A PROJECTED HIGH INTENSITY ION SOURCE BASED ON A PLASMA GUN

Abstract
This report studies the possibility of obtaining high intensity pulsed ion currents from a plasma gun.
The design aim is to produce a pulse of duration 10 microseconds and containing $10^{14}$ hydrogen ions. Such an ion source might usefully be used in the injection system of a high energy particle accelerator or in the field of thermo-nuclear physics.

I. Introduction
The magnitude of the current obtained from an ion source is dependent on the current density, and the area hole, through which the ions are extracted. In a space charge controlled plasma the current density is limited by the applied overvoltage. The area of extraction is normally limited by the vacuum conditions. In existing designs ion sources normally employ an extraction area of a few square millimetres because the gas pressure inside the tube is usually $10^{-5}$ to $10^{-6}$ mm Hg. In the design described here the extraction area is increased to about one square centimetre whilst still retaining the possibility of obtaining pressures of the order of $10^{-6}$ mm Hg.

This increase of extraction area is achieved by injecting pulses of gas which last for some $10^{-4}$ sec into the gun. Each pulse of gas is ionised, a plasma is produced, and some part of this plasma is ejected from the gun into an evacuated "expansion chamber". The volume of this chamber and its pumping rate are such that although the pulsed pressure at the gun may be from $10^{-1}$ to 1 mm Hg, the pressure in the expansion chamber may be kept at $10^{-4}$ to $10^{-5}$ mm Hg. Ions may therefore be extracted from the plasma in the expansion chamber through a relatively large hole (in fact
1 square centimetre) and injected into an accelerating structure which in its turn may be kept at a pressure of $10^{-6}$ mm Hg.

The overall arrangement is shown in fig. 1.

The details are given in a series of appendices; the advantages of this type of ion source over the more conventional design are outlined below.

II. Advantages of pulsing the amount of gas entering the plasma gun

The efficiency $\varepsilon$ of an ion source is given by the relation between the number of ions extracted to the number of neutral molecules which emerge through the extraction hole, $\varepsilon$ is thus given by

$$\varepsilon = \frac{\int_0^t J(i) \, dt}{\int_0^T \bar{n}(g) \, dt} = \frac{J(i)}{e \, n(g) \, v_Z} \frac{t_1}{T}$$

(1)

where $J(i)$ is the ion current density, $e$ is the electric charge, $n(g)$ is the number density of gas molecules, $t_1$ is the duration of the extraction ion pulse and $T$ is the time which elapses between the application of successive extraction pulses, $v_Z(g)$ is the velocity of gas molecules in the direction of current flow.

For example in the CRS machine, $t_1 = 10^{-5}$ sec and $T = 3$ sec. All the essential elements in the Linac utilise pulses except the gas input, this is in part responsible for the existing efficiency as defined by (1), of the ion source at present in use.

If the gas input is pulsed, the pressure at the gun first increases and then decreases exponentially as the gas diffuses out into the expansion chamber. If the time constant of this exponential decrease in pressure is arranged to be about $10^{-4}$ sec and discharge conditions suitable for the production of a plasma at the gun are obtained, then the time $T$ may be reduced by the efficiency correspondingly increased. In the example
quoted above as obtaining to the CPS, it should be possible with this type of gun to reduce $T$ from 3 sec to about $3 \times 10^{-4}$ sec and thus increase the present value of the efficiency by a factor of $10^3$, taking into account that $n(g)_{Z}^{-1}(g)$ is a factor 10 higher in a plasma gun than in a Thonemann ion source.

III. **Advantages of the expansion chamber**

The purpose of the expansion chamber in fig. 1 is to allow the pulsed gas pressure at the gun be damped in it and allowing a very much larger extraction hole than is normally feasible. It is shown below that with this device, it is possible to extract the ions through a hole of area $1 \text{ cm}^2$ and inject them into a region where the vacuum may be maintained at a pressure of $10^{-6} \text{ mm Hg}$. This extraction area may therefore be some hundred times larger than that previously employed in conventional ion sources.

IV. **Advantages of using a plasma gun as an ion source**

A plasma gun is capable of producing extremely large ion densities and ejecting these ions from the region of the gun at considerable velocities. The latter characteristic is particularly useful in that the ions can then be transferred from the relatively high pressure region of the gun across the intermediate pressure region of the expansion chamber, and into the low pressure region of the ion extracting device.

V. **Design Data**

The mechanical techniques of pulsing gas in times of hundreds of microseconds are described in LAMS Quarterly Report from 1960 to 1961. The data relevant to the design of such an ion source as is outlined above are given below.

Fig. 3 shows the design of the mechanical system for introducing pulses of gas from the gas container into the vacuum system of the gun. The conductivities of the individual regions are calculated below.
(a) Gas Container

The gas in the container emerges when the valve is opened with an electromagnetic hummer.

The viscous conductance $C_V$ of the exit of the valve is given by (Appendix 1)

$$C_V = \frac{\pi}{6} \left( \frac{P_{11} + P_{12}}{2} \right) \frac{r_1}{\eta l_1} a^3$$

where $a$ is the dimension of the aperture through which gas flows when the valve is open.

The molecular conductance $C_M$ is given by (Appendix 2)

$$C_M = 7.4 \times 10^5 \frac{r_1 a^2}{l_1} \text{ cm}^3/\text{sec}$$

where $r_1$, $a$ and $l_1$ are given in cm.

The total conductance is given by adding (2) and (3)

$$C_T = C_V + C_M = \frac{\pi a^3 r_1}{6 \eta l_1} \left( \frac{P_{11} + P_{12}}{2} \right) + 7.4 \times 10^5 \frac{r_1 a^2}{l_1} \text{ cm}^3/\text{sec}$$

(b) Intermediate Zone

To evaluate the conductance in the zone from valve to plasma gun is not easy because the boundary conditions are not simple. An approximation is made by assuming that the conduction is across a cylindrical tube in which the outside and inside radius are the average values obtaining to the mechanical design.

The viscous conductance across a cylindrical tube of length $l_2$ and having $r_{22}$ and $r_{21}$ as the outside and inside radius is given by (Appendix 3)
\[ C_{V_2} = \frac{\pi}{8n_2} \left[ \frac{r_{22}^4 - r_{21}^4}{r_{22}^2 - r_{21}^2} - \frac{(r_{22}^2 - r_{21}^2)^2}{\ln \frac{r_{22}}{r_{21}}} \right] \left( \frac{p_{22} + p_{21}}{2} \right) \] (5)

The molecular conductance is obtained with the same approximations and is given by

\[ C_{M_2} = 3.72 \times 10^5 \frac{(r_{22} + r_{21})(r_{22} - r_{21})^2}{l_2} \text{ cm}^3/\text{sec} \] (6)

Adding (5) and (6) results in a total conductance

\[ C_{T_2} = C_{V_2} + C_{M_2} = \frac{\pi}{8n_2} \left[ \frac{r_{22}^4 - r_{21}^4}{r_{22}^2 - r_{21}^2} - \frac{(r_{22}^2 - r_{21}^2)^2}{\ln \frac{r_{22}}{r_{21}}} \right] \left( \frac{p_{22} + p_{21}}{2} \right) + \\
+ 3.72 \times 10^5 \frac{(r_{22} + r_{21})(r_{22} - r_{21})^2}{l_2} \] (7)

(c) Plasma Gun

The plasma gun conductance is given analogously to (7) by

\[ C_{T_3} = C_{V_3} + C_{M_3} = \frac{\pi}{8n_3} \left[ \frac{r_{32}^4 - r_{31}^4}{r_{32}^2 - r_{31}^2} - \frac{(r_{32}^2 - r_{31}^2)^2}{\ln \frac{r_{32}}{r_{31}}} \right] \left( \frac{p_{32} + p_{31}}{2} \right) + \\
+ 3.72 \times 10^5 \frac{(r_{32} + r_{31})(r_{32} - r_{31})^2}{l_3} \] (8)

VI. Numerical calculations

The viscosity constant of hydrogen is taken: \( \eta = 87.6 \times 10^{-6} \) poise.
Gas Container

The following data are used:

\[
\begin{align*}
\text{Longitude} & : h_1 = 0.4 \text{ cm} \\
\text{Radius} & : r_1 = 0.5 \text{ cm} \\
\text{Volume} & : V_1 = 0.314 \text{ cm}^3 \\
\text{Longitude exit} & : l_1 = 0.1 \text{ cm}
\end{align*}
\]

(9)

Substituting in (4) the values in (9) it is found that

\[
C_{T_1} = 14.3 \times \left( \frac{P_{11} + P_{12}}{2} \right) + 3.70 \times 10^4 \text{ cm}^3/\text{sec}
\]

(10)

where \( \frac{P_{11} + P_{12}}{2} \) is given in (dynes/cm²).

Intermediate Zone

It is proposed the following parameters

\[
\begin{align*}
\text{Outside radius} & : r_{22} = 1.5 \text{ cm} \\
\text{Inside radius} & : r_{21} = 0.7 \text{ cm} \\
\text{Longitude} & : l_2 = 4.5 \text{ cm} \\
\text{Volume} & : V_2 = 25 \text{ cm}^3
\end{align*}
\]

(11)

Replacing in (7) the data of (11) we arrive at

\[
C_{T_2} = 8.18 \times 10^2 \left( \frac{P_{22} + P_{21}}{2} \right) + 1.16 \times 10^5 \text{ cm}^3/\text{sec}
\]

(12)

Plasma Gun

Taking the parameters
Outside radius \( r_{32} = 1 \text{ cm} \)
Inside radius \( r_{31} = 0.2 \text{ cm} \)
Volume \( V_3 = 12.1 \text{ cm}^3 \) (13)
Longitude \( l_3 = 4 \text{ cm} \)

Taking into account (13), (8) gives

\[
C_{n3} = 4.78 \times 10^2 \left( \frac{P_{32} + P_{31}}{2} \right) + 7.15 \times 10^4 \text{ cm}^3/\text{sec} \] (14)

It would be convenient to know the pressure distribution as a function of time, because a minimum pressure is required in the plasma gun to produce ionization, and also the diffusion time constants of the different parts of the system. To find these quantities, it is necessary to resolve mathematically the diffusion equation in the system, but this problem is very difficult because the boundary conditions are not known exactly. However, this problem may be treated with the aid of an analogous network of the form given below

\[
\text{Gas} \quad \text{Gas} \quad \text{Valve Intermediate Zone} \quad \text{Plasma Gun} \quad \text{Expansion Pumps} \quad \text{Chamber}
\]

Storage Container

Where \( \sum_{i=1}^{n} R_{ij} = R_j \) and \( \sum_{i=1}^{n} C_{ij} = C_j \), for \( j = 2, 3 \)

In the diagram the capacity \( C \) is proportional to the volume, the resistance \( R \) proportional to conductance \( -1 \) and the voltage proportional to the pressure. In this system the point at which it is necessary to
know the voltage as a function of time is 'A' because this is the equivalent position of the plasma gun begin.

It is assumed that at a given time \( t \) there is a constant flux of gas which is given by

\[
(P_{11} - P_{12})T_1 = (P_{21} - P_{22})T_2 = (P_{31} - P_{32})T_3 = Q
\]  

(15)

where \( P_{12} = P_{21} \), \( P_{22} = P_{31} \) and \( P_{32} = 0 \) as is seen from examination of fig. 1 bis) and diagram.

The minimum pressure for producing a discharge in the plasma gun is taken to be \( P_{22} \). For hydrogen at a voltage cf 1000 V it is necessary that

\[
P_{22}(r_{32} - r_{31}) = 0.25 \text{ cm x mm Hg}
\]  

(16)

Putting the data given by (13) in (16) it is found that

\[
P_{22} = 0.312 \text{ mm Hg}
\]  

(17)

Solving (10), (12) and (14) with (15) and (17) it is found that

\[
Q = 7.05 \times 10^7 \text{ dynes cm}^2/\text{sec}
\]  

(18)

\[
P_{11} = 1.86 \times 10^3 \text{ dynes/cm}^2
\]  

(19)

\[
P_{12} = 5.5 \times 10^2
\]  

(20)

\[
C_{T_1} = 5.4 \times 10^4 \text{ cm}^3/\text{sec}
\]  

(21)

\[
C_{T_2} = 5.04 \times 10^5
\]  

(22)

\[
C_{T_3} = 1.7 \times 10^5
\]  

(23)

The time constants of the different parts then become
\[ \tau_1 = \frac{V_1}{C_{T1}} = 5.82 \times 10^{-6} \text{ sec} \]  
(24)

\[ \tau_2 = \frac{2}{C_{T2}} = 50 \times 10^{-6} \text{ sec} \]  
(25)

\[ \tau_3 = \frac{3}{C_{T3}} = 71 \times 10^{-6} \text{ sec} \]  
(26)

The quantity of gas contained in the system at a time \( t_m \) is given by

\[ P_{11} V_1 + \left( \frac{P_{21} + P_{22}}{2} \right) V_2 + \frac{P_{22}}{2} V_3 = 1.45 \times 10^4 \text{ barias cm}^3 \]  
(27)

The quantity of gas which has passed out of the system into the expansion chamber is given approximately by

\[ \int_0^{t_m} P_{31}(t) C_{T3}(t) \, dt \approx P_{31}(t_m) \times C_{T3}(t_m) \times \tau_3(t_m) = 5 \times 10^3 \text{ barias cm}^3 \]  
(28)

The total gas in gas container (i.e. at \( t = 0 \)) is obtained by adding (27) and (23) and is given by

\[ (P V)_{t=0} = 1.95 \times 10^4 \text{ barias cm}^3 \]  
(29)

\[ = 1.5 \times 10^{-2} \text{ mm Hg litres} \]

and therefore the initial pressure in gas container is

\[ P_{t=0} = 47 \text{ mm Hg} \]  
(30)

**Dimensions of the pipe from gas storage to gas container**

The viscous conductance, \( C_v \), of a pipe of length \( l \) and radius \( a \) is given by

FS/3136
\[ C_{V_0} = \frac{\pi a_0^4}{8\eta l_0} \left( \frac{P_0 + P_1}{2} \right) \text{cm}^3/\text{sec} \quad (31) \]

and the molecular conductance \( C_{M_0} \) by

\[ C_{M_0} = 37.1 \times 10^4 \frac{a_0^3}{l_0} \text{cm}^3/\text{sec} \quad (32) \]

Assuming the following data of

\[
\begin{align*}
\text{length} & \quad l_0 = 1 \text{ cm} \\
\text{pressure} & \quad P_0 = 47 \text{ mm Hg} \\
\text{time constant} & \quad \tau_0 = 0.2 \text{ sec}
\end{align*}
\quad (33)
\]

and using equation (29) one finds that

\[ (P_0 - P_1)(C_{V_0} + C_{M_0}) \times \tau_0 = 1.95 \times 10^4 \text{ barias cm}^3 \quad (34) \]

If for \( P_1 \), the pressure in the gas container, one takes the average value as \( \frac{P_0}{2} \), where \( P_0 \) is the pressure in the gas storage region, then (34), with (31), (32) and (33) yields the following value for the radius of the pipe

\[ a_0 = 1.62 \times 10^{-2} \text{ cm} \quad (35) \]

and therefore a diameter of about

\[ \text{diameter} \approx 0.3 \text{ mm} \quad (36) \]
Vacuum Pumps

Every 3 sec the quantity of gas given in (29) is introduced into the system. Then the average flux is given by

\[
\dot{\Phi} = 5 \times 10^{-5} \text{ mm Hg litres/sec}
\]

(37)

If in the installation there is only the column pump (see fig. 1) with a speed \( S_c = 2,000 \text{ litres/sec} \) (CERN 60-26, page 57) the vacuum at the head of the pump is given by

\[
P_c = \frac{\dot{\Phi}}{S_c} = 2.5 \times 10^{-6} \text{ mm Hg}
\]

(38)

Vacuum Pump in the Expansion Chamber

An improvement on this pressure given by (38) can be obtained by pumping the expansion chamber.

The conductance from expansion chamber to the column is given by (Appendix 4)

\[
C_{M_c} = 34 \text{ litres/sec}
\]

(39)

if the speed of the vacuum pump connected to the expansion chamber is \( S_e = 150 \text{ litres/sec} \) the pressure is given by

\[
P_E = \frac{\dot{\Phi}}{S_e + C_{M_c}} = 2.7 \times 10^{-5} \text{ mm Hg}
\]

(40)

and the pressure in the column by

\[
P_c = \frac{P_E \times C_{M_c}}{S_c} = 4.6 \times 10^{-7} \text{ mm Hg}
\]

(41)
This value is considerably better than is required in the column so that, if it is so desired, the pressure in the gas container can be increased to obtain better discharge conditions at the gun.

**Parameters of electromagnetic hummer**

It has been found before in (25) and (26) that the time dependence of the pressure in the system has a characteristic time constant of about one hundred microseconds. It is necessary therefore that the valve should be opened in about the same time. Mechanically the problem is not easy. Existing relays have time constants of some milliseconds. For this reason it is necessary to design a fast system for opening the valve.

In fig. 3 is shown a sketch of the design. The system which opens the valve consists of a capacitor \( C_H \) charged at a voltage \( V_H \) which is discharged across a solenoid of self-induction \( L_H \).

The equation of motion of the valve in the longitudinal direction is given by

\[
\sum F = m \cdot a
\]

where \( \sum F \) takes into account all the forces i.e. electromagnetic spring and friction forces.

It is assumed that the electromagnetic forces are very much greater than the others. The electromagnetic forces are given by

\[
F_{\text{elec-mag}} = \int \int_S \left( \frac{B^2}{8\pi} \right) dS = \frac{S}{8\pi} (B^2)_a
\]

The magnetic field in the centre of \( N \) loops is given by

\[
B_c = \int \frac{NI}{r^2} \frac{dl \times \vec{r}}{r} = \frac{2\pi NI}{r}
\]
It is assumed that

$$E_c^2 = (E^2)_{av}$$  \hspace{1cm} (45)$$

Integrating (42) and taking into account (43), (44) and (45) it is found that

$$\frac{\pi^2 N^2}{2} \int_0^T I^2 dt = m v (T)$$  \hspace{1cm} (46)$$

The current $I$ across the loops is in the form

$$I = I_o e^{-t/\tau} \sin \omega t$$  \hspace{1cm} (47)$$

where $\tau = 2 \frac{L}{R}$ and $\omega = \sqrt{\omega_o^2 - \frac{1}{\tau^2}}$.

The equation (46) may be integrated (Appendix 5) and may be shown to reduce to

$$\frac{\pi^2 N^2 I_o^2}{4} T = m v (T)$$  \hspace{1cm} (48)$$

where it has been assumed that $\frac{1}{\tau^2} \ll \omega^2$ and $\tau \gg T$.

The average acceleration $G_{av}$ is given by

$$G_{av} = \frac{v(T)}{T} = \frac{\pi^2 N^2 I_o^2}{4 m}$$  \hspace{1cm} (49)$$

The time taken by the valve to open a distance $a$ is given by

$$T = \frac{\sqrt{8 m}}{\pi N I_o} \sqrt{a}$$  \hspace{1cm} (50)$$
The quantity $NI_o$ in (50) is expressed as a function of $C_H$, $V_H$ and $L_H$ by (Appendix 6)

$$NI_o = \frac{1}{10} \sqrt{\frac{C_H \text{ (farads)}}{V_H \text{ (volts)}}} \sqrt{\frac{N^2}{L_H \text{ (henrys)}}} \text{ u.e.m.}$$  \hspace{1cm} (51)

Over a length of path $a_d$ the rubber expands simultaneously with the valve and the dead time $T_d$ introduced is given by

$$T_d = \frac{\sqrt{8 m'}}{\pi NI_o} \sqrt{a_d}$$  \hspace{1cm} (52)

Taking into account that the $L_H$ self-induction may be expressed in the form

$$L_H = K \cdot r N^2$$  \hspace{1cm} (53)

where $K$ is a form factor and $r$ is the radius of the loops.

Then (50) with (51) and (53) gives

$$T = \frac{10\sqrt{8m(\text{gr})}}{\pi} \sqrt{\frac{K \cdot r \text{ (henrys)}}{2 E \text{ (joules)}}} \sqrt{a(\text{cm})} \text{ sec}$$  \hspace{1cm} (54)

**Numerical values of the project**

The mass $m$ of the valve has a value (Appendix 7)

$$m = 8.15 \text{ gr}$$  \hspace{1cm} (55)

The paths $a_d$ and $a$ are taken as

$$a_d = 0.05 \text{ cm and } a = 0.15 \text{ cm}$$  \hspace{1cm} (56)
For the factor $K \cdot r$ is found the following value (Appendix 8)

$$K \cdot r = 2.58 \times 10^{-8} \text{ henrys} \quad (57)$$

The capacitor is taken with the values

$$C = 25 \times 10^{-6} \text{ farads}; \quad V = 3.800 \text{ volts}; \quad E = \frac{1}{2} CV^2 = 180 \text{ joules} \quad (58)$$

Substituting in (54) the values (55), (56), (57) and (58), it is found that

$$T = 84 \times 10^{-6} \text{ sec}; \quad T_d = 48.6 \times 10^{-6} \text{ sec}; \quad T - T_d = 35.4 \times 10^{-6} \text{ sec} \quad (59)$$

The number $n$ of loops $N$ is taken as

$$N = 3 \text{ loops} \quad (60)$$

The selfinductance of the loops $L_H$ is obtained from (53) taking into account (57) and (60) as

$$L_H = 2.32 \times 10^{-7} \text{ henrys} \quad (61)$$

For $\omega_0$ is found

$$\omega_0 = \frac{1}{\sqrt{LC}} = 4.15 \times 10^5 \text{ sec}^{-1} \quad (62)$$

The number of half-periods $n$ is given by

$$n = \frac{\omega T}{\pi} = 11 \quad (63)$$

It is found that there is an error of less than 10 $\%$ in (59) if $T > 10 \text{ T}$, and therefore the resistance in the circuit has the value
\[ R_H = \frac{2L}{C} \leq 5.6 \times 10^{-3} \Omega \]  \hspace{1cm} (64)

The maximum current \( I_0 \) that flows in the loops is given by

\[ I_0 = \omega CV = 39.4 \times 10^3 \text{ A} \]  \hspace{1cm} (65)

After each pulse the capacitor is charged with a resistance \( R_e \) and a time constant as

\[ R_e C = 0.65 \text{ sec} \]  \hspace{1cm} (66)

With (66) and (58) it is found that

\[ R_e = 24 \text{ K} \Omega \]  \hspace{1cm} (67)

in which is dissipated an average power of

\[ P_R = \frac{V^2}{2} \frac{C}{3} = 60 \text{ Watts} \]  \hspace{1cm} (68)

The voltage \( V_H \) at which the capacitor is next discharged is:

\[ V_H(t = 3 \text{ sec}) = 0.99 V_H(t = \infty) \]  \hspace{1cm} (69)

**Discharge in the plasma gun**

A capacitor with a capacity \( C_g \) charged at a voltage \( V_g \) is discharged across the plasma gun. In the circuit there is the resistance \( R_g \) and a self-inductance \( L_g \). The current \( I_g \) is given by

\[ L_g \frac{d^2 I_g}{dt^2} + R_g \frac{dI_g}{dt} + \frac{I_g}{C_g} = 0 \]  \hspace{1cm} (70)
It is convenient that the current $I_g$ is not oscillating and this is achieved by making

$$R^2_g = 4 \frac{L}{C_g}$$  \hspace{1cm} (71)

The solution of equation (70) with (71) gives (Appendix 9)

$$I_g = \frac{V}{L_g} t e^{-t/\sqrt{L C_g}}$$  \hspace{1cm} (72)

The maximum current $I_{gm}$ occurs in the time $t = \sqrt{L C_g}$ which on substitution into (72) is to be

$$I_{gm} = \frac{V \sqrt{C_g}}{e \sqrt{L}}$$  \hspace{1cm} (73)

The ion current in the ejected plasma is approximately proportional to the $I$. This fact is observed in (LANS 2570 page 14).

Since the plasma gun may be useful as an ion source for the CPS it would be convenient if the pulse were as square as possible and with a length of 10 $\mu$sec

If the following condition is imposed

$$\frac{\Delta I_i}{I_{im}} = x$$  \hspace{1cm} (74)

then (72) gives the following values for $t$ as a function of $x$ (Appendix 10)

$$t_1 = \frac{1 - \sqrt{2x(1-x) + x^2}}{1 - x} \sqrt{L C_g}$$  \hspace{1cm} (75)

$$t_2 = \frac{1 + \sqrt{2x(1-x) + x^2}}{1 - x} \sqrt{L C_g}$$  \hspace{1cm} (76)
and

$$t_2 - t_1 = 2 \sqrt{\frac{2}{1 - x} \left( \frac{1}{1 - x} + \frac{x^2}{1 - x} \right) \sqrt{\frac{L}{g}} g \varepsilon}$$

(77)

Consider \( x = 10^{-2} \); this represents an error in \( I_1 \) given by

$$\frac{\Delta I_1}{I_1 \text{ aver}} = \pm 0.5 \, \text{o/o}$$

(78)

Substituting in (75), (76) and (77) it is obtained

$$t_1 = 0.867 \sqrt{\frac{L}{g} \varepsilon} g$$

(79)

$$t_2 = 1.152 \sqrt{\frac{L}{g} \varepsilon} g$$

(80)

$$t_2 - t_1 = 0.285 \sqrt{\frac{L}{g} \varepsilon} g$$

(81)

Finally imposing the condition that

$$t_2 - t_1 = 0.285 \sqrt{\frac{L}{g} \varepsilon} g = 10^{-5} \text{ sec}$$

(82)

it is found that

$$\sqrt{\frac{L}{g} \varepsilon} g = 35.1 \times 10^{-6} \text{ sec}$$

(83)

The voltage \( V_g \) is taken as \( V_g = 2,000 \) volts.

With condition (83) different values of \( C_g \) and \( L_g \) the maximum current \( I_{gm} \) (from (73)) varies as does the nature of plasma jet. It is necessary to find the different values of \( C_g \) and \( L_g \) which gives rise to the optimum conditions in the plasma jet.

Take first about the same values of capacity \( C_g = 100 \mu \text{f} \) that were employed in (Physics of fluids 1961, page 1065). Then the self-induction is given by

$$L_g = 12.3 \mu \text{H}$$

(84)
and the resistance $R_g$ by

$$R_g = 0.702 \, \Omega$$  \hspace{1cm} (85)

The maximum current in plasma gun by (73) as

$$I_{gm} = 2.1 \times 10^3 \text{ Ampères}$$  \hspace{1cm} (86)

The energy stored in the capacitor is given by

$$E_g = \frac{1}{2} CV^2 = 200 \text{ joules}$$  \hspace{1cm} (87)

The charging resistor is given by

$$R_g = 6.5 \times 10^3 \, \Omega$$  \hspace{1cm} (88)

with a time constant for charging the capacitor

$$\tau_g = R_g \cdot C_g = 0.65 \text{ sec}$$  \hspace{1cm} (89)

**Ion Extraction**

Fig. 4 shows the design of the extractor electrode. It is convenient to avoid the plasma that arrives in the first 30 μsec and then the pulsing network shown in figure 2 is used. During the first 30 μsec the capacitor $C_s$ is partially discharged and ions are not extracted. At 30 μsec the extraction pulse of 60 kV is applied for 10 μsec and ions pass into the accelerator column (see fig. 2).

**Divergence of the source**

If $D$ is the distance between the plasma gun and exit electrode, $r_{32}$ is the radius of the cylinder in plasma gun and $r_s$ is the radius of the exit hole, the maximum divergence angle $\alpha$ is given by
\[ \alpha = \frac{r_{52} + r_{8}}{D} \]  

(90)

If a maximum divergence of \( \alpha \lesssim 0.05 \) radians is desired, then by taking into account (13) and (39-5) it is found that

\[ D \gtrsim 31 \text{ cm} \]  

(91)

(In CPS \( \alpha \approx 0.23 \) radians).

Calculations for extracting \( 10^{14} \) ions of hydrogen in 10 \( \mu \text{sec} \)

In fig. 4 is shown a diagram of the design of the extractor electrode. If it is desired that \( N_i = 10^{14} \) ions extracted in 10 \( \mu \text{sec} \), the ion current \( I_i \) is

\[ I_i = \frac{N_i e}{t_2 - t_1} = 1.6 \text{ Ampères} \]  

(92)

Taking into account the fact that the area of extraction is of 1 \( \text{cm}^2 \).

The current density \( J_i \) is given by

\[ J_i = 1.6 \text{ Ampères/cm}^2 \]  

(93)

The distance \( D \) in (91) and the pressure \( P_{11} \) is found by experiment and may be arranged so that in the times given by (79) and (80), the ionic current in the plasma is about 1.6 Ampères/cm\(^2\).

The value of the current density from space charge considerations may be found and is given as

\[ J_{\text{sp.ch.lim.}} = \frac{1}{9} \pi \sqrt{\frac{2e}{m_i}} \frac{V_E}{d^2} \]  

(94)

Applying \( V_E = 60 \text{ kV} \) at extraction and taking into account (93), the value for \( d \) the electrodes distance, is found to be (Appendix 11)
With the object of obtaining a focusing effect in the extractor, the distance between electrodes can be 5 mm and then the electric field is pulsed of 12 kV/mm. In order to avoid space charge expansion in the beam it is convenient that the acceleration be with a high electric field, about 40 kV/cm, similar to that employed in UCRL 9743 or ORNL 3011.

VII. Discussion and Conclusions

From the design data given above, it seems possible that a plasma gun might be developed into a very high intensity ion source. Such a gun used in conjunction with an expansion chamber and an extraction device might be usefully employed to inject ions into the first stages of an accelerating structure (such as the Linac of the CPS). This design has a certain major advantage over the conventional ion source in that ions may be extracted from a hole of surface area 1 cm² or more without upsetting the vacuum conditions in the accelerating column.

It has been shown (LANS - 2444) that a plasma gun is capable of producing a 20 μsec pulse of plasma. The fundamental question as to whether or not such a plasma can be made to produce a reproducible and constant ion current of duration about 10 μsec is not easily answered except by experiment.

As is usual in problems concerning plasmas, the number of parameters available for variation is very large. This fact seems to be encouraging so far as an experimental investigation of a plasma gun ion source is concerned.

J. Lozano Campoy

Distribution: (Open)
AR Division
FS Library

PS/3136
APPENDIX I

Calling $P_{11}$ the pressure in the gas container and $P_{12}$ the pressure outside, for each film of gas of $\delta Z$ gross, in steady state, the sum of the forces of pressure and the viscosity have to be zero.

The pressure forces are given by

$$(P_{11} - P_{12}) 2\pi \tau_1 \delta Z = (P_{11} - P_{12}) \delta S$$  \hspace{1cm} (2-1)

The viscosity forces which act on the film are

$$\delta F_v = \eta \left[ \frac{\delta u(r + \delta r/2)}{\delta Z} \frac{2\pi r \tau_1}{\delta \tau} - \frac{\delta u(r - \delta r/2)}{\delta Z} \frac{2\pi r \tau_1}{\delta \tau} \right]$$  \hspace{1cm} (2-2)

Adding (2-1) and (2-2) and simplifying is obtained

$$P_{11} - P_{12} = - \eta \frac{\delta^2 u}{\delta Z^2}$$  \hspace{1cm} (2-3)

Integrating (2-3) and putting in the boundary conditions which are

$$\left( \frac{\delta u}{\delta Z} \right)_{Z=0} = 0$$  

and

$$u_{Z=a/2} = 0$$  \hspace{1cm} (2-4)

is obtained

$$u = \frac{P_{11} - P_{12}}{2\eta \tau_1} \left( \frac{a}{2} - Z^2 \right)$$  \hspace{1cm} (2-5)

The gas flux across the film is given by

$$\delta V = u \delta S = \frac{P_{11} - P_{12}}{2\eta \tau_1}$$  \hspace{1cm} (2-6)
The total volume of gas is obtained by integrating (2-6) and gives

\[ V_1 = \frac{(p_{11} - p_{12}) \pi r_1 a^3}{6\eta_1} \]  

(2-7)

The total flux is given by

\[ \left( \frac{p_{11} + p_{12}}{2} \right) V = \frac{p_{11} - p_{12}}{6\eta_1} \pi r_1 a^3 \left( \frac{p_{11} + p_{12}}{2} \right) \]  

(2-8)

From (2-8) it is found that the viscous conductance \( C_V \) is

\[ C_V = \frac{(p_{11} + p_{12})V}{2(p_{11} - p_{12})} = \frac{\pi r_1 a^3}{6\eta_1} \left( \frac{p_{11} + p_{12}}{2} \right) \]  

(2-9)
APPENDIX 2

The molecular conductance \( C_M \) is given by

\[
C_M = \frac{4}{3} \frac{V_{av}}{\int_0^1 \frac{H}{A^2} \, dl} \tag{3-1}
\]

where \( V_{av} \) is the average velocity, \( H \) is the perimeter, \( A \) is the area of cross section and \( l_1 \) is the longitude conduction.

The average velocity is given by

\[
V_{av} = 14.55 \times 10^3 \sqrt{\frac{T}{M}} \text{ cm/sec} \tag{3-2}
\]

where \( T \) is the temperature in \( ^\circ \text{K} \) and \( M \) is the molecular weight.

Taking into account that

\[
H = 4\pi r_1 \tag{3-3}
\]

and

\[
A = 2\pi r_1 a \tag{3-4}
\]

equation (3-1) is transformed in c.g.s. system

\[
C_M = 7.4 \times 10^5 \frac{r_1 a^2}{l_1} \text{ cm}^3/\text{sec} \tag{3}
\]
If \((P_{21} - P_{22})\) is the difference in pressure along a cylindrical tube of longitude \(l\), for a film of \(\delta r\) gross, in steady state, the sum of the forces must be zero, and again

\[
(P_{21} - P_{22}) \frac{2\pi \delta r}{\eta l_2} = -\eta_2 \left[ \frac{\delta u(r + \frac{\delta r}{2})}{\delta r} 2\pi \left(r + \frac{\delta r}{2}\right) - \frac{\delta u(r - \frac{\delta r}{2})}{\delta r} 2\pi \left(r - \frac{\delta r}{2}\right) \right]
\]  

(5-1)

Operating on (5-1) and simplifying it is found that

\[
-\frac{(P_{21} - P_{22}) r}{\eta l_2} = \frac{\delta u}{\delta r} + r \frac{\delta^2 u}{\delta r^2} = \frac{\delta}{\delta r} \left(r \frac{\delta u}{\delta r}\right)
\]

(5-2)

Integrating (5-2) it is found that

\[
\frac{\delta u}{\delta r} = -\frac{(P_{21} - P_{22})}{2\eta l_2} r + \frac{A}{r}
\]

(5-3)

where \(A\) is an integration constant.

Integrating (5-3) once again it is found that with an integration constant \(B\),

\[
u = A \ln r - \frac{P_{21} - P_{22}}{4\eta l_2} r^2 + B
\]

(5-4)

The boundary conditions of the problem are

\[
\begin{cases}
r = r_{21}, & u = 0 \\
r = r_{22}, &
\end{cases}
\]

(5-5)
which putting in (5-4) gives

\[ A = \frac{(P_{21} - P_{22})}{4 \eta l_2} \left( \frac{r_{22}^2 - r_{21}^2}{\ln r_{22}/r_{21}} \right) \]  \hspace{1cm} (5-6)

and

\[ B = \frac{(P_{21} - P_{22})}{4 \eta l_2} \left( \frac{r_{21}^2 \ln r_{22} - r_{22}^2 \ln r_{21}}{\ln r_{22}/r_{21}} \right) \]  \hspace{1cm} (5-7)

and therefore (5-4) is transformed in

\[ u = \frac{(P_{21} - P_{22})}{4 \eta l_2} \left( \frac{r_{22}^2 \ln r/r_{21} + r_{21}^2 \ln r/r_{22}}{\ln r_{22}/r_{21}} - r^2 \right) \]  \hspace{1cm} (5-8)

The differential volume flux across the δr is given by

\[ \delta V = u \delta S = \frac{\pi(P_{21} - P_{22})}{2 \eta l_2} \left[ \frac{r_{22}^2 \ln r/r_{21} + r_{21}^2 \ln r/r_{22}}{\ln r_{22}/r_{21}} - r^2 \right] \delta r \]  \hspace{1cm} (5-9)

Integrating (5-9) yields

\[ V = \frac{\pi(P_{21} - P_{22})}{8 \eta l_2} \left[ r_{22}^4 - r_{21}^4 - \frac{(r_{22}^2 - r_{21}^2)^2}{\ln r_{22}/r_{21}} \right] \]  \hspace{1cm} (5-10)

Multiplying (5-10) by the mean pressure \( \left( \frac{P_{21} + P_{22}}{2} \right) \) yields the flux across the cylindrical tube

\[ \left( \frac{P_{21} + P_{22}}{2} \right) V = \frac{\pi(P_{21} - P_{22})}{8 \eta l_2} \left[ r_{22}^4 - r_{21}^4 - \frac{(r_{22}^2 - r_{21}^2)^2}{\ln r_{22}/r_{21}} \right] \left( \frac{P_{21} + P_{22}}{2} \right) \]  \hspace{1cm} (5-11)
and the conductance is given as

\[
C_v = \frac{(P_{21} + P_{22})V}{2(P_{21} - P_{22})} = \frac{\pi \left( \frac{P_{21} + P_{22}}{2} \right)}{8\eta L_2} \left[ \left( \frac{X_{22}^4 - X_{21}^4}{r_{22}^2 - r_{21}^2} \right) - \frac{(r_{22}^2 - r_{21}^2)^2}{\ln \frac{r_{22}}{r_{21}}} \right]
\]  (5)
APPENDIX 4

The molecular conductance of two holes of area $A_1$ and $A_2$ in series is given by

$$\frac{1}{C_{NC}} = \frac{1}{C_{nA_1}^M} + \frac{1}{C_{nA_2}^M} \quad (39-1)$$

where $C_{nA_1}^M$ and $C_{nA_2}^M$ are functions of $A_1$ and $A_2$ of the form

$$C_{nA_1}^M = \frac{A_1 V_{av}}{4} \quad (39-2)$$

$$C_{nA_2}^M = \frac{A_2 V_{av}}{4} \quad (39-3)$$

Taking into account that in the project

$$A_1 = \pi \text{ cm}^2 \quad (39-4)$$

and

$$A_2 = 1 \text{ cm}^2 \quad (39-5)$$

then (39-1) gives for $C_{NC}^M$ the value

$$C_{NC}^M = \frac{V}{4} \frac{A_1 \cdot A_2}{A_1 + A_2} = 34 \text{ litres/sec} \quad (39)$$
APPENDIX 5

The value of \( \int_{0}^{T} 1 \frac{2}{\omega} e^{-2t/\tau} \sin^{2} \omega t \), taking \( T = n \frac{\pi}{\omega} \), where \( n \) is an integer, is given by

\[
J = \frac{1}{2} \int_{0}^{\pi} \frac{\frac{n}{\omega}}{1 \frac{2}{\tau} + 4\omega^{2}} \left[ (-2 \frac{\omega}{\tau} \sin \omega t - 2 \omega \cos \omega t) \right]^{2n} \rho \omega \sin \omega t - \omega \frac{e^{-2t/\tau}}{\omega} \bigg|_{0}^{\pi/\omega} \tag{48-1}
\]

which may be reduced to

\[
J = \frac{1}{2} \frac{\omega^{2} \tau^{3}}{4(1 + \omega^{2}\tau^{2})} \left[ 1 - e^{-2n\frac{\omega}{\tau}} \right] \tag{48-2}
\]

Developing the series \( e^{-2n\frac{\omega}{\tau}} \) gives

\[
e^{-2n\frac{\omega}{\tau}} = 1 - \frac{2n\frac{\omega}{\tau}}{} + \frac{1}{2} \left( \frac{2n\frac{\omega}{\tau}}{} \right)^{2} - \ldots \tag{48-3}
\]

Taking the two first terms, and substituting in (48-2) yields

\[
J = \frac{1}{2} \frac{\omega^{2} \tau^{3}}{4(1 + \omega^{2}\tau^{2})} \tag{48-4}
\]

with a negative error of

\[
\frac{\Delta J}{J} = \frac{n\frac{\omega}{\tau}}{\tau} = \frac{T}{\tau} \tag{48-5}
\]

If \( \omega^{2}\tau^{2} \gg 1 \) is obtained

\[
J = \frac{1}{2} \frac{\tau}{2} \tag{48-6}
\]
APPENDIX 6

It is wished to know the value $N I_0$ in (50). Taking into account that

$$\int_{0}^{\pi/2} I \, dt = Q = CV \quad (51-1)$$

it is found that

$$CV = I_0 / \omega \quad (51-2)$$

and therefore

$$NI_0 = \sqrt{CV^2} \sqrt{\frac{N_0^2}{L}} \quad (51-3)$$

As the number $NI_0$ in (50) is expressed in u.e.m. system, equation (51-3) may be written

$$NI_0 = \frac{1}{10} \sqrt{C(\text{farads})} \sqrt{V^2(\text{volts})} \sqrt{\frac{N_0^2}{L(\text{henrys})}} \text{ u.e.m.} \quad (51)$$
APPENDIX 7

The mass $m$ of the valve is given by

$$m = V_p$$  \hspace{1cm} (55-1)

The values of the project are:

**Stem**
- longitude $l_v = 3.2$ cm
- radius $r_v = 0.4$ cm
- volume $V_v = 1.61$ cm$^3$

**Platform**
- length $l_p = 0.2$ cm
- radius $r_p = 1.5$ cm
- volume $V_p = 1.41$ cm$^3$

If aluminium is employed, the density of which is $\rho = 2.7$ grs/cm$^3$, the (55-1) gives

$$m = (V_v + V_p) \rho = 8.1 \text{ grs}$$  \hspace{1cm} (55)
APPENDIX 8

The self-induction of a circular coil of circular cross section is given by

\[ L = K r N^2 = 1.257 \times 10^{-8} \left[ \ln \frac{16r}{d} - 1.75 \right] r \text{ Henrys} \tag{57-1} \]

of which \( K r \) has the value

\[ K r = 1.257 \times 10^{-8} \left[ \ln \frac{16r}{d} - 1.75 \right] r \text{ Henrys} \tag{57-2} \]

where \( r \) is the radius of the coil and \( d \) is the diameter of the winding.

For the project the following values are taken

\[ r = 1.2 \text{ cm} \]
\[ d = 0.6 \text{ cm} \tag{57-3} \]
\[ \ln \frac{16r}{d} = 3.46 \]

which substituted in (57-1) gives

\[ K r = 2.58 \times 10^{-8} \text{ Henrys} \tag{57} \]
APPENDIX 9

The equation
\[ L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = 0 \]

with the condition \( R^2 = 4 \frac{L}{C} \) and with \( V_0 \) initial voltage reduced to

\[ \frac{d^2 I}{dt^2} + \frac{2}{\sqrt{LC}} \frac{dI}{dt} + \frac{I}{LC} = 0 \]  \hspace{1cm} (72-1)

A solution of (72-2) is

\[ I = Ae^{\omega t} + Be^{\omega t} \]  \hspace{1cm} (72-2)

differentiating (72-2)

\[ \frac{dI}{dt} = Ae^{\omega t} (1 + \omega t) + Be^{\omega t} \]  \hspace{1cm} (72-3)

and

\[ \frac{d^2 I}{dt^2} = \omega Ae^{\omega t} (2A + twA + B) \]  \hspace{1cm} (72-4)

Substituting (72-3) and (72-4) in (72-1) results in

\[ (\omega^2 + \frac{2}{\sqrt{LC}} \omega + \frac{A}{LC}) t + 2A\omega + B\omega^2 + \frac{2A}{\sqrt{LC}} + \frac{2B}{\sqrt{LC}} + \frac{B}{LC} = 0 \]  \hspace{1cm} (72-5)

The (72-5) condition is for all \( t \) and thus

\[ \omega^2 + \frac{2}{\sqrt{LC}} \omega + \frac{A}{LC} = 0 \]  \hspace{1cm} (72-6)

PS/3136
\[ 2A\omega + B\omega^2 + \frac{2A}{\sqrt{LC}} + \frac{2B}{\sqrt{LC}} + \frac{B}{LC} = 0 \quad (72-7) \]

from (72-6) is obtained

\[ \omega = -\frac{1}{\sqrt{LC}} \quad (72-8) \]

and from (72-7)

\[ B = 0 \quad (72-9) \]

Putting the initial conditions \( \left( \frac{dI}{dt} \right)_{t=0} = \frac{V}{L} \) the (72-2) gives

\[ I = \frac{V}{L} e^{-\frac{t}{\sqrt{LC}}} \quad (72) \]
APPENDIX 10

The equation \( I = \frac{V}{L} t e^{-t/\sqrt{LC}} \) (72) with the condition \( \frac{\Delta I}{I_m} = x \) (74) is resolved as follows:
The relation \( \frac{\Delta I}{I_m} = x \) is put in the form

\[
I = (1 - x) I_m \quad (75-1)
\]

Substituting in (72) the (75-1) and taking into account (73) it is found that

\[
1 + \frac{\Delta t}{\sqrt{LC}} = (1 - x) e^{\Delta t/\sqrt{LC}} \quad (75-2)
\]

where \( \Delta t \) is given by

\[
\Delta t = t - \sqrt{LC} \quad (75-3)
\]

Developing in series \( e^{\Delta t/\sqrt{LC}} \) gives

\[
e^{\Delta t/\sqrt{LC}} = 1 + \frac{\Delta t}{\sqrt{LC}} + \frac{1}{2} \frac{\Delta t^2}{LC} + \quad (75-4)
\]

and taking the first three terms in (75-4) and substituting in (75-2) yields

\[
(1 - x) \frac{\Delta t^2}{LC} - 2 x \frac{\Delta t}{\sqrt{LC}} - 2x = 0 \quad (75-5)
\]

and the time \( \Delta t \) is given by

\[
\Delta t = \left( \frac{x + \sqrt{2 x (1 - x) + x^2}}{1 - x} \right) \sqrt{LC} \quad (75-6)
\]
and the times from (75-3) are given as

\[ t_1 = \left( \frac{1 - \sqrt{2 \times (1 - x) + x^2}}{1 - x} \right) \sqrt{Lo} \]

\[ t_2 = \left( \frac{1 + \sqrt{2 \times (1 - x) + x^2}}{1 - x} \right) \sqrt{Lo} \]

(75-7)
APPENDIX 11

The current density for extracting $10^{14}$ ions of hydrogen in 10 µsec in 1 cm$^2$ of surface area is given by

$$ j = \frac{I_1}{S_1} = \frac{N_1 \times e}{t_2 - t_1} S_1 = 1.6 \frac{\text{Amp}}{\text{cm}^2} = 4.8 \times 10^9 \frac{\text{u.e.s.}}{\text{cm}^2} \quad (95-1) $$

The current density space charge considerations are given by

$$ j_{\text{sp.ch.l.}} = \frac{1}{9\pi} \frac{3/2}{\sqrt{2e}} \frac{V_E}{d^2} \quad (95-2) $$

Putting $V_E = 60.000$ volts, and equating (95-1) and (95-2) yields

$$ d^2 = \frac{2e}{9\pi} \frac{1}{m_1} \frac{V_E^{3/2}}{4.8 \times 10^9} \quad (95-3) $$

where

$$ \sqrt{\frac{2e}{m_1}} = 2.40 \times 10^7 \text{ (u.e.s.)} \quad (95.4) $$

and

$$ \frac{V_E^{3/2}}{E} = 2.82 \times 10^3 \text{ (u.e.s.)} \quad (95-5) $$

and thus equation (95-3) gives

$$ d = 7.05 \text{ mm} \quad (95) $$

PS/3136
Insulator  Metal
1. Gas container  4. Electromagnetic housing (coil)
2. Intermediate zone  5. Spring

Fig. 3
1. Plasma gun
2. Expansion chamber
3. Exit hole
4. Extraction electrode
5. Acceleration

Fig 4