STRUCTURE FUNCTIONS OF PSEUDOSCALAR MESONS
IN THE SU(3) NJL MODEL

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ABSTRACT

We compute the structure functions of the pion, kaon and η mesons using a SU(3) version of the NJL model with scalar and pseudoscalar couplings. We perform the calculation directly by considering the absorptive part of the forward Compton scattering amplitude, allowing us to treat the regularization without ambiguities and to fully preserve gauge invariance at any stage of the calculation. By using the Pauli-Villars method, we find scaling and proper normalization of the structure functions in the Bjorken limit. In the chiral limit we find \( q(x) = \bar{q}(x) = \theta(x)\theta(1 - x) \). We evolve our results to higher momentum using the Altarelli-Parisi equations for three flavours. We find good agreement with the experimental data for the pion.

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1. The study of deep inelastic scattering on hadronic targets is interesting because it gives us information about the quark content of hadrons. In the Bjorken limit, the operator product expansion provides a practical way to disentangle the hard and soft pieces of the twist two contribution to the scattering amplitude. This requires an extrapolation from the region $|x| > 1$ where the OPE works to the physical region $|x| < 1$, so that much of what is theoretically known relies on the study of the absorptive parts to the forward Compton scattering amplitude of virtual photons. In fact, such an analysis makes it possible to evaluate the structure functions in terms of the off-shell quark-target scattering amplitude [Ja84], under the assumption that it vanishes sufficiently fast as a function of the quark momentum. Although perturbative QCD allows to predict the large $Q^2$ dependence of the hard piece, some particular model or lattice calculation is needed for the soft non-perturbative piece in order to make definite predictions for the hadronic structure functions. In this context, it has been suggested long ago [JR80] that quark models can be profitably used to calculate the leading twist contribution to the structure functions, assuming that they are valid at a certain low energy scale $Q_0^2$ and using the renormalization group equations [GL72,AP77] to evolve the result up to the $Q^2$-scale relevant for DIS experiments. This idea has been implemented to study deep inelastic scattering in several models [JR80,BM87,BJ88,KM90,SS91,MM91,MW94]. In some cases, the quark-target scattering amplitude has been used with the additional inclusion of hadronic form factors, or assumptions about the translational invariance or completeness of the intermediate states have been made. This generally violates the proper normalization of the structure functions or equivalently the gauge invariance of the Compton amplitude. It is then not clear whether the counterterms needed to restore gauge invariance are compatible with the assumptions underlying the derivation from the Compton amplitude to the quark-target amplitude. This makes the calculation of the structure function within the model itself ambiguous.

Relativistic and gauge invariance impose very clear constraints on the structure functions regarding their support and normalization. On the other hand, a fully covariant description of most hadrons as bound states of quarks is still lacking. From this point of view the low-lying pseudoscalar mesons are clearly distinguished from the rest of the hadronic spectrum. Although it is not known in general the role of chiral symmetry in deep inelastic scattering, the Goldstone nature of the pseudoscalar octet implies that, besides soft pion corrections, many of their properties can be determined on the basis of chiral symmetry alone. This becomes even more transparent in effective chiral quark models which implement dynamical chiral symmetry breaking, such as the Nambu-Jona-Lasinio model (NJL) [NJ61,NJL]. In this model, the pion turns out to be a deeply bound $q\bar{q}$ state. Although the NJL model does not incorporate confinement, the lack of it seems to be irrelevant in the case of the pseudoscalar mesons. Thus, we hope to learn more about the role of chiral symmetry in deep inelastic scattering by studying the lightest mesons. In addition, the pion structure function has been experimentally determined from $\pi N$ scattering using the Drell-Yan process [SM92], and the corresponding first two moments have been calculated on the lattice in the quenched approximation [MS88].

In the present work we compute the structure functions of the lightest pseudoscalar mesons, i.e. $\pi, K$ and $\eta$, by using the NJL model with SU(3) flavours. We start with the imaginary part of the forward Compton scattering amplitude which is regularized by means of the Pauli-Villars method. In the Bjorken limit, we find scaling, proper support and normalization of the structure functions. Since in the NJL model the perturbative gluons are supposed to be integrated out, we generate them using the QCD evolution equations up to higher scales. As a side remark, we would like to emphasize that starting from the Compton amplitude rather than the quark-target amplitude is not a superfluous step at all. In fact, if such an amplitude does not vanish faster than a certain power of the quark momentum, the connection between both amplitudes does not hold in the usual sense. Our way of proceeding provides us with a regularized version of the quark-target formula in this model. Finally, let us mention that deep inelastic scattering for the pion has also been studied recently in a relativistic quark model [FM94] and in the NJL model [SS93]. In no case, however, do the authors obtain consistently normalized structure functions due to a careless treatment of the Compton amplitude.
2. The SU(3) Nambu–Jona-Lasinio Lagrangian in Minkowski space is given by [NJ61, NJL]

\[ \mathcal{L}_{NJL} = \bar{q}(i\gamma^\mu - M_0)q + \frac{G_S}{2} \sum_{a=0}^{8} \left( (\bar{q}\lambda_a q)^2 + (\bar{q}\lambda_a i\gamma_5 q)^2 \right) \]  

(1)

where \( q = (u, d, s) \) represents a quark spinor with \( N_c \) colours and three flavours. The \( \lambda \)'s represent the Gell-Mann matrices of the \( U(3) \) flavour group, \( M_0 = \text{diag}(m_u, m_d, m_s) \) stands for the current quark mass matrix, and \( G_S \) is the coupling constant. In the limiting case of vanishing current quark masses the NJL-action is invariant under the global \( U(3)_R \otimes U(3)_L \) group of transformations\(^1\). For the rest of the paper we neglect isospin breaking effects, i.e. we set \( m_u = m_d \).

The vacuum to vacuum transition amplitude in the presence of external bosonic \((s, p, v, a)\) fields of the NJL Lagrangian can be written, after bosonization [Eg75], integration of the quarks and Pauli-Villars regularization [PV49] as a path integral

\[ Z[s, p, v, a] = \langle 0 | \exp \left\{ i \int d^4x \left[ \bar{q} \left( \not\!p - m_0 \right) q - \frac{1}{4G_S + i\epsilon} \right] \right\} | 0 \rangle = \int \mathcal{D}SDP \exp \{ iS \}. \]  

(2)

The normal parity \((\gamma_5\text{-}even)\) contribution to the effective action is

\[ S_{\text{even}} = -\frac{iN_c}{2} \sum_{i} c_i \text{Splog}(DD_i + A_i^2 + i\epsilon) - \frac{1}{4G_S + i\epsilon} \int d^4x \text{tr}(S^2 + P^2), \]  

(3)

where the Dirac operators

\[ iD = i\gamma^\mu - M_0 - (S + i\gamma_5 P) + \not\!p + \not\!S - (s + i\gamma_5 p) \]

\[ iD_5 = i\gamma^\mu - M_0 - (S - i\gamma_5 P) + \not\!p - \not\!S - (s - i\gamma_5 p) \]  

(4)

have been introduced. \((S, P)\) are dynamical internal bosonic SU(3) fields. Two subtractions are needed to regularize the quadratic divergence, and we take the limit \( \Lambda_1 = \Lambda_2 = \Lambda \). The Pauli-Villars regulators are then defined in practice by the identity \( \sum_{i} c_i f(A_i^2) = f(0) - f(A^2) + A^2 f'(A^2) \). (See refs [BV89,SR02] for more details as well as the connection to the Proper-Time regularization.) Any mesonic correlation function can be obtained from this gauge invariantly regularized effective action by suitable functional differentiation with respect to the relevant external fields. In practice we work in the limit \( N_c \to \infty \), or equivalently at the one quark loop level.

3. To fix the parameters in the Pauli-Villars regularization we consider the calculation of several mesonic properties. The effective potential leads to dynamical chiral symmetry breaking yielding dynamical quark masses \((M_u, M_d, M_s)\) and condensates given by

\[ \frac{\langle \bar{u}u \rangle}{M_u - m_u} = \frac{\langle \bar{d}d \rangle}{M_d - m_d} = \frac{\langle \bar{s}s \rangle}{M_s - m_s} = -\frac{1}{2G_S}. \]  

(5)

with the condensate given by the integral

\[ \langle \bar{q}\cdot Q \rangle = -4N_c M_a J''_2 = 4N_c M_a \sum_{i} c_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{-k^2 + M_a^2 + A_i^2 - i\epsilon}. \]  

(6)

\(^1\) To account for the \( U_A(1) \) anomaly we consider the Witten-Veneziano [Wi79,Ve79] term in the strong coupling limit. This allows to determine \( \eta \) as a non-mixing \( \eta_8 \) state. A full discussion of other breakings, introduction of vector couplings as well as more details of our calculation will be presented elsewhere [DR94].
Calculation of the pertinent correlation functions yields the following expressions for the pseudoscalar meson masses

\[
\begin{align*}
    m_x^2 &= \frac{2I_2}{I_2} \frac{m_u}{F_{uu}(m_2^u)}(M_u - m_u), \\
    m_K^2 &= \frac{1}{F_{uu}(m_K^2)} \left\{ \frac{m_u}{M_u - m_u} I_2^u + \frac{m_s}{M_s - m_s} I_2^s \right\}, \\
    m_n^2 &= \frac{2}{F_{uu}(m_n^2) + 2F_{us}(m_s^2)} \left\{ \frac{m_u}{M_u - m_u} I_2^u + \frac{m_s}{M_s - m_s} I_2^s \right\},
\end{align*}
\]

(7)

which in the SU(3) limit reproduce the Gell-Mann-Okubo formula \(4m_K^2 - m_x^2 = 3m_n^2\). The pseudoscalar decay constants are

\[
\begin{align*}
    f_x &= 4N_c M_u F_{uu}(m_2^u) g_{xuu}, \\
    f_K &= 2N_c \left\{ [(M_u - M_s)T_{uu}(m_K^2) + (M_u + M_s)F_{uu}(m_K^2)] g_{Kuu} \right\}, \\
    f_n &= 2N_c \left\{ g_{nuu} F_{uu}(m_n^2) + g_{nus} F_{us}(m_s^2) \right\},
\end{align*}
\]

(8)

with the meson-quark-quark coupling constants given by

\[
\frac{1}{g_{F_{\alpha\beta}}} = 4N_c \frac{d}{dp^2} \left. \left\{ p^2 F_{\alpha\beta}(p^2) \right\} \right|_{p^2 = m_P^2}.
\]

(9)

Here we have introduced the following short hand notations

\[
T_{\alpha\beta}(p^2) = \int_0^1 dx(2x - 1) F_{\alpha\beta}(p^2, x); \quad F_{\alpha\beta}(p^2) = \int_0^1 dx F_{\alpha\beta}(p^2, x),
\]

(10)

in terms of the Pauli-Villars regularized one loop integrals

\[
F_{\alpha\beta}(p^2, x) = -i \int \frac{d^4k}{(2\pi)^4} \sum \frac{1}{c_i \left[ -k^2 + x(1-x)p^2 + (1-x)M_\alpha^2 + xM_\beta^2 + \Lambda_i^2 + i\epsilon \right]^2},
\]

(11)

which fulfill the symmetry relation \(F_{\alpha\beta}(p^2, x) = F_{\beta\alpha}(p^2, 1 - x)\). The parameters are fixed as usual; we adjust the cut-off to reproduce the physical weak pion decay constant \(f_x = 93.3\) MeV and the physical pion \((m_\pi = 139.6\) MeV) and kaon \((m_K = 494\) MeV) masses. This leaves only one free parameter which we choose to be the constituent quark mass \(M_u\). The values of \(f_K\), \(f_n\) and \(f_x\) are then predicted. For \(M_u = 280\) MeV we obtain \(m_u = 7\) MeV; \(m_s = 175\) MeV; \(M_s = 527\) MeV; \(\Lambda = 870\) MeV; \(m_\mu = 501\) MeV (exp. 549 MeV) \(f_K = 102\) MeV (exp. 113 MeV). We will mainly take this value of \(M_u\) in our calculations, which turns out to maximize the current quark mass corrections.

4. The hadronic tensor \(W_{\mu\nu}(p, q)\) can be obtained as the imaginary part of the forward Compton amplitude for virtual photons as follows

\[
\begin{align*}
    W_{\mu\nu}(p, q) &= \frac{1}{2\pi i} \text{Im} T_{\mu\nu}(p, q) \\
    &= W_1(q^2, p^\cdot q) \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2(q^2, p^\cdot q)}{m_P^2} \left( p_\mu - \frac{p^\cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p^\cdot q}{q^2} q_\nu \right),
\end{align*}
\]

(12)

where

\[
T_{\mu\nu}(p, q) = i \int d^4x e^{iq \cdot x} \langle \pi(p) | T\left\{ J_{\mu}^{\text{em}}(x) J_{\nu}^{\text{em}}(0) \right\} | \pi(p) \rangle,
\]

(13)

\(m_P^2 = p^2\) is the mass of the pseudoscalar meson, and \(q\) the momentum of the virtual photon. In the limit \(N_c \to \infty\) only quark loop diagrams survive. Due to gauge invariance one has to consider, in addition to the
usual box diagrams, the process $\pi \gamma \rightarrow \pi \rightarrow \pi \gamma$, which involves the off-shell electromagnetic form factor of the pseudoscalar meson. The latter turns out to be a higher twist contribution in the NJL model as obtained from the Pauli-Villars regularized effective action (3). This point is not entirely trivial since we are dealing with a theory with a finite cut-off, where naive counting rules do not necessarily work. In the Bjorken limit, we obtain after straightforward manipulations ($x = -q^2/(2p \cdot q)$)

$$W_{\mu \nu}(p, q) = \frac{1}{2\pi} \text{Im} T_{\mu \nu}(p, q) = F(x) \left[ -g_{\mu \nu} - \frac{q_{\mu} q_{\nu}}{q^2} - \frac{1}{q^2} (p_{\mu} - q_{\mu}) (p_{\nu} - q_{\nu}) \right],$$

with

$$F(x) = \frac{1}{2} \sum_{i = u, d, s} \epsilon_i^2 \left[ q_i(x) + q_i(x) \right].$$

The $\pi$, $K$ and $\eta$ structure functions may be conveniently written as

$$u_u(x) = \tilde{d}_u(1 - x) = 4N_c g_{\pi uu}^2 \frac{d}{dp^2} \left( p^2 F_{uu}(p^2, x) \right) \bigg|_{p^2 = m_\pi^2},$$

$$u_K(x) = \tilde{s}_K(1 - x) = 4N_c g_{\pi ss}^2 \frac{d}{dp^2} \left( p^2 F_{ss}(p^2, x) \right) \bigg|_{p^2 = m_K^2},$$

$$u_\eta(x) = \tilde{u}_\eta(x) = \tilde{d}_\eta(x) = 4N_c \left( \frac{1}{g_{\eta uu}^2} + \frac{2}{g_{\eta ss}^2} \right) \frac{d}{dp^2} \left( p^2 F_{uu}(p^2, x) \right) \bigg|_{p^2 = m_\eta^2},$$

$$s_\eta(x) = \tilde{s}_\eta(x) = 8N_c \left( \frac{1}{g_{\eta uu}^2} + \frac{2}{g_{\eta ss}^2} \right) \frac{d}{dp^2} \left( p^2 F_{ss}(p^2, x) \right) \bigg|_{p^2 = m_\eta^2},$$

in the interval $0 < x < 1$. All other distribution functions are exactly zero, in accord with the fact that we do not have gluons or sea quarks in the model. As one can see, for the pion and the kaon these distribution functions are normalized to one. For the $\eta$ only the total sum is normalized to one, due to the different quark species content. Furthermore, using (11) and (16) the momentum sum rule can be easily verified. Notice also that in the chiral limit both the pion and kaon structure functions coincide and give a constant equal to one, independent of the parameters and cut-off. Our result for the pion structure function is different from previous calculations [SS93,FM94]. In one case, [FM94], the authors formally work in the infinite cut-off limit and reinterpret the light-cone pion wave function in terms of a non-relativistic quark model. This is certainly inconsistent with the original Compton amplitude the authors started with. In another case, [SS93], the cut-off is introduced ad hoc, and in a way that the symmetry under the operation $x \rightarrow 1 - x$ gets lost, obtaining the unphysical result that the up and down quark momentum fractions are different even for $m_u = m_d$. Moreover, in both cases the normalization is introduced by hand, and the momentum sum rule is not satisfied. We would like to emphasize that ours is a clear-cut calculation, once the effective action has been gauge-invariantly regularized, the rest follows.

5. With the parameters fixed as indicated above, we show in fig. 1 our computed pion structure function for a constituent up quark mass of $M_u = 280$ MeV, together with the calculation of ref [SS93]. Following the suggestion of ref [JR80] we assume our result to be valid at some small $Q_0$ scale, and evolve the computed structure functions by using the Altarelli-Parisi equations [AP77]. This certainly makes sense in our model as the valence quarks carry all the momentum at $Q_0$. The sea quark and the gluons are then generated by the QCD evolution. In addition, since we want to compare to experimental data at about the charm threshold $Q^2 = 4$ GeV$^2$ [SM92], we have found it more appropriate to do the evolution with three flavours and have matched $\alpha_S(N_f = 3) = \alpha_S(N_f = 4)$ at 4 GeV$^2$. The evolved result for the pion is also shown in fig. 1, together with the experimental fit of ref [SM92]. To determine the evolution ratio, $r = \log(Q^2_0/\Lambda_{\text{QCD}}^2)/\log(Q^2/Q^2_{\text{QCD}})$ we match the valence quark momentum fraction of our calculation with that determined by [SM92] at $Q^2 = 4$ GeV$^2$. We find $r = 0.15$, which for $\Lambda_{\text{QCD}} = 0.226$ GeV yields $Q_0 = 0.312$ GeV. This value is surprisingly close to the constituent quark mass $M_u$ and agrees with the $Q_0$ found for the nucleon structure function [MM91, MW94]. We have found that finite pion mass corrections are negligible. Our results for the kaon $u$ and $s$ distributions before and after evolution are depicted in fig. 2. The difference to the pion case mainly lies in the asymmetry induced by the strange quark and kaon masses. In fig. 3 we show the
ratio of the up valence quark distribution in the kaon compared to the corresponding quantity in the pion at \( Q^2 = 20 \text{ GeV}^2 \). We do not show the structure function for the \( \eta \) but simply mention that for \( M_\eta = 280 \text{ MeV} \) we get 71% of up and down quarks and 29% of strange quarks. We have also checked that our results are rather insensitive against changes in the constituent up quark mass in the region between 250 and 450 MeV.

An interesting aspect of the present investigation is the influence of the regularization method. We have found that the conventional Proper-Time method [NJL] does not regularize the imaginary part of the Compton amplitude, leading to a scaling violation in the model proportional to \( \log(Q^2/M^2) \) in the Bjorken limit. Such behaviour is also obtained in the Pauli-Villars scheme in the regime \( 4M^2 << Q^2 < 4(M^2 + \Lambda^2) \). This scaling violation, however, does not coincide with the QCD result, since there are no explicit gluonic degrees of freedom in the model, and hence cannot be considered as an effective way of mimicking QCD evolution. In this sense we do not consider this regularization suitable for the study of the leading twist contribution to the structure function, in particular eqs.(16) cannot be reinterpreted within a conventional Proper-Time scheme.

6. In the present paper we have evaluated the structure functions of pseudoscalar mesons within a SU(3) NJL model. In contrast to other hadrons, this model provides a fully covariant description of pseudoscalar mesons as deeply bound \( \bar{q}q \) states. Therefore we do not expect a better theoretical understanding of a deep inelastic process than in the case of the pseudoscalar mesons. Most of their low energy properties are governed by the spontaneous breaking of chiral symmetry, and confinement effects are not expected to play an essential role. Desirable features of a structure function are proper normalization, proper support, scaling and \( F_1(x) = 2xF_2(x) \) as dictated by gauge invariance, relativistic invariance, scale invariance and the fact that quarks are spin one-half particles. Our calculation fulfills automatically all these requirements, and leads to the remarkable result that in the chiral limit the pion and kaon structure functions are a constant equal to one. In fact, after QCD evolution we are able to fit experimental data for the pion rather well. We have also found that the finite pion mass corrections induce negligible corrections, whereas the effects of the finite kaon mass corrections are, after evolution, at the 10% level. Our calculation can also be extended to the octet of vector mesons, which due to the \( A - \pi \) mixing ought to change the \( x \) dependence of the pseudoscalars' distribution functions away from the chiral limit. Work along these lines is presently in progress. Finally, let us mention that our calculation strongly suggests that the present results in the chiral limit may in fact be more general than the model used. We believe that this point deserves further investigation, and might help to understand the role of chiral symmetry in deep inelastic scattering.

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REFERENCES


FIGURE CAPTIONS

Figure 1 Valence quark distribution function for the pion $x u_+(x) = x d_+(x)$. The dashed line represents the present calculation. The dashed-dotted line is the calculation of ref.[SS93]. The full line represents our evolved valence quark distribution up to $Q^2 = 4\text{GeV}^2$ with three flavours. The fit to the experimental result as given in ref.[SM92] is represented by the dotted line.

Figure 2 Quark distribution functions for the kaon. The dotted and dashed lines represent the up quark distribution before and after evolution, whereas the solid and dashed-dotted lines stand for the $s$ before and after evolution respectively.

Figure 3 Ratio of the $u$ valence quark distribution of the kaon and the pion $u_K(x)/u_+(x)$ at $Q^2 = 20\text{GeV}^2$. The experimental data are taken from [Ba80].