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Interacting Boson Model

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ΔI = 4 Bifurcation and the sdg Interacting Boson Model

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Abstract

We show that the superdeformed nuclear states can be described in the framework of the interacting boson model with the g-bosons being taken into account in this letter. The superdeformed rotational bands with ΔI = 4 bifurcation can be reproduced in the SU(5) limits of the sdg IBM. The perturbation causing the ΔI = 4 bifurcation to emerge in the ΔI = 2 superdeformed rotational band is shown to possess the SU(5) symmetry.

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In the spectroscopy of superdeformed nuclei, rotational level sequences have been observed in which the ΔI = 2 rotational band is perturbed and split into two branches1,2. Because in each of the branchs the energy levels differ in angular momentum by four, this phenomenon is called ΔI = 4 bifurcation, or ΔI = 2 staggering, since the states differing by two in angular momentum are staggered in energy. The occurrence of such a staggering in the rotational energy suggests that the strongly prolate spheroidal deformation has been perturbed by hexadecapole deformation3 (for example, the deformation parameter β4 of 129Gd may be up to 0.05%). In other words, a perturbation holding symmetry V₄ with respect to the symmetry axis of the nucleus emerges in the superdeformed nuclear states4. Thus, the experiments are regarded as evidences that superdeformed nuclei have C₄v symmetry5–8 or intrinsic vortical motion9,10. In the framework of interacting boson model (IBM)11, the nucleus with quadrupole and hexadecapole deformations consists of s-, d- and g-bosons. The approach to describe the nucleus including s-, d- and g-bosons is usually referred as sdg interacting boson model (sdg IBM)12,13. With the sdg IBM we will show in this letter that the perturbation may probably possess much higher symmetry SU(5) in the sdg IBM.

In sdg interacting boson model, the collective nuclear states (with quadrupole and hexadecapole deformations) are generated as states of a system with N s-, d- g-bosons. Since the total single boson space is 15 dimensional, the symmetry group is U(15), and the states of N-bosons belong to the totally symmetric irreducible representation (irrep) [N]₁ of the U(15). Moreover, it has been shown that the sdg IBM has strong coupling dynamical symmetries SU(3), SU(5), SU(6), O(15) and weak coupling dynamical symmetries U₄(6) ⊗ U₄(9), U₄(14), U₄(5) ⊗ U₄(10). Numerical calculations show that the sdg IBM is quite successful in describing nuclear states with large deformation and E₂ transitions14. On the other hand, with the projective coherent state scheme15 being exploited, Devi and collaborators showed12,13,16 that the potential energy surface of the nucleus with dynamical symmetry U₄(6) ⊗ U₄(9), U₄(14), U₄(5) ⊗ U₄(10), SU(3) or O(15) has one minimum, which is just the same as that in the sd IBM. However, the energy surface of a nucleus with SU(6) symmetry has three degenerate minima and that of the SU(5) symmetry has two minima that are displaced in energy (see Fig. 5 of Ref.12 and Figs. 1b–1f of Ref.13). It indicates that the sdg IBM admits shape coexistence and shape phase transformation which can be driven by angular momentum17,18,19. Considering this fact and the view that superdeformed states are generated in the second minimum of the potential energy surface17, we know that the superdeformed nuclear states can be described with the SU(5) limits of the sdg IBM.

As a nucleus has the SU(5) symmetry in the sdg IBM, the states of the nucleus can
be classified by the irreps of the group chain
\[ U(1) \supset SU(3) \supset SO(3) \supset SO(3). \] (1)

The wave function can then be written as
\[ \psi(I) = \left[ \text{Y}_{13} J \left( n_1, n_2, n_3, n_4 \right) \psi \left( \tau_1, \tau_2 \right) \alpha \right] \text{Y}_{13} J \left( n_1, n_2, n_3, n_4 \right) \psi \left( \tau_1, \tau_2 \right) \alpha I, \] (2)

where \( J \) is the additional quantum number to distinguish the same \((\tau_1, \tau_2)\) belonging to the same \([n_1, n_2, n_3, n_4]_J\), and \( \alpha \) is the additional quantum number to distinguish the same \( J \) belonging to the same \((\tau_1, \tau_2)\). They are the integers satisfying the relations \( 1 \leq \delta \leq J_{\text{max}}, \frac{1}{2} \leq \alpha \leq \alpha_{\text{max}} \), respectively, in which \( J_{\text{max}} \) is the multiplicity of irrep \((\tau_1, J)\) belonging to the irrep \([n_1, n_2, n_3, n_4,J]_J\), and \( \alpha_{\text{max}} \) is the multiplicity of \( J \) belonging to the irrep \((\tau_1, J)\). All the irreps and the multiplicities can be determined from the branching rules of the irreps which have been discussed by Sun et al.\(^{11}\). For a given \([N]_J\), the irreps \([n_1, n_2, n_3, n_4]_J\) can be all the possible \([2N - 2p - 4q - 6r - 6s, 2(p + q + r), 2(q + r), 2r]\), with restriction \(2p + 3q + 4r + 5s \leq 4N\). For a given \([n_1, n_2, n_3, n_4]_J\), the irrep \((\tau_1, \tau_2)\) can be determined with the Young tableaux technique\(^{12}\) or Schur function method\(^{13}\), for instance,
\[ [n, 0, 0, 0]_J = (n, 0) \oplus (n - 2, 0) \oplus (n - 4, 0) \oplus \cdots \oplus \left\{ \begin{array}{ll}
0, 0 & (\text{for } n \text{ even}), \\
1, 0 & (\text{for } n \text{ odd}).
\end{array} \right. \] (3)

\[ [n_1, n_2, 0, 0]_J = \left\{ \begin{array}{ll}
F(n_1, n_2) \oplus F(n_1 - 2, n_2) \oplus \cdots \oplus F(n_1 - 2, n_2) & (\text{for } n_1 = n_2), \\
F(n_1, n_2) \oplus F(n_1 - 2, n_2) \oplus \cdots \oplus F(n_1 - 2, n_2) & (\text{for } n_1 = n_2), \\
F(n_1 + 1, n_2) & (\text{for } n_1 = n_2).
\end{array} \right. \] (4)

where
\[ \left( n_1, n_2 \right) = \sum_{i, j} \bar{\delta}(n - 2i, n - 2j - 2), \]

\[ F(n_1, n_2) = \sum_{i, j} \bar{\delta}(n - 2i, n - 2j - 2). \]

For a given \((\tau_1, \tau_2)\), the reduction of \( SO(3) \supset O(3) \) can be obtained with the multipartition technique\(^{19}\). For example, for a given \((\tau_1, 0)_J\), the corresponding \( I \) can be \( 2\tau_1, 2\tau_1 - 2, 2\tau_1 - 4, \ldots \). For a given \((\tau_1, \tau_2)\), the corresponding \( I \) can be \( 2\tau_1 + \tau_2, 2\tau_1 + \tau_2 - 2, \ldots \). The interaction Hamiltonian of the bosons in a nucleus with the \( SU(5) \) symmetry can be written as
\[ H = E_0 + \Delta C_{\text{SUD(5)}} + BC_{\text{SU(5)}} + CC_{\text{SU(5)}}. \] (5)

in which \( C_{\Delta} \) is the quadratic Casimir operator of the group \( g \). The energy of the state \( |\psi(n_1, n_2, n_3, n_4)\rangle \) can be given as
\[ E(I) = E_0 + \Delta \left( n_1(n_1 + 1) + n_2(n_2 + 1) + n_3(n_3 + 1) + n_4(n_4 + 1) - \frac{1}{2} \left( n_1 + n_2 + n_3 + n_4 \right) \right) \]
\[ + \left( B\left( n_1(n_1 + 1) + n_2(n_2 + 1) + n_3(n_3 + 1) + n_4(n_4 + 1) \right) + C\left( n_1(n_1 + 1) + n_2(n_2 + 1) + n_3(n_3 + 1) + n_4(n_4 + 1) \right) \right). \] (6)

where \( \Delta \) determines the energy difference between the lowest \( I = 0 \) states with different irrep \([n_1, n_2, n_3, n_4]_J\). \( B \) decides the energy difference between the different lowest \( I \) states with different \([n_1, n_2, n_3, n_4]_J\) but the same \([n_1, n_2, n_3, n_4]_J\). \( C \) gives the energy difference between the states with different \( I \) but the same \([n_1, n_2, n_3, n_4]_J\) and \([n_1, n_2, n_3, n_4]_J\). In order to keep the \( I = 0 \) state with irrep \([2N, 0, 0, 0]_J \) \((N = \text{the total number of the bosons})\) and \((\tau_1, \tau_2)_J = (0, 0)\) being the ground state, the parameters should be taken as \( A < 0, B > 0, C > 0 \). Thus, the energy bands generated by the totally symmetric irrep \([2N, 0, 0, 0]_J \) are usually lower than those generated by the nontotally symmetric irreps \([2N - 2, 2, 0, 0]_J \) and others. On the other hand, the branch rules of the irrep reduction show that the totally symmetric irrep \([2N, 0, 0, 0]_J \) of the \( SU(5) \) generates the energy bands with level sequences \([0, 0, \ldots, 0, 4N - 2, 4N - 2, 4N - 6, 4N - 10, \ldots, 4N - 12, \ldots, 4N - 4, 4N - 6, 4N - 8, 4N - 10, \ldots, 4N - 12, \ldots, 4N - 4, 4N - 6, 4N - 8, 4N - 10, \ldots, 4N - 12, \ldots] \) respectively (see Fig. 2 of Ref. [13]). We know then that the irrep \([2N, 0, 0, 0]_J \) of \( SU(5) \) can reproduce the low-lying energy bands with level sequence \( I_0, I_0 + 2, I_0 + 4, \ldots \), but can not generate the superdeformed energy band with level sequence \( I_0, I_0 + 2, I_0 + 4, \ldots \). For the irrep \([2N - 2, 2, 0, 0]_J \) of \( SU(5) \), since the irreps of \( SO(5) \) can be \([2N - 2, 2), (2N - 4, 2), (2N - 6, 2), \ldots, (2N - 4, 0), (2N - 6, 0), \ldots, (2, 0), (0, 0)\), of which the corresponding largest angular momenta are \( 4N - 2, 4N - 6, 4N - 10, \ldots, 4N - 12, \ldots, 4N - 4, 4N - 6, 4N - 8, 4N - 10, \ldots, 4N - 12, \ldots \) come to naturally, and couple to one band with level sequence \( I_0, I_0 + 2, I_0 + 4, \ldots \) of which the states differing by \( 2 \) in angular momentum are staggered in energy. Figure 1 is an example of the energy spectrum built on the irrep \([2N - 2, 2, 0, 0]_J \) of the \( SU(5) \). From the irrep reduction rule in the group chain \( SO(3) \supset SO(3) \) and figure 1 we know that many bands with \( \Delta I = 2 \) staggering in energy can appear in one system. Therefore, one nucleus can have more than one superdeformed rotational bands with \( \Delta I = 4 \) bifurcation, such as the ones reported in Ref. [2].

To show the \( \Delta I = 4 \) bifurcation more explicitly, we discuss the energy differences \( \Delta E_4 \) between two consecutive gamma-ray transitions of the energy band generated by the irrep \([2N - 2, 2, 0, 0]_J \) of the \( SU(5) \) as a function of angular momentum and rotational frequency respectively after subtraction of a smooth reference energy \( \Delta E_{4}'(I) \). Since \( \Delta E_4(I) \) is defined as \( \Delta E_4(I) = E_4(I + 2) - E_4(I) \) and the \( \Delta E_{4}'(I) \) is given as \( \Delta E_{4}'(I) = \Delta E_4(I - 2) + \Delta E_4(I) + \Delta E_4(I + 2))/4 \), we have \( \Delta E_4(I) - \Delta E_{4}'(I) = E_4(I - 2) - 3E_4(I) + E_4(I + 2) - E_4(I + 4))/4 \). Figures 2 and 3 illustrate the changing feature of
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the $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ versus the angular momentum $I$ and that against the rotational frequency $\omega = \frac{1}{2} E_i(I)$ respectively with the parameters in eqs. (5) and (6) are taken as $E_0 = 0, A = 0, B = 5$ keV, $C = 0.1$ keV for a nucleus with $1s - 1s, d - g$ bosons. In figures 4 and 5 we show the two kind variation features with parameters $E_0 = 0, A = 0, B = 0.03$, $E = 0.25$ keV for the same nucleus. Since the parameters $E_0$ and $A$ determine only the energy of the lowest state belonging to $\{2N - 2, 2N, 0\}$ with respect to the ground state, we take $E_0 = 0$ and $A = 0$. And the choice of parameters in this way does not change the band structure being considered.

As the parameters are taken as $B = 5$ keV, $C = 0.1$ keV, i.e., $B \gg C$, the Hamiltonian (3) has the $SU(5)$ symmetry. Figure 2 shows that the $\Delta I = 2$ staggering in energy generated by the $SU(5)$ symmetry is very obvious (for negative energy differences $\Delta E_i(I) - \Delta E_i^{\nu}(I)$, the amplitude is in the same order of magnitude as the $E_i(I)$, for the positive energy differences, the amplitude is in the same order of magnitude as the neighboring transition energy $E_i(I - 2)$ or $E_i(I + 2)$) and the staggering amplitudes get large and large as the angular momentum $I$ increases. Figure 3 indicates that energy differences $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ versus the rotational frequency are separated into two isolated branches for the two $\Delta I = 4$ level sequences. For one $\Delta I = 4$ level sequence the $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ is always positive and the rotational frequency of the state $i$ is very small (less than 0.03 MeV). For another $\Delta I = 4$ level sequence $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ is definitely negative and the rotational frequency is in the usual situation (from 0.03 MeV to 0.32 MeV). Since the difference between the rotational frequencies of the states with angular momentum $I$, $I + 2$ is quite large, the positive $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ and the negative $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ does not appear alternately but split into two branches. It suggests that if the energy of state $i$ in a band with level sequence $I_0, I_0 + 2, I_0 + 8, \ldots$ is close to the energy of the state $I + 2$ in the band with level sequence $I_0 + 2, I_0 + 6, \ldots$, the $\Delta I = 2$ staggering of the $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ washes out and two segregated branches emerge. It shows also that to describe well the observed $\Delta I = 4$ bifurcation theoretically, the staggering of energy differences as a function of the rotational frequency is a more reliable characteristic than that versus the angular momentum.

When the parameters are chosen as $B = 0.03$, $C = 0.25$, i.e., $B \ll C$, the interacting Hamiltonian is expressed as an axial rotational interaction and a perturbation with the $SU(5)$ symmetry. The calculated results of the energy differences between two consecutive $\gamma$-ray transitions $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ as a function of the angular momentum and that as a function of the rotational frequency are illustrated in figures 4 and 5 respectively. The figures show that the energy differences $\Delta E_i$ between two consecutive $\gamma$-ray transitions after subtraction of a smooth reference are really very small even though they do not agree with the experimentally data precisely (we are not attempt to fit the data in this letter at all). Meanwhile the variation characteristic of $\Delta E_i(I) - \Delta E_i^{\nu}(I)$ as a function of rotational frequency represents the observed changing feature of the staggering versus the rotational frequency, and is consistent with that as a function of angular momentum. Comparing figures 2, 4 with figures 4, 5 respectively, we know that, as the interaction with the $SU(5)$ symmetry is handled as a perturbation, the $\Delta I = 2$ staggering in $\gamma$-ray energy differences can be described better than in the case that the interaction is taken as a dominant. It suggests that, even though the $SU(5)$ symmetry is imperative in generating energy levels with $\Delta I = 4$ sequence, the interaction generating the superdeformed nuclear states with $\Delta I = 4$ bifurcation is not governed by the $SU(5)$ symmetry, but still regulated by the rotational interaction. Nevertheless, the perturbation causing the $\Delta I = 2$ rotational band to split into $\Delta I = 4$ bifurcation may possess $SU(5)$ symmetry. We have also investigated the changing feature of the energy differences $\Delta E_i$ introduced as the manifestation of the $\Delta I = 4$ bifurcation by Codreanu[9]. The same result as shown above is obtained.

In summary, we have shown in this letter that the superdeformed nuclear states can be described in the framework of the interacting boson model as the $\gamma$-bosons are taken into account. In experiment, the representation to show the $\Delta I = 4$ bifurcation is the staggering in energy differences $\Delta E_i$ between two consecutive $\gamma$-ray transitions after subtraction of a smooth reference[10] as a function of the rotational frequency; since the angular momenta are not assigned. In theoretical description, although the angular momentum can be assigned, the changing feature of the energy differences $\Delta E_i$ as a function of angular momentum is not consistent with that versus the rotational frequency when the difference of rotational frequencies between the two $\Delta I = 4$ branches is large. Then, to describe the observed $\Delta I = 2$ staggering well, one should consider the same changing characteristic as in experiment as fully as possible. Otherwise, we have not attempted to fit the experimental data in the sdg IBM in this letter. However, preliminary calculation indicates that the general feature of the staggering in energy differences $\Delta E_i$ in a $\Delta I = 2$ superdeformed rotational band can be described in the sdg IBM as the Hamiltonian is taken as a rotational interaction plus a perturbation with $SU(5)$ symmetry. In this scheme, one nucleus can naturally have more than one superdeformed rotational band with $\Delta I = 4$ bifurcation. We then come to a conclusion that the perturbative interaction making the $\Delta I = 2$ superdeformed rotational band split into $\Delta I = 4$ bifurcation may possess $SU(5)$ symmetry.

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References

[12] for review see, for example, Y. D. Devi and V. K. B. Kota, Pramana J. Phys. 39 (1992), 413.

Figure Captions:

Figure 1. A part of the energy spectrum generated by the irrep $[8, 2, 0, 0]$ of the $SU(5)$ (the parameter $a$ are taken as $E_0 = A = 0$, $B = 25$ keV, $C = 5$ keV. The labels at the left side of the levels are the corresponding irrep of the $SO(5)$ group).

Figure 2. The energy differences $\Delta E$, between two consecutive $\gamma$-ray transitions in the energy band generated by the irrep $[2N+2, 2, 0, 0]$ of the $SU(5)$ as a function of angular momentum after subtraction of a smooth reference $\Delta E^{\text{ref}}_r^2(1)$. The nucleus is taken as the one with $18$ s., d- and g-bosons and the parameters are taken as $E_0 = 0$, $A = 0$, $B = 5$ keV, $C = 0.1$ keV.

Figure 3. The energy differences $\Delta E$, between two consecutive $\gamma$-ray transitions in the energy band belonging to the irrep $[2N+2, 2, 0, 0]$ of the $SU(5)$ as a function of rotational frequency after subtraction of a smooth reference $\Delta E^{\text{ref}}_r^2(1)$. The nucleus is also taken as the one with $18$ s., d- and g-bosons and the parameters are taken as $E_0 = 0$, $A = 0$, $B = 5$ keV, $C = 0.1$ keV.

Figure 4. The same as figure 2 but for parameters $E_0 = 0$, $A = 0$, $B = 0.01$ keV, $C = 6.25$ KeV.

Figure 5. The same as figure 3 but for parameters $E_0 = 0$, $A = 0$, $B = 0.01$ keV, $C = 6.25$ KeV.
Fig. 1.

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\begin{align*}
(8,2) & \quad 18 & \quad 17 \\
(8,0) & \quad 16 & \quad 15 \\
(6,2) & \quad 14 & \quad 13 \\
(6,0) & \quad 12 & \quad 11 \\
(4,2) & \quad 10 & \quad 9 \\
(4,0) & \quad 8 & \quad 7 \\
(2,2) & \quad 6 & \quad 5 \\
(2,0) & \quad 4 & \quad 3 \\
\end{align*}

\[ \ldots \quad 3, \quad V = 2, \quad \ldots \quad 0 \]

Fig. 2.

Rotational Frequency (MeV)

Fig. 3.
Fig. 4.

Rotational Frequency (MeV)

Fig. 5.