Resummation Effects at Small $x^1$

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Abstract

The motivation is recalled for investigating higher-order terms in QCD perturbation theory at small $x$, and the results of recent resummed calculations for structure functions are discussed.


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At present and future high-energy hadron colliders, a large set of data receive contribution from events at small \( x \), that is, events in which the momentum transferred in the hard scattering of partons is much greater than \( \Lambda_{QCD} \) but much smaller than the total energy available to the system. Reliable predictions for these processes are therefore necessary, for both carrying out tests of QCD and estimating the background to searches for new physics.

The evaluation of these processes is complicated by the presence of potentially large corrections to all orders in perturbation theory, associated with multiple emission of gluons over a large rapidity interval, disordered in transverse momentum and polarized with eikonal helicities. For inclusive variables, the resummation of the leading logarithmic accuracy is accomplished by the BFKL equation. The coupling of the BFKL Green's functions to physical hard-scattering processes, such as deep inelastic lepton scattering or large-\( p_T \) events in hadron-hadron collisions, is determined by the high-energy factorization. This provides a method to identify systematically classes of logarithmic contributions, and match them with finite-\( z \) non-logarithmic corrections in perturbation theory.

In this framework, a resummed calculation of the deep inelastic structure function \( F_2(x, Q^2) \) has been recently performed by Ellis, Webber and the author. In addition to the full one-loop and two-loop contributions to \( F_2 \), this calculation resums to all orders in \( \alpha_s \), the small-\( z \) leading logarithms, and a class of the next-to-leading logarithms, namely the ones associated with the contribution of quark operators. Results for \( F_2 \) are then obtained by assuming a set of input parton distributions at some low mass scale.

The outcome of this calculation indicates that the size of resummation effects varies depending on the shape of the parton distributions assumed as inputs. The flatter the input distributions at small \( x \), the bigger the impact of resummation on their evolution. Fig. 1 shows the \( Q^2 \)-dependence of \( F_2 \) at small values of \( x \) according to resummed (leading, \( L(x) \), and next-to-leading, \( NLQ(x) \)) and fixed-order (1-loop and 2-loop) theory in the case of flat distributions MRSD0' at \( Q_0^2 = 4 \) GeV\(^2\). One can see that in this case corrections from higher-order logarithmic terms are rather large. It is also worth noting that the most important contribution to these corrections does not come from the leading resummation but from the next-to-leading one. This is because \( F_2 \) couples directly to quarks (and not gluons), and it is only at the next-to-leading level that small-\( x \) contributions to the quark channels set in.

![Figure 1. The structure function \( F_2 \) at small \( x \).](image)

Measurements of \( F_2 \) in the small-\( x \) range are currently being carried out at the HERA ep collider, and the 1993 data of the Zeus collaboration are plotted in Fig. 1. Comparison with the theoretical curves suggests...
that the enhancement effects encompassed by perturbative resummation could provide a mechanism to account for the scaling violation observed at HERA, without need for a very steep input gluon distribution or a very low input mass scale. This can be contrasted with standard analyses based on fixed-order perturbation theory, in which the observed behaviour of $F_2$ at small $x$ is traced back primarily to properties of the non-perturbative input, that is, either a steep input gluon or a low input scale. Since $F_2$ is not very sensitive to the leading BFKL dynamics, it is rather difficult to discriminate between these different possible explanations of the small-$x$ behaviour on the basis of data on $F_2$ only. From this point of view, investigations of other observables could be useful, both at the inclusive level and at the level of the distributions associated to the final states.

A qualitative estimate of the order of magnitude of the resummation effects one may expect in the longitudinal structure function $F_L$ can be obtained using the results in Ref. 4 for the corresponding coefficient function. One may consider the relationship between the moments of the resummed coefficient functions for $\partial F_2/\partial \ln Q^2$ and $F_L$. The ratio $R$ between these two quantities is plotted in Fig. 2 as a function of the parameter $\gamma$, which is the anomalous dimension determined by the BFKL equation, and increases up to the saturation point $1/2$ for asymptotically small values of $x$. Fig. 2 shows that this ratio is about 3 at $\gamma = 1/2$. This leads one to expect the order of magnitude of the resummation effects in $F_L$ to be about 30% of those in $F_2$. Also, one may note that, whilst the Born approximation to this curve is far away from the resummed value around $\gamma = 1/2$, the first-order approximation is rather close, suggesting that an evaluation at fixed perturbative level may do pretty well in this case.

In conclusion, one should observe that, because corrections to $F_2$ from quark evolution are fairly large, it is likely that small-$x$ resummation may prove to be helpful in improving our understanding of the HERA data. To pursue this, it is important to enlarge the spectrum of resummed analyses to additional observables. At the same time, it is highly desirable to enhance the accuracy of the resummed theory by gaining a better control of the subleading terms, in the first place the next-to-leading corrections to the gluon channels.

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References

1. For a review see for instance A.H. Mueller, plenary talk at this Conference.