$1/m$ Corrections to Heavy Baryon Masses in the Heavy Quark Effective Theory Sum Rules

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Abstract

The $1/m$ corrections to heavy baryon masses are calculated from the QCD sum rules within the framework of the heavy quark effective theory. Numerical results for the heavy baryons are obtained. The implications of the results are discussed.

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Heavy baryons provide us a testing ground for the Standard Model (SM), especially to QCD in some aspects. With the accumulation of the experimental data on the heavy baryons, more reliable theoretical calculations are needed, although some of them are rather complicated. Within the framework of the heavy quark effective theory (HQET), which is a model-independent method, the theoretical analysis to the heavy baryons containing a single heavy quark is comparatively simple because of the heavy quark symmetry [1]. However, there are still quantities in this framework which need to be determined from nonperturbative QCD.

QCD sum rule [2], which is regarded as a nonperturbative method rooted in QCD itself, has been used successfully to calculate the properties of various hadrons. For instance, besides the light mesons [2], light baryons were first considered in Ref. [3]. Heavy meson properties were systematically analyzed within the HQET [4, 5]. Heavy baryons were first discussed in Ref. [6], where masses and form factors for heavy baryons were calculated in the HQET to the leading order heavy quark expansion in Refs. [7] and [8]. In Ref. [9], the calculation for the heavy baryons began with the full theory and results of the calculation were expanded by heavy quark masses. In this paper, within the framework of the HQET, we study the heavy baryonic two-point correlators to the subleading order of the heavy quark expansion by QCD sum rules and obtain results for the heavy baryon masses to that order.

In the HQET, the heavy quark mass $m_Q$ which is defined perturbatively as the pole mass has been removed by the field redefinition. The heavy quark field $b_c$ is defined by

$$P_c Q(c) = \exp(-im_Qc) b_c(c)$$

(1)

where $P_c = \frac{1}{2} [1, c]$. To the order of $1/m_Q$, the effective Lagrangian for the heavy

\[\text{Mailing address}\]
\[ L_{\text{eff}} = \frac{1}{2} u_i \bar{u}_i D u_i + \frac{1}{2 m_q} \bar{u}_i D^2 u_i - \frac{g}{4 m_q} \bar{u}_i \gamma_5 \omega u_i e^{-i \frac{\omega}{m_q} \frac{\tau}{4} \tau_3} \]  \]  \tag{1.1}

As for the $1/m_q$ terms, the first one still conserves heavy quark spin symmetry. It is the last term which violates the spin symmetry. The heavy baryon mass $M$ is expanded in $\frac{1}{m_q}$ \[ M = m_q + \frac{\delta M}{m_q} + \frac{\delta M}{m_q} \gamma_\mu \gamma_5 \gamma_\mu > + O\left( \frac{1}{m_q^2} \right) \] \tag{1.3}

where $\delta M$ is the heavy baryon mass in the heavy quark limit, which has been calculated in Ref. [3]. $\delta M^K$ and $\delta M^G$ parameterize the spin-conserved and spin-violated $1/m_q$ corrections respectively. All of them characterize the properties of the light degrees of freedom. $\gamma_q$ denotes the heavy quark spin, and $\mu$ stands for the total angular momentum of the light degrees of freedom. For $\Sigma^Q_q$ baryons, both $\delta M^K$ and $\delta M^G$ terms are nonvanishing with $\gamma_q \gamma_5 \gamma_\mu \gamma_\mu > \neq 0$ for $\Sigma^Q_q$ and $\gamma_q \gamma_\mu \gamma_\mu > \neq 1$ for $\Sigma^Q_q$. The heavy baryonic currents $J^{\mu}$ have been given in Refs. [1] and [17] in the rest frame of the heavy baryons. Generally they can be expressed as:

\[ J^{\mu} = e^{\alpha i} e^{\beta i} e^{\gamma i} \Gamma^{\mu} e^{\alpha i} e^{\beta i} e^{\gamma i} \] \tag{1.4}

where $\alpha$, $\beta$, $\gamma$ is a color index, $\Gamma$ and $\Gamma'$ are some gamma matrices, and $\alpha$, $\beta$, $\gamma$ denote the color indices. $\Gamma$ and $\Gamma'$ can be chosen consistently as:

\[ \Gamma_3 = \gamma_5, \quad \Gamma'_{\alpha} = 1 \] \tag{1.15}

for $\Sigma^Q_q$ baryons:

\[ \Gamma_{i=1,2} = \gamma_i \quad \Gamma'_{i=1,2} = (\gamma_i + \gamma_5) \gamma_i \] \tag{1.16}

The two-point correlator $\Gamma(\omega)$ which we choose for isospin analyzing in the HQET is:

\[ \Gamma(\omega) = i \int d^4 x e^{i \omega x} < 0 | \bar{\psi}^{i=1,2} (0) \psi^{i=1,2} (0) | 0 > \] \tag{1.10}

\[ \omega = 2 e \cdot k \]  \tag{1.10}

The hadronic representation of this correlator is:

\[ \Gamma(\omega) = e^{i \frac{\omega}{m_q} \frac{\tau}{4} \tau_3} \chi^{1,2} (2 \omega - \omega') - e^{i \frac{\omega}{m_q} \frac{\tau}{4} \tau_3} \chi^{1,2} (2 \omega - \omega') + \frac{1}{2} + \text{res} \] \tag{1.11}

where $\delta \Lambda$ and $\delta f$ stand for the $1/m_q$ corrections in Eqs. (3) and (9). On the other hand, $\Gamma_{i=1,2}$ can be calculated in terms of quark and gluon language with vacuum condensates. The establishes the sum rule: We use the commonly adopted quark-hadron duality for the resonance part of Eq. (11),

\[ \text{res} = \int_\omega^{\infty} d \omega' \text{Im} \left( \frac{\Gamma(\omega)}{\omega - \omega'} \right) \] \tag{1.12}

where $\Gamma(\omega)$ denotes the perturbative contribution, and $\omega$ is the continuum threshold. In this work, we shall consider only the diagonal correlators ($i = j$).
The calculations of $F_{2n}$ are straightforward. The usual point gauge is used [11]. All the condensates with dimensions lower than 6 are retained. We also include the dimension 6 condensate $\langle \bar{q}q \rangle$ in our analysis which is a main contribution. We use the gaussian ansatz for the distribution in spacetime for this condensate [12].

In the heavy quark limit, we have double checked the analysis of Ref. [7]. We use the following values of the condensates:

\begin{align}
\langle \bar{q}q \rangle &\sim -0.23 \text{ GeV}^3, \\
\langle qGq \rangle &\sim 0.04 \text{ GeV}^4, \\
\langle q\bar{q}G^\mu \eta \rangle &\equiv m_0^2 \sim 0.5 \text{ GeV}^4.
\end{align}

(13)

When $\omega_c$ lies between $2.1 - 2.7$ GeV for $\Lambda_0$ and between $2.3 - 2.9$ GeV for $\Sigma_0^q$, the stability window exists. We obtain

\begin{align}
\Lambda_0 &\sim 0.79 \pm 0.01 \text{ GeV,} \\
f_0^2 &\sim (10.3 \pm 0.04) \times 10^{-4} \text{ GeV}^3.
\end{align}

(14)

for $\Lambda_0$ baryon with $\omega_c \sim 2.3$ GeV.

\begin{align}
\Lambda_0 &\sim 0.96 \pm 0.01 \text{ GeV,} \\
f_0^2 &\sim (11.7 \pm 0.1) \times 10^{-4} \text{ GeV}^3.
\end{align}

(15)

for $\Sigma_0^q$ baryon with $\omega_c \sim 2.7$ GeV. The normalization $T_{FF} = 1$ has been used in the analysis. Compared with Ref. [7], the numerical results are slightly smaller, the range of the Borel parameter is the same $T = 0.1 - 0.7$.

The $1/m_q$ corrections to the two point correlator $F(x)$ can be calculated by including insertions of the $1/m_q$ operators of the Lagrangian [21] with standard method which is shown in Fig. 1. The insertions of spin-conserved and spin-violated operators are calculated separately. The final form of the sum rules are obtained by performing Borel transformation. With some simple tricks [13], the sum rules for the mass and $f$ can be separated. The results for the mass of $\Lambda_0$ baryon come from the spin-conserved operators only ($\delta \Lambda_0 = \delta \Lambda_0^S$):

\begin{align}
\delta \Lambda_0 &= -\frac{T^2}{16\pi f^2} f_{\Lambda_0}, \\
\frac{\delta \Lambda_0}{\Lambda_0} &= -\frac{T^2}{16\pi f^2} f_{\Lambda_0},
\end{align}

(16)

where

\begin{align}
l_{\Lambda_0} &= \frac{3}{2\pi^4} \int_0^{\omega_c} d\omega \omega^2 e^{-\omega T} + \frac{3m_0^2 - m_0^4}{T} e^{-m_0^2/T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T},
\end{align}

(17)

and the subscripts 1 and 2 denote $f_1^S$ and $f_2^S$ respectively. The sum rule for $f$ of $\Lambda_0$ is

\begin{align}
\delta f_{\Lambda_0}^S &= -\frac{1}{3} (1 + \frac{d}{d\ln T}) f_{\Lambda_0},
\end{align}

(18)

The masses of baryons $\Sigma_0^q$ and $\Sigma_0^q$ are given in terms of $\delta \Lambda^S$ and $\delta \Lambda^G$. They are determined by the following sum rules:

\begin{align}
\delta \Lambda_0^S &= -\frac{T^2}{16\pi f^2} f_{\Lambda_0}^S, \\
\delta \Lambda_0^G &= -\frac{T^2}{16\pi f^2} f_{\Lambda_0}^G,
\end{align}

(19)

where

\begin{align}
l_{\Lambda_0}^S &= \frac{3}{2\pi^4} \int_0^{\omega_c} d\omega \omega^2 e^{-\omega T} + \frac{3m_0^2 - m_0^4}{T} e^{-m_0^2/T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T} + \frac{12}{2\pi^4} <q < q^2 \frac{e^{-q^2 T}}{T},
\end{align}

(20)

The sum rules for $f$ are given by

\begin{align}
\delta f_{\Sigma_0}^{S,G} &= -\frac{1}{3} (1 + \frac{d}{d\ln T}) f_{\Sigma_0}^{S,G},
\end{align}

(21)
It can be seen that while the two diagonal sum rules coincide with each other at the leading order, they are no longer the same for the spin-conserved $1/m_Q$ corrections.

The numerical sum rule results for the $1/m_Q$ corrections are given in Table 1 and Figs. 2-4. The numerical differences resulting from the different choices of $a$ are not significant. The values of $a$ are generally smaller than the leading order results but still lie in the allowed range of the leading order results. The lower limit of the Borel parameter $T = 0.4$ is determined by requiring that the condensates in Eqs. (17) and (20) have less than 40% contribution. The upper limit $T = 0.8$ is obtained by requiring that the pole contribution is over 70%. This window is narrower than the leading order one. In the window $T = 0.4 - 0.6$ the results for $\Delta A_2$ and $\Delta A_2'$ are comparatively stable. However, from Fig. 1, we see that $\Delta A_2$ has no good stability in this window. This is because we have not included the Feynman diagrams with internal gluon lines which are expected to be important for the spin-violated term. Therefore the value $\Delta A_2$ in Table 1 is not reliable.

From $m_{L}$ and $m_{K}$, we determine the heavy quark masses $m_c = 1.14$ GeV and $m_b = 4.83$ GeV. These values give the following results:

$m_{L} = 2.53$ GeV, $m_{Q} = 2.56$ GeV, $m_{W} = 5.84$ GeV, $m_{K} = 5.84$ GeV

From the discussion above we know the individual mass value in Eq. (22) suffers from the inaccuracy of $\Delta A_2$. The quantities

$$\frac{1}{2} (m_{L} + 2m_{Q}) = 2.55 \text{ GeV} \quad \frac{1}{3} (m_{L} + 2m_{K}) = 2.52 \text{ GeV}$$

are independent of $\Delta A_2$, therefore more reliable. Experimentally $m_{L} = 2.53 \pm 0.2$ MeV [14]. There is an experimental evidence for $m_{Q}$ at $m_{Q} = 2.530 \pm 0.030$ MeV [14]. If we take this value for $m_{Q}$, we have $\frac{1}{2} (m_{L} + 2m_{Q}) = 2.50 \pm 0.2$ GeV. The result is in reasonable agreement with the theoretical value in Eq. (23). The second quantity in Eq. (23) can be checked by the experiment in the near future.

To conclude, we have calculated the $1/m_Q$ corrections to the heavy baryon masses from the QCD sum rules within the framework of the HQET. This study refines the leading order analysis [7]. Furthermore, within this framework, we can study the three-point correlators which will give the form factors for the weak transitions of the heavy baryons $B_J$ to the order of $1/m_Q$. It is also visible to include the QCD radiative corrections in the leading order and subleading order calculations. Both of these two aspects are under our studying.

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References


Figure captions

Fig. 1. The subleading operator insertions relevant to our analysis.

Fig. 2. Sum rules for $\Delta M$ with (a) $\hat{\lambda}_f$ and (b) $\hat{\lambda}_g$. The sum rule window is $T = 0.4 - 0.6$ GeV.

Fig. 3. Sum rules for $\Delta M$ with (a) $\hat{\lambda}_f$ and (b) $\hat{\lambda}_g$. The sum rule window is $T = 0.4 - 0.6$ GeV.

Fig. 4. Sum rule for $\Delta M$. The sum rule window is $T = 0.4 - 0.6$ GeV.
Table 1. Numerical results for \( \alpha \Lambda \) and \( \rho \).

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<thead>
<tr>
<th>( \alpha \Lambda_0 (\text{GeV}^2) )</th>
<th>( \alpha \Lambda_0^D (\text{GeV}^2) )</th>
<th>( \alpha \Lambda_0^P (\text{GeV}^2) )</th>
<th>( \rho_1 (10^{-11} \text{GeV}^3) )</th>
<th>( \rho_1^D (10^{-11} \text{GeV}^3) )</th>
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<td>( 0.09 \pm 0.02 ) &amp; ( 0.22 \pm 0.02 ) &amp; ( 0.03 \pm 0.02 ) &amp; ( -0.30 \pm 0.05 ) &amp; ( 0.7 \pm 0.2 ) &amp; ( -0.1 \pm 0.1 )</td>
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<td>( 0.09 \pm 0.03 ) &amp; ( 0.21 \pm 0.02 ) &amp; ( 0.03 \pm 0.02 ) &amp; ( -0.3 \pm 0.1 ) &amp; ( 0.8 \pm 0.1 ) &amp; ( -0.1 \pm 0.1 )</td>
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