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PREFACE TO VOLUME IV

The 1967 CERN School at Råttvik was the sixth in a series that began in 1962. The lectures of the 1967 School started on the morning of 22 May and were attended by 84 students from 21 countries in Western Europe, Eastern Europe, the Middle East, and India. The School closed on 2 June with the traditional banquet.

The purpose of the School was to familiarize young post-graduate students of experimental physics with the current theoretical and experimental situation in elementary particle studies. Eleven lecturers contributed to this end by giving a total of 34 seminars, lectures, or after-dinner talks.

This volume contains the lectures given by Dr. K.G. Steffen on "Beam Optics" and by Dr. M. Vivargent on "K⁰ Decay", and the seminar talk by Dr. M. Jacob on "Polarized Targets in Particle Physics". In the interests of speed we have photographed the typescripts given to us by the authors and used the photo-offset reproduction process to produce the four volumes of the Proceedings. All errors, corrections, illegible formulae, etc., are therefore the responsibility of the individual authors and not of the editors or the CERN Scientific Information Service, who have not proofread the texts. We trust that what has been lost in beauty of presentation will be compensated by the fact that the Proceedings will be available to the scientific community in a much shorter time than previously after the end of the School. We would be pleased to receive comments from readers on this change in our publishing policy.

Our thanks are due to the authors who have worked very hard to provide us with their manuscripts either at the School itself or a very few weeks afterwards, and to the Scientific Information Service for their careful and rapid work of publication.

Editorial Board.
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BEAM OPTICS

K.G. Steffen

Deutsches Elektronen-Synchrotron DESY
Brief Summary of a Lecture on

**Beam Optics**

by

K.G. Steffen

Deutsches Elektronen-Synchrotron DESY

Introduction: **Beam optics as a tool for precision experiments**

The general experimental trend leans toward more accurate measurements employing more precise magnets, beams and spectrometers. With well designed linear magnets the beam shaping system and spectrometers are capable of a much higher precision than has commonly been used in the past. Their optical behaviour can quite accurately be described by simple linear beam optics theory, provided that it is correctly applied. I will try in these 3 lectures to give a concentrated survey of concepts and methods in high energy beam optics, hoping that they will aid the experimentalist in inventing and designing his systems while having in mind the points most crucial for performance. Although I have tried to prepare the subject matter in a way which differs from my book, I found it, of course, hard to avoid using it quite freely, and especially more than half of the figures are taken from there.
I. Trajectory Optics

1) General Trajectory equations in Cartesian Coordinates.

From the Lorentz force equation
\[ \frac{d}{dt} (mv) = e(E + v \times B) \]

the exact trajectory equations in a Cartesian coordinate system \((z,x,s)\) (HEBO Fig. 1-1\(^+\)) are derived to be
\[ z'' = \frac{e}{p} \sqrt{1 + z'^2 + x'^2} \left[ x'B_s - (1+z'^2)B_x + x'z'B_z \right] \]
\[ x'' = -\frac{e}{p} \sqrt{1 + z'^2 + x'^2} \left[ z'B_s - (1+x'^2)B_z + x'z'B_x \right] \]

2) Curved coordinate system.

Restricting our considerations to magnetic fields which have an equipotential symmetry plane \(z = 0\), we choose a reference trajectory in this plane and define a right-handed curved coordinate system \((z,x,s)\) as follows (HEBO Fig. 2-1):
- \(s\) is the coordinate measured along the reference trajectory
- \(z\) is the distance from the field symmetry plane
- \(x\) is the coordinate measured in the symmetry plane as normal distance from the reference trajectory

The \(x\)-direction is that of the radius of curvature \(\xi\) of the reference trajectory; we choose \(\xi > 0\) if the center of curvature has a positive \(x\)-coordinate.

All formulae given below will refer to the curved coordinate system.

2a) Field expansion with symmetry plane.

The general field expansion with equipotential symmetry plane in the curved coordinate system is shown to be of the form
\[ \frac{e}{p_o} B_z = h + kx + \frac{1}{2}r x^2 - \frac{1}{2}(h'' -hk + r)z^2 + O(3) \]
\[ \frac{e}{p_o} B_x = kz + r zx + O(3) \]

\(^+\) Reference is made to the figures of K.G. Steffen, "High Energy Beam Optics", Wiley, 1965
$$\frac{e}{p_0} B_s = h'z + (hh' + k')zx + O(3)$$

where
$$h(s) = \frac{e}{p_0} \frac{\partial \mathcal{Z}}{\partial s}, \quad k(s) = \frac{e}{p_0} \frac{\partial \mathcal{Z}}{\partial x} \quad \text{and} \quad r(s) = \frac{e}{p_0} \frac{\partial \mathcal{Z}}{\partial x^2}$$

characterize the field and its transverse derivatives at the reference orbit $z = x = 0$ as functions of $s$.

2b) Linear trajectory equations.

The relations for the curvature
$$\frac{1}{\rho(s)} = h(s) = \frac{e}{p_0} B_z(s) \bigg|_{z=x=0}$$

or, integrated, for the deflecting angle
$$\delta(s) = \int_0^s \frac{1}{\rho} \, dt = \frac{e}{p_0} \int_0^s B_z (\tau) \, d\tau$$

and the linear trajectory equations
$$z'' + kz = 0,$$
$$x'' - \left( k - \frac{1}{\rho^2} \right) x = -\frac{1}{\rho} \frac{\Delta p}{p_0}$$

are obtained. For large momentum spread, these equations are improved by substituting
$$\frac{1}{\rho p} = \frac{p_0}{p} \frac{1}{\rho} = \frac{e}{p} B_z \quad \text{and} \quad k_p = \frac{p_0}{p} k = \frac{e}{p} \frac{\partial \mathcal{Z}}{\partial x}$$

instead of $\frac{1}{\rho}$ and $k$, respectively.

3) Quadrupole transformation.

In the hard-edged model, we have $k(s) = k = \text{const}$, $\frac{1}{\rho} = 0$ inside the quadrupole magnet and $k = \frac{1}{\rho} = 0$ outside. With $\mathcal{Y}(s) = s \sqrt{k}$, the trajectories inside the quadrupole are given by the following expressions

a) for $y''/y < 0$, i.e. ($*k > 0$ (focusing case):
$$y(s) = y_o \cos \mathcal{Y} + y'_o \frac{1}{\sqrt{k}} \sin \mathcal{Y}$$
$$y'(s) = -y_o \sqrt{k} \sin \mathcal{Y} + y'_o \cos \mathcal{Y}$$
b) for \( y''/y > 0 \), i.e. \((+k) < 0\) (defocusing case):

\[
\begin{align*}
y(s) &= y_o \cosh \varphi + y_o' \frac{1}{\sqrt{k}} \sinh \varphi \\
y'(s) &= y_o \sqrt{k} \sinh \varphi + y_o' \cosh \varphi
\end{align*}
\]

For \( \varphi = \sqrt{k} \) (1 = quadrupole length), we obtain for the transformation between quadrupole ends in matrix notation

a) focusing case

\[
\begin{pmatrix}
y \\
y'
\end{pmatrix}
= \begin{pmatrix}
\cos \varphi & \frac{1}{\sqrt{k}} \sin \varphi \\
-\sqrt{k} \sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
y \\
y'
\end{pmatrix}_o
= N + \begin{pmatrix}
y \\
y'
\end{pmatrix}_o
\]

b) defocusing case

\[
\begin{pmatrix}
y \\
y'
\end{pmatrix}
= \begin{pmatrix}
\cosh \varphi & \frac{1}{\sqrt{k}} \sinh \varphi \\
\sqrt{k} \sinh \varphi & \cosh \varphi
\end{pmatrix}
\begin{pmatrix}
y \\
y'
\end{pmatrix}_o
= M + \begin{pmatrix}
y \\
y'
\end{pmatrix}_o
\]

The transformation for a field free drift space of length 1 is

\[
\begin{pmatrix}
y \\
y'
\end{pmatrix}
= \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
y \\
y'
\end{pmatrix}_o
= N^0 \begin{pmatrix}
y \\
y'
\end{pmatrix}_o
\]

The transformation through a system of quadrupoles drift spaces is obtained by subsequent matrix multiplication. In the normalized \((y, y')\)-phase plane, the quadrupole transformation can be geometrically interpreted by moving on concentric circles or hyperbolas, respectively (HEBO Figs. 1-10 and 1-11).

4) Sector magnet transformation.

A homogenous field deflecting magnet with orthogonal entry and exit of the reference trajectory, called "sector magnet" (HEBO Fig. 2-4) can in the hard-edged model be approximated by assuming

\[
\begin{align*}
\frac{1}{y(s)} &= \frac{1}{y} = \text{const}, k = 0 \text{ inside the magnet of length } l \\
\frac{1}{y} &= k = 0 \text{ outside. With } \varphi = \frac{1}{y}, \text{ its linear transformation in } x \text{ is given by}
\end{align*}
\]

\[
\begin{pmatrix}
x \\
x'
\end{pmatrix}
= \begin{pmatrix}
\cos \varphi & \varphi \sin \varphi \\
-\frac{1}{\varphi} \sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
x \\
x'
\end{pmatrix}_o
+ \frac{\Delta p}{p_o} \begin{pmatrix}
-\varphi(1-\cos \varphi) \\
-\sin \varphi
\end{pmatrix}
\]

The first term represents a focusing in \( x \), the second term the so-called "momentum dispersion". In \( z \), the sector magnet with hard edges acts like a drift space. The \( x \)-focusing leads to the theorem that an object point, its image point and the center of curvature of the trajectory are
on a straight line (HEBO Fig. 2-5).

5) Synchrotron magnet transformation (hard-edged model).

A hard-edged magnet with $\frac{\gamma}{\beta} = \text{const} \neq 0$ and $k = \text{const} \neq 0$ is called a "synchrotron magnet". With $K = k - \frac{\gamma}{\beta}$ and $\varphi = 1 \sqrt{K}$, its $x$-transformation is given by

a) focusing case with $K < 0$, i.e. $k < \frac{1}{\beta^2}$:

$$
\begin{pmatrix}
x \\
x'
\end{pmatrix}
= 
\begin{pmatrix}
\cos \varphi & \frac{1}{\sqrt{K}} \sin \varphi \\
-\frac{1}{\sqrt{K}} \sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
x \\
x'
\end{pmatrix}_0 + \frac{\Delta \beta}{\beta_0} \begin{pmatrix}
-\frac{1}{\beta^2} \sin \varphi \\
\frac{1}{\beta^2} \cos \varphi
\end{pmatrix}
$$

b) defocusing case with $K > 0$, i.e. $k > \frac{1}{\beta^2}$:

$$
\begin{pmatrix}
x \\
x'
\end{pmatrix}
= 
\begin{pmatrix}
\cosh \varphi & \frac{1}{\sqrt{K}} \sinh \varphi \\
\frac{1}{\sqrt{K}} \sinh \varphi & \cosh \varphi
\end{pmatrix}
\begin{pmatrix}
x \\
x'
\end{pmatrix}_0 + \frac{\Delta \beta}{\beta_0} \begin{pmatrix}
\frac{1}{\beta^2} \sinh \varphi \\
\frac{1}{\beta^2} \cosh \varphi
\end{pmatrix}
$$

In $z$, the synchrotron magnet with hard edges acts like a quadrupole of strength $k$.

6) Exact linear transformation of a general magnet (including end fields) in terms of effective magnetic length and strength.

Assuming a general magnet which is symmetric about its mid-plane $s = m$, it is shown that, in each of the coordinates $x$ and $z$, its exact linear transformation can be given in terms of five effective parameters $l_z, k_z, l_x, k_x, \frac{1}{\beta}$ and that this transformation is obtained by inserting these parameters into the respective synchrotron magnet transformation. Therefore, in calculating the optical behaviour of magnet systems, the effective parameters should be accurately known for each magnet. However, the accurate determination of these parameters is cumbersome in practice necessitating elaborate field mapping and trajectory tracing techniques, especially if the end fringe field configurations do not have a linear shape and vary with magnet excitation. This is the reason why optical systems are commonly calculated with rather approximate parameters only, which may lead to inaccurate results (see e.g., HEBO Fig. 1-22). Here, the increasing demand for optical precision calls for future improvement, and it may be envisaged that at some time in the future accurate effective magnet parameters will be provided for standard magnets as a general laboratory service.
7) Analog computer.
An analog computer method for simultaneous display of x- and z-trajectory components is described. The computer solves the linear trajectory equations in the hard edged model. The effective magnet parameters and lengths of drift spaces are introduced by a step function generator (HEBO Fig. 1-14). An overall accuracy of the order of 10^{-3} can be achieved. The method is especially useful as a suggestive tool for systems design.

8) Quadrupole approximation with linear end fields.
The end fringe field of a quadrupole magnet can be linearized by means of hyperbolically rounded pole ends in conjunction with magnetic end mirror plates, approximating the form

\[ B_z = g^s x \]
\[ B_x = g^s z \]
\[ B_s = g^s z x \]

Then, in the fringe field region, the transverse field components have a quadrupole distribution in every plane \( s = \text{const} \), with the quadrupole strength \( k(s) \) increasing linear with \( s \) (HEBO Figs. 1-21, 1-24, 1-23, 1-27). For such quadrupoles, the effective length and strength parameters can be obtained by simple integration of the trajectory equations. The results obtained for two quadrupole examples demonstrate that the effective parameters depend on the quadrupole strength \( k(m) \) in the mid-plane and may, especially for quadrupoles with large relative influence of fringe fields, substantially deviate from \( k(m) \) and from the so-called "integrated magnetic length"

\[ l_{\text{magnetic}} = \frac{1}{k(m)} \int_{-\infty}^{+\infty} k(s) \, ds \]

respectively (HEBO Fig. 1-22).

9) Linear end field approximations for deflecting magnets with orthogonal and with nonorthogonal entry and exit.
The end fringe field of a homogeneous field deflecting magnet can be linearized employing hyperbolically rounded pole ends in conjunction with magnetic end mirror plates (HEBO Fig. 2-2). Then, the field component \( B_z \) will rise linearly with \( s \) in the fringe field region. An approximate trajectory integration through the fringe field, performed for orthogonal as well as nonorthogonal entry (HEBO Figs. 2-6 and 2-7), yields the following result:

If the reference trajectory forms an angle \( \varepsilon \) with the normal to the magnet face (sign convention see HEBO Fig. 2-8), the effect of the fringe field may be approximated by a thin lens which is represented by the following transformation matrices

\[ x\text{-coordinate: } \begin{pmatrix} 1 & 0 \\ \frac{1}{\tan \varepsilon} & 1 \end{pmatrix} \]
z-coordinate: \[
\begin{pmatrix}
1 & 0 \\
\frac{1}{5}(\tan e + \frac{b}{6 \gamma \cos e}) & 1
\end{pmatrix}
\]

Here, \( b \) is the length of the fringe field, measured in the direction normal to the magnet face.

10) **Principal trajectories and focal properties.**

The "cosinelike trajectory" \( C(s) \) and the "sinelike trajectory" \( S(s) \) are two linearly independent solutions of the homogeneous part of the linear trajectory equations, starting at \( s=0 \) with the initial conditions

\[
\begin{pmatrix}
C \\
C'
\end{pmatrix}
= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad
\begin{pmatrix}
S \\
S'
\end{pmatrix}
= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

The "dispersion trajectory" or "dispersion" \( D(s) \) is that particular solution of the inhomogeneous linear trajectory equation

\[
D'' - (k - \frac{1}{\gamma^2}) D = - \frac{1}{\gamma^2}
\]

which starts at \( s = 0 \) with initial conditions

\[
\begin{pmatrix}
D \\
D'
\end{pmatrix}
= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

In terms of the "principal trajectories" \( C, S \) and \( D \), the linear transformation between points \( s = 0 \) and \( s \) may therefore be written

\[
\begin{pmatrix}
z \\
z'
\end{pmatrix}
_s = \begin{pmatrix}
C_z(s) & S_z(s) \\
C_z'(s) & S_z'(s)
\end{pmatrix}
\begin{pmatrix}
z \\
z'
\end{pmatrix}
\]

\[
\begin{pmatrix}
x \\
x'
\end{pmatrix}
_s = \begin{pmatrix}
C_x(s) & S_x(s) \\
C_x'(s) & S_x'(s)
\end{pmatrix}
\begin{pmatrix}
x \\
x'
\end{pmatrix}
+ \frac{\Delta \rho}{\rho_o}
\begin{pmatrix}
D(s) \\
D'(s)
\end{pmatrix}
\]

with \( CS' - SC' = |M| = 1 \).

The focal properties of a system are discussed by finding the planes \( s = s_i \) where one of the matrix elements vanishes and interpreting these planes in standard optical terms (e.g. HEBO Fig. i-15).
11) Non-dispersive deflecting systems.
In terms of the sinelike and cosinelike trajectories, the dispersion trajectory may be written
\[
D = -S \int_0^1 \frac{1}{P} C d\sigma + C \int_0^1 \frac{1}{P} S d\sigma
\]
In many experimental situations it is required that this dispersion be removed again further downstream so as to have, at a certain point \( s = s_1 \)
\[
D_1 = D'_1 = 0
\]
We call a system with this feature non-dispersive between points 0 and \( s_1 \). For this, a necessary and sufficient condition is
\[
\int_0^{S_1} \frac{1}{P} C d\sigma = \int_0^{S_1} \frac{1}{P} S d\sigma = 0
\]
Seven examples of non-dispersive systems with and without beam deflection or translation, respectively, are discussed (e.g. HEBO Figs. 3-1, 3-2, 3-3, 3-4).

12) Isochronous systems.
The length difference between a trajectory \( x(s) \) and the reference trajectory is, in linear approximation, given by
\[
\Delta l = -\int_0^s \frac{1}{P} x d\tau
\]
We call a system isochronous between \( s = 0 \) and \( s = S_1 \) if \( \Delta l(s) = 0 \) for all linear trajectories. An isochronous system is always non-dispersive and, in addition, satisfies the equation
\[
\int_0^{S_1} \frac{1}{P} D ds = 0
\]
An example of a symmetric non-dispersive deflecting system is given.

13) Second order chromatic aberrations.
Extending the linear solution of the trajectory equation by including second order chromatic aberration terms, we may write for the x-coordinate
\[
x(s) = \left( \frac{C(s) + \tau(s) \Delta P}{P_0} \right) x_o + \left( \frac{S(s) + \sigma(s) \Delta P}{P_0} \right) x'_o + \left( \frac{D(s) + \nu(s) \Delta P}{P_0} \right) \Delta P
\]
In a system composed of quadrupole magnets only, we have \( v(s) = 0 \), and the chromatic aberration functions \( \tau(s) \) and \( \sigma(s) \) are given by

\[
\tau = S \int_0^S C' \cdot d\sigma - C \int_0^S C'S' \cdot d\sigma
\]

\[
\sigma = S \int_0^S S' \cdot C' \cdot d\sigma - C \int_0^S S'^2 \cdot d\sigma
\]

Considering a parallel beam at \( s = 0 \) and looking at its chromatic aberration in the focal plane \( s = s_1 \) with \( C(s_1) = 0 \) and \( S(s_1) \neq 0 \), we have

\[
\tau(s_1) = S(s_1) \int_0^{s_1} C'^2(\sigma) \cdot d\sigma \neq 0
\]

which means that it is impossible to produce an achromatic focus (HEBO Fig. 1-16). Similarly, it can be seen that it is as well impossible to produce an achromatic image in any type of quadrupole system (HEBO Fig. 1-17).

14) Sextupole transformation.

Inserting the sextupole field

\[
B_x = g'zx
\]

\[
B_z = \frac{1}{2}g'(x^2 - z^2)
\]

with \( g' = \left. \frac{\partial^2 B_z}{\partial x^2} \right|_{x=z=0} \)

into the general trajectory equations, one obtains for \( z' \ll 1 \) and \( x' \ll 1 \)

\[
z'' = -m z x
\]

\[
x'' = \frac{1}{2}m(x^2 - z^2)
\]

with \( m = \frac{e}{P_0g'} \)

The solution of these equations cannot be written in a closed form; they can, however, easily be solved in an analog computer. Looking at the field component \( B_z \) in the plane \( z = 0 \), the sextupole may be considered as a lens in which the strength varies linearly with the distance from the axis. When placed in a point with large dispersion, where particles of different momentum are separated in \( x \)-direction, the sextupole may therefore be used to submit particles of different momentum to a different focusing strength which is chosen such that the chromatic aberration of previous or subsequent quadrupole magnets is compensated.
II. Magnet requirements for precision optics.

What can the experimentalist expect from good spectrometer magnets?

1.) The field distribution should be linear to within at least a few tenth of a percent over the entire aperture, local as well as integral including the end fields.

2.) The field map should be independent of excitation over a wide range, and changes of the field distribution due to saturation should become noticeable at very high excitations only. Especially, the integrated magnetic length should not vary with field level.

Requirements 1. and 2. can be achieved by careful magnet design including shaping of end fields.

3.) The effective length and strength parameters $l_z$, $k_z$, $l_x$, $k_x$ and $\frac{1}{\sigma}$ describing the linear optics of the magnet should be known with an accuracy of about $10^{-3}$.

This, in general, requires accurate point-to-point field measurements as a basis for precise trajectory calculations, from which the effective magnet parameters can be derived. If the end fields closely resemble a linear field model, the effective parameters may, however, be fairly accurately calculated from this model without directly referring to field measurements.

4.) As an inherent design feature, each magnet should carry a rigid gadget carefully aligned with respect to the magnetic symmetry planes, into which a precision alignment target can be reproducibly inserted.

Representative alignment tolerances, which can be achieved by careful application of standard surveying techniques, are:

- twist of quadrupole $\leq \frac{1}{2} \cdot 10^{-3}$ rad
- transverse displacement of quadrupole $\leq 0.1$ mm
- twist of deflecting magnet $\leq 10^{-4}$ rad

Realizing that all of the features stated above will certainly not be relevant in every particular application, I nevertheless feel in view of the trend toward more accurate experiments that they should be incorporated in standard laboratory magnets, especially since they do not involve much extra cost.

III. Composite Spectrometers.

1.) Sloped window spectrometer.

In a spectrometer, the particle displacement

$$x(s) = x_0 C(s) + x' S(s) + \frac{\Delta P}{P_0} D(s)$$

assumes an especially simple form in the focal plane $s = s_1$ with $C(s_0) = 0$. There, a point counter on the axis will be intercepted by all particles which satisfy the linear relation

$$\frac{\Delta P}{P_0} = \sum x' - x_0$$

with $\sum = -\frac{s_1}{D_1}$.
between momentum and angle of emission independent of their origin $x_0$ in the target, which often is of no physical interest anyhow. This linear relation represents a sloped line in the $(p, \theta)$-plane which, in studying a two-body reaction, can be matched to the line representing the dependence of momentum on the production angle $\theta$, thus giving optimum discrimination against particles originating from reactions involving more than two secondaries (HEBO Fig. 3-8).

According to

$$\frac{1}{\Sigma} = -\frac{D_1}{S_1} = \int_0^1 C\sigma \, d\sigma,$$

the slope $\Sigma$ of a spectrometer can be varied by

1. Changing the deflecting angles of the magnets, i.e. changing $\frac{1}{\Sigma}$
2. Changing the focusing strengths of the quadrupoles, thus changing the amplitude of $C(s)$ within the magnets.

A third, approximate method of slope variation consists in moving the counters slightly off the focal plane or, equivalently, moving slightly the focal length of the preceding quadrupole.

Some properties of the sloped window spectrometer, including the limitation of resolution due to second order chromatic aberrations, are discussed and illustrated by a numerical example.

2.) **Image Spectrometer.**

The same spectrometer example, when operated with the counters in the image plane, is used to illustrate the properties of an image spectrometer (HEBO Figs. 3-7 to 3-12).

The following examples of spectrometers composed of deflecting magnets and quadrupoles are described:

3.) DESY 2GeV/c sloped window spectrometer
4.) DESY 6GeV/c sloped window spectrometer
5.) SLAC 8GeV/c spectrometer with vertical deflection
6.) SLAC 20GeV/c spectrometer with vertical deflection.

**IV Envelope optics.**

1.) **Liouville's Theorem.**

A proof is given of Liouville's theorem, which states that the particle density in phase space does not change in the vicinity of a particle moving in an external electromagnetic field. In an optical system with no coupling between z- and x-coordinates, the "beam area" in each of the phase planes $(z,z')$ and $(x,x')$ therefore stays constant, as expressed by the fact that all transformation matrices have unit determinant.

2.) **Magnet acceptance area.**

The acceptance area of a synchrotron magnet of length $l$ and constant aperture width $a$, referred to the magnet mid-plane $s=m$, is given, including quadrupole, sector magnet and slit collimator as special cases (HEBO Fig. 4-1). From this the acceptance area of a composite magnet system can be obtained by
linearly transforming all single magnet acceptance areas to a common point.

3.) Beam envelope formalism.

A most useful scheme for describing the acceptance of a system or the emittance of a beam, respectively, is the beam envelope formalism. It approximates the acceptance or emittance area in phase plane by an ellipse as given by

\[ A^2 y^2 - 2EE'yy' + E^2 y'^2 = \epsilon^2 \]

(HEBO Figs. 4-2, 4-3) and describes the linear transformation of the ellipse parameters \( E(s) \) and \( A(s) \) through the system.

\( E(s) \) is the maximum excursion from the axis within the family of particles enclosed by the ellipse and is called "envelope function".

\( A(s) \) is the maximum angular deviation and is called "angular envelope".

\( \epsilon \) is the constant area of the ellipse, divided by \( \pi \), and is called the "beam emittance".

A pair of "conjugated trajectories" \( y_1(s); y_2(s) \) on the ellipse is given by

\[ y_1 = E \cos(\phi - \psi) \]
\[ y_2 = E \sin(\phi - \psi) \]

with

\[ \phi(s) = \int_0^s \frac{\epsilon}{E^2} \, dt \]

being called the "phase function"

and \( \psi \) being an arbitrary phase constant. Consequently, the envelope is given by

\[ E(s) = \sqrt{y_1^2 + y_2^2} \]

Similarly, one obtains for the angular envelope

\[ A(s) = \sqrt{y_1'^2 + y_2'^2} \]

These formulas permit simple analog computation of these functions (e.g. HEBO Figs. 4-4, 4-5).

V Fundamentals of beam shaping systems.

1.) Emittance filter.

In order to produce a momentum- or mass- analysed beam the beam emittance must be defined. An emittance filter consisting of two slit collimators with proper focusing between them is described.

2.) Momentum filter.

Demanding that all particles having a momentum deviation greater than a given \( \Delta p \) are to be removed from a beam by \( P_0 \).
means of a given deflecting magnet, the maximum beam emittance that can be cleaned by this magnet is calculated in phase plane (HEBO Fig. 4-8).

The result is

$$\epsilon = \frac{\Delta p}{p_0} \frac{aL}{4\gamma^2}$$

with \(a\) being the aperture width and \(L = 2f \sin \frac{\theta}{2}\) the "straight" magnet length. Spatial separation of the momentum analysed beam is obtained, using subsequent focusing, at a point \(s_1\) with

$$\phi(s_1) - \phi(m) = \int_m^{s_1} \frac{\epsilon}{E^2} ds = \frac{\pi}{2}$$

\((s = m \text{ location of magnet mid-plane})\)

3.) Mass filter.

In analogy to the previous paragraph, the acceptance of an electrostatic mass separator (HEBO Figs. 4-9, 4-10) is calculated for given particle momentum \(p\) and velocities \(v_0\) and \(v\), respectively. With

$$f = \frac{e}{p} \left( \frac{1}{v} - \frac{1}{v_0} \right) \quad \text{and} \quad g = \frac{|f|}{2} \frac{Vl^2}{a^2}$$

\((V \text{ being the voltage, } l \text{ the length and } a \text{ the gap height of the separator}), \text{ the separator acceptance is obtained to be}

$$\epsilon = \frac{1}{2} \frac{a^2}{l} g \sqrt{1-g^2} \quad \text{for} \quad g < \frac{1}{\sqrt{2}} \quad \text{and}$$

$$\epsilon = \frac{1}{4} \frac{a^2}{l} \quad \text{for} \quad g \geq \frac{1}{\sqrt{2}}.$$
$K^0$ DECAY

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K$^0$ Decay
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It is in two directions that experimental physicists are looking at K$^0$ decays:
i) The violation of CP and T
ii) The validity of the selection rules.

Before relating the results concerning the two fields, let us start with a brief review of the theory of the K$^0$ - $\overline{K}^0$ system.

1. Theoretical Review

We take the same notations as T.D. Lee and C.S. Wu$^1$). Without any assumption concerning CPT invariance, we consider a mixture of $|K^0>$ and $|\overline{K}^0>$ whose amplitude in its proper time is described by:

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}.$$ 

$|\overline{K}^0>$ is defined to be:

$$|\overline{K}^0> = CP|K^0>.$$ 

$|K^0>$ : eigenstate of $(H_{st} + H_Y)$ with strangeness + 1

$|\overline{K}^0>$ : eigenstate of $(H_{st} + H_Y)$ with strangeness - 1.

$\psi(t)$ is a solution of the time-dependent Schrödinger equation

$$-\frac{d\psi}{dt} = (\Gamma + iM)\psi.$$ 

Γ and M are $(2 \times 2)$ Hermitian matrices $(\Gamma = \Gamma^+, M = M^+)$. Γ is the so-called decay matrix and M the mass matrix. Their elements are expressed as a function of the weak interaction Hamiltonian by the relations:
\[ \Gamma_{11} = \pi \sum_F \rho_F |< F | H_{wk} | K^0 >|^2 \]

\[ \Gamma_{22} = \pi \sum_F \rho_F |< F | H_{wk} | \bar{K}^0 >|^2 \]

\[ \Gamma_{21} = \Gamma_{12}^* = \pi \sum_F \rho_F < \bar{K}^0 | H_{wk} | F > < F | H_{wk} | K^0 > \]

\[ M_{11} = m_K^0 + < K^0 | H_{wk} | K^0 > + \sum_n P [ |< n | H_{wk} | K^0 > | \gamma (m_K^0 - m_n^0)] \]

\[ M_{22} = m_K^0 + < \bar{K}^0 | H_{wk} | \bar{K}^0 > + \sum_n P [ |< n | H_{wk} | \bar{K}^0 > | \gamma (m_K^0 - m_n^0)] \]

\[ M_{21} = M_{12}^* = < \bar{K}^0 | H_{wk} | K^0 > + \sum_n P [ |< \bar{K}^0 | H_{wk} | n > < n | H_{wk} | K^0 > | (m_K^0 - m_n^0)] \]

\[ m_K^0 = < K^0 | H_{st} + H_{\gamma} | K^0 > = < \bar{K}^0 | H_{st} + H_{\gamma} | \bar{K}^0 > . \]

P stands for the principal value and the sum n represents the appropriate sums and integrations over unperturbed eigenstates of \((H_{st} + H_{\gamma})\) whose \(m_n^0\) are the corresponding unperturbed energies.

A solution \(\psi(t)\) of the Schrödinger equation may be written:

\[ \psi(t) = |K_j^0 > e^{-\gamma_j t + i m_j} \]

where

\[ \gamma_j = \text{total decay width of } |K_j^0 > \]

\[ m_j = \text{mass of } |K_j^0 > \]
\[ |K_j^0 > \text{ are defined by:} \]
\[ (\Gamma + iM) |K_j^0 > = \left(\frac{1}{2} \gamma_j + i m_j \right) |K_j^0 > \quad j = 1,2. \]

The two solutions are:

\[ |K_1^0 > = \left[2(1 + |\epsilon_1|^2)\right]^{-\frac{1}{2}} \begin{pmatrix} 1 + \epsilon_1 \\ 1 - \epsilon_1 \end{pmatrix} \]

\[ |K_2^0 > = \left[2(1 + |\epsilon_2|^2)\right]^{-\frac{1}{2}} \begin{pmatrix} 1 - \epsilon_2 \\ (1 - \epsilon_2) \end{pmatrix} \]

\[ \epsilon_1 \text{ and } \epsilon_2 \text{ are related to the matrix elements of } \Gamma \text{ and } M \text{ by the relations:} \]

\[ \epsilon = \frac{1}{2}(\epsilon_1 + \epsilon_2) = \frac{\Gamma_{12} - \Gamma_{12}^* + i(M_{12} - M_{12}^*)}{(\gamma_1 - \gamma_2) + 2i(m_1 - m_2)} , \]

\[ \delta = \frac{1}{2}(\epsilon_1 - \epsilon_2) = \frac{\Gamma_{11} - \Gamma_{22} + i(M_{11} - M_{22})}{(\gamma_1 - \gamma_2) + 2i(m_1 - m_2)} . \]

In the calculations, quadratic terms in \( \epsilon \) and \( \delta \) are neglected. If CPT is conserved for any Hamiltonian \( H_{st} \), \( H_{\gamma} \) and \( H_{wk} \)

\[ \Gamma_{11} = \Gamma_{22}, \quad M_{11} = M_{22} \quad \text{and} \quad \epsilon_1 = \epsilon_2 = \epsilon . \]

In the \( |K^0 > |K^0 > \) representation CP may be expressed by the matrix:

\[ \text{CP} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

which commutes with itself and the matrix unity only. Its proper values are +1 and -1 corresponding to the eigenstates
\[ |K_+^0 \rangle = \frac{|K^0 \rangle + |\bar{K}^0 \rangle}{\sqrt{2}} \]
\[ \epsilon = 0 \]
\[ |K_-^0 \rangle = \frac{|K^0 \rangle - |\bar{K}^0 \rangle}{\sqrt{2}} . \]

If CP were conserved in weak decays $K^0_+$ and $K^0_-$ should only decay into eigenstates of CP. Before the discovery of Christenson et al.\(^2\)), it was believed that CP was conserved and that $|K^0_+ \rangle$ and $|K^0_- \rangle$ were the two states of the $K^0$ corresponding to CP = +1 and CP = -1. They found the existence of $K^0_-$ decay into $\pi^+ \pi^-$ which is always a pure CP = +1 state.

If we consider that CPT is valid, and there is no reason to suppose the contrary, the two $K^0$ states have to be expressed by

\[ |K^0_+ \rangle = [2(1 + |\epsilon|^2)]^{-1/2} [(1 + \epsilon)|K^0 \rangle + (1 - \epsilon)|\bar{K}^0 \rangle] \]
\[ |K^0_- \rangle = [2(1 + |\epsilon|^2)]^{-1/2} [(1 + \epsilon)|K^0 \rangle - (1 - \epsilon)|\bar{K}^0 \rangle] . \]

2. **EXPERIMENTAL STATUS**

2.1 **Lifetimes of $K_1^0$ and $K_2^0$**

As we know from the experiment by Christenson et al., $\epsilon$ is very small \(\sim 10^{-3}\) and we can identify $K^0_1$ with $K^0_+$ and $K^0_2$ with $K^0_-$. Only $K^0_1$ can decay into $2\pi$. $K^0_2$ can decay into three-body decays ($3\pi$, leptonic decays). Due to the ratio between the phase spaces the lifetime of $K^0_1$ must be shorter than the lifetime of the $K^0_2$.

Experimentally, the two lifetimes are:

\[ \tau_1 = (0.87 \pm 0.009) \times 10^{-10} \text{ sec} \]
\[ \tau_2 = (5.6 \pm 0.26) \times 10^{-9} \text{ sec} . \]

2.2 **Mass difference between $K_1^0$ and $K_2^0$**

If transitions between $|K^0 \rangle$ and $|\bar{K}^0 \rangle$ were allowed by normal weak interaction with $|\Delta S| = 2$, the mass difference between $K^0_1$ and $K^0_2$ should be
of the order of $10^3$ eV. If weak interactions allow only $|\Delta S| = 1$ transition this mass difference was evaluated to be $\sim 3 \times 10^{-5}$ eV.\(^1\).

Different methods have been used to measure the absolute value of this mass difference and its sign.

2.2.1 **Regeneration methods.** Schrödinger's equation was written for $K^0$ in vacuum. If we are in some material, we need to add to $\Gamma + iM$ another matrix and $\Gamma + iM$ becomes $\Gamma' + iM'$:

$$
\Gamma' + iM' = \Gamma + iM - \frac{i2\pi N \nu}{(1 - v^2)^{1/2} k} \begin{pmatrix}
 f(0) & 0 \\
 0 & \bar{f}(0)
\end{pmatrix}
$$

where

$N$: Number of nuclei by cm$^3$ of material

$\nu$: velocity of the $K^0$ ($c = 1$)

$k$: wave number of the $K^0$

$f(0)$: $K^0$ scattering amplitude

$\bar{f}(0)$: $\bar{K}^0$ scattering amplitude.

The effects are the following:

i) We have new eigenstates $|K'_1^0\rangle$ and $|K'_2^0\rangle$ characterized by $\epsilon'$, $m'_1$, $m'_2$, $\gamma'_1$, $\gamma'_2$.

ii) Neglecting quadratic terms in $\epsilon$, $\epsilon'$

$$
|K'_1^0\rangle = |K^0_1\rangle + r|K^0_2\rangle
$$

$$
|K'_2^0\rangle = |K^0_2\rangle - r|K^0_1\rangle
$$

The regeneration parameter $r$ is expressed by

$$
r = \frac{i\pi N A[f(0) - \bar{f}(0)]}{k[\frac{1}{2} + i\delta]}
$$

with
\[ \delta = \frac{m_1 - m_2}{\gamma_1 - \gamma_2} \]

and \( \Lambda \) is the mean decay distance for \( K_2^0 \) in the medium. Practically we can take \( \delta = (m_1 - m_2)/(\gamma_1 - \gamma_2) \) and for \( \Lambda \) the value in the vacuum.

In putting material in a pure beam of \( K_2^0 \) we will obtain a coherent regenerated amplitude of \( K_1^0 \) at the end of this material. If the amplitude for the transmitted \( K_2^0 \) at the end of the regenerator is normalized to the unity, the amplitude for \( K_1^0 \)'s regenerated will be:

\[
|K_1^0 > = \frac{iN\Lambda \delta f_{21}(0)}{i\delta + \gamma_2} [1 - e^{-(i\delta + \gamma_2)\ell}] \]

where

\[
f_{21}(0) = \frac{f(0) - \bar{f}(0)}{2}
\]

\[\lambda = 2\pi/k\]

\[\ell = \text{length of the regenerator}/\Lambda.\]

Next to this coherent regeneration in the forward direction there is an incoherent regeneration whose intensity in any \( \Theta \) direction is given (neglecting multiple scattering regeneration and diffracted coherent regeneration) by

\[
I(\Theta) = \frac{dN(\Theta)}{d\Omega} = |f_{21}(\Theta)|^2 N\Lambda(1 - e^{-\ell}).
\]

i) Good et al. \(^3\) used the fact that the ratio between coherent and incoherent regeneration at \( 0^\circ \) is given by:

\[
R = \frac{I(0)}{(dN/d\Omega)_0} = N\lambda^2 \Lambda \varphi / (1 - e^{-\ell})
\]

with

\[
\varphi = \frac{|1 - e^{-(i\delta + \gamma_2)\ell}|^2}{\delta^2 + \gamma_2^2}.
\]
The only parameter in $R$ is $\delta$. They put targets of Pb and Fe inside a propane bubble chamber and measured the coherent and incoherent regeneration by looking at the $\pi^+\pi^-$ decay of the $K_1^0$ around 0° relative to the beam direction. They obtained: $|\delta| = 0.84 \pm 0.29$.

ii) Christenson et al. (Christenson, Thesis) obtained a value of $|\delta|$ in measuring the $\pi^+\pi^-$ decay intensity from $K_1^0$ after two regenerators separated by variable distance as shown below.

The expression for these intensities is given by:

$$I(\delta) = a_1^2 e^{-\delta} + a_2^2 + 2a_1 a_2 e^{-\delta/2} \cos(\delta + \Delta\delta)$$

where

$a_1$, $a_2$, $\Delta\delta$ are constants depending only on the thickness of the regenerator, and of its composition;

$g$ : distance between the regenerators expressed in A units.

Their result is:

$$|\delta| = 0.50 \pm 0.10.$$  

iii) T. Fujii et al. 4) have measured the intensity of $K_1^0 \rightarrow \pi^+\pi^-$ decays after one regenerator in changing the thickness and they obtained:

$$|\delta| = 0.82 \pm 0.12$$

2.2.2 Strangeness oscillation method. Starting with a pure $K^0$ or $\bar{K}^0$ beam at time $t = 0$, a mixture of $K^0$ and $\bar{K}^0$ will result at time $t \neq 0$. 
With our notation

\[ |K^0 > = \frac{\sqrt{2(1 + |\epsilon|^2)}}{2(1 + \epsilon)} \left[ |K_1^0 > + |K_2^0 > \right] \]

\[ |\bar{K}^0 > = \frac{\sqrt{2(1 + |\epsilon|^2)}}{2(1 - \epsilon)} \left[ |K_1^0 > - |K_2^0 > \right]. \]

At time \( t \neq 0 \), the wave function representing the evolution of the pure \( |K^0 > \) state at \( t \neq 0 \) will be:

\[ \psi(t) = \frac{1}{2(1 + \epsilon)} \left[ (1 + \epsilon)|K^0 > \left( e^{-[(\gamma_1/2) + i\mu_1]t} + e^{-[(\gamma_2/2) + i\mu_2]t} \right) \\
+ (1 - \epsilon)|\bar{K}^0 > \left( e^{-[(\gamma_1/2) + i\mu_1]t} - e^{-[(\gamma_2/2) + i\mu_2]t} \right) \right]. \]

The \( |K^0 > \) amplitude \( A_{|K^0 >}(t) \) is:

\[ A_{|K^0 >}(t) = \frac{1}{2} \left[ e^{-[(\gamma_1/2) + i\mu_1]t} + e^{-[(\gamma_2/2) + i\mu_2]t} \right], \]

and the \( |K^0 > \) intensity:

\[ N_{|K^0 >}(t) = \frac{1}{4} \left[ e^{-\gamma_1 t} + e^{-\gamma_2 t} + 2^{-t(\gamma_1 + \gamma_2)/2} \cos \Delta m t \right]. \]

By identifying \( K^0 \) and \( \bar{K}^0 \) by their strong interactions with nucleons, one can determine \( N_{|K^0 >}(t) \) and \( N_{|\bar{K}^0 >}(t) \) and obtain a value for \( |\Delta m| \).

These methods gave large errors and are subject to large corrections\(^5\)\(^-\)\(^7\). From them, \( |\delta| \) ranges from \( 1.9 \pm 0.3 \) to \( 0.62 \pm 0.33 \) - \( 0.27 \).

If \( \Delta S = \Delta Q \) is valid in leptonic decays \( K^0 \) and \( \bar{K}^0 \) may be identified in their leptonic decays. If we call \( A_1 \) and \( A_2 \) the decay in one mode for \( K_1^0 \) and \( K_2^0 \) we will obtain for the amplitude at time \( t \) (starting from pure \( |K^0 > \) beam)

\[ A(t) = \frac{\sqrt{2(1 + |\epsilon|^2)}}{2(1 + \epsilon)} \left[ A_1 e^{-[(\gamma_1/2) + i\mu_1]t} + A_2 e^{-[(\gamma_2/2) + i\mu_2]t} \right] \]
And the decay intensity in this mode at \( t \) will be:

\[
N(t) = \frac{(1 + |\epsilon|^2)}{2(1 + \epsilon)} \left[ |A_1|^2 e^{-\gamma_1 t} + |A_2|^2 e^{-\gamma_2 t} + 2 \Re(A_1 A_2^*) \cos \Delta m t \right. \\
+ \left. 2 \Im(A_1 A_2^*) \sin \Delta m t \right],
\]

where

\[
\Delta m = m_1 - m_2.
\]

Aubert et al.\(^8\) have used this method by identifying electrons and positrons in the Ecole Polytechnique Heavy Liquid Bubble Chamber and have obtained:

\[
|\delta| = 0.47 \pm 0.21.
\]

2.2.3 **Interference effects between \( \pi^+ \pi^- \) decays from \( K^0_S \) and regenerated \( K^0_L \).** After the discovery of the \( K^0_S + \pi^+ \pi^- \) decay it was necessary to reject some theories\(^9\) to see if there was interference between \( K^0_S \) and \( K^0_L \) regenerated. One interesting product of these experiments is the value of \(|\delta|\).

The experiment consists in putting a piece of regenerator in a pure \( K^0_S \) beam and measuring the rate of \( \pi^+ \pi^- \) decays after the regenerator in the proper time system of the \( K^0 \)'s.

After the regenerator the expected time distribution is given by:

\[
N(t) = c \left[ R^2 e^{-t} + 1 + 2R e^{-\frac{t}{2}} \cos (\delta t + \phi) \right]
\]

\[
\gamma_2 << \gamma_1, \quad e^{-\gamma_2 t} \sim 1.
\]

where

\[
R = \frac{|A|}{|\eta_+|}
\]

\( t \) is measured in units of \( K^0_S \) lifetime

\[
A = \frac{i \lambda \Delta N f_{231}(0)}{1 - e^{-(i\delta + \frac{1}{2})t}} \left[ 1 - e^{-(i\delta + \frac{1}{2})t} \right]
\]

\[
\phi = \arg \eta_+ = - \arg A.
\]
Two experiments were performed at CERN using a similar set-up but different regenerators. The set-up used by M. Bott-Bodenhausen et al. is given in Fig. 1. M. Bott-Bodenhausen et al. obtained \(|\Delta| = 0.480 \pm 0.024\) which is the best determination for \(|\Delta|\) until now and C. Alff-Steinberger et al. obtained \(|\Delta| = 0.445 \pm 0.034\) which is a very compatible result.

The following points were deduced from these experiments:

i) The mass difference \(|\Delta m|\) is of the order of \(7 \times 10^{-6} \text{ eV}\) excluding \(|\Delta S| = 2\) weak transitions.

ii) There is really an interference between \(K_2^0\) and \(K_1^0\) in their \(\pi^+\pi^-\) decay (Fig. 2).

iii) The hypothesis from \(\pi\)'s weakly interacting has to be rejected.

What is the sign of the mass difference?

The more significant result was given by Piccioni at the last Berkeley conference. He used interference in the \(\pi^+\pi^-\) decay mode between the regenerated \(K_1^0\) beam and the originally produced but attenuated \(K_1^0\) beam coming from pure \(K^0\) beam (Fig. 3).

His result is

\[ \Delta m = m_2 - m_1 = +0.6 \pm 0.2 \text{ (Fig. 4).} \]

Other experiments seem to confirm these results \(^{12,13}\).

2.3 \textbf{CP and T violation in }K^0\textbf{ decays}

2.3.1. Non leptonic decays

a) \(2\pi\) decays

The \(K_2^0\) decays into \(\pi^+\pi^-\) and \(\pi^0\pi^0\) are characterized by the parameters

\[ \eta_{+-} = \frac{<\pi^+\pi^-|H_{wk}|K_2^0>}{<\pi^+\pi^-|H_{wk}|K_1^0>} = \epsilon + \epsilon' \]

\[ \eta_{00} = \frac{<\pi^0\pi^0|H_{wk}|K_2^0>}{<\pi^0\pi^0|H_{wk}|K_1^0>} = \epsilon - 2\epsilon' \]
\[ \epsilon' = \frac{1}{\sqrt{2}} \frac{\text{Im} A_2}{A_0} e^{i(\pi/2) + \delta_2 - \delta_0} \]

where

- \( A_2 \) : amplitude of the \( I = 2 \) state
- \( A_0 \) : amplitude of the \( I = 0 \) state
- \( \delta_2 - \delta_0 \) : phase shift between \( I = 2 \) and \( I = 0 \) states in the interaction between the two pions in the final state.

If CPT is conserved

\[ \epsilon = \left[ \frac{(T_{12} - T_{12}^*) + i(M_{12} - M_{12}^*)}{(\gamma_1 - \gamma_2) + 21(m_1 - m_2)} \right]. \]

\( \Gamma \) may be decomposed into its components coming from leptonic and non-leptonic decays in neglecting very rare decays

\[ \Gamma = \Gamma_{2\pi(I=0)} + \Gamma_{2\pi(I=2)} + \Gamma_{3\pi} + \Gamma_{\text{leptonic}} \]

with

\[ \Gamma_{2\pi(I=0)} = A_0^2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \]

\[ \Gamma_{2\pi(I=2)} = \begin{pmatrix} |A_2|^2 & A_2^2 \\ A_2^2 & |A_2|^2 \end{pmatrix} \]

\[ \Gamma_{3\pi} = \begin{pmatrix} \alpha_{3\pi} & x_{3\pi} + iy_{3\pi} \\ x_{3\pi} - iy_{3\pi} & \alpha_{3\pi} \end{pmatrix} \]

\[ \Gamma_{\text{lep}} = \begin{pmatrix} \alpha_\ell & x_\ell + iy_\ell \\ x_\ell - iy_\ell & \alpha_\ell \end{pmatrix} \]
M may be written as:

\[
M = \begin{pmatrix}
M_{11} & M_{12} + iM_{13} \\
M_{21} - iM_{31} & M_{22}
\end{pmatrix}
\]
with \(M_{11} = M_{22}\).

\(\epsilon\) is then expressed by

\[
\epsilon = \frac{2i(y_{3\pi} + y_{2} + 2\text{Re} A_{2} \text{ Im} A_{2}) - 2M}{\gamma_{1} - \gamma_{2} + 2i(m_{1} - m_{2})}
\]

Since \(\gamma_{1} >> \gamma_{2}\) and that \(K_{0}^{0}\) decay almost entirely into \(2\pi\),

\[
\frac{1}{\gamma_{1}} = \gamma_{1} \neq \frac{4\Lambda_{0}^{2}}{\epsilon},\text{ we can write:}
\]

\[
\frac{2i(y_{3\pi} + y_{2} + 2\text{Re} A_{2} \text{ Im} A_{2}) - 2M_{1}}{4\Lambda_{0}^{2}(1 + 2i\delta)} \approx \frac{2i(y_{3\pi} + y_{2} + 2\text{Re} A_{2} \text{ Im} A_{2}) - 2M_{1}}{4\Lambda_{0}^{2}(1 - i)}
\]

We may decompose \(\epsilon\) into its components

\[
\epsilon_{A_{2}}(I=2) = \frac{4i \text{ Re} A_{2} \text{ Im} A_{2}}{4\Lambda_{0}^{2}(1 - i)}
\]

\[
\epsilon_{3\pi} = \frac{2i y_{3\pi}}{4\Lambda_{0}^{2}(1 - i)}
\]

\[
\epsilon_{1} = \frac{2i y_{1}}{4\Lambda_{0}^{2}(1 - i)}
\]

\[
\epsilon_{M} = \frac{-2M_{1}}{4\Lambda_{0}^{2}(1 - i)}
\]

It has been shown that

\[
|\epsilon_{A_{2}}| \approx 3.2 \times 10^{-4}
\]

\[
|\epsilon_{3\pi}| \approx 4.0 \times 10^{-4}
\]
\[ \sum_{i} |\epsilon_{lep}| \approx 1.70 \times 10^{-4} \]

\[ |\epsilon_{A2}| + |\epsilon_{3\pi}| + |\epsilon_{lep}| \approx 9 \times 10^{-4} \]

that has to be compared with

\[ |\eta_{+-}| = (1.94 \pm 0.09) \times 10^{-3}. \]

To explain the value of \(|\eta_{+-}|\) we have to introduce \(\epsilon_{M}\) or \(\epsilon'\).

If we suppose that

\[ |\eta_{+-}| = |\epsilon_{M}| \]

then

\[ \phi_{\eta_{+-}} = \phi_{\epsilon} = \phi_{\epsilon_{M}} = 45^\circ. \]

In that case, CP is violated only in the mass matrix and there should be a super-weak interaction with \(|\Delta S| = 2\) responsible for this violation, as proposed by Wolfenstein.

i) Determination of \(|\eta_{+-}|\)

Christenson et al.\(^2\) measured for the first time \(|\eta_{+-}|\) for \(K^0\)'s of 1.1 GeV/c momentum. It was proposed by several theoreticians\(^{14,15}\) that this apparent CP violation in weak interaction may be due to some new interaction connected with the strangeness or the hypercharge of the \(K^0\) and \(\bar{K}^0\). By supposing that this field is a vector one, the effects would give a \(\gamma_2\) dependence of the rate of the \(K^0 \rightarrow \pi^+\pi^-\) decay. Table 1 gives the results of experiments performed to measure \(|\eta_{+-}|\) at different energies.

The up-to-date value for \(\eta_{+-}\) is \((1.94 \pm 0.09) \times 10^{-3}\).

From our result at 10.7 GeV/c, by fitting the branching ratios at different \(K^0\) momenta we can obtain an upper limit on the energy splitting between \(K^0\) and \(\bar{K}^0\) due to gravitational effects of less than \(10^{-20} m_{K^0}\).
Table 1

<table>
<thead>
<tr>
<th>Authors</th>
<th>$K^0$ momentum</th>
<th>Ratio $\frac{K^0 \rightarrow \pi^+ \pi^-}{K^0 \rightarrow \text{all charged}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christenson et al. $^2$</td>
<td>1.1 GeV/c</td>
<td>$(2.0 \pm 0.4) \times 10^{-3}$</td>
</tr>
<tr>
<td>Princeton I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Galbraith et al. $^4$</td>
<td>3.15 GeV/c</td>
<td>$(2.24 \pm 0.23) \times 10^{-3}$</td>
</tr>
<tr>
<td>de Bouard et al. $^7$</td>
<td>10.7 GeV/c</td>
<td>$(2.12 \pm 0.18) \times 10^{-3}$</td>
</tr>
<tr>
<td>Princeton II $^9$</td>
<td>1.5 GeV/c</td>
<td>$(1.97 \pm 0.18) \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Clearly $|\eta_{+-}|$ is found to be constant.

ii) Determination of the phase of $\eta_{+-}$

The CERN experiments which investigated interference in $\pi^+ \pi^-$ decay of $K^0$ and $K^0$ regenerated do not give directly the $\eta_{+-}$ phase, but the composite phase:

$$\phi = \phi_{\eta_{+-}} - \phi_A$$

where $\phi_A = \text{arg } f_2(0)$.

If we suppose $m_c > m_1$, the values obtained are

- M. Bott-Bodenhausen et al. $^{10}$: $\phi = (1.54 \pm 0.17)$ rad.
- C. Alff-Steinberger et al. $^{11}$: $\phi = (1.41 \pm 0.18)$ rad.

From their results and experimental data on total $K^+$ and $K^-$ cross-sections on copper, Rubbia and Steinberger $^{19}$ derived for $\phi_{\eta_{+-}}$

$$\phi_{\eta_{+-}} = (1.47 \pm 0.30) \text{ rad.}$$

M. Bott-Bodenhausen et al. $^{12}$ taking into account total cross-section on carbon obtained

$$\phi_{\eta_{+-}} = (1.22 \pm 0.36) \text{ rad.}$$

Mischke et al. $^{31}$ have obtained

$$\phi_{\eta_{+-}} = (0.44 \pm 0.44) \text{ rad.}$$
iii) Determination of $|\eta_{00}|$.

Recently two beautiful experiments, one at CERN, the other one at Princeton, have been done measuring the absolute value for $\eta_{00}$. Geillard et al.\textsuperscript{23)} measured the momentum and the invariant mass of the $2\pi^0$'s coming from the $K_2^0$ (Figs. 5a, 5b). They found

$$R = \frac{K_2^0 \rightarrow 2\pi^0}{K_2^0 \rightarrow \text{all decays}} = \left(3.3 \pm 1.8\right) \times 10^{-3} \text{ at 2.1 GeV/c},$$

giving

$$|\eta_{00}| = \left(4.8 \pm 1.2, -0.9\right) \times 10^{-3}.$$  

J.W. Cronin et al.\textsuperscript{23)} obtained at 250 MeV/c, in measuring the $\gamma$ spectrum, in c.m. system of $K_2^0$ with high precision (Figs. 6a, 6b)

$$|\eta_{00}| = \left(4.9 \pm 0.5\right) \times 10^{-3}.$$  

iv) Phase of $\eta_{00}$

One experiment is now starting at CERN to measure the composite phase $\varphi = \varphi_{\eta_{00}} - \varphi_A$, by looking at interference effect after a regenerator of copper put in a $K_2^0$ beam.

Another one is being performed by Cronin at Princeton.

b) $3\pi$ decays

Without CP invariance violation $K_2^0$ may decay into $3\pi$'s with charged $\pi$'s ($\pi^+ \pi^- \pi^0$) because $\text{CP} = (-)^{l+1}$ where $l$ is the angular momentum between the $\pi^+ \pi^-$ system and the last $\pi^0$ as shown below

```
\pi^+ \quad \text{L} \quad \pi^0
\downarrow \quad l
\pi^-
```


But the centrifugal barrier forbids the $K^0_L$ decay and the ratio

$$\frac{\text{Rate } K^0_L \rightarrow \pi^+ \pi^- \pi^0}{\text{Rate } K^0_S \rightarrow \pi^+ \pi^- \pi^0} \sim 10^{-5}.$$  

$3\pi^0$ is a pure CP = -1 state and $K^0_L$ cannot decay into $3\pi^0$ without CP violation. If CP is violated, $K^0_L$ may decay with a more appreciable rate into $3\pi^+$'s.

Experimentally\(^{14}\) it was found that, if the pion state is a pure symmetric $I = 1$ and if $|M| = \frac{1}{2}$ is valid

$$\frac{K^0_L \rightarrow \pi^+ \pi^- \pi^0}{K^0_S \rightarrow \pi^+ \pi^- \pi^0} \leq 0.45$$

$$2\gamma_{3\pi} \tau_1 = \begin{pmatrix} -1.46 & + & 3.12 \end{pmatrix} 10^{-4}$$

$$\epsilon_{3\pi} = \begin{pmatrix} -1.03 & + & 2.2 \end{pmatrix} 10^{-4}.$$  

This result is compatible with no CP violation but the errors are too big to come to a significant conclusion.

2.3.2 Leptonic decays

If $\Delta Q = \Delta S$ is valid and even if CP is violated in these decays, there will be non-contribution from them to $\epsilon$. We need $\Delta Q = -\Delta S$ amplitudes to connect $|K^0 >$ to $|\bar{K}^0 >$ in the decay and mass matrices.

There is, up to now, no strong indication that CP is violated in leptonic decays.

Young et al.\(^{25}\) have recently measured the transverse muon polarization relative to the decay plane of the $K^0_L$ in the $K^0_{\mu3}$ decay. If CPT is valid, $T$ violated and CP also, there should be a transverse polarization.

Their results give

$$F_{T}^{c.m.} = 0.003 \pm 0.014$$

$$\text{Im} \xi = \text{Im} \frac{f^-}{f^+} = 0.012 \pm 0.059$$

compatible with no polarization, but mixing between $\Delta Q = \Delta S$ and $\Delta Q = -\Delta S$ amplitudes may give difficulties in interpreting this result.
If $\Delta Q = \Delta S$ is not valid, we can write for the amplitudes for $K^0_{e3}$ decay

$$
\begin{align*}
    f &= |K^0 > \rightarrow \pi^- e^+ \nu \\
    f^* &= |\bar{K}^0 > \rightarrow \pi^+ e^- \bar{\nu} \\
    g &= |\bar{K}^0 > \rightarrow \pi^- e^+ \nu \\
    g^* &= |K^0 > \rightarrow \pi^+ e^- \bar{\nu} \\
\end{align*}
$$

$\Delta Q = \Delta S$

$\Delta Q = -\Delta S$.

If, with

$$
\begin{align*}
    \alpha &= \frac{2\text{ Re } f^* g}{|f|^2 + |g|^2} \\
    \beta &= \frac{2\text{ Im } f^* g}{|f|^2 + |g|^2} \\
    \gamma &= \frac{|f|^2 - |g|^2}{|f|^2 + |g|^2}.
\end{align*}
$$

The time distribution of $e^+$ and $e^-$ is given, in proper time of $K^0$, if we start with a pure $K^0$ beam by (neglecting $e$)

$$
\begin{align*}
    N^+(t) &= \frac{|f|^2 + |g|^2}{4} \left[ (1+\alpha) e^{-\gamma t} + (1-\alpha) e^{\gamma t} + e^{-(\gamma_1 + \gamma_2) t/2} \times \\
            &\quad \times (2\gamma \cos \Delta m t + 2\beta \sin \Delta m t) \right] \\
    N^-(t) &= \frac{|f|^2 + |g|^2}{4} \left[ (1+\alpha) e^{-\gamma t} + (1-\alpha) e^{\gamma t} + e^{-(\gamma_1 + \gamma_2) t/2} \times \\
            &\quad \times (-2\gamma \cos \Delta m t + 2\beta \sin \Delta m t) \right].
\end{align*}
$$

Putting $X = \frac{|g|}{|f|}$, $\frac{g}{f} = X e^{i\phi}$

$$
\begin{align*}
    \alpha &= \frac{2X \cos \phi}{1 + X^2} \\
    \beta &= \frac{2X \sin \phi}{1 + X^2} \\
    \gamma &= 1 - \alpha^2 - \beta^2.
\end{align*}
$$

$\alpha$ is reflecting $\Delta S = \Delta Q$ violation and $\beta$ CP violation.

Many experiments have been performed to measure these distributions. Cabibbo gave a summary of the results at the last Berkeley Conference (Table 2 and Fig. 7). The errors are big; nevertheless, it seems possible that there is a contribution of $\Delta Q = -\Delta S$ amplitude in the $K^0_{e3}$ decay and that this amplitude may strongly violate CP conservation.
| Reference                        | Technique | $|\Delta m|$ | $X \cos \phi$ | $X \sin \phi$ | $X$   | $\phi$  |
|---------------------------------|-----------|-------------|---------------|---------------|-------|--------|
| Paris [Physics Letters 17, 59 (1965)] | FrBC:     | $0.47 \pm 0.21$ | $0.035 + 0.11$ | $0.215 + 0.15$ | $0.22 + 0.16$ | $+79^\circ + 37^\circ$ |
| Padua                           | FrBC:     | $0.15 \pm 0.50$ | $0.035 + 0.11$ | $0.215 + 0.15$ | $0.22 + 0.16$ | $+79^\circ + 37^\circ$ |
| [Nuovo Cimento 38, 684 (1965)]  | HBC:      | $0.15 \pm 0.35$ | $0.035 + 0.11$ | $0.215 + 0.15$ | $0.22 + 0.16$ | $+79^\circ + 37^\circ$ |
| Columbia                        | HBC:      | $0.5$        | $0.0$          | $0.0$          | $0.0$   | $0.0$   |
| [Phys.Rev. 140, 127 (1965)]     | SprK:     | $0.187 \pm 0.16$ | $0.0 + 0.25$   | $0.0 + 0.25$   | $0.0 + 0.25$ | $0.0 + 0.25$ |
| Pennsylvania                    | D_{2}BC:  | $0.58$       | $0.18 + 0.10$  | $0.19 + 0.14$  | $0.27 + 0.12$ | $+43^\circ + 33^\circ$ |
| Carnegie Tech.                  | HBC:      | $0.54$       | $0.18 + 0.10$  | $0.19 + 0.14$  | $0.27 + 0.12$ | $+43^\circ + 33^\circ$ |

\[ \begin{align*}
\frac{1 + X \sin \phi}{1 - X \sin \phi}^2 &= -0.7 \pm 0.8; \\
\frac{2X \sin \phi}{1 - X \sin \phi} &= +0.6 \pm 0.5
\end{align*} \]
Improved results confirm this tendency. It is found that

\[ X \sin \varphi = 0.20 \pm 0.08 \]
\[ X \cos \varphi = 0.09 \pm 0.09 \]
\[ 2y_{\text{lep}} r_{1} = 5.3 \pm 2.2 \times 10^{-4} \]
\[ \epsilon_{\text{lep}} = 3.75 \pm 1.61 \times 10^{-4} \]

From the values obtained for \( y_{3\pi}, y_{\text{lep}}, \text{Re} A_{2}, \text{Im} A_{2} \), it is seen that the total contribution to \( \epsilon \) should be small. Pascual concluded that

\[ 2(y_{3\pi} + y_{\text{lep}}) r_{1} = 0.38 \pm 0.38 - 0.25 \times 10^{-3} \quad 5.7 \times 10^{-5} < \left| \frac{\text{Re} A_{2}}{A_{0}^2} \right| < 1.4 \times 10^{-4} \]

\[ |\epsilon_{3\pi} + \epsilon_{\text{lep}}| = (0.27 \pm 0.27) \times 10^{-3} \quad |\epsilon_{A_{2}}| < 10^{-4} \]

If \( M_{1} \) is also small, \( |\epsilon| < |\eta_{+}| \).

From their measurement on interference in \( K^{\mu 3} \) decay, Bott-Bodenshausen et al. have deduced a value of \( \text{Re} \epsilon \)

\[ \text{Re} \epsilon = 0.5 \pm 6.0 \times 10^{-3} \]

Once more the error is too big to introduce strong experimental limitation.

Conclusions about CP, T

From \( |\eta_{+}| \neq |\eta_{00}| \) it is proved that \( \epsilon' \) is contributing to CP violation and that this contribution is due to amplitudes with \( \Delta I \geq \frac{3}{2} \) and is perhaps the only contribution in \( 2\pi \)'s decay.

Until now there has been no evidence that CP violation is present in \( 3\pi \) decays.

There is also no strong evidence for CP violation in leptonic decays. Only more precise results on \( \varphi_{\eta_{+}} \) and \( \varphi_{\eta_{00}} \) may allow us to determine graphically \( \epsilon \) and \( \epsilon' \). (Fig. 8.)
2.4 Selection rules

2.4.1 Non leptonic decays of the $K^0$'s

i) $2\pi$ decays

CP being practically valid (only small violation $\sim 10^{-3}$) $K^0_i$ can only decay into $2\pi$, the system having CP = +1 with two values for $I$, $I = 0$ and $I = 2$.

The rule $|\Delta I| = \frac{1}{2}$ forbids $I = 2$ decays.

As a consequence the ratio between the rate of the decays of $K^0_i \to \pi^+\pi^-$ and $K^0_i \to \pi^0\pi^0$ should be two. Experimentally the two branching ratios are (mean value):

\[
K^0_i \to \pi^+\pi^- = 69.1 \pm 2.2\%
\]
\[
K^0_i \to \pi^0\pi^0 = 30.9 \pm 2.2\%.
\]

The rule is violated at the maximum in the $K^+ \to \pi^+\pi^0$ decay.

We can deduce the ratio of the amplitudes $a_3(\Delta I = \frac{3}{2})$ and $a_1(\Delta I = \frac{1}{2})$, if we forget the possible phase shift between them and the possible contributions of $a_3(\Delta I = \frac{5}{2})$.

\[
\frac{a_3}{a_1} = (4.5 \pm 0.5)10^{-2}
\]

violation is small.

ii) $3\pi$ decays

For the $3\pi$ decays only states with PC = -1 are favoured because of the G parity and of centrifugal barrier arguments. Consequently, the decay of the $K^0_i$ into $3\pi$ is favoured. The comparison between theory and experiments concerning the $K^0_2$ and $K^+ \to 3\pi$ decays is given in Table 3.

The violating $a_3$ has been determined as given by:

\[
|a_3/a_1| = (3 \pm 5)10^{-2}
\]
\[
|a_3/a_1| = (2 \pm 9)10^{-2}
\]

from two different methods by comparing decays of $K^0_2$ and $K^+$. 
2.4.2 Leptonic decays

a) $|\Delta t| = \frac{1}{2}$ rule

If this rule is valid, $\Delta S = \Delta Q$ should also be valid. The rate of the $K_2^0$ decays into $\pi^+\pi^-\nu$ or $\pi^-\pi^+\bar{\nu}$ compared to the rate of the $K^+$ decay into $\pi^0e^+\bar{\nu}$ should be the same. The ratio between the two experimental values is

$$\frac{\Gamma^+}{\Gamma^0} = \frac{(3.90 \pm 0.15) \times 10^6}{(3.29 \pm 0.15) \times 10^6}$$

compatible with 1

b) $\Delta Q = \Delta S$ rule

This rule does not imply the $|\Delta t| = \frac{1}{2}$ rule but it is interesting to test it because it should be strictly observed from the current algebra conclusions.

i) Three-body decays

As we said above the experimental status presented by Cabibbo\textsuperscript{26} has not allowed one to give an answer to this problem until now.
ii) Four-body decays

Results with $K^+$ decays give for the rate of $\Delta Q = -\Delta S$ and $\Delta Q = \Delta S$ decays

$$K^+ \rightarrow \pi^+ \pi^+ e^- \nu$$

$$K^+ \rightarrow \pi^+ \pi^+ e^+ \nu$$

< 0.02

This result is not precise enough to reach a conclusion.

c) Neutral currents interdiction

Experiments seem to prove that neutral currents are strongly suppressed in leptonic decays. Until now, no event has been observed. Bott-Bodenhausen et al.\textsuperscript{27} have put severe limitations on them. They quoted:

$$\frac{\Gamma(K^0 \rightarrow \mu^+ \mu^-)}{\Gamma(K^0 \rightarrow \text{all})} = 1.6 \times 10^{-6} \quad (90\% \text{ confidence})$$

$$\frac{\Gamma(K^- \rightarrow e^+ e^-)}{\Gamma(K^- \rightarrow \text{all})} < 1.8 \times 10^{-5} \quad (90\% \text{ confidence})$$

$$\frac{\Gamma(K^0_S \rightarrow \mu^+ \mu^-)}{\Gamma(K^0_S \rightarrow \text{all})} < 7.3 \times 10^{-5} \quad (90\% \text{ confidence})$$

$$\frac{\Gamma(K^0 \rightarrow \mu^+ e^-)}{\Gamma(K^0 \rightarrow \text{all})} < 9 \times 10^{-6} \quad (90\% \text{ confidence}).$$
REFERENCES


Figure captions

Fig. 1 : Detection apparatus for the experiment of Bott-Bodenhausen et al. [Phys. Letters 20, 212 (1966)].

Fig. 2 : Interference term of $K_S^0$ and $K_L^0$ amplitudes in the $\pi^+\pi^-$ decay mode as determined from the data. Theoretical $\cos(\delta t + \varphi)$ curve is shown $^{20}$. 

Fig. 3 : Experimental set-up of Mehlop et al. (Piccioni experiment).

Fig. 4 : Rate of $K_S^0$ decays versus distance between target where $K^0$ are produced and the regenerator. Best fit to the data in the interference region assuming $\delta = +0.5$, $\delta = -0.5$ or no interference effect (Piccioni experiment).

Fig. 5a : Experimental apparatus of Gaillard et al. $^{22}$.

Fig. 5b : (1) Experimental mass distribution for regenerated events.
(The dotted histogram is the $2\pi$ spectrum shape calculated with the Monte Carlo programme.)

(2) Experimental mass plot for free decay events.
(The dotted line is the background spectrum shape calculated with the Monte Carlo programme for $3\pi^0$.)

(3) Experimental mass plot for free decay events with the Monte Carlo spectrum subtracted.
(The dotted line is a fit by eye to the residual background.)

Fig. 6a : Plan view of the experimental set-up of Cronin et al. $^{23}$

Fig. 6b : Distribution in $E_{c.m.}$ for events measured with the fine analysis for $\gamma$-ray energies greater than 154 MeV in the laboratory. There is an event at $E_{c.m.} = 365$ MeV not shown on the figure. The solid lines are the expected distributions from the Monte Carlo calculations.

Fig. 7 : Diagram of experimental results on $X = g/T$ measurements $^{26}$.

Fig. 8 : Diagram for CP violating amplitudes in $K^0 \rightarrow 2\pi$ decay.
Fit to 2cm C, 4cm C and 30cm C Regenerator

\[ \delta = 0.480 \frac{h}{\tau S c^2} \]

\[ \varphi = -87.5^\circ \]

\[ \tau = 0.900 \]
$|F_2| = 10.61$ F AT $P = 775$ MeVc
$\varphi = 155^\circ$

**Fig. 4**
Fig. 5b
\[ \Delta S = \Delta Q \text{ RULE EXPERIMENTS} \]

\[ x = \frac{A(K^0 \rightarrow e^+ \pi^- \nu)}{A(K^0 \rightarrow e^+ \pi^- \nu)} \]

Fig. 7
iF\Im A_2/\sqrt{2} A_0 = 1/2 \epsilon' 

\delta_1 

\eta_{00} 

\epsilon/2 

\delta_S 

\phi_{00} 

\eta_{+-} 

\phi_{+-}
POLARIZED TARGETS IN PARTICLE PHYSICS

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I - INTRODUCTION

Polarized proton targets represent a very valuable tool in particle physics. They have been developed recently at Saclay, by Abagram and his collaborators [1], and at Berkeley, by Jeffries and his collaborators [2]. They have been already used in several laboratories, making available most interesting results.

Present targets have proton polarization which are of the order of 60 %. Nevertheless, their proton content is rather low and of the order of 3 % only. They contain heavy nuclei [3]. This makes them not as amenable to experiment as many particle physicists would wish them to be. Development in target technology is however now progressing very rapidly and their use will soon become easier and more diversified [4]. They are however, already, extremely useful in providing most valuable insight into the spin structure of reactions.

This spin structure is at present most interesting to analyze for many reasons. Limiting here ourselves to particle physics [5], one may quote three general topics in which polarized targets appear as a necessary tool for collecting enough information in order to reach an understanding of the pertinent phenomena.

The first one is the rapidly growing number of the hadrons. Their outstanding variety is at present suggestive of a new type of spectroscopy. This makes it the more important to know their quantum numbers and, in particular, their spin and parities. It is also important to carefully analyze elastic amplitudes not to miss any resonance which could be an expected candidate for the many open multiplets. In so doing one rapidly realizes that polarized targets are extremely useful. The second one is the spin complication of high energy reactions. A few years ago, it was generally and erroneously assumed that, at high energy, the many open channels would average out most spin effects and rapidly lead to simple diffraction-like amplitudes. We know that such an asymptotic domain is at least not yet reached with present accelerators. The persistence of spin effects provides on the other hand valuable insights in the analysis of production reactions. For both reasons polarized targets are extremely interesting in the analysis of high energy scattering or
production experiments. A third and important reason one may quote is the present imperious need to test discrete symmetries predictions in many reactions. In so doing polarized targets should provide the necessary tool.

The purpose of this talk is to show on a few examples, chosen in each of these particular topics, how spin effects present themselves in the theoretical analysis of the reactions considered. We shall also show how polarized targets may then provide the necessary information

We shall here take the existence of polarized proton targets for granted.\[4\]. We may simply say that an electronic polarization (carried by a paramagnetic impurity), obtained with conditions of the type: 10° Oersted, 1.5 degree K, is transferred to the proton in saturating a radio-frequency line \[1,2\].

2 - DEFINITION AND DETERMINATION OF SPIN AMPLITUDES

We focus first on the most simple case, that is two body scattering. The reaction is described in terms of two angles as shown on figure 1.

![Figure 1.](image)

In order to get the outgoing particle direction, one first rotates \( y \) through an angle \( \theta \) around \( z \) and then \( z \) through an angle \( \phi \) around \( y' \).
We define two sets of axes respectively associated to the incident (x, y, z) and outgoing (x', y', z') particle. If all four particles are spinless, one defines a single scattering amplitude, a function of two variables, the total energy and the cosine of the scattering angle say. In the Center of Mass system it is usually written in terms of partial wave amplitudes

$$F(W, \cos \theta) = 2 \sum_{J} (J + \frac{1}{2}) f_{J}(W) P_{J}\cos \theta \quad ,$$

(1)

where J stands for the total angular momentum.

When the particle have spin one needs to introduce a priori as many amplitudes as there are possible spin configurations, namely \((2 \, S_{1}\uparrow \downarrow) (2 \, S_{2}\uparrow \downarrow) (2 \, S_{3}\uparrow \downarrow) (2 \, S_{4}\uparrow \downarrow)\). As well known however invariance properties and, in particular, parity invariance may highly reduce this number. Instead of (1) one may then write a similar expression for each of the helicity amplitudes:

$$F_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(W, \theta, \phi) = \sum_{J} (J + \frac{1}{2}) f_{J}^{J}(W) d_{\lambda_1 \lambda_2}^{J}(\theta) \exp(i(A_{1} - A_{4})\phi) \quad ,$$

(2)

where \(A_{1} = \lambda_1 - \lambda_2\) and \(A_{4} = \lambda_3 - \lambda_4\).

\(\lambda_i\) stands for the helicity of particle "i", that is the component of its spin along its momentum. The \(\phi\) dependence of the amplitudes is readily given. As simple a \(\phi\) dependence would hold for amplitudes defined in the laboratory system, where (1) and (2) refer of course to Center of Mass amplitudes.

Let us now consider the simplest non-trivial case, that is spin 1/2, spin 0 scattering (\(\pi - \text{nucleon scattering say}\)). There are a priori four different amplitudes, labelled with the final and initial helicity of the spin 1/2 particle.

$$F_{++}, F_{+-}, F_{-+}, \text{ and } F_{--} .$$

Under space inversion all helicities change sign (the momentum is reversed and the polarization remains unchanged). \(F_{++}\) and \(F_{--}\) on the one hand, \(F_{+-}\) and \(F_{-+}\) on the other.
and $F_{-+}$ on the other hand are therefore related when parity invariance holds. With one particular choice of phases \([7]\) one would have $F_{++} = \eta F_{--}$; $F_{+-} = \eta e^{2\Phi} F_{-+}$ where $\eta = \pm 1$ is the relative intrinsic parity of the initial and final state. We shall here assume parity invariance and keep only $F_{++}$ and $F_{-+}$ as independent amplitudes. It is then an easy matter to calculate the differential cross section and polarizations obtained for an arbitrary initial polarization (figure \(1\)). At $\Phi = 0$, that is with the $y$ (or $y'$) axis chosen normal to the reaction plane, this reads

$$
\frac{d\sigma}{d\Omega} (\theta, \phi) = |F_{++}|^2 + |F_{-+}|^2 - 2\eta \, p_{y}^{(1)} \, \text{Im} \{F_{++} F_{-+}^*\}
$$

$$
\frac{d\sigma}{d\Omega} \, p_{x'} = \eta \, p_{x}^{(1)} \left(|F_{++}|^2 - |F_{-+}|^2\right) + 2 \, p_{z}^{(1)} \, \text{Re} \{F_{++} F_{-+}^*\}
$$

$$
\frac{d\sigma}{d\Omega} \, p_{y} = -2 \, \text{Im} \{F_{++} F_{-+}^*\} + \eta \, p_{y}^{(1)} \left(|F_{++}|^2 + |F_{-+}|^2\right)
$$

$$
\frac{d\sigma}{d\Omega} \, p_{z'} = -2 \, \eta \, p_{x}^{(1)} \, \text{Re} \{F_{++} F_{-+}^*\} + p_{z}^{(1)} \left(|F_{++}|^2 - |F_{-+}|^2\right)
$$

\[(3)\]

The initial (and final) polarization is defined as the expectation value of the three spin components along the axis $x$, $y$, and $z$ ($x'$, $y$ and $z'$). Relation (3) holds as well for laboratory and for center of mass amplitudes and polarizations.

This relation shows a very important result. This is the simple connection between the polarized cross section, that is the part of the cross section which changes sign with the initial polarization, and the polarization (normal to the production plane) observed with an unpolarized initial baryon, changing the sign of the initial polarization (a rotation of $\pi$ of the initial state around the incident $z$ axis) is equivalent to changing $\Phi$ into $\Phi + \pi$ in (3-a), keeping the same initial polarization.

We obtain therefore a relation between the left right asymmetry with initial polarization, $A$, and the polarization $P$, observed with an unpolarized
initial state

\[ \Delta = \frac{L-R}{L+R} = \frac{2\eta \text{ Im} \left\{ \frac{F_{++}^{*}}{F_{++}} \right\}}{|F_{++}|^2 |F_{+-}|^2} p^{(1)} \]

\[ p = -\frac{2 \text{ Im} \left\{ \frac{F_{++}^{*}}{F_{++}} \right\}}{|F_{++}|^2 |F_{+-}|^2} \]

(4)

This makes it possible to determine the intrinsic parity of any spin 1/2 baryon. Taking for instance the reaction

\[ \pi^- p \rightarrow \Sigma^+ + K^- \]

with (by definition) same \( \pi \) and \( K \) parity, one may obtain the \( \Sigma^+ p \) relative parity. The measurement of the \( \Sigma^+ \) polarization is in principle easy to obtain from the weak decay asymmetry. This experiment has been done at Berkeley, giving the same parity for both \( p \) and \( \Sigma^+ \). A similar experiment, on \( \Xi \) parity, is under way at CERN.

It is possible to give a simple semi-classical argument which makes this particular and most interesting result more easily understood. To this end, let us introduce new amplitudes, labelled this time with spin (up or down) components along the normal to the reaction plane. We associate again the left right asymmetry of the final state to a rotation of angle \( \pi \) on the initial state. One readily writes:

\[ \Delta = \frac{L-R}{L+R} = p^{(1)} \left| \frac{|e_{+-}|^2 - |e_{-+}|^2 - |e_{++}|^2 - |e_{--}|^2}{|e_{++}|^2 |e_{+-}|^2 |e_{--}|^2 |e_{-+}|^2} \right| \]

when the final polarization with no initial polarization, which is normal to the
reaction plane, readily reads as follows:

\[
p^{(t)} = \frac{|g_{++}|^2 - |g_{+-}|^2 + |g_{+--}|^2 - |g_{--+}|^2}{|g_{++}|^2 + |g_{+-}|^2 + |g_{-+-}|^2 + |g_{--+}|^2}
\]

(5)

We now turn to a semi-classical argument illustrated by figure 2.

\[\vec{J} = \vec{L} + \vec{s}\]

**Figure 2.**
Semi classical picture \(\vec{J}\) and \(\vec{s}\) are all normal to the scattering plane.
For each total angular momentum value $J$, the spin should (or should not) flip according to the necessary change (or constant value) of the orbital angular momentum. It must change by one unit if the relative parity is odd. It must remain constant if the relative parity is even. As obvious on (5), one readily reaches the announced result since $g_+^* = g_- = 0$ in the first case (change in intrinsic parity), when $g_+^* = g_+ = 0$ in the second case (same intrinsic parity).

3 - $\pi$ - NUCLEON SCATTERING

If we now go back to (3), we see that the information which can be reached with a target, polarized normally to the selected reaction plane, (the simplest experimental set up for obvious reasons), namely $\text{Im} \left\{ F_+^* F_- \right\}$, is identical to the information obtained without a polarized target but now measuring the final baryon polarization. Nevertheless it is now often easier to analyze an angular distribution with a polarized proton target than to reach a recoil proton polarization through a second scattering experiment. Beautiful sets of experiments have been carried out already at Berkeley, Argonne and at the Rutherford Laboratory and the results arrived at this way have been most valuable in clarifying the resonance structure of pion nucleon scattering. Among the new resonances which have been ascertained, one may quote the $N^*$ (1674) $5/2^-$, the $N^*$ (1688) $5/2^+$ (which has now been known for a long time), the $N^*$ (1920) $7/2^+$ and the $N^*$ (2190) $7/2^-$. Separating the two first resonances, with opposite parities is to be put to the credit of polarized targets.

In view of (3), one may think that all possible experiments are necessary in order to determine both amplitudes, up to a common phase. This is correct but, when the energy is not too high, unitarity can be used in a rather simple way through a phase shift analysis. This makes it possible to determine completely the amplitude with only a partial information. Nevertheless, if this is possible up to 2 GeV say, at higher energies, unitarity is not longer readily used. One then needs to perform all experiments in (3) in order to determine both amplitudes.
In order to do so it is necessary to polarize the target in the reaction plane and further measure the recoil proton polarization. Such experiments are referred to as $A$ and $R$ measurements. Relation (3-b) is written as

$$P_x^{(f)} = R P_x^{(1)} + A P_z^{(1)} \quad .$$

This polarization experiment will soon be performed at CERN.

One will obtain this way detailed information on the high energy (spin flip and non spin flip) $\pi$ - Nucleon amplitudes. As easily verified from (3) one obtains both amplitudes $[8]$, up to a common phase factor which may depend on the scattering angle! This is the price one has to pay for not using unitarity, that is taking only elastic scattering out of the many possible reactions. Nevertheless, the knowledge of both absolute values as well as the relative phase of the $\pi$-Nucleon amplitudes is already a very valuable piece of information.

Experiments have been so far limited to polarization normal to the reaction plane. This has however already given extremely interesting results. At present High Energy amplitudes are generally parametrized in terms of a few Regge trajectories $[9]$. Experimental results on $\pi^\pm$ proton differential cross sections have been extremely well described in terms of only three trajectories, the Pomeronchuk $P$ trajectory, the $P'$ trajectory and the $P'$ trajectory. It is then possible to predict the polarization effects which are expected with protons polarized normally to the reaction plane. Such results have been beautifully verified experimentally $[10]$. It is furthermore possible to determine the $\pi^-p \rightarrow \pi^-n$ charge exchange amplitudes spin flip and non spin flip which should be dominated by the $P$ trajectory. The angular distribution was found to agree with such prediction and the further consequence was an a priori absence of polarization effects since with one single trajectory, both amplitudes have the same phase. Nevertheless the use of a polarized target has made the pertinent test possible at 5.9 GeV/c and then at 11.2 GeV/c a few months ago. Contrary to expectation, polarization effects were found as large as in $\pi^\pm$ scattering, and not to vary significantly with energy.
Making possible such a test, polarized targets have shown that the simple and rather elegant picture arrived at, is still far from the actual physical situation. It would be extremely valuable to carry out this experiment at higher energies. Polarized targets with a higher proton content might then be necessary since it becomes then more difficult to separate events occurring on the free and polarized protons from those occurring on the bound, and unpolarized, nucleons.

We have considered here only the most simple case of $\pi^-$ nucleon scattering. This is generalized in a straightforward way to

$$\pi N \rightarrow \eta N \text{ or } KN \rightarrow KN \text{ or } \pi Y$$

collisions, where $Y$ is a hyperon and studies of such reactions on polarized targets should be done in the near future. A similar, though more complicated analysis is possible in the case of nucleon nucleon scattering. All the necessary information is available in the excellent review articles of Wolfenstein and Bilenkii, Lapidus and Ryndin \(^\text{(11)}\). The present experimental status, as well as the interest of possible experiments, is reviewed in reference (6).

It is obvious that polarized targets could be extremely interesting in studying resonance production in order to get information about the production mechanism or on the final polarization state. Information on the polarization state would then help spin and parity analysis. Nevertheless most resonances are too wide to allow a neat separation between production on polarized free protons on the one hand and on bound unpolarized nucleons on the other hand. Fast progress in target technology may however soon bring such experiments into the realm of possibility. We shall not mention them here and simply refer to reference 6 for the pertinent information.

4. **POLARIZED TARGETS AND TIME REVERSAL INVARINACE**

In order to illustrate how polarized targets can be used in order to test invariance principles, we shall briefly describe a possible test of time reversal invariance in electromagnetic interactions which will probably be soon
performed. The interest of such tests is stressed in Professor Nillson's lectures. The experiment has been proposed by Christ and Lee and we refer to their article for a more detailed discussion [12]. The proposed experiment is electroproduction of a $\pi$-nucleon resonance on a polarized target

$$e + N \rightarrow e + N^*$$

Such a process is considered to lowest order in electromagnetic interactions as shown on figure 3.

![Figure 3 Diagram]
Experimental evidence for a term which would change sign under time
reversal in the amplitude, would then imply a term which changes sign under time
reversal in the interaction Hamiltonian. It therefore would prove that time rever-
sal invariance, or charge conjugation invariance, assuming PCT invariance, is vi-
olated in electromagnetic interaction.

Such a term may be chosen as :
\[ \vec{P} \cdot \vec{k} \times \vec{k}' \]
where \( P \) is the polarization of the target and \( \vec{k} \)
and \( \vec{k}' \) are the momenta of the initial and final electron. A variation in diffe-
rential cross section at fixed angle, obtained under reversal of the target polariza-
tion, would prove the existence of a time reversal violating interaction. Estimates
have been made on what effect to expect if time reversal invariance is violated
in a "maximal" way, that is if time reversal violating amplitudes are as large as
they could be. A net effect of 30% (to be multiplied of course by the 2% net
proton polarization) may be obtained \[13\]. We shall not go here into the details of
such an analysis \[12\]. We shall simply recall that a possible violation of \( C \) (or \( T \))
invariance in electromagnetic interaction is a priori difficult to detect \[14\] and
that a production or decay amplitude must be studied for that purpose. No effect
could show up in standard elastic scattering form factor analysis.

Now if the non observation of any effect in \( \eta \) decay (\( \pi^+\pi^- \) asymmetry or
\( \eta^0 \) Dalitz pair decay) should be explained as a consequence of a particular selection
rule \( \Delta I = 0 \) associated with \( C \) violating electromagnetic amplitudes \[15\], no
effect should also be observed in \( N^* \) (1240) production. In order to get evidence
for \( C \) (or time reversal) non invariance in electromagnetic interaction, it would
then be necessary to go at least to the \( N^* \) (1520), a much more difficult experiment
with the very high Bremsstrahlung spectrum associated with the heavy nuclei inside
present targets.

We just recall also that in order to test for time reversal invariance
one must not try to observed the \( \pi^- \) nucleon system associated to the \( N^* \). All
hadronic states at a given center of mass energy should be summed over \[16\] and
a bias could easily creep in through the observation of the $N \pi$ system, even though the interesting sample might seem more selectively reached. The reason for choosing a resonance is simply due to the fact that otherwise the many different amplitudes present might average out any interesting effect. The presence of a rather large background, at the resonance level, is nevertheless not a priori troublesome.

I hope to have shown hereby why polarized targets are so interesting a tool in particle physics. The few examples given have been selected according to present possible experiments. No doubt that many more as well as more diversified experiments will be possible in the near future with higher proton content targets.
REFERENCES AND FOOTNOTES


[3] - The LMN crystal target is a double nitrate of Lanthanum and Magnesium doped with a paramagnetic impurity (Neobidium). The polarized protons are the protons of the water molecules embedded in the crystal lattice.

[4] - A thorough and up to date review of target technology is presented in the proceedings of the Saclay International Conference on polarized targets and ion sources (1966).

[5] - Polarized targets are also extremely interesting in nuclear physics. In connection with this one should consult the Proceedings of the 2$^{nd}$ International Symposium on polarization phenomena of Nucleon, Karlsruhe (1965). See also P. NOYES' contribution to reference [4].


[7] - Even though we may make some arbitrary phase conventions, the results arrived at are quite general. We follow here reference [6], it is however not mandatory to get into any particular phase convention to understand the following.
[8] - One needs \( |F_{++}|^2 + |F_{--}|^2, |F_{++}|^2 - |F_{--}|^2, \)
\[
\text{Re} \{F_{++} F_{--}^*\} \text{ and Im} \{F_{++} F_{--}^*\} \text{ in order to obtain both amplitudes with the exact relative sign.}
\]

[9] - This topics is covered in details by Professor Swenson in his series of lectures. We merely list here a few results.

   MÜHLER et al. Phys. letters 22 203 (1966)

   S.M. BILENKII, L.J. LAPIDUS, R.M. RYNDIN, Soviet Physics Uspekhi 7 721 (1965)

   For a brief but more detailed discussion see reference 6.
   See also J.D. BJÖRKEN and J.D. WALECKA, Annals of Physics 38 35 (1966)

[13] - See for instance possible C-violation in electromagnetic interaction


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