THE COMPENSATION OF THE EARTH FIELD IN THE EXPERIMENTAL HALL FOR THE STORAGE RING

1. In connection with the storage ring the problem arose of protecting the stored beam against the earth magnetic field, stray fields and variations thereof. The maximum tolerable value is $\Delta B = 30$ m G. This very low value indeed can be obtained in essentially two ways:

   a. the free sections outside the bending magnets and the quadrupole lenses will have to be screened by highly permeable materials at low field strengths.

   b. a compensation of the earth field by a suitable arrangement of Helmholtz coils to compensate for the vertical and horizontal components of the earth field.

      \[ B_v \sim 430 \text{ m G}, \ B_h \sim 180 \text{ m G} \] *

      We shall compare the merits and disadvantages of the two systems and include a computation of the homogeneity of a field created by the Helmholtz coils.

2. Screening of the vacuum chamber.

   The screening of the vacuum chamber must be so efficient as to attenuate the DC magnetic field and $\Delta B$ to the desired value of $30$ m G. AC fields will be sufficiently weakened if we calculate the attenuation needed for DC fields.

   If we consider the vacuum chamber a cylinder, we can express the attenuation of the magnetic field $H_0$ existing outside the vacuum chamber by the following equation [1]:

* In Adam's Hall $B_v \sim 190$ m G. This low value is probably due to iron structures around the measuring place.

\[ \eta = \frac{H_i}{H_0} \approx \frac{4R^2}{\mu_A (R^2 - r^2)} \]

\[ \mu_A \left[ \text{G/Oe} \right] \]

\[ R \ [\text{cm}] = \text{external radius of cylinder} \]
\[ r \ [\text{cm}] = \text{internal radius of cylinder} \]
\[ d = R - r = \text{wall thickness} \]

For \( \mu_A \gg 1 \)

Requiring \( \eta \approx \frac{H_i}{H_0} = \frac{30}{430} \approx 1/15 \) we can easily calculate \( d \) for various kinds of screening materials.

With a mean radius of \( r = 5 \text{ cm} \) for the vacuum chamber we find the wall thickness \( d \) of the screening cylinder.

\[
\text{attenuation factor } \eta = \frac{H_i}{H_0} \quad \text{material} \quad \text{permeability} \quad \text{wall thickness } d \ [\text{cm}]
\]

\[
\{ \begin{aligned}
\mu \text{-metal (has to be heat treated after machining)} &\approx 20000 & 0.075 \\
\text{ordinary soft iron (can be easily machined, no special heat treatment required)} &\approx 200 & 7.5 \\
\text{cobalt soft iron (can be easily machined, no special heat treatment required)} &\approx 4000 & 0.38
\end{aligned} \]

According to the permeability the thickness \( d \) varies within wide limits and determines the practical solution of the screen. In the case of \( \mu \)-metal the screening could be efficient by using a \( \mu \)-metal foil 7.5 mm thick which could be simply wrapped around the vacuum chamber. However, since \( \mu \)-metal has to be heat-treated after having been exposed to mechanical stress to regain its properties this application could only be used if a material could be found that keeps its high permeability even after mechanical stress has been applied. There is hope that an American firm could supply us with such material.
A use of ordinary soft iron could be envisaged too, although the screening becomes more complicated since, due to its thickness the screening would have to be made in form of boxes that are put around the vacuum chamber in sections between bending magnets and lenses; and at the same time the screening should not make access to the vacuum chamber difficult.

3. Compensation of earth field.

In connection with the calibration of the measuring coil for the quadrupole lenses the necessity arose to compensate the earth field (vertical component) before the calibration field be applied. This solution stimulated the idea of examining the possibility to use a set of Helmholtz coils to compensate the vertical and horizontal component of the earth field. For a serious application it would be interesting to study the homogeneity of a field created by Helmholtz coils as well as the power that would be needed. Actual values of the magnetic field in the new Experimental Hall would have to be measured as soon as the building is ready.

3.1. Theory of the Helmholtz coil.

A short summary is given on the magnetic field created by a simple loop.

The vector potential can be written in the general form.

\[ \mathcal{A} = \frac{1}{4\pi} \int \mathbf{g} \cdot \frac{dv}{r} \]

where \( g \) = current density \hspace{1cm} (1)

\[ \mathcal{Q} = \frac{I}{4\pi} \oint \frac{dl}{r} \]

\[ g = I.F.d\zeta \] \hspace{1cm} (2.1)

the induction is then given by

\[ \mathcal{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\zeta}{r} \]

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\[ r = \sqrt{a^2 + x^2 + z^2 - 2ax \cos \phi} \]

For symmetry reasons the magnitude of \( \mathcal{L} \) does not depend on \( \phi \). Therefore it is simpler to chose \( P \) where \( y = 0 \).

\[ ds_\phi = a \cdot d\phi \cdot \cos \phi \]

Then \( \mathcal{L} \) becomes

\[ \mathcal{L}_\phi = \frac{\mu I}{4\pi} \ \text{ycot} \int_0^\pi \frac{a \cos \phi \ d\phi}{\sqrt{a^2 + x^2 + z^2 - 2ax \cos \phi}} \] \quad (4)

After several substitutions \([2]\) using \( \phi = \pi + 2\theta \), \( \cos \phi = 2 \sin^2 \theta - 1 \)

\[ d\phi = 2d\theta \]

and introducing \( k^2 = 4ax \left[ (a+x)^2 + z^2 \right] - 1 \)

we find

\[ B_x = \frac{\mu I}{2\pi} \cdot \frac{x}{x \sqrt{(a+x)^2 + z^2}} \left[ \frac{1}{K + \frac{a^2 + x^2 + z^2}{(a-x)^2 + z^2}} \right] \] \quad (6)

\[ B_z = \frac{\mu I}{2\pi} \cdot \frac{1}{\sqrt{(a+x)^2 + z^2}} \left[ K + \frac{a^2 - x^2 - z^2}{(a-x)^2 + z^2} \right] \] \quad (7)

\( K \) and \( E \) stand for the complete elliptic integrals of the first and second kind, respectively.

\[ K = \frac{\pi}{2} \left[ 1 + \frac{1}{2}k^2 + \frac{3}{8}k^4 + \cdots \right] \quad \text{for } k^2 < 1 \]

\[ E = \frac{\pi}{2} \left[ 1 - \frac{1}{2}k^2 + \frac{3}{8}k^4 - \cdots \right] \]

Normalizing the $B_z$ and $B_x$ components by setting

$$Z = z/a, \ X = x/a, \ k^2 = \frac{4x}{(1 + x)^2 + z^2}$$

$$B_x = \frac{\mu I}{2\pi a} \cdot \frac{Z}{X\sqrt{(1 + X)^2 + Z^2}} \left[ -K + \frac{1 + X^2 + Z^2}{(1 - X)^2 + Z^2} E_1 \right]$$

(8)

$$B_z = \frac{\mu I}{2\pi a} \cdot \frac{1}{(1 + X)^2 + Z^2} \left[ K + \frac{1 + X^2 - Z^2}{(1 - X)^2 + Z^2} E_2 \right]$$

(9)

Helmholtz's idea is to place two loops at a distance that is equal to their radius and we find the field due to these loops by superposing the field components of 2 loops whose centres are situated at

$$Z = \pm 0.5, \ X = 0$$

$$B_{x,t} = \frac{\mu I}{2\pi a} \left\{ \frac{Z - 0.5}{X\sqrt{(1 + X)^2 + (Z + 0.5)^2}} \left[ -K_1 + \frac{1 + X^2 + (Z - 0.5)^2}{(1 - X)^2 + Z^2} E_1 \right] \right. \right.$$

$$+ \frac{Z + 0.5}{X\sqrt{(1 + X)^2 + (Z + 0.5)^2}} \left[ K_2 + \frac{1 + X^2 + (Z + 0.5)^2}{(1 - X)^2 + Z^2} E_2 \right] \right\}$$

(10)

$$B_{z,t} = \frac{\mu I}{2\pi a} \left\{ \frac{1}{\sqrt{(1 + X)^2 + (Z - 0.5)^2}} \left[ K_1 + \frac{1 - X^2 - Z^2}{(1 - X)^2 + Z^2} E_1 \right] \right. \right.$$

$$+ \frac{1}{\sqrt{(1 + X)^2 + (Z + 0.5)^2}} \left[ K_2 + \frac{1 - X^2 - Z^2}{(1 - X)^2 + Z^2} E_2 \right] \right\}$$

(11)

3.2. Dimensioning the Helmholtz coil.

3.2.1. Vertical Component of Earth Field.

If we assume the radius of the machine to be $r = 4$ m and we would choose $\left[ ^1 \right]$ the radius $a$ of the Helmholtz coil $a = 8$ m and the distance $A$ between

FS/2265 $\left[ ^1 \right]$ this depends on space available in the hall.
the two loops, \( \Delta = 8 \, \text{m} \left[ \pm 4 \, \text{m from the centre plane of the vacuum chamber.} \right] \)

These parameters give us

\[ X = \frac{4}{5} = 0.5 \quad \text{and} \quad Z = 0 \]

and we can find the vertical component \( B_z \) along the vacuum chamber.

\[ \frac{B_z}{\mu_1/2\pi a} = 4345 \quad \frac{B_x}{\mu_1/2\pi a} = 0 \]

\[ \begin{cases} X = 0.5 \\ Z = 0 \end{cases} \]

Under the assumption of \( B_{v\text{earth}} \approx 200 \, \text{m G} \) we easily find the current \( I_v \) necessary to compensate \( B_{vE} \).

\[ I_v \approx 188 \, \text{A}. \]

We can approximate the one-turn-loop by a bundle of wires and taking 50 t of copper wire (1 mm cross-section) the required current is

\[ I_v = 3.76 \, \text{A}. \]

The power consumption of 2 coils can be expressed in terms of number of turns \( W \) per coil:

\[ N = I^2 \cdot R = 2 \frac{K^2}{W} \cdot \varphi \cdot \frac{1}{F} \]

\[ K = \text{Amp-turns} \]

\[ W = \text{number of turns per coil} \]

\[ N \approx 1.3 \, \text{KW} \ (1 \, \text{mm}^2 \, \text{wire}). \]

\[ l = \text{total length} = 2\pi a \]

\[ F = \text{cross-section of wire} \]

\[ \varphi = \text{specific resistance} \]

3.2.2. Horizontal Components of Earth Field.

In order to compensate \( H_D \), the arrangement shown in fig. 4 would have to be chosen. The plane of the Helmholtz coils should be
perpendicular to \(B_E\). Then we must investigate how the field varies along the circumference of the vacuum chamber. For this we chose 5 points lying in one quadrant and determine the \(B_z\) and \(B_x\) for these points, which will give a picture of the field constellation. The curves showing \(B_z\) and \(B_x\) as functions of \(P_1\) to \(P_5\) are represented in fig. 3 and 4.

We shall first examine the variation of \(B_z\) which must compensate \(B_E\). We shall chose \(I_H\) such as to compensate \(B_E\) exactly for the point \(P_0\). This means that \(B_z > B_E\) and \(B_z < B_E\). We shall calculate the deviations \(\Delta B_z\) from \(B_E\) in order to find out if \(\Delta B_z\) is an admissible error (error \(\leq 30 \text{ m G}\)). See fig. 4.

With \(B_E = 160 \text{ m G}\) (exact values will have to be determined in the new experimental hall) we find for \(B_z\) the current \(I_H = 145 \text{ A}\).

Then \(\Delta B_z = 7.2 \text{ m G}\).

This means that around the storage ring the maximum horizontal component is

\[
\Delta B_z = 7.2 \text{ m G}
\]

This value is well within the desired tolerance of 30 m G.

It remains to examine the variation and the absolute value of \(B_x\).

The absolute value of \(B_x = 5.8 \text{ m G} / \text{ for } I = 145 \text{ A}\).

Since \(P_0\) falls in the region of maximum amplitude of \(B_x\) the total variation \(\Delta B_x = B_x \text{ for } P_0 \approx 5.8 \text{ m G}\). [\(\pm 3.4\)]

For convenience the coil arrangement for the horizontal component can be chosen similarly to the one for the vertical component. This implies a total power consumption for the earth field compensation of

\[N \approx 2.6 \text{ KW}\]
3.3. Conclusions.

From our calculations we understand that a compensation of the earth field well below the given admissible value is perfectly feasible. The system has the following advantages: It can be easily adapted to the machine and does not intervene with the accessibility of the machine. It needs no specially stabilised power supply since $B \sim I$ and being for $B$ well within the tolerance we can easily allow up to 10% stability for the current supply.

The only disadvantages of the system are: We assume a homogeneous and time-stable field distribution in the hall. Only actual measurements of the field in the new hall could give a clear answer to this. And finally the space requirements of the Helmholtz coil are not modest and it remains to check if enough space can be found to accommodate the coils.

F. A. Ferger

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