DESIGN OF SOME EXPERIMENTS WITH PROTON STORAGE RINGS

B. de Raad

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1. Introduction

The purpose of this report is to consider a few simple storage ring experiments in somewhat more detail than has been done previously, especially regarding the background due to residual gas.

We shall base ourselves on a design for concentric storage rings (CSR), with separate focusing and bending magnets that has been discussed in a previous report\(^1\). The interaction region with adjacent CSR magnets is shown in Fig. 1. The beam characteristics in the interaction region are as follows:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton energy</td>
<td>25 GeV</td>
</tr>
<tr>
<td>Beam width</td>
<td>2 cm</td>
</tr>
<tr>
<td>Beam height</td>
<td>1 cm</td>
</tr>
<tr>
<td>Total energy spread</td>
<td>2.5 o/o</td>
</tr>
<tr>
<td>Intersection angle</td>
<td>15°</td>
</tr>
<tr>
<td>Circulating beam current</td>
<td>20 A</td>
</tr>
<tr>
<td>Interaction rate ((\sigma^* = 40) mb)</td>
<td>(1.6 \times 10^5) events/sec.</td>
</tr>
</tbody>
</table>

The quadrupoles \(Q_1, Q_2\) etc. have an open median plane as shown in Fig. 2. To obtain a small beam width we have assumed the use of a Terwilliger\(^2\) or Hereward\(^3\) scheme for superposing the closed orbits of different momenta in the interaction region. The total number of stacked protons per CSR is \(N_t = 3.6 \times 10^{14}\), which corresponds to stacking 600 beam pulses from the CPS with an intensity of \(6 \times 10^{11}\) protons/pulse. The radius \(R\) of the CSR is 135 m.

2. Characteristics of colliding beam interactions

Predictions of the angular distribution of elastically scattered protons in 25 GeV colliding beam reactions, based on extrapolations of measurements
at the CPS, have been made by Taylor. In Table 1 we have reproduced his figures in a form which is convenient for our purposes. The first column gives the scattering angle $\theta$ and the second column gives the number of elastically scattered protons $\frac{dN}{d\omega}$ per steradian per colliding beam event. Since $\frac{dN}{d\omega}$ changes so rapidly with $\theta$ the angular resolution of a scattering experiment should not be worse than 1 mrad. The third column therefore shows the total number of protons per colliding beam event that are elastically scattered into a solid angle $2\pi \sin^2 \theta$ with $d\theta = 1$ mrad. The latter figure is a measure for the maximum obtainable counting rate in an experiment. Since the CSR magnets severely restrict the accessible solid angles the counting rate in an actual experiment will in general be an order of magnitude lower. The largest angle at which useful counting rates can be obtained is between 40 mrad and 50 mrad.

**Table 1**

<table>
<thead>
<tr>
<th>$\theta$ (mrad)</th>
<th>$\frac{dN}{d\omega}$ (per ster. per colliding beam event)</th>
<th>$\frac{dN}{d\omega} \times 2\pi \sin^2 \theta \times 10^{-3}$ (per colliding beam event)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>400</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>375</td>
<td>$1.7 \times 10^{-2}$</td>
</tr>
<tr>
<td>10</td>
<td>275</td>
<td>$9 \times 10^{-3}$</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>$3 \times 10^{-4}$</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>$6 \times 10^{-5}$</td>
</tr>
<tr>
<td>30</td>
<td>0.3</td>
<td>$5 \times 10^{-8}$</td>
</tr>
<tr>
<td>40</td>
<td>$2 \times 10^{-4}$</td>
<td>$6 \times 10^{-11}$</td>
</tr>
<tr>
<td>50</td>
<td>$2 \times 10^{-7}$</td>
<td>$4 \times 10^{-12}$</td>
</tr>
<tr>
<td>60</td>
<td>$1 \times 10^{-8}$</td>
<td>$4 \times 10^{-13}$</td>
</tr>
<tr>
<td>70</td>
<td>$1 \times 10^{-9}$</td>
<td>$5 \times 10^{-14}$</td>
</tr>
<tr>
<td>80</td>
<td>$1 \times 10^{-10}$</td>
<td></td>
</tr>
</tbody>
</table>
Angular distributions for π-meson production in the cms for 29.5 GeV stationary target experiments have been given by Cool 5) and turn out to be very anisotropic. Table 2 gives the approximate ratio between the production cross-section at θ = 0° and θ = 90° estimated by extrapolating Cool's graph down to θ = 0°.

**Table 2**

<table>
<thead>
<tr>
<th>p_π (GeV/c)</th>
<th>( \frac{dσ}{dw} (0°) / \frac{dσ}{dw} (90°) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>10^2</td>
</tr>
<tr>
<td>1.40</td>
<td>3 x 10^2</td>
</tr>
<tr>
<td>1.75</td>
<td>10^3</td>
</tr>
</tbody>
</table>

The total kinetic energy in the cms in Cool's case is 5.7 GeV whereas in our case it is 48 GeV. It is well established that the average transverse momentum of secondaries from high energy collisions is about 0.45 GeV/c, independent of the primary energy. Since the average longitudinal momentum in the cms increases roughly as \( E^{1/2} \), where \( E \) is the kinetic energy in the cms of the colliding protons, the angular distributions of π-mesons in our case will be even more anisotropic than those given by Cool.

The energy required to create a proton-antiproton pair in Cool's case is \( 1/3 \) of the total available cms kinetic energy. Nevertheless the transverse momentum of the antiprotons is not appreciably different from that of the π's. If new heavy particles would be produced in the CSR there is a good chance that this production will also occur at rather small angles.

Cocconi et al. 6) have proposed that the distribution of the transverse momentum \( p_\perp \) of secondaries is independent of the longitudinal momentum distribution and is given by
\[ \sigma(p_\perp) \, dp_\perp = \frac{p_\perp}{p_0^2} \exp\left(-\frac{p_\perp}{p_0}\right) \, dp_\perp \tag{1} \]

where \(2p_0\) is the average transverse momentum, that we shall take as 0.45 GeV/c. Using this formula we find that half of the secondaries are produced with \(p_\perp < 1.68 \, p_0 = 0.38 \text{ GeV/c}\) so that their production angle is smaller than \(0.38/p\), where \(p\) is the total momentum in GeV/c (which is about equal to the longitudinal momentum). For momenta of a few GeV these angles are considerably larger than for elastic p-p scattering but they still fall in a range where serious interference with the CSR magnets occurs.

3. Background due to residual gas.

We shall assume that the residual gas in the CSR vacuum chamber is \(N_2\) at a pressure of \(10^{-10} \text{ mm Hg}\). This pressure is within the limits of present ultra high vacuum technique\(^7\), but little is known about the composition of the residual gas. With cryogenic pumping the pressure could be made even lower and the residual gas would probably be mainly \(H_2\). Therefore the assumptions made above are probably on the conservative side.

The total cross-section of 25 GeV protons on nitrogen nuclei\(^8\) is about 380 mb. With a current of 20 A the number of beam-gas interactions is then \(3.4 \times 10^2\) per cm of vacuum chamber per sec.

In the following we shall be especially concerned with elastic and nearly elastic p-p scattering. The two questions about background are then the following:

1) How many elastic beam-gas scatterings look like elastic beam-beam scatterings?

2) What is the total counting rate, irrespective of particle type or momentum in a counter placed close to the CSR vacuum chamber?
To answer 1) we assume that the ratio between elastic and total cross-section and the angular distribution of the elastically scattered protons from nitrogen nuclei is the same as from free target protons. Using Taylor's data 4) we have calculated the number of protons per beam-gas event per ster. that are elastically scattered at an angle \( \theta \). The result is shown in the second column of Table 3. The third column of this table gives the momentum difference \( \Delta P \) between incident and scattered proton. Up to 40 mrad \( \Delta P \) is smaller than the momentum spread of the circulating beam so that one cannot distinguish between elastic beam-beam and elastic beam-gas scattering by means of momentum analysis. As discussed in section 5 this difficulty can be overcome by detecting the two elastically scattered protons from the colliding beam events in coincidence.

**TABLE 3**

<table>
<thead>
<tr>
<th>( \theta ) (mrad)</th>
<th>( \frac{dN}{d\omega} ) (per ster per beam-gas event)</th>
<th>( \Delta P ) (GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>375</td>
<td>0.008</td>
</tr>
<tr>
<td>10</td>
<td>325</td>
<td>0.032</td>
</tr>
<tr>
<td>15</td>
<td>150</td>
<td>0.075</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
<td>0.134</td>
</tr>
<tr>
<td>25</td>
<td>13</td>
<td>0.208</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>0.30</td>
</tr>
<tr>
<td>40</td>
<td>0.16</td>
<td>0.53</td>
</tr>
<tr>
<td>50</td>
<td>8 \times 10^{-3}</td>
<td>0.83</td>
</tr>
<tr>
<td>60</td>
<td>2 \times 10^{-3}</td>
<td>1.20</td>
</tr>
<tr>
<td>70</td>
<td>6 \times 10^{-4}</td>
<td>1.63</td>
</tr>
<tr>
<td>80</td>
<td>2 \times 10^{-4}</td>
<td>2.13</td>
</tr>
</tbody>
</table>
The flux of elastically scattered protons in a counter telescope with angular acceptance $\omega$, looking at an angle $\vartheta$ at the beam is approximately $\frac{6.8 \times 10^2 \omega}{\vartheta} \times \frac{dN}{dw}$.

To answer 2), we shall make a number of simplifying assumptions. The elastically scattered protons hit the vacuum chamber at such a small angle that their average path in it is close to 1 mmfp. We shall therefore assume that all beam-gas interactions are inelastic and concentrated in a line source with $q$ interactions per unit length per sec located in the centre of the vacuum chamber. The influence of the CSR magnetic field is neglected. Let us call $\left( \frac{dN}{dw} \right)_t$ the total number of secondaries produced per ster. per beam-gas interaction at an angle $\vartheta$. One can readily show, that the total flux in an annular counter of width $w$, concentric with the vacuum chamber is

$$\phi = 2 \pi w Q \left( \int_0^{\pi/2} \frac{dN}{dw} \right)_t \cos \vartheta d\vartheta$$  \hspace{1cm} (2)

independent of its radius. The main contribution to the integral occurs at small $\vartheta$, where $\cos \vartheta \approx 1$ so that a good approximation is

$$\phi = 2 \pi w Q \left( \int_0^{\pi/2} \frac{dN}{dw} \right)_t d\vartheta$$ \hspace{1cm} (3)

The total number of particles produced at an angle $\vartheta$ is

$$2 \pi \sin \vartheta \left( \frac{dN}{dw} \right)_t d\vartheta$$

so that the efficiency of particles in contributing to the background flux is inversely proportional to their production angle.

Reasonably complete angular distributions$^5,9$ have only been measured for $\pi^-$. Using these and Cocconi's universal curves$^{10}$, we have integrated the $\pi^-$ momentum spectra at different angles. Fig. 3 shows a graph of $\left( \frac{dN}{dw} \right)_{\pi^-}$. Integrating this curve gives
\[ \int_{0}^{\pi/2} \frac{dN}{dw} \pi^{-} \ d\Theta = 2.23 \] (4)

so that the total \( \pi^- \) flux in an annular counter with \( w = 1 \) cm becomes

\[ \rho^- = 5 \times 10^3/\text{sec} \] (5)

The total number of \( \pi^+ \) and \( \pi^0 \) is the same as that of the \( \pi^- \). Moreover, the \( \pi^0 \) decays into 2 \( \gamma \)'s which have a good chance of being converted in the vacuum chamber wall. We shall therefore take \( \rho_+ = \rho^- \) and \( \rho_0 = 4 \rho^- \). Inspection of the angular distribution of secondary protons shows that their average energy is much higher than that of the \( \pi^- \) but that their total number is roughly equal to that of the \( \pi^- \) at most angles. We therefore take \( \rho_p = \rho^- \). Adding up we find \( \rho = 3.5 \times 10^4/\text{sec} \) for \( w = 1 \) cm.

The CSR magnets should have rather little influence on the numbers calculated above. Their main effect is to create an extra flux of degraded nuclear particles, low energy electrons and \( \gamma \)'s. The degraded particles are more isotropic and therefore contribute less to the background flux than the first generation secondaries considered above. Moreover, shielding with some suitably placed lead bricks may eliminate a large fraction of the degraded particles. A reliable calculation is therefore very difficult and we shall quite arbitrarily multiply the flux calculated above by a factor 2. The total flux in a 1 cm wide annular counter is then

\[ \rho_{\text{tot}} = 7 \times 10^4/\text{sec} \] (6)

and we shall use this number in the following to estimate background fluxes.
4. **Measurement of the total p-p cross-section**

The number of colliding beam events per sec. is

\[ N_{bb} = \int_{-\frac{1}{2}h}^{\frac{1}{2}h} \frac{N_{t_1}(z) N_{t_2}(z)}{(2\pi R)^2} \frac{c\sigma}{\tan \frac{\alpha}{2}} \, dz \quad (7) \]

where \( N_{t_1}(z) \, dz \) and \( N_{t_2}(z) \, dz \) are the total number of protons between \( z \) and \( z+dz \) independent of their radial position in the two CSR, \( h = \) beam height, \( \alpha = \) crossing angle of beams, \( c = \) velocity of light and \( \sigma' = \) total p-p cross-section.

For simplicity we shall here and in the following sections assume that the vacuum chamber is a 5 cm diameter thin walled pipe, although in reality it will have a complicated shape with special exit windows to minimize absorption and scattering.

A possible counter setup to measure \( N_{bb} \) is shown in Fig. 4. \( S_1, S_2, S_1', S_2' \) are 80 cm diameter counters placed at 0.6 and 6 m from the interaction region. The vacuum chamber passes through holes in the counters. Secondaries from colliding beam events which pass through the hole in \( S_1 \) will be counted by \( S_2 \) unless their emission angle \( \Theta \) is smaller than 5 mrad. The same holds for \( S_1' \) and \( S_2' \). The outputs of \( S_1 \) and \( S_2 \) and of \( S_1' \) and \( S_2' \) are in parallel. A coincidence between \( (S_1 + S_2) \) and \( (S_1' + S_2') \) is considered as a colliding beam event.

Elastic scattering mainly occurs at very small angles and has nearly constant differential cross-section for \( \Theta < 5 \) mrad. By making measurements with different holes in the counters one can separate the inelastic and elastic cross-section and extrapolate down to \( \Theta = 0 \).

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In the layout described above the main source of background are secondaries from beam-gas reactions, which pass through $S_1$ and $S'_1$. To eliminate these it is necessary to use directional Cerenkov counters, arranged in such a way that $S_1$ and $S_2$ count only particles travelling from left to right and $S'_1$ and $S'_2$ only particles travelling in the opposite direction.

Using the $\phi_{\text{tot}}$ calculated in the preceding section we find that the particle flux in $S_1 + S_2$ is $5.6 \times 10^6$/sec. However, the counting rate is much smaller, since practically all secondaries from each beam-gas event are relativistic and will arrive at the same time in the counter, giving rise to one large pulse. We shall therefore take the counting rate in $S_1 + S_2$ as equal to the total number of beam-gas events in 10 m of vacuum chamber. The average velocity of light in a plastic scintillator, due to its index of refraction and increase of the light path by reflections is about $\frac{1}{3}$ of $c$. In view of the size of the counters we shall assume a time resolution $\tau = 5 \times 10^{-9}$. The accidental coincidence counting rate $(S_1 + S_2) + (S'_1 + S'_2)$ is then

\[
2\tau N (S_1 + S_2) N (S'_1 + S'_2) = 10^3/\text{sec}
\]

which is only $6 \times 10^{-3}$ times the colliding beam event rate. An independent measurement of the secondaries can be made, of course, by shifting the relative delays of $(S_1 + S_2)$ and $(S'_1 + S'_2)$.

The absolute counting rates of several times $10^5$/sec are high but well within the possibilities of fast electronics, especially since the duty cycle is 100 o/o. On the other hand the ratio signal to accidental coincidences is independent of the number of stacked protons, since both counting rates are proportional to the square of the circulating beam current. For total cross-section measurements a beam current of 1 A would be entirely sufficient.
The simplest and most accurate method to measure $N_{t_1}(z)$ and $N_{t_2}(z)$ is to insert a thick target from above or below into the beam and to measure the surviving circulating beam current as a function of target position. This is a destructive measurement that can only be made after the counting run has been completed. Since the whole experiment could be done in a few hours, during which beam blow up due to gas scattering or other undesirable effects can certainly be kept very small, this is not a serious restriction.

5. **Elastic p-p scattering.**

Orbits of elastically scattered protons are shown in Fig. 5. This drawing assumes that the gaps of the CSR bending magnets $B_1$ and $B_1'$ (see Fig. 1) have been enlarged so that the scattered protons pass through a homogeneous field and are deflected in the same way as the circulating beam. The vacuum chamber center line has been drawn as a straight line. The stray field of the quadrupoles was taken from measurements on an analogue model but could, of course, be modified by shimming to suit a specific experiment. We shall now describe in some detail, how the differential cross-section for elastic p-p scattering at 30 mrad could be measured. This angle is representative for the range from 15 mrad to 40 mrad. Afterwards we shall make a few comments on scattering measurements between 5 mrad and 15 mrad.

It is not sufficient to observe only one of the scattered protons and to extrapolate back its trajectory to see if it came from the interaction region. With a beam width of 2 cm there are $2/0.030 = 67$ cm of beam that can give elastic beam-gas scatterings with the orbit of the scattered proton (or its prolongation) passing through the interaction region. Comparison of Tables 1 and 3 shows that $\sqrt{= 30}$ mrad, $\frac{dN}{d\omega}$ for elastic beam-gas scattering is 10 times larger than $\frac{dN}{d\omega}$ for elastic beam-beam scattering so that the ratio signal to background would be about one to one. Therefore it is necessary to detect the two elastically scattered protons in coincidence. Momentum analysis is also necessary in order to discriminate against
inelastic pp scattering or the production of two \( \pi \)'s in opposite directions.

The elastically scattered protons pass so close to the CSR magnets that it is practically impossible to analyse them with separate magnets. The natural solution is to extend \( B_1 \) and \( B'_1 \) radially and to use their field for momentum analysis. Fig. 5 shows, that all scattering angles up to 50 mrad, beyond which the counting rate is too low anyhow, can be covered by extending the homogeneous field region of \( B_1 \) up to \( \Delta r = +60 \text{ cm} \) and that of \( B'_1 \) up to \( \Delta r = -60 \text{ cm} \).

The maximum angles of proton orbits in the circulating beam, due to horizontal betatron oscillations are \( \pm 0.6 \text{ mrad} \). Suppose that the scattered proton trajectory is defined by two spark chambers, 4 m apart and with a \( \pm 0.5 \text{ mm} \) accuracy in spark location. The angular definition of these trajectories is then \( \pm 0.25 \text{ mrad} \). It looks reasonable to accept scattered protons over an angular interval comparable to the experimental resolution. Let us assume therefore that we want to count all protons which are scattered in the angular interval \( 29 \text{ mrad} < \vartheta < 31 \text{ mrad} \). The vertical solid angle is limited by the poles of \( Q_1 \) and \( Q'_1 \) to about \( \pm 7 \text{ mrad} \). To accept these vertical angles the gap height of \( B_1 \) and \( B'_1 \) must be 20 cm.

The interaction region is a particle source of nonuniform density. In principle this could be corrected for but it is better to design the experiment in such a way, that the detection efficiency is 100 o/o for all protons scattered within the desired angular interval from any point in the interaction region. A possible layout for the experiment is then as follows. Spark chambers \( SC_1 \) to \( SC_4 \) detect all protons scattered within the desired angular interval and within the \( \pm 1.25 \text{ o/o} \) momentum spread of the circulating beam. The deflection in \( B_1 \) and \( B'_1 \) is 67 mrad so that the angular resolution of the spark chambers is sufficient for this purpose. \( S_2 \) and \( S_4 \) are triggering counters with equal area as the adjacent spark chambers. The proton going in the opposite direction is detected by a similar set of spark chambers \( SC'_1 \) to \( SC'_4 \) and counters \( S'_2 \) and \( S'_4 \). The spark
chambers are triggered by coincidences \( S_4 S_4 S'_4 S'_4 \). An event is accepted if a proton in the specified angular interval passing through \( SC_1 \) to \( SC_4 \) is accompanied by a proton in \( SC'_1 \) to \( SC'_4 \) going in the opposite direction within \( \pm 2 \) mrad. horizontally and \( \pm 1.5 \) mrad. vertically. In order to have a 100 o/o detection efficiency for the second proton the sizes of \( SC'_1 \) to \( SC'_4 \) must be about twice the sizes of \( SC_1 \) to \( SC_4 \). In choosing the dimensions and positions of \( SC_3 \) etc. the deflection and focusing effect of the CSR quadrupoles must be taken into account. The stray field of \( Q_1 \) and \( Q'_1 \) is horizontally defocusing with a gradient of 5CC gauss/cm at the position of the central trajectory of our setup. The total accepted solid angle is \( 2.8 \times 10^{-5} \) ster. and using Table 1 we find 1.3 good events/sec. The total triggering rate will be about twice as large due to elastically scattered protons which do not fall within the specified angular interval. An actual experiment could, of course, be made much more elaborate in order to collect data at many angles simultaneously, but we shall not discuss this here.

We see that the purpose of the spark chambers is to obtain good angular and momentum resolution over a "large" solid angle in a geometry over which the experimenter has little control. The analysis of the spark chamber data may appear a laborious undertaking. However, it looks feasible now to locate spark positions by means of acoustic pickups whose information can be recorded on magnetic tape in a form that can be used directly by a computer. The speed of recording could probably be several tens of events per sec. The computer must be supplied with an accurate map of the in general rather inhomogeneous fields of the CSR quadrupoles and bending magnets in order to work out the momentum. Since the geometry is simple the computer programme should be easy to make.

To obtain some idea about the background problem we shall calculate

a) The probability of finding a spurious track in one of the spark chambers when they are triggered.
b) The probability that the spark chambers are triggered on wrong events due to accidental coincidences in the counters.

The fluxes due to beam-gas events follow from eq. 6 and are shown in Table 4. For the spark chambers close to the interaction region we have also included an estimate of the particle fluxes due to colliding beam events.

<table>
<thead>
<tr>
<th>spark chamber</th>
<th>area in cm²</th>
<th>particle flux due to beam gas events per sec.</th>
<th>particle flux due to colliding beam events per sec.</th>
<th>total particle flux per sec.</th>
<th>spurious tracks in 0.2 μsec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC₁</td>
<td>9</td>
<td>$1.8 \times 10^4$</td>
<td>$2.4 \times 10^4$</td>
<td>$4.2 \times 10^4$</td>
<td>0.009</td>
</tr>
<tr>
<td>SC₂</td>
<td>35</td>
<td>$2.2 \times 10^4$</td>
<td>$0.6 \times 10^4$</td>
<td>$2.8 \times 10^4$</td>
<td>0.006</td>
</tr>
<tr>
<td>SC₃</td>
<td>125</td>
<td>$3.6 \times 10^4$</td>
<td>-</td>
<td>$3.6 \times 10^4$</td>
<td>0.007</td>
</tr>
<tr>
<td>SC₄</td>
<td>200</td>
<td>$4.5 \times 10^4$</td>
<td>-</td>
<td>$4.5 \times 10^4$</td>
<td>0.009</td>
</tr>
<tr>
<td>SC’₁</td>
<td>13</td>
<td>$2.6 \times 10^4$</td>
<td>$3.5 \times 10^4$</td>
<td>$6.1 \times 10^4$</td>
<td>0.012</td>
</tr>
<tr>
<td>SC’₂</td>
<td>73</td>
<td>$4.6 \times 10^4$</td>
<td>$1.2 \times 10^4$</td>
<td>$5.8 \times 10^4$</td>
<td>0.012</td>
</tr>
<tr>
<td>SC’₃</td>
<td>300</td>
<td>$8.8 \times 10^4$</td>
<td>-</td>
<td>$8.8 \times 10^4$</td>
<td>0.018</td>
</tr>
<tr>
<td>SC’₄</td>
<td>500</td>
<td>$11.6 \times 10^4$</td>
<td>-</td>
<td>$11.6 \times 10^4$</td>
<td>0.023</td>
</tr>
</tbody>
</table>

The last column of Table 4 shows the probability of a spurious track occurring in any of the spark chambers, assuming a 0.2 μsec sensitive time. The probability of observing a spurious track in all 8 spark chambers together is about 10 c/o. The proton trajectory is overdetermined by its horizontal and vertical coordinates in each of the 4 spark chambers, so that it will nearly always be possible to identify and reject the spurious track. Since each spark chamber would have, say 10 gaps with 0.02 mm

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TABLE 5

<table>
<thead>
<tr>
<th>Material Description</th>
<th>Scattering Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm beryllium vacuum chamber window</td>
<td>0.03 mrad</td>
</tr>
<tr>
<td>spark chamber, equivalent to 1 mm aluminum</td>
<td>0.06 &quot;</td>
</tr>
<tr>
<td>20 m air</td>
<td>0.16 &quot;</td>
</tr>
<tr>
<td>1 cm scintillator</td>
<td>0.09 &quot;</td>
</tr>
</tbody>
</table>

The air scattering could be reduced and the spark chamber end windows suppressed by enclosing the whole setup in a pipe filled with helium gas, that at the same time serves as spark chamber gas.

6. Inelastic p-p scattering

Experiments by Cocconi et al.\textsuperscript{13} at the CERN have shown that the energy spectrum of protons scattered at small angles has 2 bumps somewhat below the elastic peak. This is interpreted as the formation of two excited isobars via the reaction

\[ p + p \rightarrow p + \bar{p} \]

(8)

The masses of the excited isobars are \( W_1 = 1.51 \text{ GeV} \) and \( W_2 = 1.69 \text{ GeV} \). We shall now try to see how reaction (8), if it should occur, could be measured with colliding beams.

The momentum loss of the proton in this case is

\[ \Delta p = \left( \frac{W^2}{M^2} - 1 \right) \frac{N_0}{4\gamma} \]

(9)
where \( M \) = proton rest mass and \( \gamma M c^2 \) = total energy of the colliding protons. Substitution gives

\[
\Delta P_1 = 14 \text{ MeV/c for } W_1 \\
\Delta \beta_2 = 20 \text{ MeV/c for } W_2
\] (10)

These momentum differences are, of course, far too small to detect reaction (8) by momentum analysis of the scattered protons. The latter must therefore be detected in coincidence with at least one of the decay products of the \( W \). The decay modes of the \( W_1 \) and \( W_2 \) are not known. For simplicity we shall neglect three-body decays and assume that both \( W_1 \) and \( W_2 \) can decay according to

\[
W \rightarrow p + \pi^0 \\
W \rightarrow n + \pi^+
\] (11)

with equal probability.

If the total energy of the colliding protons is 25 GeV, the total energies of the decay products are for \( W_1 \)

\[
E_\pi = 7.40 \cos \varphi + 7.77 \text{ GeV} \\
E_p = 7.40 \cos \varphi + 17.25 \text{ GeV}
\] (12)

and for \( W_2 \)

\[
E_\pi = 8.45 \cos \varphi + 3.73 \text{ GeV} \\
E_p = 8.45 \cos \varphi + 16.29 \text{ GeV}
\] (13)

where \( \varphi \) is the decay angle in the rest system of the \( W \) with respect to its direction of flight. We restrict ourselves to detecting the charged decay products. It appears best to detect the \( p \) or \( \pi^+ \) near \( \varphi = 0 \).
since at small angles \( E_p \) and \( E_\pi \) vary slowly with \( \varphi \) and the transformation of solid angles from the \( W \) rest system to the laboratory system is most favourable. For \( \varphi = 0 \) we find \( E_\pi = 15.17 \) GeV or \( E_\pi = 17.18 \) GeV and 
\[
E_p = 24.65 \text{ GeV} \quad \text{or} \quad E_p = 24.74 \text{ GeV}
\]
The relative energy spread in the circulating beam produces an approximately equal relative spread in \( E_\pi \) and \( E_p \) and therefore measurements on the decay proton appear marginal. Fortunately the values of \( E_\pi \) have a comfortable difference and are far below the elastic proton peak. The laboratory angle \( \psi \) with respect to the flight direction of the \( W \) at which the \( \pi^+ \) is produced is approximately

\[
\psi \approx \varphi/33 \quad \text{for } W_1
\]
\[
\psi \approx \varphi/29 \quad \text{for } W_2
\]

Let us now consider, how the elastic proton scattering experiment at 30 mrad could be modified to detect the scattered proton and the decay \( \pi^+ \) in coincidence. The branch \( SC_1 \) to \( SC_4 \), which detects the proton remains unchanged while the branch \( SC'_1 \) to \( SC'_4 \) now accepts all \( \pi^+ \) produced within 7 mrad horizontal angular acceptance and the dispersion in \( B'_1 \).

By measuring the direction of the scattered proton one knows the direction of the \( W \) within 2 mrad horizontally and 1.5 mrad vertically. From \( \psi \) we can calculate \( \varphi \) and \( \cos \varphi \) to correct the measured \( \pi^+ \) momentum for the decay angle \( \varphi \). Using eq. 12 we find for \( W_1 \)

\[
dE_\pi = 7.4 \sin \varphi \: d\varphi
\]

The maximum value of \( \psi \) accepted by the detectors is 14 mrad. Taking the angular uncertainty \( d\psi = 2 \) mrad we find \( dE_\pi = 0.23 \text{ GeV} \). Even allowing for the momentum spread of the circulating beam we see that the peaks belonging to the two excited isobars should be very well separated in the \( \pi^+ \) energy spectrum.
Assuming isotropic decay of the W in its rest frame we find that SC\textsubscript{1}' to SC\textsubscript{4}' have a probability of \(1.2 \times 10^{-2}\) for detecting the \(\pi^+\). The measurements of Cocconi et al. show that the probability for formation of an excited isobar is roughly the same as for elastic scattering. Since we assumed that half of the W's decay into \(n + \pi^+\) the total counting rate of good events is \(6 \times 10^{-5}\) times the counting rate for elastic p-p scattering and amounts to \(8 \times 10^{-3}/\text{sec}\).

Due to the large counter sizes a triggering system consisting of S\textsubscript{2}' and S\textsubscript{4}' alone is not very selective. It looks better therefore to place additional triggering counters S\textsubscript{1}' and S\textsubscript{3}' adjacent to SC\textsubscript{1}' and SC\textsubscript{3}'. The large horizontal solid angle leads to a bad momentum resolution of the triggering counters so that the spark chambers are also triggered by colliding beam elastic p-p scatterings at a 100 times faster rate. To eliminate these it is necessary to subdivide S\textsubscript{2}' and S\textsubscript{4}' horizontally into a number of smaller counters and reject coincidences between certain pairs S\textsubscript{2i}' and S\textsubscript{4j}'. In this way one can also reject against lower energy particles and make the triggering system only sensitive to particles in the momentum range 10 GeV/c to 20 GeV/c.

The particle fluxes in the \(\pi^+\) detecting spark chambers are listed in Table 6. A 14 o/o probability of a spurious track in any of the spark chambers SC\textsubscript{1} to SC\textsubscript{4}' is quite acceptable, especially since apart from the other rejection criteria one can also record through which of the counters S\textsubscript{2i}' and S\textsubscript{4j}' the \(\pi^+\) has passed.
### TABLE 6

<table>
<thead>
<tr>
<th>spark chamber</th>
<th>area in cm²</th>
<th>particle flux due to beam gas events per sec.</th>
<th>particle flux due to colliding beam events per sec.</th>
<th>total particle flux per sec.</th>
<th>spurious tracks in 0.2 µsec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC'₁</td>
<td>20</td>
<td>$4 \times 10^4$</td>
<td>$5.3 \times 10^4$</td>
<td>$9.3 \times 10^4$</td>
<td>0.018</td>
</tr>
<tr>
<td>SC'₂</td>
<td>125</td>
<td>$8 \times 10^4$</td>
<td>$2.1 \times 10^4$</td>
<td>$10^5$</td>
<td>0.020</td>
</tr>
<tr>
<td>SC'₃</td>
<td>1000</td>
<td>$2.3 \times 10^5$</td>
<td>-</td>
<td>$2.3 \times 10^5$</td>
<td>0.046</td>
</tr>
<tr>
<td>SC'₄</td>
<td>1800</td>
<td>$2.8 \times 10^5$</td>
<td>-</td>
<td>$2.8 \times 10^5$</td>
<td>0.056</td>
</tr>
</tbody>
</table>

As in the case of the elastic p-p scattering accidental triggering will mainly be caused by background particles which pass through all triggering counters of one branch of the setup. Due to the addition of the extra triggering counters the number of these particles in the $\pi^+$ branch is certainly smaller than 1 o/o of the total flux in $S'_2$. The number of accidental triggerings is therefore smaller than $1.4 \times 10^{-3}$/sec which is 0.17 times the good event rate.

One might fear that the bumps in the $\pi^+$ spectrum could be drowned in secondaries, mainly protons, from other inelastic colliding beam events. It looks improbable, however, that in such events the proton momentum in SC₁ to SC₄ would be equal to that of the circulating beam. If necessary the selectivity of the triggering system could be greatly improved by detecting the $\pi^+$ with a threshold gas Cerenkov counter behind SC'₄. Since its cross-section would have to be about $20 \times 100$ cm² this would be a rather difficult instrument to build and it would certainly be worthwhile to try first a run without Cerenkov counter.

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7. 4π detector

The usefulness of a 4π detector in the form of a magnetic spark chamber surrounding the interaction region is quite evident. We shall try to give some figures for the detection efficiency of such a device. A possible type of magnet is shown in Fig. 7. With a proper choice of magnet dimensions the return flux which also crosses the circulating beam, can be made to cancel the effect of the main flux. The gap height of the main pole has been drawn rather arbitrarily as 75 cm. In principle it would be sufficient to give the return poles the same gap height as the CSR bending magnets. Since most secondaries, especially those with higher momenta, are produced at rather small angles to the circulating beam, the return flux is probably more useful than the main flux for their momentum analysis. It looks advisable, therefore, to give the return poles at least the same gap height as the main pole. By also placing coils around the return poles their stray field and its influence on the CSR quadrupoles is kept small. The magnet shown in Fig. 7 has massive poles since we assume that sparks can be located with acoustic pickups or alternately that stereo photographs can be made from the side by means of suitably disposed mirrors. Due to its large gap height the main pole has an important stray flux and its steel will already be completely saturated (30 kgauss) for 16 kgauss in the interaction region. We shall use the latter figure in the calculations below. Higher fields would require excessive amounts of power. The magnet of Fig. 7 would consist of 250 tons steel, 55 tons copper and consume 5 MW.

Two extreme modes of spark chamber operation are possible, with of course many variants in between. Firstly one may want to record a large number of colliding beam events, without any distinction regarding the type of event, in the same way as is done with bubble chambers at present. In general one would be satisfied to record a few events/sec. By triggering with a counter setup of the type described for the total cross-section measurement a nearly 100 o/o triggering efficiency could be reached, so
that a colliding beam interaction rate of a few events/sec would be enough. By scraping off most of the beam and stacking not too many pulses one could make an interaction region of say 1 mm high, 3 mm wide and 10 mm long, before the interaction rate became too low. The small size of the interaction region would be of considerable help in the data analysis.

The alternative is to run the CSR at full beam intensity and to use a very selective counter set-up for triggering. To search for the production of an intermediate boson \(^{14}\) one could e.g. trigger on large electron showers. As argued in sec. 4 the spark chamber would pick up background from about 10 m vacuum chamber in both CSR, each beam-gas reaction producing up to about 10 secondaries. With an 0.2 \(\mu\)sec sensitive time the probability of recording a beam gas event when the spark chamber is triggered on something else is 0.14. The probability of recording 2 colliding beam events is 0.03. Since the latter will be very difficult to analyze and since they are only a small fraction of the total, the easiest is to mark them with additional counters and simply discard them. Two events that are simultaneous for the spark chamber (0.2 \(\mu\)sec sensitive time) are seen separated in time by the counters because of their excellent time resolution.

Like the circulating beam the orbits of secondaries will have a kink so that they emerge from the return pole in the same direction as at production but laterally displaced by a distance \(d\). If \(d\) is large enough the orbit can never fit inside the vacuum chamber and must always show up somewhere in the spark chamber. Table 7 shows for different momenta the value of \(\Delta d\), the lateral displacement with respect to the circulating beam.
TABLE 7

<table>
<thead>
<tr>
<th>momentum (GeV/c)</th>
<th>(\Delta d) for pos. particles (cm)</th>
<th>(\Delta d) for neg. particles (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0</td>
<td>6.2</td>
</tr>
<tr>
<td>20</td>
<td>0.8</td>
<td>7.0</td>
</tr>
<tr>
<td>15</td>
<td>2.1</td>
<td>8.3</td>
</tr>
<tr>
<td>10</td>
<td>4.7</td>
<td>10.9</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
<td>18.7</td>
</tr>
</tbody>
</table>

No negative particles and no positive particles below 10 GeV/c can remain undetected in the spark chamber.

To allow a good momentum measurement a particle must have a reasonably long visible path in the magnetic field. For high energy secondaries we shall assume that the momentum is "well measurable" if their orbit is at 3 cm or more from the vacuum chamber center at 1.5 m from the interaction region. In that case their full path in the return field is visible. The fraction \(F\) of particles which does not satisfy this condition, has been calculated with eq. 1 and is shown in Table 8. The small values of \(F\) for negative particles result from the large angle at which they must be produced to be unmeasurable.

TABLE 8

<table>
<thead>
<tr>
<th>momentum (GeV/c)</th>
<th>(F) for pos. particles</th>
<th>(F) for neg. particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.82</td>
<td>0.05</td>
</tr>
<tr>
<td>20</td>
<td>0.49</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>0.26</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>0.12</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>
The values of $F$ are not too good for positive particles but very low for negatives. However, even $F = 0$ does not solve all problems, since neutrals will escape undetected anyhow.

It would be of great value if strange particles could be identified either because they decay outside the vacuum chamber or because one can extrapolate back the orbits of their decay products. The mean free path for decay $\lambda$ of various unstable particles at a momentum of 10 GeV/c is given in Table 9. Using eq. 1 one can readily show that the probability that a particle will decay outside the vacuum chamber, considered as a straight pipe of radius $r$ is

$$P_{\text{out}} = \frac{M^2}{p_0 c^2} \int_0^\infty y \exp \left( -\frac{M v}{p_0 c} - \frac{r}{c \tau' y} \right) \, dy \quad (16)$$

independent of its longitudinal momentum. In this equation $M =$ particle rest mass, $\tau' =$ particle life time. Taking $p_0 = 0.225$ GeV and $r = 3$ cm we have given in the last column of Table 9 the value of $P_{\text{out}}$.

<table>
<thead>
<tr>
<th>particle</th>
<th>$\lambda$ (cm) at 10 GeV/c</th>
<th>$P_{\text{out}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>61</td>
<td>0.26</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>70</td>
<td>0.31</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>44</td>
<td>0.17</td>
</tr>
<tr>
<td>$\Sigma^+  $</td>
<td>21</td>
<td>0.16</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>41</td>
<td>0.06</td>
</tr>
</tbody>
</table>

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Of the particles listed in Table 9 only the $\Omega(75\ o/o)$ and $\Lambda(58\ o/o)$ can
decay into two charged particles. The others always have at least one
neutral decay product so that their decay can only be recognised from a sudden
change in direction of the track. This is only possible if the particles
decay reasonably far outside the vacuum chamber.

Assuming a $\pm\ 0.5\ mm$ accuracy in spark location we estimate that for
50 o/o of the $\Lambda$'s and for 90 o/o of the $\Omega$'s decaying inside the vacuum
chamber the point of decay can be found by extrapolating back the tra-
jectories of the secondaries with an accuracy of 0.1 $\lambda$. This is roughly
independent of momentum since $\lambda$ is proportional to $p$ but the angle between
the decay products is proportional to $1/p$. We conclude therefore, that the
detection efficiency for $\Omega$ and $\Lambda$ is quite good but that for the other
particles the identification of their decay will in general be very difficult
indeed.

8. Monitors and absolute cross-section measurements

In the previous sections we have seen that secondaries from colliding
beam events can be detected with a known efficiency in well defined solid
angles. To normalise angular distribution measurements one must also know
the total number of interactions and the most straightforward method is to
measure this directly with a monitor counter. The latter only needs to count
something which is proportional to the total number of colliding beam
interactions and can be calibrated separately.

It looks probable that the vertical beam size and intensity distribution
in each of the two CSR is the same in all interaction regions. This can of
course be checked independently. The interaction rates in all interaction
regions would then be the same if the beams in both CSR were at exactly
the same height. This can be checked by moving the beam in one of the CSR
up and down with vertical kicker magnets and looking when the colliding beam
event rate is a maximum. In this way one can, by temporarily installing one
total cross-section set-up as described in sec. 4, calibrate the secondary
monitors everywhere else.

Except at points very close to the interaction region, which may often
not be accessible because of other detectors the flux of particles from
beam-gas events exceeds that from colliding beam events. This is true even
for particles produced at large angles to the beam. Therefore it is always
necessary to pick out colliding beam events by means of coincidences and
that can only be done with two counter telescopes in opposite directions
looking at small angles to the beam. These must then be fitted in as well
as possible with the rest of the experimental equipment. In the case of a
magnetic 4π detector it might be convenient to count γ's with the monitor,
in order to be independent of the magnetic field in the interaction region.
If the vertical positions of the beams would turn out to be extremely stable,
an experiment in one interaction region could rely upon the monitor in
another interaction region.

9. Implications on storage ring design.

From the discussion in the previous sections emerge a number of points
that are important for the design of the CSR. Most of these are not new but
it may be useful to list them here together.

**High vacuum.** With $10^{-10} \text{ mm H}_2\text{O}$ one can suppress the background in many cases
but often a better vacuum would be very convenient.

**Superposition of closed orbits in the interaction region.** A small size of
the interaction region is often essential to get a reasonable geometry in
a scattering experiment and to extend it to small angles.

**Vertical and horizontal beam stability.** The position of the interaction
region should be stable within 1 mm.
**Vertical beam adjustment.** If beams with 1 mm height are used they may completely miss each other unless their vertical position in the interaction regions can be adjusted.

**Large beam current.** To extend p-p scattering experiments to larger angles the circulating beam current should be as large as possible.

**Use of CSR magnets as analysing magnets.** It must be easy to exchange the CSR magnets near the interaction region for others with a larger gap height or gap width. Such magnets will in general need separate power supplies with, of course, excellent current stabilisation.

**Large beam height and large floor loading capacity.** The analysing magnets and the magnets of the 4m magnetic spark chambers around the interaction region have weights ranging from 200 ton to 400 ton and might be quite large. The CSR alignment should not be upset by moving around such large magnets.

**Minimum interference between CSR magnets and particles produced at small angles.** This means long straight sections in the interaction regions, quadrupoles with open median plane and some more straight sections downstream of the interaction region.

The main question that remains open is the amount of experimental space required at large angles to the beam. This is an important factor that heavily influences the design and cost of the shielding and buildings\(^{15}\). The present evidence points almost exclusively in the direction of small angle experiments, but it is always possible, of course, that something interesting turns up at large angles.

**Acknowledgement.** I thank Drs. A. Schoch and K. R. Symon for several stimulating discussions.

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B. de Raad

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Magnet layout of CSR.
Fig. 2  Quadrupole with open median plane
Fig. 3. Number of $\pi$-mesons of all energies per beam-gas interaction produced at various angles $\theta$. 

\( \frac{dN}{d\Omega} \) (per ster, per beam-gas interaction)
Fig. 4. Counters for p-p total cross-section measurement.
Fig. 5. Vertical and horizontal trajectories of elastically scattered protons and location of spark chambers.
Fig. 6. Horizontal trajectories of $\pi^+$ with different momenta, produced at various angles $\theta$. 
Fig. 7 Magnet for 4π spark chamber.