}} \log \frac{kb}{\pi d} \]  \hspace{1cm} (2)

for d/b < 0.55 and s > 2d; s, b and d are defined in Fig. 1.
For $s \to \infty$, one obtains the characteristic impedance of a single rod

$$Z = \frac{60}{\sqrt{\varepsilon_r}} \log \frac{b}{\pi d} \quad \text{(3)}$$

In Ref. 1 (p. 193) the coupling capacitance between two round rods is related to the even and odd mode impedances. For identical rods, one finds:

$$C_{ab} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \frac{Z_e - Z_0}{2\varepsilon Z_0} \quad \text{(4a)}$$

From equations (1) and (2) one obtains:

$$Z_e = \frac{60}{\sqrt{\varepsilon_r}} \left( \log \frac{b}{\pi d} + \log \coth \frac{\pi s}{2b} \right)$$

and

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \left( \log \frac{b}{\pi d} - \log \coth \frac{\pi s}{2b} \right) \quad \text{(4b)}$$

Hence

$$C_{ab} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \left( \frac{\log \coth \frac{\pi s}{2b}}{60} \right)$$

For rods which are not too close ($s/b > 1$), the second term of the denominator is very small compared to the first one, and can be dropped out. One obtains finally the following relation:
log coth \( \frac{\pi s}{2b} = \frac{C_{ab}}{\varepsilon} \frac{60}{\pi_0} \left( \log \frac{4b}{\pi d} \right)^2 \). \hspace{1cm} (5)

The coth curve is plotted in Fig. 2.

3. APPLICATION TO THE COMB FILTER DESIGN

For relatively small fractional bandwidths (<10%), the coupling between rods is small, and therefore their characteristic impedance is not modified by the presence of adjacent rods. This is confirmed by the exact calculations for rectangular bars following the method of Ref. 1.

Therefore, the filter can be built with identical rods, the optimum diameter being given by the condition \( Z = 70 \, \Omega \) (this characteristic impedance gives the best \( Q \) for the resonators, see Ref. 1, p. 502 for example).

From equation 3, one obtains:

\[
b/d = 2.52 \quad \text{for} \quad Z = 70 \, \Omega.
\]

The normalized coupling capacitances between bars number 1 to \( n \) are given by the conventional design (Ref. 1, p. 498-499). Applying equation (5) which for \( b/d = 2.52 \) can be rewritten:

\[
\log coth \frac{\pi s}{2b} = 0.216 \frac{C_{ab}}{\varepsilon}
\]

(6)

gives the distances between rods \( (s/b) \) corresponding to the \( C_{ab} \)'s required. This is not valid for the coupling bars which show a much tighter coupling, and applies simply to the \( n \) resonators.

4. DESIGN OF THE COUPLING ELEMENTS

Instead of using additional bars for the coupling to the 50 \( \Omega \) input and output ports of the filter, we propose to determine convenient tapping points on the first and the last rods as proposed in Ref. 3.
The Q of the first resonator, when loaded by the 50 Ω input impedance is determined by the conventional filter design procedure (see for instance Ref. 2, chapter 8). One obtains

\[ Q_1 = Q_n = q_1 \frac{f_0}{b_{3dB}} = q_n \frac{f_0}{b_{3dB}} \]  \quad (7)

\( b_{3dB} \) being the total 3 dB down bandwidth of the filter, and \( q_1 \) and \( q_n \) are given by the usual filter tables.

Remark \( b_{3dB} \) used here is slightly different from the bandwidth used in Ref. 1 for the design of the filter (ripple bandwidth).

In Appendix 1, the calculation of the Q of a resonating line having an electrical length of 0 radians, and loaded by a resistor \( R \) tapped at a distance \( \theta_1 \) from the short-circuit is given. The result is simply:

\[ Q = \frac{R}{2Z} \frac{1}{\sin^2 \theta_1} (\theta + \sin \theta \cos \theta) \]  \quad (8)

From (7) and (8) one can calculate the position \( \theta \) of the tapping point, which completes the design of the filter.

5. NUMERICAL EXAMPLE

One wants to design a filter having the following characteristics:

- center frequency \( f_0 = 200 \text{ MHz} \)
- equal ripple bandwidth: \( b_w = \pm 2 \text{ MHz} \)
- number of poles: 4
- ripple: 0.1 dB

The frequency response is fully defined by the curve 4 page 8.8 (Ref. 2).

From that one deduces the 3 dB down bandwidth: \( b_{3dB} = \pm 2.42 \text{ MHz} (4.84 \text{ kHz}) \).
We choose the electrical length of the filter: \( \theta = \pi/5 \) (15 cm), which determines also the tuning capacitor: \( C = 15.57 \text{ pF} \) for the optimum 70 Ω lines.
Following the design procedure of Ref. 1, one finds the normalized coupling capacitances:

\[
\frac{C_{12}}{e} = \frac{C_{34}}{e} = 0.103 \quad \text{and} \quad \frac{C_{23}}{e} = 0.082
\]

which give, using Equation (6) and curve in Fig. 2:

\[
\left(\frac{\pi a}{2b}\right)_{1.2} = 2.28 \quad \text{and} \quad \left(\frac{\pi a}{2b}\right)_{2.3} = 2.4
\]

We choose \( b = 10 \) mm, and then obtain:

\( s_{12} = s_{34} = 14.5 \) mm and \( s_{23} = 15.28 \) mm.

The diameter of the rods, given by \( b/d = 2.52 \), is: \( d = 3.96 \) mm.

According to Ref. 2, page 8-26, one finds: \( q_1 = q_4 = 1.34 \), and therefore

\[ Q_1 = Q_4 = 55.37. \]

It follows from Equation (8) (\( R = 50\Omega, Z = 70\Omega, \theta = \pi/5 \)) that \( \theta_1 = 4.83^\circ \).

The tap will therefore be located at 20 mm from the short-circuit end of the bar. This filter has been built using standard 4 mm brass rods between 2 mm thick brass plates. The tuning capacitors (1 to 10 pF) are shunted by normal 6.8 pF ceramic capacitors. 10 \( \mu \)m thick silver plating was necessary to achieve the 3 dB insertion loss figure. The measured performance of this filter is given on the attached photos.

# References


APPENDIX I

CALCULATION OF THE Q OF THE OUTER LINES

The line is short-circuited at one end and tuned by a capacitor C at the other end (Fig. 3). We want to calculate the impedance at an intermediate point defined by the electrical lengths $\theta_1$ and $\theta_2$. ($\theta_1 + \theta_2 = \theta$).

The admittance of the portion of line $\theta_1$, is given by:

$$Y_1 = Y_0 \frac{1}{j \tan \theta_1}$$

$Y_0$ is the characteristic admittance of the line.

The admittance of the portion of line $\theta_2$ (loaded by C) is given by:

$$Y_2 = Y_0 \frac{C w + \frac{Y \tan \theta_2}{Y_0 - C w \tan \theta_2}}{Y_0 - C w \tan \theta_2}.$$

The admittance at the intermediate point is therefore:

$$Y = Y_1 + Y_2 = j Y_0 \left( \frac{C w + \frac{Y \tan \theta_2}{Y_0 - C w \tan \theta_2} - \frac{1}{\tan \theta_1}}{Y_0 - C w \tan \theta_2} \right). \quad (9)$$

One can verify that $Y = 0$ for $C = Y_0 / \omega_0 \tan \theta$ which is the resonant condition of the line. In the vicinity of the resonance the function $Y(\omega)$ can be represented by the first term of its development:

$$Y = Y(\omega_0) + \frac{dY}{d\omega} \delta \omega = \frac{dY}{d\omega} \delta \omega \quad \omega = \omega_0$$

(10)

One has:

$$\frac{dY}{d\omega} = \frac{2Y + 2Y \frac{d\theta}{d\omega} + 2Y \frac{d\theta_1}{d\omega} + 2Y \frac{d\theta_2}{d\omega}}{2}$$

and

$$\frac{d\theta}{d\omega} = \frac{\theta_1}{c} \quad ; \quad \frac{d\theta_1}{d\omega} = \frac{\theta_1}{c} \quad ; \quad \frac{d\theta_2}{d\omega} = \frac{\theta_2}{c} \quad ; \quad C = Y_0 / \omega_0 \tan \theta.$$
After some manipulations, one obtains finally:

\[
\frac{dY}{d\omega} = jY_0 \frac{1}{\omega_0 \sin^2 \theta_1} (\theta + \sin \theta \cos \theta) .
\]  \hspace{1cm} (11)

If we put a resistor \( R \) at the intermediate point, the 3 dB down frequencies shall verify

\[
\frac{1}{R} = |Y(\delta \omega)| = Y_0 \frac{1}{\omega_0 \sin^2 \theta_1} (\theta + \sin \theta \cos \theta) \delta \omega .
\]

The \( Q \) of the resonator is thus:

\[
Q = \frac{\omega_0}{2\delta \omega} = \frac{R(\theta + \sin \theta \cos \theta)}{2Z_0 \sin^2 \theta_1} .
\]  \hspace{1cm} (12)

Fig. 3
1. Transfer function - polar coordinates

2. Amplitude transfer function log scale (2.5 dB/div and 10 dB/div vertical, 2 MHz/div horizontal).

The upper line of the graticule corresponds to 0 dB on the 2.5 dB/div scale (insertion loss ≈ 3 dB).
3. Amplitude transfer function
log scale 10 dB/div. 0 to 2 GHz
The second pass-band occurs above 2 GHz. The spike below 200 MHz is due to the measuring instrument.

4. Input impedance. Smith chart display
The VSWR is below 1.4 in all the pass-band.
Remark: This is the most sensitive display to tune the filter.