Complete set of Feynman rules for the MSSM – erratum

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Abstract

This erratum contains clarifications of some statements and the list of corrections of errors found in the paper Complete set of Feynman rules for the Minimal Supersymmetric Standard Model [1].

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Most of the expressions for mass matrices, mixing angles and vertices listed in [1] have been checked during the calculations of the 1-loop radiative corrections in the gauge and Higgs sectors of the MSSM [2, 3]. The 1-loop corrections were calculated in on-shell renormalization scheme, which provide a very strict test of correctness of all formulae entering the expressions for the renormalised quantities: most of the errors in Feynman rules lead immediately to non-cancellation of the divergencies. Only the most exotic vertices like 4-sfermion couplings, several rarely used 2 Higgs boson-2 sfermion couplings and some vertices proportional to the strong coupling were not used and did not pass this test yet. Other vertices can be with good probability considered as checked. Below I list the errors encountered in [1] until now. I point out also some unclear statements and conventions. The page numbers refers to the version published in Phys. Rev. D.

1. Section II, p. 3465, first line. The expression for Yukawa interactions should read as

\[ -\frac{1}{2} \left( \frac{\partial^2 W}{\partial A_i \partial A_j} \Psi_i \Psi_j + \text{H.c.} \right) \]

2. Section III, p. 3465, point (2). In general the soft SUSY breaking gaugino mass parameters may be complex. Redefinition of the phases of all fermions in the model allows to remove the complex phase only from one gaugino mass parameter. Therefore the most general form of the expression for the gaugino mass terms is

\[ L_{\text{mass}} = (m_1 \lambda_G^a \lambda_G^a + \text{H.c.}) + (m_2 \lambda_A^i \lambda_A^i + \text{H.c.}) + m_3 (\lambda_B \lambda_B + \text{H.c.}) \]

where \( m_1, m_2 \) are complex and I have chosen \( m_3 \) (gluino mass parameter) to be real. One should note that with this definition of \( L_{\text{mass}} \) the diagonal entries in the chargino and neutralino mass matrices and the physical gluino mass are equal \( 2m_1, 2m_2 \) and \( 2m_3 \) respectively.


\[ L_{\text{soft}} = k^{IJ} H_2^i L_i^L R^J + e^{IJ} H_2^i Q_i^L D^J + w^{IJ} H_1^i Q_i^L U^J \]

cannot be generated perturbatively if set once to 0. Still those terms are perfectly allowed from the point of view of construction of the consistent soft broken SUSY model and rarely (or never) taken into account.

4. Section III, p. 3466. Kobayashi-Maskawa matrix \( C = V_{Q_2}^+ V_{Q_1} \) introduced in the end of Sec. III is unfortunately defined as the Hermitian conjugation of the standard definition [4]. This convention is consistently used through the paper, so one should only remember about this problem assigning numerical values to the elements of the \( C \) matrix.

5. Section IV, p. 3467, 3468. In all expressions for the sfermion mass matrices the \( D \)-terms have the wrong sign. Correct expressions can be obtained by the replacement

\[ (v_1^2 - v_2^2) \rightarrow -(v_1^2 - v_2^2) \]

6. Section VI, p. 3470 and Appendix B, p. 3486. Correct expression for the chargino-chargino-Z\(^0\) vertex reads as \( (P_{L,R} = \frac{1+i\tau_3}{2}) \):

1
7. Appendix A, p. 3478, point (7). Squark mass matrices and trilinear squark couplings appearing in this point are defined already including the redefinition of quark-squark multiplets described in the end of Section III. For example, if the initial Yukawa and trilinear scalar couplings are \( u^{(0)IJ} \) and \( u^{(0)IJ}_S \) respectively, the redefinition of quark and squark fields lead to diagonal Yukawa coupling \( u_I^\delta = V_{KI}^\dagger Q_1 u^{(0)KL} V_L^J \) and new trilinear scalar coupling \( u^{(0)IJ}_S = V_{KI}^\dagger Q_1 u^{(0)KL}_S V_L^J \), which appears in point (7). In general transformation diagonalizing \( u^{(0)IJ}_S \) (Yukawa) coupling does not diagonalize scalar coupling \( u^{(0)IJ}_S \). The same holds for other squark couplings and mass matrices of point (7). The procedure of the redefinition of the squark mass matrices remains some freedom. I have chosen the redefined \((m^2_Q)^{IJ}\) in such a way that the Kobayashi-Maskawa matrix multiplies \((m^2_Q)^{IJ}\) in the up-squark mass matrix.

8. Appendix B, p. 3486. The neutralino-neutralino-\( Z^0 \) vertex is not properly symmetrized. Correct form is:

\[
\chi_j^0 \gamma^\mu\left( \frac{ie}{2\sin \theta_W \cos \theta_W} (Z^{i*}_i Z^j_N) P_L + Z^{(i* + j)N} P_R + \delta^{ij} \cos 2\theta_W \right)
\]

9. Appendix B, p. 3486 (last line). Expression for the up squark–down quark–chargino vertex contains typographical error. Correct form reads as:

\[
i \left( \frac{-e}{\sin \theta_W} Z^{i*}_i Z^{j*}_j + u^{(J+3)*} Z^{j*}_j P_L - d^l Z^{i*}_i Z^{jo}_j P_R \right) C^{IJ*}
\]

10. Appendix B, p. 3487. Vertices of the CP-even and CP-odd Higgs bosons with neutralinos are not properly symmetrized. Correct expressions are:

\[
\frac{ie}{2\sin \theta_W \cos \theta_W} \left\{ \left[ (Z^I_R Z^j_N - Z^j_R Z^i_N) (Z^{i*}_N \sin \theta_W - Z^{j*}_N \cos \theta_W) \right] P_L + \left[ (Z^I_R Z^j_N - Z^j_R Z^i_N) (Z^{i*}_N \sin \theta_W - Z^{j*}_N \cos \theta_W) \right] P_R \right\}
\]
11. Appendix B, p. 3497. Also the four-scalar vertices \( H^0_i H^0_j H^+_k H^-_l \) and \( H^0_{i+2} H^0_{j+2} H^+_k H^-_l \) are not properly symmetrized. Correct forms are:

\[
\begin{align*}
\frac{e}{2\sin\theta_W \cos\theta_W} & \left\{ \left[ \left( Z^i_H Z^j_N - Z^j_H Z^i_N \right) \left( Z^{i_1}_N \sin\theta_W - Z^{j_1}_N \cos\theta_W \right) \right] P_L \\
& + \left( \left( Z^i_H Z^{j^*}_N - Z^{j^*}_H Z^i_N \right) \left( Z^{i_1^*}_N \sin\theta_W - Z^{j_1^*}_N \cos\theta_W \right) \right) P_R \right\} \\
\end{align*}
\]

12. Appendix B, p. 3500. Expression for the gluon-gluino-gluino vertex contains unnecessary factor \( 1/2 \). It should be:

\[
G_{\mu c} \rightarrow \left( \frac{1}{\cos^2\theta_W} A^{i j}_H A^{k l}_H + A^{i k}_H A^{j l}_H + A^{i l}_H A^{j k}_H \right)
\]

13. Appendix B, p. 3500. Expression for the 4-gluon vertex contains typographical error. Correct form is:

\[
- g_3 f_{abc} \gamma^\mu
\]
14. Appendix B, p. 3499. Vertices gluon-gluon-squark-squark are not properly symmetrized in gluon indices. Correct forms are:

\[ G_{\mu a} G_{\nu b} G_{\rho c} G_{\sigma d} \]

\[ -i g_3^2 \left[ f_{abc} f_{def} \left( g^{\mu\lambda} g^{\nu\rho} - g^{\mu\rho} g^{\nu\lambda} \right) \right. \]

\[ + f_{aef} f_{bce} \left( g_{\mu\nu} g^{\lambda\rho} - g_{\mu\rho} g^{\lambda\nu} \right) \]

15. Appendix B, p. 3500 and 3501. 4-sfermion vertices \( UUUU \) and \( DDDD \) contain errors and are not correctly symmetrized. Correct forms are:

\[ U_{\beta \gamma} \]

\[ G_{\mu a} G_{\nu b} \]

\[ i g_3^2 (Y^a Y^b + Y^b Y^a)_{\alpha \beta} g^{\mu \nu} \delta^{ij} \]

\[ D_{\beta \gamma} \]

\[ G_{\mu a} G_{\nu b} \]

\[ i g_3^2 (Y^a Y^b + Y^b Y^a)_{\alpha \beta} g^{\mu \nu} \delta^{ij} \]
where

\[
\begin{align*}
R_{ij}^U &= Z_U^{i*} Z_U^j \\
X_{ij}^U &= \delta_{ij} - 2R_{ij}^U \\
Y_{ij}^U &= 4\delta_{ij} - 5R_{ij}^U \\
V_{ij}^{kl} &= (u^I Z_{U}^{(I+3)i*} Z_{U}^j)(u^J Z_{U}^{(J+3)j*} Z_{U}^k Z_{U}^{(J+3)k})
\end{align*}
\]

\[
D_{\delta}^- \\
D_{\beta}^- \\
D_{\gamma}^-
\]

\[
\begin{align*}
&-i \left( \frac{g_s^2}{6} X_{ij}^U X_{ij}^D - X_{ij}^U X_{ij}^D \right) + \frac{e^2}{4s_W^2} R_{ij}^U R_{ij}^D \\
&+ \frac{e^2}{36c_W^2} Y_{ij}^U Y_{ij}^D + V_{ij}^{kl} + V_{ij}^{kl} \right) \delta_{\alpha\beta} \delta_{\gamma\delta} \\
&+ \left( \frac{g_s^2}{6} (3X_{ij}^U X_{ij}^D - X_{ij}^U X_{ij}^D) + \frac{e^2}{4s_W^2} R_{ij}^U R_{ij}^D \right) \\
&+ \frac{e^2}{36c_W^2} Y_{ij}^U Y_{ij}^D + V_{ij}^{kl} + V_{ij}^{kl} \right) \delta_{\alpha\beta} \delta_{\gamma\delta}
\end{align*}
\]

where

\[
\begin{align*}
R_{ij}^D &= Z_D^{Ij} Z_D^{Ij*} \\
X_{ij}^D &= \delta_{ij} - 2R_{ij}^D \\
Y_{ij}^D &= 2\delta_{ij} - R_{ij}^D \\
V_{ij}^{kl} &= (d^I Z_{D}^{(I+3)i} Z_{D}^{Ij*})(d^J Z_{D}^{(J+3)j} Z_{D}^{(J+3)j})
\end{align*}
\]

16. Section VI, p. 3476 and Appendix B, p. 3501. 4-sfermion vertex \( UDDD \) contains errors. Using the symbols defined above, correct form can be written down as:

\[
\begin{align*}
D_{\delta}^- \\
D_{\beta}^- \\
D_{\gamma}^-
\]

\[
\begin{align*}
&-i \left( \frac{g_s^2}{6} X_{ij}^U X_{ij}^D - X_{ij}^U X_{ij}^D \right) + \frac{e^2}{4s_W^2} R_{ij}^U R_{ij}^D \\
&+ \frac{e^2}{36c_W^2} Y_{ij}^U Y_{ij}^D + V_{ij}^{kl} + V_{ij}^{kl} \right) \delta_{\alpha\beta} \delta_{\gamma\delta} \\
&+ \left( \frac{g_s^2}{6} (3X_{ij}^U X_{ij}^D - X_{ij}^U X_{ij}^D) + \frac{e^2}{4s_W^2} R_{ij}^U R_{ij}^D \right) \\
&+ \frac{e^2}{36c_W^2} Y_{ij}^U Y_{ij}^D + V_{ij}^{kl} + V_{ij}^{kl} \right) \delta_{\alpha\beta} \delta_{\gamma\delta}
\end{align*}
\]

Formulae for the diagonalization of mass matrices and most of the vertices listed in [1] are accessible also as the ready FORTRAN codes. They are parts of the bigger library for calculation of the 1-loop radiative corrections in on-shell renormalization scheme to the MSSM neutral Higgs production and decay rates [2]. This library can be found at the URL address [http://www.fuw.edu.pl/~rosiek/neutral_higgs.html](http://www.fuw.edu.pl/~rosiek/neutral_higgs.html).
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References


