CCD PHOTOMETRY OF THE δ-SCUTI STAR κ²BOO

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Submitted to: Astronomy & Astrophysics
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Abstract. CCD time-series differential photometry of the binary star κ²Boo has been carried out from a small in-town situated telescope. One component of the system is known to be a δ-Scuti pulsator. Due to the high S/N data, four frequencies have been detected even though the window function is quite complicated. A search for a standard model that fits all available data, including the observed mode frequencies, produces a model, which fits everything well, but with an unlikely combination of modes. This demonstrates the necessity of redundant data of very high quality, if the seismic studies shall lead to progress in the understanding of stellar evolution. An interesting by-product is that the analysis leads to precise prediction for observables that we expect to have in the near future, such as the distance to be published in the Hipparcos catalog.

Key words: Stellar evolution. Open clusters. δ-Scuti stars. CCD-photometry

1. Introduction

The δ-Scuti stars have attracted much attention as objects, for which good observations can provide critical tests of evolutionary models of stars. The reason for this is that they belong to one of the groups of stars in the HR-diagram pulsating in more than one or two modes. The δ-Scuti stars are found in the lower part of the instability strip on the Main Sequence or just above on their way to the red giant state. They form a rich set in different evolutionary stages, which means that the cores must be quite different. The number of excited modes, measured so far in any δ-Scuti star, is limited to less than ten, but this could turn out to be caused only by the noise level of the observations.

In order to profit from the information contained in the frequencies of the modes one must resolve the often dense mode spectra. Multisite observations have proved superior (e.g. Breger et al. 1993; Belmonte et al. 1993) to single-site observations and give clean spectra allowing correct identification of all frequencies above the noise level (no alias problem). At the same time very low noise levels are obtained giving more detected modes.

Observations from just one site can provide data of interest as well, but will of course be affected seriously by alias problems. An example is the extensive work by the italian group (see, e.g., Mantegazza & Poretti 1993).

The observational status as well as the implications for stellar evolution studies can be obtained in the proceedings from a series of recent meetings (Breger 1990b; Mantegazza & Poretti 1990; Mantegazza et al. 1993; Belmonte et al. 1993; Matthews 1993; Dziembowski 1993; Däppen 1993). Also a recent review by Brown & Gilliland (1994) should be consulted.

In their own right δ-Scuti stars present a list of puzzling phenomena: Only some of the stars in the instability strip become variables and only fairly rapidly rotating \((V_{\text{rot}} > 40 \text{ km s}^{-1})\) stars show multiple modes. When many modes are excited, they appear in a pattern which is not easily explained. There seems to be two types of variables (Michel 1993b), where one type has a wide range of frequencies well separated, and the other type has frequencies in a more narrow range and often one or more pairs of closely spaced frequencies. The latter type must oscillate in non-radial modes to generate such close frequencies with separation of only a few \(\mu\text{Hz}\).

At the end of the main sequence, models predict a very complicated behaviour of the mode spectrum (mixed g- and p-modes). The spectrum becomes very dense (Dziembowski et al. 1990). This fact plus rotational splitting makes mode identification very difficult, the more so as the basic parameters like mass and luminosity are not very well known. The previously mentioned two types could be a group of unevolved and a group of evolved stars, but stars of both types occur in the same region in the HR diagram (Michel 1993b). A link between rotation and
mode excitation clearly exists and has been studied by Dziembowski et al. (1988). But so far no clear picture has emerged, explaining why exactly the modes observed are excited for a given star.

Related to the excitation is also the amplitude changes reported by Breger (1990a) in 4 CVn. This is just one example out of a handful. The excitation seems to be a very delicate process in these stars.

The present contribution to the subject was spawned by an interest in stellar evolution, combined with the wish to see how well one can observe stars under rather hostile conditions. Could one make contributions to the subject with a CCD-camera on a small (50 cm) telescope close to the center of town at a university observatory? From experience with differential photometry (Kjeldsen & Frandsen 1992; Gilliland & Brown 1992) we saw no obstacle in the case of bright objects.

One necessary condition was to locate a suitable object. The use of CCD cameras with small (1 cm²) CCD chips to study selected, single, bright stars often fails due to the lack of a reference star in the small field of view of the camera. Fortunately the binary $κ^2$ Boo fulfilled all conditions: The primary component is bright and the secondary works as a good reference star. Scintillation dominates at the magnitudes involved so that no deterioration takes place by having a reference, which is fainter than the principal object.

The observations have been presented before, and the first interpretation (Jones et al. 1993) have led to some criticism due to the numerous frequencies listed (Kurtz 1994). The results reported in this paper supersede those obtained from our preliminary analysis presented in Jones et al. (1993).

2. The star $κ^2$ Boo

The main component $κ^2$ Boo of the binary system $κ$ Boo has spectral type A8 IV, it is a variable star and is located above the main sequence. The other component $κ^2$ Boo is an F1 IV star and is potentially a variable star as well. Nevertheless we have used it as reference star. Oscillations in the faint component will therefore be visible if present. The parameters for both components are given in Table 1. This table gives our best estimate from current literature. Note the high rotational velocity of $κ^2$ Boo.

The star $κ^2$ Boo has attracted attention before. It is known to oscillate with a dominant period in the neighbourhood of $P = 0.066$ days (Vallier 1972; Elliot 1974; Breger 1979). Dasiskachary et al. (1971) prefer an alias and quote 0.07306 days (the alias of 0.06810 days). The short period is the correct main period as shown later. A set of observations covering 8 nights is presented by Moreno (1980) without interpretation.

Elliot (1974) and Desikachary et al. (1971) note that a beat (16 days?) is present, which indicates that more than one mode is present. This is shown to be true later, but the frequency differences do not directly correspond to 16 days.

That $κ^2$ Boo is a non-radial pulsator (NRP) is shown by Walker et al. (1987) and by Kennelly et al. (1991) from an analysis of line profile variations. Such observations are useful for a fast rotator. A high l-mode is detected with a tentative value of $|m| = 12$ and a period ($P = 0.071$ days) similar to the photometric period. The $m$-value is somewhat uncertain, as it depends on the value of the inclination of the rotational axis ($\sin i$).

In total this looks like a very interesting binary system, where new high quality data would be able to unravel a fair set of NRP modes.

3. The observations done from Århus

The observations took place at the Ole Rømer Observatory with a high-quality CCD camera using a Tk512 thinned chip (27 $\mu$m pixel size) mounted on the 50 cm telescope. The image scale is 1" = 33$\mu$m. A V filter was used, but the combination of the filter and the CCD response had not been optimized to reproduce the standard V response. The observatory is situated in the outskirts of Århus with plenty of city lights around. The telescope has no guiding system and was at the time of observations not tracking properly. The telescope had to be manually set to the right position every 5 minutes. This was done by inspecting the last frame of each series. The exposure time had to be as short as 2 s to avoid saturating the chip, even though we had defocused the telescope to make the images of the two stellar components of $κ$ Boo larger. We obtained a frame every 5 s giving us a duty cycle of 40%. The fast readout was possible by binning and by reading out only a rectangle of 66 x 48 pixels. The whole operation was in fact only possible due to a lot of enthusiasm of the observing team. There was no third comparison star nearby, so we have assumed that the fainter star is constant. There is a small chance that it could be a variable too, but we

| $κ$ Bootis | $κ^2$ | $κ^3$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td>$δ$(2000)</td>
<td>32.81</td>
<td>27.37</td>
</tr>
<tr>
<td>SC</td>
<td>A8 IV</td>
<td>F1 V</td>
</tr>
<tr>
<td>$V$</td>
<td>4.54</td>
<td>6.69</td>
</tr>
<tr>
<td>$(B-V)$</td>
<td>+0.20</td>
<td>+0.39</td>
</tr>
<tr>
<td>$V\sin(i)$</td>
<td>140 km/s</td>
<td>40 km/s</td>
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<td>$L/L_\odot$</td>
<td>43</td>
<td>6.4</td>
</tr>
<tr>
<td>$T_{eff}$</td>
<td>8150 K</td>
<td>6800 K</td>
</tr>
<tr>
<td>$R/R_\odot$</td>
<td>3.29</td>
<td>1.82</td>
</tr>
<tr>
<td>$M/M_\odot$</td>
<td>2.20</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Table 1. Stellar parameters

| dist | 60.8 (±5.9) pc |
| age  | 0.8 (±0.3) Gyr |
assume that it would not have frequencies mixed with the bright component.

The periods where the telescope was allocated to the project was characterized by unusually good weather. The first period 13/5/92 - 26/5/92 was continuously clear and only one night was lost due to a telescope failure. The second period 16/6/92 - 27/6/92 was not quite as good, and a few nights were lost due to a malfunctioning shutter.

An idea about the data can be had looking at Fig. 1, where the data for one night before and after removal of the signal (see later) is displayed. The r.m.s. noise level for the first period is 20 mmag per data point. This should lead to a noise level in the amplitude spectrum around 0.1 mmag. We do not do quite that well due to instrumental problems. As an example the lack of a guiding system introduces a fluctuation in the signal due to flat field problems. Also the bad window function increases the detection level over the 0.5 mmag one would expect from pure noise considerations.

We decided to analyse the data from the two periods separately as the first period was superior in quality to the second, so that the second period serves as a test of the results from the first period. The power spectrum of the window functions for the two periods are displayed in Fig. 2. These window functions have been computed using the weights mentioned later.

The observing procedure introduced a few typical periodicities in the data stream: The daily cycle, the length of the night of about 6h and finally a 3 mm break while the telescope position was reset. The importance of this will be discussed later.

4. Data reduction techniques

The data reduction constitutes of a series of steps. First we have to derive the brightness difference between the two objects for each exposure. This is done by applying a set of standard CCD frame operations to calibrate each image for flat field variations etc. Then we apply a photometric reduction program. Next we analyse the light curve to try to locate as many modes as possible. Taking the large data set into account (nearly 59000 data points), we found it justified to try a number of techniques. The methods are described in separate subsections.

4.1. From CCD frames to relative brightness

After doing standard CCD image reductions we derive the brightness difference between k’ Boo and k1 Boo using a software package MOMF (Kjeldsen & Frandsen 1992; Kjeldsen 1992) written especially to analyse time series of CCD frames.

When we compute power spectra we apply weights to the data points. These weights are found by calculating the deviation of each point from a local mean of the data. The weight is the inverse of the square of the deviation.

Keeping the observing procedure in mind, we have checked the data for correlation of the brightness difference with other parameters. But strangely enough, we did not detect any correlation with position on the chip, seeing or other observables. Thus no de-correlation was applied.

4.2. Calculation of the power spectrum

The power spectrum was calculated by a method described in detail by Kjeldsen (1992), to produce a weighted power spectrum. This is a least-squares sine-wave fit to the data using statistical weights found in the time series analysis, which can reduce the noise in the power spectrum at low frequencies. For each frequency, ωi, in a given frequency interval, the harmonic content in the data is estimated by a least squares fit to \( A_i \sin(\omega_i t + \phi_i) \), where \( A_i \) is the amplitude, \( t \) the time and \( \phi_i \) the phase. The power is then given by

\[ P_i = A_i^2 = \alpha^2 + \beta^2, \]

where \( \alpha \) and \( \beta \) is given by

\[ \alpha = (s c_2 - c x)/(s c_2 - x^2), \]
\[ \beta = (c s_2 - s x)/(s c_2 - x^2) \]

and

\[ c = \sum w_j x_j \cos(\omega_i t_j), \]
\[ s = \sum w_j x_j \sin(\omega_i t_j), \]
\[ c_2 = \sum w_j \cos^2(\omega_i t_j), \]
\[ s_2 = \sum w_j \sin^2(\omega_i t_j), \]
\[ x = \sum w_j \sin(\omega_i t_j) \cos(\omega_i t_j), \]

where \( (t_j, x_j) \) are the data points and \( w_j \) the corresponding weights. The power spectra for the two separate data strings are presented in Fig. 3. The two low frequency peaks, at the daily alias or half of it, are due to a near singularity in equation (2) for the specific sampling. It reflects no real oscillation. The additional noise at low frequencies is due to a non-white noise source in the data.

4.3. The CLEAN algorithm

Normally one only has a vague idea of the pulsation periods for a given star. From the power spectrum (see Fig. 3) it is easy to see that a number of frequencies are present in the interval 100 - 200 μHz. But the spectrum is too complicated to give a precise estimate of the frequencies. To proceed one has to apply some cleaning procedure to the data.

Given a table of brightness differences and weights as functions of time, we search for oscillation modes. By using the CLEAN algorithm one locates the highest amplitude mode, removes it and looks for the next mode and iterates.
Fig. 1. The lightcurve from one night. After the removal of the stellar signal some variation is still left as explained in the text. The observing pattern is clearly seen in the light curve.

Fig. 2. Window functions for the first period (a) and for the second period (b). The origin of the window functions has arbitrarily been set to 500 $\mu$Hz.
this scheme until only noise is present. We have used two types of the CLEAN algorithm, the traditional one where we work only on the power spectrum, and one where we go back to the time string, remove the modes detected and recompute the power spectrum. The latter is maybe better described as iterative sine-wave fitting (ISF, cf. §4.5), and it has been widely used in studies of multiperiodic oscillating stars.

The first method we apply is a CLEAN procedure, where we suppress the sidebands in the power spectrum due to the window function. Due to the 1/day aliasing the power spectrum is dominated by the window function. After about 1000 CLEAN iterations we have a power spectrum almost completely without side lobes. Despite its simplicity this method is very efficient.

4.4. The Maximum Entropy Method (MEM)

As an alternative to CLEANing the obtained power spectra we have considered the use of methods based on the principles of statistical inference. In the present paper we only consider MEM applied to the power spectrum, just as we use the simple CLEAN algorithm. In principle, the advantage of MEM compared to CLEAN is that it rests on a somewhat different theoretical framework and that it, during the deconvolution, exploits the information contained in the entire window function, not only its peak.

The implementation of MEM that we use is the one described by Alhasid, Agnon & Levine (1978). Consider a positive, additive distribution \( \{p_i\} \), \( i = 1, \cdots, N \), \( p_i \geq 0 \), normalized,

\[
1 = \sum_{i=1}^{N} p_i, \tag{4}
\]

and subject to constraints

\[
o_r = \sum_{i=1}^{N} \psi_{r,i} p_i, \quad r = 1, \cdots, M; \quad M < N - 1. \tag{5}
\]

Here \( \{p_i\} \) may be identified with the ‘true’ power spectrum (i.e., unaffected by the window function), \( \psi_{r,i} \) with the window function (cf. Fig. 2), and the constraints \( o_r \) with the data, i.e., the ‘observed’ power spectrum that we wish to deconvolve (Fig. 3).

We now seek the distribution which maximizes the entropy (cf. Narayan & Nityananda 1980)

\[
S[p] = -\sum_{i=1}^{N} p_i (\ln p_i - 1) \tag{6}
\]

under the constraints (4) and (5). The method of Lagrange multipliers shows that the solution is of the form

\[
p_i = \exp \left( -\lambda_0 - \sum_{r=1}^{M} \lambda_r \psi_{r,i} \right). \tag{7}
\]

Hence, the problem is recast to one of determining the Lagrange multipliers \( \lambda_0 \) and \( \lambda_r \) associated with the constraints (4) and (5) respectively. It may be shown that this can be done iteratively by minimizing a convex ‘work function’ which has a unique minimum (Alhasid et al. 1978). Recently Hollis et al. (1992) have suggested that a fast, yet reliable, way of performing this minimization is through a ‘logarithmic’ updating scheme. We have verified that this is a useful algorithm in a number of applications to spectra and images (see, e.g., Hjorth & Jensen 1993)—hence this method is also adopted in the present work. We typically used 1000 iterations to deconvolve a power spectrum.
4.5. Iterative Sinewave Fitting (ISF)

CLEAN and MEM as described up to now only acts on the power spectrum, i.e., the information contained in the phases is not exploited. To do so one must work on the original time string.

ISF is a method to find amplitudes and phases of given frequencies in the time string. ISF simultaneously fits a Fourier series of known frequencies to a given time string. A so-called merit function,

$$\chi^2 = \sum_{j=1}^{N} \left( y_j - \sum_{k} w_j (y(x_j; a_1, a_2, ..., a_k)) \right)^2$$

is defined. $N$ is the number of data points, $x_j$, $y_j$ the data and $w_j$ the statistical weights. $y(x_j; a_1, ..., a_k)$ is the model function. The merit function is then minimized to get the best-fit parameters $a_1$, ..., $a_k$.

The solution is based on Marquardt's method (cf. Press et al. 1989), which is an iterative nonlinear least-squares routine. After phases and amplitudes have been found by ISF these frequencies are removed from the time string. The procedure is the following: Find the frequency of the highest peak in the power spectrum. Fit a Fourier series with this particular frequency to the time series. Subtract this fit from the time series, make a new power spectrum and find once more the highest peak. Now fit a Fourier series of these two frequencies to the time series and subtract the fit from it. Iterate this procedure until the residual power spectrum is a pure noise spectrum.

5. Analysis of spectra

The results using the methods just described will now be presented and the significance discussed.

5.1. Simulations

To test the reduction methods we have made a blind search for modes in a simulated time string, which had exactly the same window function as the observed data. The random noise was modeled to agree with the data by adding a 'white' noise component plus a 1/f (random walk) noise component to the signal (Kjeldsen & Frandsen 1992).

5.1.1. CLEAN and MEM

The CLEAN method has to be used with care when applied to multifrequency analysis as pointed out by Michel (1993a). The same is true for MEM. In Fig. 4 we show the deconvolved spectra of the simulated time string with the window function of the first period, using either CLEAN or MEM. The two spectra looks similar although there are differences. MEM finds fewer frequencies and thus appears to be slightly more conservative. Also the suppression of the side bands (daily aliases) is different in the two methods. Finally, as discussed below, the detection of false modes differs significantly in the two approaches. This opens up for the possibility to discriminate against detected false modes in a time string by simply applying different mode detection techniques.

<table>
<thead>
<tr>
<th>Table 2. Search on simulated data</th>
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<tbody>
<tr>
<td>ID $^1$</td>
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<tr>
<td></td>
</tr>
<tr>
<td>S1</td>
</tr>
<tr>
<td>S2</td>
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<tr>
<td>S3</td>
</tr>
<tr>
<td>S4</td>
</tr>
<tr>
<td>S5</td>
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<tr>
<td>X1</td>
</tr>
<tr>
<td>X2</td>
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<td>X3</td>
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<td>X8</td>
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<td>X9</td>
</tr>
<tr>
<td>X10</td>
</tr>
<tr>
<td>X11</td>
</tr>
</tbody>
</table>

$^1$ The ID's of S1–S5 refer to the input modes in the signal and the ID's X1–X11 to false modes not present in the input signal. $^2$ The headings 1 and 2 refer to the two observing periods.

The result of the test is summarized in Table 2 where the result of the search for pulsation modes is given. We have as for the real data worked on the two periods independently and therefore display two columns for each method. The first period leads to the best values due to the better coverage and lower noise. All modes are rediscovered by CLEAN and MEM. Unfortunately, also several false modes are found. The two methods seem to function equally well given a power spectrum and the corresponding window function.

The amplitudes have been calculated as the square root of the total power in the main frequency plus all its aliases. In the deconvolutions there is an ambiguity in the assignment of modes since the peak at 173.75±0.2 mHz may be an alias of either the S4 or S5 frequency.

Due to the vast possibilities for a coupling between the window function and different choices of input data, this test does not provide more than an idea about the typical result of a data reduction with the two methods. So the following uncertainties are what we expect to see. The frequency determination is good, with errors of the order 0.2 mHz for period 1 and up to 0.5 mHz for period 2. Working with data from one site, we cannot in each case select among a frequency and one of its one-over-a-day aliases.
We find false peaks up to a level of 1.5 mmag. Evidently some of the power in the input modes is transferred to false modes. The error on the amplitude is in the range 1-2 mmag, closer to 1 mmag for period 1 and closer to 2 mmag for period 2. Roughly speaking one can say that only modes with amplitude greater than 1.5 mmag are significant. On the other hand, modes with amplitude greater than 1 mmag are potentially real, so they can be spotted by e.g. CLEAN and then hopefully verified by some other means. False modes may be discerned from real modes in that they only show up in one of the two strings, and interestingly, are often specific for each of the two deconvolution methods. Unfortunately, X1 is detected both by CLEAN and MEM with amplitudes of 1.3-2.0 mmag, and X7 with an amplitude of 1.2 mmag, but in different time strings. Thus, it seems that a safe detection threshold is between 1.0 and 1.5 mmag.

5.1.2. Iterative mode detection

The ISF method needs a number of input frequencies. We now describe how these are found. The CLEAN or MEM deconvolutions give quite a useful set of frequencies and amplitudes for the strongest modes. These can therefore be used as input for ISF. Specifically, the S1 mode with the frequency given by CLEAN or MEM is fitted to the time series and subtracted. Then a new power spectrum is generated, the CLEAN/MEM algorithm is applied, and the procedure is repeated. When the strongest peaks have been subtracted ambiguities may arise. In this case, several frequencies may be included in the ISF fit, but only the one with the largest amplitude is subsequently subtracted.

In Fig. 5 we show the steps of such a procedure using the CLEAN method for detection of modes. In Fig. 5a the CLEAN spectrum of the original time series (first string) is shown (cf. Fig. 4) marked with the positions of the input frequencies S1-S5. Notice that the input frequency S4 is
Fig. 5. The CLEAN spectra for the first simulated time series after removing successively more and more modes using CLEAN and ISF. In panel (a) the CLEAN spectrum before prewhitening and the positions of the input frequencies S1-S5 are shown. In panels (b) to (d) the modes S1 to S3 have been subtracted and in panel (e) S5 has been subtracted.

not visible in this spectrum, at least not with the correct frequency.

In Figs. 5b–d the modes S1 to S3 have been subtracted from the time series using ISF. In Fig. 5d the highest peak now is S5, and mode S4 still is not present at the correct frequency. However, its 1/day alias at 150.4 μHz begins to be significant. The result of subtraction of mode S5 is shown in panel Fig. 5e. The mode S4 is now present still together with its 1/day alias. Thus a remarkable feature of this sequence is that new frequencies may show up, when the strongest are subtracted. This shows the great advantage of the ISF method. Another advantage is that the opposite is also true: A mode detected with a high amplitude in the original power spectrum using CLEAN or MEM (e.g., mode X1 in Fig. 4 and Table 2) may almost disappear using ISF.

The results of the ISF method on the simulated time series (first string), using either CLEAN or MEM for detection of input frequencies, are summarized in Table 3. We conclude that we reliably detect all the input modes and that the two methods yield similar results.

5.2. The real data

By use of the reduction techniques described in the previous section, we have found a set of frequencies for κ²Boo with high significance.

Figure 6 shows the power spectra after successive removal of more and more modes in the combined time series. In the bottom panel we find it hard to tell where to find yet another mode, and thus we have a set of 4 modes (A-D) of which the fourth (D) is marginal. It is included due to arguments presented below.

The three frequencies A, B, and C are detected with high confidence and the position can probably not be changed as a result of a 1/day aliasing problem. The C frequency was not present in the original CLEAN spectrum for the second period, but turns up with a very high
The power spectra for the entire $\kappa^2$ Boo time series after successively removing the modes A to D using CLEAN/MEM and ISF. Prewhitening with mode A is given in panel (a) while panel (b) shows the power spectrum after prewhitening with both mode A and B. In (c) also mode C has been subtracted while in (d) all four modes have been subtracted.

In addition to the modes A–D with high significance, we give a list of high-amplitude modes E–H that CLEAN/MEM suggest could be present. The mean amplitude of the two methods before any prewhitening are given in Table 4.

We think that E may be a real mode because it is detected in both time strings. But we still regard it as a tentative mode and do not include it among the modes.
Table 4. Mode data for $\kappa^2$ Boo for the two distinct observing periods

<table>
<thead>
<tr>
<th>ID</th>
<th>$\nu$ ($\mu$Hz)</th>
<th>ISF</th>
<th>CLEAN/MEM</th>
<th>Phase (HJD)$^3$</th>
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<tr>
<td>A</td>
<td>178.6</td>
<td>5.8</td>
<td>6.6</td>
<td>5.9</td>
</tr>
<tr>
<td>B</td>
<td>180.3</td>
<td>5.7</td>
<td>5.2</td>
<td>5.7</td>
</tr>
<tr>
<td>C</td>
<td>168.0</td>
<td>3.5</td>
<td>2.6</td>
<td>2.9</td>
</tr>
<tr>
<td>D</td>
<td>183.0 (171.3)</td>
<td>2.3</td>
<td>2.2</td>
<td>3.0</td>
</tr>
<tr>
<td>E</td>
<td>189.1</td>
<td>0.7</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>F</td>
<td>175.5</td>
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<td>-</td>
</tr>
<tr>
<td>G</td>
<td>173.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>H</td>
<td>175.0</td>
<td>-</td>
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<td>1.9</td>
</tr>
</tbody>
</table>

$^1$ A–H identify potential modes. $^2$ The headings 1 and 2 refer to the two observing periods. $^3$ The phase is the phase in Heliocentric Julian Date determined from the first time string.

to be used when matching models (cf. §6). Modes F and G are only present during the second period, and we suspect that this is a result of energy leaking from B and C to these frequencies given a fairly bad window function. Instrumental and weather influence could also have introduced spurious signals, but this is hard to trace. Mode H is only present in the first period, so we consider this as a probable noise peak.

After prewhitening with the modes A to D the mean noise levels in the residual spectra for the first and second period, calculated between 100–300 $\mu$Hz, are 0.77 mmag and 1.11 mmag, respectively. Taking these values as the $1\sigma$ noise level in the two amplitude spectra we have estimated the probability that a mode is a real detection by use of Figure 4 in Kjeldsen & Frandsen (1992). The probabilities are given in Table 5 together with the estimated amplitudes from the total time string using ISF. We find that the modes A to C are real modes with a probability greater than 95% whereas the corresponding probability for mode D is around 95%.

6. Comparison with an evolutionary model

If it were a single star, we would not be able to find a good model for $\kappa^2$ Boo as there are too many degrees of freedom. But we can assume that the two components are on the same isochrone. Using standard models (Christensen-Dalsgaard 1993) and solar values for the Helium content $Y$ and the mixing length $\alpha$ we need to find a combination of luminosity $L$ and heavy element abundance $Z$ that satisfies this demand. We select a set of models with different $Z$ that place the first component near the main sequence. As we have a good determination of the relative luminosities and effective temperatures of the two stars, we can find the parameters (mass $M$, $Z$) for the more massive component that make the model evolve through the center of the error box in the HR diagram. The exact $L$ depends on the distance. The evolutionary track for a solution with close to solar abundance is shown in Fig. 7.

Fig. 7. The evolutionary track for a 2.15 $M_\odot$ star with the error box indicating the location of $\kappa^2$ Boo. The position of the error box is determined after fixing the small companion on the isochrone. The error in the temperature is based on the photometric calibration.

For each parameter set $(Z, M)$ we compute the adiabatic eigenfrequencies of the pulsation modes as function of time. In the slot of time, where the evolutionary model is located within the error box we look for the best fit with the observed frequencies. The choice of solution gives us an estimate of the radius of the star (and knowing the $V \sin i$ an upper limit on the rotational splitting). We also calibrate the luminosity and thus the distance to the binary system, and we get an estimate of the $Z$ parameter.

In Fig. 8 we display the evolution of the modes. We show two panels, one which represents a close fit in the set of models and one with a large value of $M$ for comparison. The solution with mass $M = 2.15 M_\odot$ fits surprisingly well. Four observed frequencies are indicated in the figure. From such diagrams we have interpolated to a very
Table 5. Frequency and classification table for $\kappa$ Boo

<table>
<thead>
<tr>
<th>ID</th>
<th>$\mu$Hz</th>
<th>c/d</th>
<th>Period (days)</th>
<th>mmag</th>
<th>$Q \times 10^3$</th>
<th>Probability</th>
<th>(n, l, m)</th>
<th>Model frequency ($\mu$Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>178.6</td>
<td>15.43</td>
<td>0.06479</td>
<td>6.4</td>
<td>1.607</td>
<td>&gt; 95%</td>
<td>(3, 0, 0)</td>
<td>178.7</td>
</tr>
<tr>
<td>B</td>
<td>180.3</td>
<td>15.58</td>
<td>0.06418</td>
<td>5.0</td>
<td>1.592</td>
<td>&gt; 95%</td>
<td>(1, 2, 0)</td>
<td>180.2</td>
</tr>
<tr>
<td>C</td>
<td>168.0</td>
<td>14.52</td>
<td>0.06889</td>
<td>3.7</td>
<td>1.708</td>
<td>&gt; 95%</td>
<td>(0, 2, 0)</td>
<td>166.0</td>
</tr>
<tr>
<td>D</td>
<td>183.0</td>
<td>15.81</td>
<td>0.06325</td>
<td>2.9</td>
<td>1.507</td>
<td>~ 95%</td>
<td>(2, 1, 0)</td>
<td>183.0</td>
</tr>
</tbody>
</table>

best solution, which has parameters as listed in Table 6. In Table 5 the computed frequencies for the best matching modes are indicated along with the observed frequencies. We also give the classification of the modes in terms of the 'quantum' numbers. The labeling of modes is not without problems for the evolved stars, where modes change their characteristics from p to g modes (see Fig. 8). We have chosen to keep the labeling of modes as it is at the main sequence, where it is trivial to find the right classification. We have on purpose ignored the frequency changes due to rotation and searched for a non-rotating model with frequencies that would fit the observations. It is a bit surprising that such a good fit is possible. One would expect to see rotationally split modes, not only a 'random mix' of low quantum numbers ($l$ and $n$). It probably reflects the fact, that with the freedom to select a model we have, due to the few restrictions on the basic parameters, one can find solutions (only using standard models) which match four frequencies very well. But, our solution correspond to a particular set of parameters, and better values for the distance (Hipparcos) or $Z$ (spectroscopy) may very well show this model not to be correct. Included in Table 6 is the probability that a mode is a real detection as described in section 5.2.

A comforting finding is that the dominant mode is a radial pulsation. Some of the other modes have to be non-radial to fall so closely together as we observe.

Table 6. Resulting basic parameters for the binary system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Comp. 1</th>
<th>Comp. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (pc)</td>
<td>47.9</td>
<td></td>
</tr>
<tr>
<td>Age (Myr)</td>
<td>655</td>
<td></td>
</tr>
<tr>
<td>[heavy elements ($Z$)]</td>
<td>0.019</td>
<td>2.12</td>
</tr>
<tr>
<td>Mass ($M_\odot$)</td>
<td>1.38</td>
<td>2.12</td>
</tr>
<tr>
<td>Luminosity ($L_\odot$)</td>
<td>3.94</td>
<td>26.8</td>
</tr>
<tr>
<td>Radius ($R_\odot$)</td>
<td>1.48</td>
<td>2.78</td>
</tr>
<tr>
<td>Effective $T$ (K)</td>
<td>6695</td>
<td>7875</td>
</tr>
</tbody>
</table>

7. Summary and discussion

The modes that we have detected match previous results. The dominant period found earlier could be the combination of A and B, as the resolution of earlier measurements...
was not sufficient to separate these two modes. We have
analysed the data given by Moreno (1980), and we find two
possible modes, \( f_1 = 180.35 \mu \text{Hz} \) and \( A_1 = 8.0 \text{mmag} \) and
\( f_2 = 177.03 \mu \text{Hz} \) and \( A_2 = 5.4 \text{mmag} \), where the strongest
mode corresponds to our mode B and the other has no
parallel. The second mode may not be real as one can
judge oneself looking at Fig. 9.

![Power spectrum images](image)

**Fig. 9.** Power spectrum of the Moreno (1980) data, first row and then with one and two modes subtracted

The distribution of modes that we see is typical of the
closely spaced \( \delta \)-Scuti spectrum type, where frequencies
only occur in a narrow band and some very close modes
exist. \( \kappa^2\) Boo is in this respect very similar to the slightly
brighter star \( \theta^2\) Tau, which has been observed by Breger et
al. (1989). Had we detected a couple of frequencies more
close to the detected modes, we could have repeated the
conclusion of that paper: Only by having a set of non-
radial rotationally split modes is it possible to fit the ob-
served spectrum. This was based on a comparison with the
computations by Fitch (1981). Comparing the Q-values
from Table 5 with the Fitch tables for \( l = 0, 1, 2 \) modes
in the model 2.0M43, we find that we should be looking
at \( p_3 \) or \( p_4 \) overtone oscillations. This is consistent with
the results for the radial modes in the new model that we
have analysed. The classification of the non-radial modes
depends on the scheme used to determine the overtone
number and we do not do this the same way as Fitch,
who determines the mode by looking at the mode set at
a given time. We follow the evolution and retain the clas-
sification obtained at the zero main sequence age with
the mode switching that comes with this (Fig. 8). On the
other hand our results also show, that the models can pro-
duce a small set of close modes, even when rotation is not
invoked. Thus one should be careful about claiming to see
rotationally split modes.

8. Conclusions

A short list of the most important result obtained from
these observations is given here.

- 1. Under good weather conditions one can get good sig-
nal to noise data with a small telescope equipped with
a CCD camera. This allows one to extract frequencies
from a power spectrum that is quite complicated due
to a bad window function from one-site data. Multi-
site data would of course be superior, but our results
are comparable in S/N to some multisite results ob-
tained with photoelectric photometers. In the recent
STEPHI campaign (three weeks, three sites) aimed at
BN and BU Cancri in Praesepe, 5 and 6 modes were
detected with the lowest amplitude being 1.6 and 2.1
mmag respectively (Belmonte et al. 1994). In our case,
probably more frequencies would have been found in a
multisite operation, and the modeling of the star not
as uncertain as the case is with only four significant
modes.

- 2. \( \kappa^2\) Boo is a typical multi-mode pulsator, where we
see non-radial modes. They may or may not be rotation-
ally split modes, but the model we find produce a set of modes with \( m = 0 \) that match the observed
frequencies. With only four frequencies detected it is
possible to find other solutions, if rotation is taken into
account. The identification of modes in these stars is
probably not possible without additional constraints
provided by observations of phase delays or amplitude
ratios between observables (different photometric
colours and/or velocity measurements). Detailed
spectroscopic data would be even better, but costly
in terms of telescope allocation (see, e.g., Mantegazza
et al. 1994).

- 3. The CLEAN algorithm or MEM technique are well
suited to locate possible mode frequencies, but both
methods find peaks that need not be real modes. The
distribution of power among the real and artificial
peaks in the spectrum depend strongly on the window function and the weights allocated to the data points.

- 4. An evolutionary model is presented, that give frequencies that fit the observed frequencies. More modeling including the effects of rotation is needed before any final mode identification is possible. Also the basic parameters need to be known with higher precision to decrease the present very large error boxes for the fundamental parameters of the star. HIPPARCOS data would be very helpful in this respect. The difficulties due to the very complicated mode spectrum occurring at the end of the main sequence phase means that stars in stellar systems (open clusters or eclipsing binaries) where more is known about the distance etc. are simpler to analyse.

The time series dataset, too large to be included here, is available in an anonymous ftp account on obs.aau.dk in the file pub/srf/kappa.Boo.

Acknowledgements. We would like to thank the many people who spent long nights to help obtain the observations described in this paper. We would also like to express our appreciation of the enthusiasm of everybody at the IFA which made for a very exciting time. Andrew Jones participated in the project thanks to a grant from the Danish Research Council. Jens Hjorth and Hans Kjeldsen received support from the Carlsberg Foundation. We also thank Luis Balona and Don Kurtz for extensive comments on a first version of the paper. This research has made use of the Simbad database, operated at CDS, Strasbourg, France.

References

Breger M., 1979, PASP 91, 5
Elliot J.E., 1974, AJ 79, 1082
Kjeldsen H., 1992, Ph.D. Thesis, University of Aarhus, Denmark
Kjeldsen H., Frandsen S., 1992, PASP 104, 413
Michel E., 1993a, Delta Scuti Star Newsletter (ed. Breger M.), Vienna, No. 6, 19
Michel E. 1993b, private communication
Valtier J.C., 1972, A&A 16, 38

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