ON THE PRODUCTION OF COMPRESSED BUNCHES IN THE C.P.S.,

USING THE PHASE FOCUSING EFFECT

Procedure using phase focusing.

Table of contents.

Introduction.
The compression scheme.
Quantitative relations.
Reduction of the energy spread.
The phase focusing step.
Limitations.
Discussion and numerical example.
Immediate use of the transient bunches.
Capture step.
The problem of removing unstable particles.
Some supplementary remarks.
Appendix.
References.
Acknowledgements.
Introduction.

The procedures which have been proposed for bunch shortening in the C.P.S. are based on the adiabatic change of the amplitude of the synchrotron oscillations. The compression of the phase range occupied by a bunch in normal operation of the machine is effected by raising the voltage amplitude in the accelerating gaps. Thus the restoring force on particles, which are displaced from the stable phase, is increased.

As the bunch width decreases in an adiabatic change with \( V^{1/4} \) (linear range inside a bucket in the phase plane), it needs very high voltage amplitudes in the accelerating system for getting an appreciable bunch compression. Therefore by practical limitations it is impossible to go very far in this procedure.

For example, the total accelerating voltage amplitude being \( V_o = 108 \text{ kV} \), it would need about 1.7 MV for merely to reach half the original bunch width.

However, there is another effect, which combined with a preliminary adiabatic change of voltage can improve in some respects the situation for shorter bunch production. This is the phase focusing effect, which occurs when the R.F. force is suddenly raised to a higher amount. Then after a quarter of a synchrotron oscillation one obtains in the phase plane, in which the elliptic trajectory, comprising the particles in the bunch, rotates, the situation where the ellipse is vertically orientated and the longitudinal bunch extension is minimum, \( (\Delta \phi_{tr} \) in figure 1).

The smaller the energy range \( \Delta E \) (maximum) at the beginning of the transient motion, the smaller also \( \Delta \phi_{tr} \). Therefore a preliminary process for reducing \( \Delta E \) (from the value in standard operation of the machine to a smaller amount) will be included in the procedure.

The previous proposals for bunch compressing provide also changes to higher R.F.-frequencies. This means will also be used here in combination with the phase focusing process. It results in the possibility to obtain very short bunches without being obliged (as one was in the previous proposals) to increase the number of bunches. Thus one may obtain a very small duty cycle, that means small ratio of bunch width to distance between successive bunches.

\[ \text{See the note on page 15.} \]
The compressing procedure we want to describe comprises several steps, therefore, and requires in the general performance two bunching stations (cavities).
The Scheme of Compression.

It is supposed that the procedure will be applied at top energy of the C.P.S. and with the beam control in operation, but with non-accelerating phase, (flat top, \( \dot{B} = 0 \)).

The following steps are distinguished:

1) Reduction of the standard accelerating voltage \( V_0 \), down to \( V_1 \), thus bringing the energy spread \( \Delta E_0 \) of the bunched beam down to \( \Delta E_1 \).

This change is made adiabatically and the R.F. frequency is unchanged.

1a) (for a special version of the procedure)

Releasing the beam from the synchrotron forces for some period (say 10 ms) in order to get an approximately continuous beam.

2) The phase focusing step, where synchrotron forces of higher amplitude and frequency are applied. This state is switched on rapidly.

3) Capture of the "transient bunches" by providing a bucket structure corresponding to application of a considerably higher frequency, so that the longitudinal bucket width is only slightly larger than the bunch width reached in step 2).

4) There may then follow, or be included in step 3), some period of acceleration (or deceleration) for cleaning the beam, that means losing the particles not trapped in the capture step.

For realising step 2 and step 3 generally two separate bunching cavities are needed, operating on different harmonics of the beam revolution frequency.

The Quantitative Relations.

We want now to consider the relations governing the different steps of the above scheme, and assume for simplicity that the switching of the voltage and frequency steps can be made rapidly in comparison with the respective synchrotron motions.

Reduction of the energy spread, 1)

Within the "linear" range of a bucket in the phase plane, for non-accelerating phase of the stable point, and with the R.F. frequency unchanged (Lit. 1),
we have:  
\[
\frac{\Delta \phi_1}{\Delta \phi_0} \cdot \frac{\Delta E_0}{\Delta E_1} = \frac{4}{\sqrt{\frac{V_0}{V_1}}} = \frac{\Delta \phi_0}{\Delta \phi_1}.
\]
(1)

\(\phi\) is here the angle which increases by 2\(\pi\) in a bucket period.

Take for example the standard values at top energy in the P.S.

\[
\Delta \phi_0 = 25^0 \quad \text{and} \quad \Delta E_0/E = 0.5 \times 10^{-3},
\]

then one has, choosing \(V_0/V_1 = 10\), \((\sqrt[4]{10} = 1.77)\),

\[
\Delta \phi_1 = 44.3^0 \quad \text{and} \quad \Delta E_0/E = 0.282 \times 10^{-3}.
\]

This reduction process can in principle be pushed as far, as one will be able to keep the beam control system operating when reducing the voltage. We assume that a reasonable limit in this respect is actually, for top energy and non-accelerating phase, the value chosen above.

There are two ways in devising the further procedure, depending on whether one aims to keep the standard number of bunches as delivered from the machine, or to change it.

In the first case, A, the bunches, as obtained at the end of the reduction step 1), are taken as initial particle configuration for the phase focusing process, which is initiated immediately following the end of step 1). Then the number of bunches in the ring is not changed, and no unwanted stray particles are caused in the spacing between the bunches. So far no further cleaning operation will be necessary.

In the second case, B, a period of release from synchrotron forces is next provided. The purpose is to lose the bunch structure and to get the beam continuous in the limit. Then one may construct in the following step a new number of bunches on the ring.

However, in this case, the transient bunches formed in the phase focusing step have tails and it will not be possible to avoid that, in the subsequent capture step, the spacings between the "main" bunches remain filled with stray.
particles. Though one may think about a supplementary effort for again cleaning these spacings, this point puts the procedure B at a disadvantage with respect to A.

We shall discuss in the sequel merely the first version, A, of a bunch compression scheme.

The phase-focusing step, 2).

This step means a period in operation, where the synchronising R.F. forces are again switched on (case B), or are switched from the initial to a higher level (case A).

Let \( V_2 \) hereby be the new maximum accelerating voltage over the whole ring, and \( f_{rev} \) the new R.F. frequency.

In a phase plane with the coordinates \( \dot{\phi} \) and \( \phi \) one has for this change the following situation where the bunch is concerned:

To the initial voltage \( V_1 \) and frequency \( f_{rev} \) corresponds the initial bunch width \( \Delta \phi_1 \) and the height of the bunch \( 2\phi_1 \).

When the new bucket structure is imposed, then at the first moment the longitudinal position and the energy of any particle is conserved,

\[
\delta \phi_{2o} = \delta \phi_1 \quad \text{and} \quad \delta E_{2o} = \delta E_1.
\]

For the extreme particles of the bunch, which make the bunch width and height, therefore also

\[
\Delta \phi_{2o} = \Delta \phi_1 \frac{h_2}{h_1} \quad \text{and} \quad \Delta E_{2o} = \Delta E_1
\]  

(2)

As one sees from equation (6), appendix, \( \hat{\phi} \) changes as follows:

\[
\hat{\phi}_{2o} = \hat{\phi}_1 \frac{h_2}{h_1}.
\]  

(3)

Finally the synchrotron frequency changes, according to eq. (5), appendix, where we want to have immediately the step from the state "o" to "2",

\[
\Omega_2 = \Omega_o \sqrt{\frac{h_2 V_2}{h_o V_o}}.
\]  

(4)
Now, from $\hat{\phi}_{20}$ and $\Omega_2$ we are able to calculate the bunch width arising one quarter of a synchrotron oscillation later, $\Delta \phi_{2tr}$, using relation (3), appendix,

$$\Delta \phi_{2tr} = 2 \frac{\hat{\phi}_{20}}{\Omega_2}.$$  (5)

Substituting $\hat{\phi}_{20}$ from (3) and there $\hat{\phi}_1$ from (1), and taking further into account that

$$h_1 = h_0 \quad \text{and} \quad 2 \frac{\hat{\phi}_0}{\Omega_0} = \Delta \phi_0,$$

one obtains the bunch width ratio, if the bunch width is expressed with respect to the whole ring

$$\frac{\Delta \phi_{2tr}}{\Delta \phi_0} = \frac{h_0}{h_2} \frac{\Delta \phi_{2tr}}{\Delta \phi_0} = \frac{2}{V_1} \sqrt{\frac{V_1}{V_0}} \sqrt{\frac{h_0 V_0}{h_2 V_2}}$$  (6)

This relation shows, that making $V_2 h_2 > V_0 h_0$ one may push the reduction in bunch width to a higher order of magnitude, but that only at a moment during the transient process. There is the further task to keep this transient bunch width, as will be discussed later on.

Eq. (6) shows also the contribution of the preceding adiabatic reduction step for the resulting bunch compression.

Limitations.

a) Certain eventual limitations must be envisaged. One such limitation may consist in the interdiction, that applying the new frequency, $h_2 f_{rev}$, the bucket width should in any case be larger than the bunch width. This means an upper limit for the parameter $h_2$, expressed by the relation

$$\frac{h_2}{h_0} \Delta \phi_1 = \Delta \phi_{20} = 2 \pi \rho, \quad \text{where} \quad \rho < 1, \quad \text{and such}$$  (7)

---

Alternative "A" (under 1)).

PS/2953
that \( h_2 \) is a whole number, as is \( h_0 \).

If \( \Delta \phi_1 \) is expressed in terms of \( V_1 \) and the initial data, using the preceding formulae, one obtains

\[
\frac{h_0}{h_2} = c_1^2 \left( \frac{V}{V_0} \right) \frac{V_1}{V_0}, \quad c_1 = \sqrt{\frac{1}{h_2}} \left( \frac{V_1}{V_0} \right) \frac{V}{V_0},
\]

(8)

with

\[
c_1^2 = \frac{1}{2\pi Q} \left( \frac{2\pi h_0 E}{e V_0} \right)^{1/2} \frac{\Delta E}{E}.
\]

(9)

Thus one has the reduction ratio

\[
\frac{\Delta \phi_{2\text{tr}}}{\Delta \phi_{20}} = c_1 \left( \frac{V_1}{V_0} \right)^{1/2} \left( \frac{V}{V_2} \right)^{1/2}
\]

(10)

b) A second eventual limitation is implied with respect to radial width, (the height in the phase plane) of the transient bunch. Certain requirements in connection with the use of the beam may impose a narrow limit. Otherwise there is the limit, when particles are lost reaching the unstable excentric region in the vacuum chamber.

The maximum height of the transient bunch occurs one quarter of a synchrotron oscillation after application of the change in voltage and frequency. It can be calculated from \( \Delta \phi_{20} \). One has

\[
\hat{\phi}_{2\text{tr}} = \frac{\pi}{2} \Omega_2 \Delta \phi_{20}.
\]

(11)

Operating the formulae used already above, one may transform the right side of this relation for introducing the original height \( \hat{\phi} \). One obtains, putting as above \( h_1 = h_0 \),

---

**Note:** Here in particular we shall fix an upper limit for \( \rho \) such, that for all particles in a bunch the frequency will still be practically the same, (restriction to the "linear" range in the bucket).
\[ \phi_{2tr} = \tilde{\phi}_o \left( 4 \sqrt{\frac{v_2}{v_l}} \sqrt{\frac{v_2}{h_0 h_2}} \right)^{3/2}. \]  

(12)

It is more convenient to make this comparison in terms of the energy spread connected with the bunch height. One has thus (cf. eq. (6), appendix),

\[ \frac{\Delta E_{2tr}}{\Delta E_0} = 4 \sqrt{\frac{v_1}{v_o}} \sqrt{\frac{h_2 v_2}{h_0 v_1}} = 4 \sqrt{\frac{v_o}{v_1}} \sqrt{\frac{h_2 v_2}{h_0 v_0}} \]  

(13)

and introducing the limitation for \( h_2 \),

\[ \frac{\Delta E_{2tr}}{\Delta E_0} = \frac{1}{c_1} \frac{v_o^{7/8} v_2^{1/4}}{v_1^{1/8} v_2^{1/4}}. \]  

(14)

Comparing (10) with (14) one sees that

\[ \frac{\Delta E_{2tr}}{\Delta E_0} = \frac{\Delta \phi}{\Delta \phi_{2tr}} = \frac{\Delta \phi / h_{2rev}}{\Delta \phi_{2tr} / h_{2rev}} = \frac{T_o}{T_2} \]  

(15)

which is a check for conservation of the bunch surface in a phase plane with energy and longitudinal displacement as coordinates. Thus (15) is checked immediately from that theorem.

**Discussion.**

Inspecting the parameters relevant for the bunch compression up to this point of the procedure, \( v_1, v_2, h_2 \), we find that it is necessary to make \( h_2 \) as high as allowed, because then the value of \( v_2 \) necessary will be the lowest possible one. Therefore the first one of the above limitations is to be observed.

One states from (10) that the influence of the adiabatic reduction step, expressed by \( v_1 \), is weak compared with the one of the focusing step, expressed by \( v_2 \). Nevertheless, for making savings in necessary \( v_2 \), one will in any way make use of the first step, the reduction, as much as one can.
Thus we shall assume for a numerical discussion, following the example at the beginning,

\[ \frac{V_0}{V_1} = 10. \]

From eq. (8) we obtain \( c_1^2 \) for various numbers of \( h_2/h_0 \). The following table shows some calculated values.

<table>
<thead>
<tr>
<th>( h_2/h_0 )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0.376</td>
<td>0.356</td>
<td>0.3065</td>
<td>0.284</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.48</td>
<td>0.6</td>
<td>0.72</td>
<td>0.84</td>
</tr>
</tbody>
</table>

For determining \( \rho \) from (9), a set of initial values is assumed. We have taken the standard values which are valuable for the CPS at top energy (\( \beta = 1 \)),

\[ E = 24 \text{ GeV}, \quad \Delta E/E = 0.5 \cdot 10^{-3}, \quad eV_c = 108 \text{ kV}, \]
\[ h_0 = 20, \quad Q = 6.25, \quad x_c = 1/39. \]

The numerical relation for checking \( \rho \) (operating range in the bucket) is then,

\[ \rho = 0.0676/c_1^2. \]

Then, for a range of parameters, the reduction ratio \( \Delta \theta_{2tr}/\Delta \theta_o \) has been calculated from relation (10) and plotted in figure 2 as a function of the voltage ratio \( V_2/V_0 \).

Considerable bunch compression can be obtained as fig. 2 shows. For example, one has a longitudinal compression by the factor \( \sqrt{10} \), using \( h_2/h_0 = 5 \), (R.F. frequency 50 Mc/s), with an R.F. peak voltage

\[ V_2 = 6.4 \cdot 108 = 692 \text{ kV}, \]

a value, which could be realistic eventually using several acceleration gaps.
The reduction factor of $\sqrt{10}$ means on the basis of $\Delta \phi = 25^\circ$ a bunch length expressed in time; 0.7 ns. The spacing between the bunches, however, is kept from the standard operation, namely 100 ns between centres.

Also $\Delta r$, twice the maximum radial particle excursion from the stable orbit, as corresponds to $\Delta E_{2tr}$, is given in Fig. 2. It is calculated, with the compaction factor $\sqrt{39}$ and with $r_o = 10^4$ cm, as

$$\Delta r = r_o \frac{1}{39} \frac{\Delta E}{\Delta E_0} \frac{\Delta E_{2tr}}{\Delta E_0}$$

For the above example of operating parameters one should have

$$\Delta r = 1.25 \text{ cm}$$

for a particle, whereas for the beam the extreme radial width by the bunches will be larger, the original extension of the beam being of the order of $3 - 4 \text{ mm}$. So about 2 cm clearance for the beam is required in this example.

**Immediate use of the transient bunches.**

Immediate use of the bunch compression during the transient process may be provided in connection with a fast ejection system. The ejection system must enable to remove the bunches completely or partially from the orbit in the synchrotron at the most favourable instant, which is $\sqrt{4}, 3/4, 1 + \sqrt{4}, 1 + 3/4$, and so on synchrotron periods after the change in voltage and R.F. frequency.

The ejection period must be short compared with the synchrotron period $T_{s2}$, which is

$$T_{s2} = T_{so} \left( \frac{h \frac{V}{2}}{\hbar \frac{V}{2}} \right)^{\sqrt{2}}$$

(16)

In the case of the above example it is about

$$T_{s2} = 3.5 \text{ ms} \cdot \frac{1}{\sqrt{5} \cdot 6.4} = 618 \mu s \approx 1600 \text{ Hz}$$

excluding the Betatron motion.
Figure 3 explains that one should not spend more than at most a few per cent of the synchrotron period for the ejection process, if one does not want to lose in bunch shortness.

For producing target bursts with a smaller percentage of the beam in this short period, also a very strong R.F. knockout excitation might be successful.

For conserving the short bunches obtained in step 2 for some period, a new bucket structure must be instantaneously provided by applying a higher R.F. frequency and an appropriate voltage in the accelerating cavity which is operated for this purpose. This case is given, when for instance along series of short target bursts should be produced using an appropriate procedure for consuming the beam slowly in connection with the target.

It is clear, the transient bunch as produced in step 2 must be enveloped by the new bucket structure, if one does not want to lose particles out of the synchronous binding. Thus the conditions are for the change

\[ \Delta E_{b3} = b \Delta E_{2tr} \quad \text{and} \quad \Delta \phi_{b3} = a \Delta \phi_{2tr}, \quad (17) \]

where \( h_2/h_3 \) = rational fraction, \( h_3 \) a whole number, \( a \) and \( b \) larger than 1, and the suffix \( b \) referring to a bucket.

From the second condition (17) one finds, since \( \Delta \phi_{b3} = \frac{2\pi}{h_3} \)

\[ h_3 = \frac{1}{a} \cdot \frac{2\pi}{\Delta \phi_0} = \frac{1}{\Delta \phi_{2tr}/\Delta \phi_0} \]

or written with \( \Delta \phi_0 = h_0 \Delta \phi_0 \)

\[ \frac{h_3}{h_0} = \frac{1}{a} \cdot \frac{2\pi}{\Delta \phi_0} \cdot \frac{\Delta \phi_0}{\Delta \phi_{2tr}} \quad (18) \]
In order to get \( V_3 \) necessary from the first condition, one introduces
relation (8, appendix) for the bucket height
\[
\frac{\beta_{b3}}{\beta_3} = 2 \Omega_3
\]
Expressing the left and the right term by the corresponding relations one finds:
\[
\sqrt{\frac{e^{2} V_3}{E}} = \frac{1}{2 \Omega} \sqrt{\frac{\pi}{2} h_3} \frac{\Delta E_{b3}}{E}
\]  
(19)

Then from (19) and using (9)
\[
\sqrt{\frac{V_3}{V_0}} = \frac{\pi}{2} \rho_1 \beta h \sqrt{\frac{h_3}{h_0}} \frac{\Delta \phi}{\Delta \phi_{2tr}}
\]  
(20)

where \( \rho_1 \beta h \) comprises only initial parameter values.

We now first want to obtain the order of magnitude of the R.F. peak voltage, which would be necessary to establish a bucket of sufficient height for capturing the whole bunch.

Assume for example \( a = 1.1 \) and \( b = 1.5 \). Then basing on the values taken in the example of a focusing operation, of the preceding chapter, one has
\[
\frac{h_3}{h_0} = \frac{1}{1.1} \frac{360^0}{250} 10 = 131
\]
which corresponds to an R.F. frequency of \( 1310 \text{ Hz} / a \), and one has
\[
\frac{V_3}{V_0} = \frac{\pi}{2} 0.6 \cdot 0.336^2 \cdot 1.5 \cdot 11.5 \cdot 10 = 18.4
\]
hence \( V_3 = 108 \text{ kV} \cdot 338 = 3615 \text{ MV} \).

That is not a realistic figure for a bunch compressing device. We see that one is unable to capture the whole bunch in this way. It is in practice unavoidable, to lose a good part of the particles.

An idea how much of the bunch one could keep in this example, shall be given by supposing 1 MV as R.F. peak voltage. One obtains in this case
\[
b \sim 0.25
\]

This situation for the phase plane is sketched in figure 4.
The Problem of Removing Unstable Particles.

The particles not included in the capturing bucket envelope will be non-synchronous and therefore smeared out over the ring of the machine, but they will theoretically not be removed from the ring without appropriate additional arrangement, since we have the case of non-acceleration.

Normally moving the R.F. phase for getting acceleration and keeping this state for some period, will perform that cleaning action \(^\#\). However, in our special case such a procedure brings a new loss of particles. The reason is that the bucket in the non-accelerating state is already entirely filled with particles. Varying the size of the bucket by transition to an accelerating stable phase means losing a part of the particles, (those in the region of the unstable fixed point in the set of trajectories in the phase plane).

Another possibility (theoretical, at present) of removing unwanted non-synchronous particles from the ring would consist in using the procedure of fractional R.F. knockout (Lit. 2).

Supplementary remarks: The phase focusing effect can of course also be produced when the new situation in R.F. force will be applied for example during a period much shorter than \(1/4\) of a synchrotron oscillation. After an appropriate subsequent drift period, the maximum particle compaction occurs. However, it can be shown, that the bunch width obtainable will be in any way inferior to that according to the \(1/4\) synchrotron oscillation procedure.

There is also another method proposed in USA (reported by Johnsen), where by phase shifting of the bunch to the unstable point in the phase plane the particle configuration is stretched along the separatrix. Shifting then back to the stable point that particle configuration moves in the phase plane through a state of minimum in bunch width.

\(^\#\) Not perfectly, but to some extent.
Synchrotron Oscillation in the Linear Range in a Bucket.

The gap voltage is taken as

\[ u = V \cos \phi , \]

where \( \phi \) is set

\[ \phi = \phi_0 + \delta \phi \quad (\phi_0 : \text{stable phase}) , \]

The synchrotron oscillation is then described by

\[ \dot{\phi} = \frac{1}{2} \Delta \phi \cos (\Omega t + \varphi) . \quad (A \ 1) \]

One has in \( (A \ 1) \)

\[ \Omega = 2 \pi f_\infty \sqrt{\frac{eV}{2\pi hE}} \left( \alpha - \frac{E_0}{E} \right) \sin \phi_0 , \quad (A \ 2) \]

where \( \beta f_\infty = f \) is the applied accelerating frequency,

\( \beta = v/c \), the relative particle velocity,

\( h = \hbar/f_{\text{rev}} \), \( f_{\text{rev}} \) beam revolution frequency

\( \alpha \propto \sqrt{Q^2} \), the momentum compaction factor

\( E \), the energy of the stable phase particle,

and \( E_0 \), the rest energy.

The time derivative of \( \dot{\phi} \), which is needed for the phase diagram of the synchrotron oscillation, is from \( (A \ 1) \)

\[ \ddot{\phi} = \delta \dot{\phi} = -\Omega \frac{1}{2} \Delta \phi \sin (\Omega t + \varphi) . \quad (A \ 3) \]

On the other hand exists the connection between \( \dot{\phi} \) and the energy spread

\[ \dot{\phi} = -2 \pi f_\infty \left( \alpha - \frac{E_0}{E} \right) \frac{1}{\beta} \frac{\delta E}{E} . \quad (A \ 4) \]
In the case that \( E \) is large compared with \( E_0 \) and that we have non-accelerating condition \( \phi_0 = \pi/2 \), and using the approximation \( \alpha = 1/q^2 \), we get

\[
\Omega = \frac{f_{\text{rev}, \infty}}{Q} \sqrt{2 \pi} \hbar \sqrt{\frac{eV}{E}},
\]

and

\[
\dot{\phi} = 2 \pi f_{\text{rev}, \infty} \frac{\hbar}{Q^2} \frac{\delta E}{E}
\]

\[
= \frac{\Omega}{f_{\text{rev}}} \frac{E}{eV} \frac{\delta E}{E}
\]

Corresponding to the maximum energy spread \( \Delta E/2 = \delta E_{\text{max}} \), occurring for the extreme particles in the bunch, one has in Eq. (A 6) the maximum value

\[
\hat{\phi} \]

and in Eq. (A 3),

\[
|\dot{\phi}| = \gamma \Omega \Delta \phi_{(\text{max})}
\]

Bucket height.

From the non-linear relation for the synchrotron motion one has for the maximum \( \hat{\phi} \) on the separatrix

\[
\hat{\phi}_{b, \text{max}}^2 = \frac{4 eV}{A L}
\]

with

\[
A = \frac{E}{2 \pi c f_\infty (\alpha_c - (E_0/E)^2)}
\]

and

\[
L = \frac{e \hbar}{f_\infty}
\]

That makes, using (A 2)

\[
\hat{\phi}_b = 2 \Omega
\]
References:


Acknowledgement

The author wishes to thank Dr. H.G. Hereward for discussions and corrections and H. Fischer for communications and discussions.

M.R. Geiger.

Distribution: (open)
A.R. Division
P.S. Library.

/kt

PS/2953