The displaced closed orbits in a structure

with achromatic insertions.

Collins (1961) has proposed to put long straight sections into a circular machine by using quadrupoles to match the properties of the straight sections to the properties of the rest of the machine. Holt and Newns (1962) have shown that this leads to a distortion of displaced closed orbits, and have shown that this can be serious by computing through particular cases. Edwards (1962) has made nice analytical treatment of this. His result can be summed up roughly as follows:

If one introduces matched straight sections into a structure which unperturbed would have a maximum closed orbit deviation $x_{0\ max}$ due to a momentum deviation $\Delta p$, the closed orbit would get an additional maximum deviation of approximately

$$\Delta x_{\ max} \sim x_{0\ max} \left| \frac{\sin (\mu/2)}{\sin (\mu/2)} \right|$$

where $\mu$ is the phase shift over a superperiod and $\mu_1$ is the phase shift over the insertion. This is an approximate formula. For the detailed expression I refer to the original report.

By careful choice one could make $\sin (\mu/2) \gtrsim 1$. However, resonant and stop-band consideration will more likely put one in the neighbourhood of $\sin (\mu/2) \lesssim 0.7$. A reasonable straight section would be considerably shorter than $\mu_1 \sim 2\pi$. It is concluded by Edwards that it is much more likely to have $\mu_1$ in the range $60^\circ$ to $120^\circ$. This means that $\left| \sin (\mu_1/2) / \sin (\mu/2) \right|$ is in practical cases of the order of unity, which leads to a doubling of the closed orbit deviation.

In large A.C. proton synchrotrons this is not serious. The parameters normally come out such that the space needed for the synchrotron oscillations is much smaller than the aperture and one can fairly easily tolerate a doubling of this space, especially in view of what one gains.

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This, however, is quite different in a set of storage rings, for instance for the C.P.S. In such a device most of the horizontal aperture is taken by momentum spread and momentum shift and any increase in the closed orbit deviation reduces the interaction rate by about the square of the inverse of the factor by which the closed orbit deviation is increased.

Edwards has in his report made no assumption about the type of focusing used. In this respect his treatment is general. However, he makes the two limiting assumptions that his insertions are matched and that the insertions contribute nothing to the bending. This makes it rather difficult to see from his results how to improve on the situation in the storage rings, by for instance sacrificing somewhat on the matching (i.e. sacrificing transverse phase space). One might generalised Edwards' treatment, but this would give more cumbersome expressions which would be more difficult to analyse.

In order to get expressions that are simple to interpret the following analysis is made on the basis of smoothed approximations, i.e. sinusoidal oscillations in all parts of the machine. In this way it has been easy to include the interesting case of the insertions to be mismatched and also to have bending. In fact there is no fundamental difference between the main part of the machine and the insertions. We, therefore, treat the case of a superperiod consisting of two different section lengths, each with different betatron wavelength and different momentum compaction as indicated in Fig. 1.

If the machine consists of only type 1 focusing the closed orbit deviation is supposed to be $\tilde{x}_1$, and similarly for type 2. With the two types together the closed orbit must enter section 1 with the same excursion and angle as it leaves section 2, and there must be no discontinuity in the closed orbit or its derivative between the two sections. The closed orbit is a sine curve about $\tilde{x}_1$ in section 1 and a sine curve about $\tilde{x}_2$ in section 2, and it is symmetric about the midpoints of
the sections (with the assumptions made). Taking a mid-point as a starting point of \( z \) we can write for the closed orbit in a section

\[
x = \hat{x} \cos \frac{2\pi z}{\lambda} + \bar{x}
\]

and the requirement of no discontinuity in \( x \) or \( x' \) gives

\[
\hat{x}_1 \cos \frac{\mu_1}{2} + \bar{x}_1 = \hat{x}_2 \cos \frac{\mu_2}{2} + \bar{x}_2
\]

\[
- \frac{2\pi}{\lambda_1} \hat{x}_1 \sin \frac{\mu_1}{2} = \frac{2\pi}{\lambda_2} \hat{x}_2 \sin \frac{\mu_2}{2}
\]

where

\[
\mu_1, 2 = 2 \pi L_1, 2/L_1, 2
\]

Solving with respect to \( \hat{x}_1 \) and \( \hat{x}_2 \) we get

\[
\hat{x}_1 = \frac{(\bar{x}_2 - \bar{x}_1) \sin \frac{\mu_2}{2}}{\sin \frac{\mu_2}{2} + (\lambda_2/\lambda_1 - 1) \sin \frac{\mu_1}{2} \cos \frac{\mu_2}{2}}
\]

\[
\hat{x}_2 = \frac{(\bar{x}_1 - \bar{x}_2) \sin \frac{\mu_1}{2}}{\sin \frac{\mu_1}{2} + (\lambda_1/\lambda_2 - 1) \sin \frac{\mu_2}{2} \cos \frac{\mu_1}{2}}
\]

We notice that formula (5) reduces to (1) for the special case of matched straight sections.

From the above expressions we can consider various ways of minimizing the "mismatch" of the closed orbit.

1) The trivial way is to make \( \mu_2/2 \ll 1 \) and keep \( \lambda_1 \sim \lambda_2 \), i.e., keeping the matched conditions for the betatron oscillations. This, however, leads to so small straight sections that they are of little practical interest.

2) Making \( \mu_2 \sim 2\pi \) and still \( \lambda_1 \sim \lambda_2 \). This leads to impractically long straight sections.

Both these methods were discussed by Edwards.

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If we are willing to sacrifice somewhat on transverse phase-space matching we open up more possibilities. This is of particular interest in the case of storage rings as already mentioned. The following possibilities are worth mentioning:

3) We make $\lambda_2 \gg \lambda_1$, in which case we can still satisfy the condition $\sin(\mu/2) \ll 1$ with practical lengths of straight sections. The extreme of this case is being used in both the AGS and the CFS, where long straight sections with no lenses (i.e., $\lambda_2 = \infty$) have been inserted. Since we can afford more mismatch in the storage rings than in the synchrotrons, we shall be able to go further in this direction than has been practice so far. Detailed calculations only can show how far we can go with the structures we are considering, and these calculations will be carried out.

4) Approximately concentric storage rings will look as sketched in Fig. 2.

Where a ring is on the inside we can have two extreme situations: either the inside has the same bending radius as the outside but longer straight sections or it can have the same bending length as the outside but larger bending radius. The first method is covered by points 1) and 2) above and it is seen that we must expect difficulties.

The method of different bending radii gives the possibility of "matching" the closed orbit by making

$$\bar{x}_1 = \bar{x}_2$$

(8)

This also leads to a mismatch in the transverse phase space, but normally less than by method 3), as can be seen from the following:

$$\bar{x} = \left(\frac{\Lambda}{2nR}\right)^2 R \Delta p/p$$

inserted into (1) gives

$$\lambda_1/\lambda_2 = (R_1/R_2)^{1/2}$$

(9)
A typical numerical example would be

\[ \frac{R_1}{R_2} \sim 3/5 \]

giving

\[ \frac{\lambda_1}{\lambda_2} \sim 0.8 \]

Before definite conclusions can be drawn from the considerations presented in this report, one shall have to consider specific structures. Even complete matching in the smooth approximation may mean a certain mismatch in a wiggly structure. The above very simplified treatment indicates, however, in which directions one might seek to obtain the best compromise, and it indicates that there is an incompatibility between matching of the transverse phase space and the minimisation of the wiggles on the closed orbit.

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References.


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