PROPOSAL FOR INCREASING THE STABILITY OF THE VAN DE GRAAFF

I. Introduction

It has been suggested that it would be an advantage to stabilize the 2 MV Van de Graaff to one part in ten to the four. The advantages to be gained by achieving such a stability are twofold: firstly, the tolerances on the injection scheme are so rigorously maintained that the use of a beam spreader for easing such tolerances becomes unnecessary. Secondly, by eliminating the beam spreader, the probability of capturing nearly all the beam is much increased.

The two stabilizers supplied with the Van de Graaff, a belt current control, and a feedback amplifier used in conjunction with a capacitative liner, are not capable of controlling the voltage to one part in ten to the four, nor would they be capable of recovering in time from the transients which will be imposed on the top terminal. Moreover, it does not seem as if any possible modification can result in their meeting the necessary requirements.

However, a stabilizer which controls the voltage by its ability to vary the current drawn from one of the Van de Graaff's two beams will meet the requirements. Since the maximum steady current which it is prudent to carry on the belt is 1/2 mA and we can reduce the control-beam current to practically nothing we can in effect supply 1/2 mA to the top terminal to increase its potential. The top terminal capacity is 100 pF and thus we can recover from a 100 kV transient in some 20 ms. It will be shown that in order to hold the potential to one part in ten to the four (i.e. 200 V in 2 MV) the system must have a response time of less than 70 µs. The beam which is used for exerting this control is also used for locking the top terminal potential to the storage ring field, by passing it into a magnetic spectrometer (as was suggested by M. J. Pentz and D. A. Swenson).
The stabilizer is described first summarily in the following pages and in greater detail in the appendices.

II. Spectrometer Probe

The magnet for the spectrometer is to be similar to those used for "bending" the electrons around the storage ring and this, with reasonable spectrometer dimensions (8 metres between Van de Graaff and probe) yields a value for the resolution of about 800 V per mm deflection of the nominally 2 MeV beam. Since it is necessary to monitor the beam energy to 200 eV, the spectrometer probe must be able to resolve deflections of the order of 0.25 mm.

If a metal probe is used to detect this beam, the vacuum problems would be relatively easy, but a simple calculation shows that the average current must not exceed 1 or 2 µA if the metal of the probe is not to evaporate. Measurement of average currents of this order of magnitude which may consist of pulses of charge arriving at the probe at the required repetition rates (see appendix I) of about $10^6$ p.p.s. would be difficult, especially as the background "pick-up" and noise level in the region of the Van de Graaff is an unknown quantity at present. It was therefore decided to employ a scintillating crystal coupled with a photo multiplier to detect the electrons. This scintillator is to be mounted outside the vacuum system and the electrons strike the crystal after transmission through a suitable vacuum tight "window". With this method the number of electrons falling on the metal "window" can be reduced to negligible proportions so that no heating and therefore negligible outgassing of the metal occurs. Even with only a hundred or so electrons passing into the scintillator the output signal from the photomultiplier can amount to volts. This large output signal can then be conveniently used to close the loops of the servo system controlling the amount of charge drawn in some appropriate manner from the top terminal of the Van de Graaff. A diagram of the probe is given at the end of this report.
The window and flange is machined out of one solid piece of aluminium. The 2 MeV electrons will be transmitted through 0.3 mm to 0.5 mm of aluminium. This thickness is still relatively strong from the vacuum point of view and further, electrons will not be scattered by more than 0.3 to 0.5 mm to one side of the point of incidence (see diagram at end of report) so that the resolution of the spectrometer is not upset. The actual resolution of such a probe is to be tested by means of a β-emitter and an arrangement of collimators.

Examination of the diagram of the probe shows that it will only give out a signal when the electrons, after having been deflected by the spectrometer magnet, pass into the scintillator or, because of scattering in the aluminium window, when they pass within about 0.3 mm of the discriminating edge of the crystal. The scintillator will be positioned so that the electrons falling within 0.25 mm on either side of its discriminating edge will have the required energy to within ±1 part in $10^4$.

III. Feedback Control System

The control-beam may consist of D.C. or pulsed current. A direct current would require modulation of the grid of the electron gun in the top terminal. If the control-beam is to be capable of coping with fast transients (with time constant in the range 10 to 100 µsec), the modulation of grid of the electron gun would require complicated electronics in the top terminal. This is undesirable. It would be better to have a simple standard-pulse generator in the top terminal triggered by means of a series of light-signals. This means a pulsed control-beam which can be made to cope with fast transients by simply choosing a suitable light signal generator. It must also be remembered that if a spectrometer is to be used as a monitor of the top terminal voltage then it is necessary that there be a minimum control-beam current flowing from the Van de Graaff.

It now remains to decide on a method of using the pulsed-beam, or rather using pulses obtained from the probe, to control the discharging rate of the top terminal so that a steady state can be reached wherein
the voltage of the Van de Graaff never overshoots the required voltage by more than 1 part in $10^4$.

There are two obvious ways of controlling the discharging rate i.e. increasing the average control-beam current. These are

(i) increasing or decreasing the charge per pulse
(ii) increasing or decreasing the repetition rate of the pulses.

Method (i) involves the more complicated electronics in the top terminal. However, a standard pulse generator in the top terminal could easily be used to control the discharging rate by method (ii).

If method (ii) is adopted, with the charging rates, tolerances on the voltage and response times given in the appendices, then a maximum repetition rate of one pulse per microsecond is found to be sufficient. The maximum average control-beam current is found to be about twice the belt charging current when the minimum beam current is about one tenth of the belt current.

The actual charging and discharging rates at any given time can be controlled so that the top terminal voltage always returns to within ± 1 part in $10^4$ of its required value. The largest voltage transient that the stabilizer can handle within the 20 ms between injection pulses is of the order of 100 kV and this limit is set, not by the stabilizer, but by the maximum current, stated to be 1/2 mA, that can be carried continuously by the belt.

The electronic and mechanical details of the method of controlling the charging and discharging rates, or in other words, controlling the pulsed repetition rate, are given in a series of appendices to this report.

Eifionydd JONES
Julian DOW

Distribution:
AR Division
Library

PS/2658
APPENDIX I

Feedback Equations

The feedback system may be described by the following block diagram:

![Block Diagram]

The top terminal voltage $V_T$ either decreases or increases or is constant according to the whether $I_b > I_o$, $I_b < I_o$ or $I_b = I_o$. $I_o$ is the belt charging current. $I_b$ is the average beam current drawn off the top terminal and is dependent upon the pulsing rate produced by the generator. The pulsed beam current is triggered by light signals.

When the electrons in the pulsed beam pass into the probe, the output $(V_p)$ from the photomultiplier may be described as follows:

\[ V_p = V_o \left( 1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{K(V_T - V_R)}{\sigma^2}} e^{-x^2} dx \right) \]

$V_R$ is the voltage at which the top terminal is to be held to 1 part in $10^4$. A sufficient approx. dependence of $V_p$ on $V_T$ is given by the error function:

\[ V_p = V_o \left( 1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{K(V_T - V_R)}{\sigma^2}} e^{-x^2} dx \right) \]

(1)
The constant $K$ in the upper limit is given by the discrimination of the probe, e.g. if $\Delta V_T \sim 200$ volts then $K \sim \frac{1}{50}$. The pulses $V_p$ are fed into a filter network which yields an output voltage $V_I$ which is independent of the repetition rate but is proportional to the magnitude of $V_p$. In other words the output $V_I$ of the filter describes the envelope of the input pulses.

The time of response of the filter is governed by a time constant $\lambda$ and the magnitude of this response by the gain factor $g$ in the following manner:

$$g \frac{dV_I}{dt} = V_I + \lambda \frac{dV_I}{dt}$$  \hspace{1cm} (2)

The generator produces pulses of constant height at a rate governed by the magnitude of the input signal $V_I$. The greater $V_I$ the greater the rate of generation of output pulses and consequently the larger the average beam current $I_b$ drawn from the top terminal. Thus $V_I$ may be related to $I_b$ as

$$I_b = \sigma V_I + I_{b\text{ min}}.$$  \hspace{1cm} (3)

where $\sigma$ is some arbitrary constant and $I_{b\text{ min}}$ is the minimum average beam current produced by the minimum pulsing rate of the generator. This minimum pulsing rate is so arranged that $I_{b\text{ min}}$ is about $1/10$th $I_o$. There is also a maximum pulsing rate such that $I_{b\text{ max}} = 2 I_o$. This value of $I_{b\text{ max}}$ is chosen primarily because the top terminal capacity may be discharged at least as quickly as it can be charged by $I_o$. The relation between $I_b$, $I_o$ and $V_T$ is, of course,

$$C \frac{dV_T}{dt} = I_o - I_b = CV_T.$$  \hspace{1cm} (4)

Thus when the feedback loop is closed, the equation for $V_T$ may be found from equations (1), (2), (3) and (4) and is given as
\[ \sigma g V_o \left\{ 1 + \frac{2}{\sqrt{\pi}} \int_0^K (V_T - V_R) e^{-x^2} \, dx \right\} = I_o - I_{b \text{ min}} - CV_T^* + \lambda I_o - \lambda CV_T^* \] (5)

when \( V_T \gg V_R \), equation (3) gives

\[ V_I = \frac{1}{\sigma} (I_{b \text{ max}} - I_{b \text{ min}}) = 0 \]

since \( I_{b \text{ max}} \) and \( I_{b \text{ min}} \) are constants imposed by the electronics.

Thus

\[ V_{I \text{ max}} = \frac{1}{\sigma} (I_{b \text{ max}} - I_{b \text{ min}}) \]

and therefore

\[ gV_p \text{ max} = g2V_o = V_{I \text{ max}} \]

and

\[ \sigma gV_o = 1/2 \left( I_{b \text{ max}} - I_{b \text{ min}} \right) \]

Now first assume \( K \sim 1 \) then when \( V_T > V_R \)

\[ (I_{b \text{ max}} - I_{b \text{ min}}) = I_o - I_{b \text{ min}} - CV_T^* + \lambda I_o - \lambda CV_T^* \] (6)

and when \( V_T < V_R \)

\[ 0 = I_o - I_{b \text{ min}} - CV_T^* + \lambda I_o - \lambda CV_T^* \] (7)

Putting

\[ L = I_{b \text{ max}} - I_o \text{ for } V_T > V_R \text{ and } L = I_{b \text{ min}} - I_o \text{ when } V_T < V_R \]
Then the following equation is to be solved:

\[
\lambda \dddot{V} + \ddot{V} - \frac{\lambda}{C} \dot{I}_o + \frac{L}{C} I_o = 0
\]  

(8)

where \( V = \bar{V} \) at \( t = 0 \)

and \( \frac{dV}{dt} = \frac{d\bar{V}}{dt} \) at \( t = 0 \)

Putting \( \frac{\lambda}{C} I_o = F(t) \) and \( \frac{L}{C} I_o = Q - \frac{I_o}{C} \); \( Q = \) constant

we have \( \lambda \dddot{V} + \ddot{V} + Q = \dddot{F}(t) + \frac{1}{\lambda} F(t) \)

put

\[
V = \frac{1}{2\pi i} \int_{X - i\infty}^{X + i\infty} v(s) e^{st} ds - Qt + \bar{V}
\]  

(9)

and

\[
F(t) = \frac{1}{2\pi i} \int_{X - i\infty}^{X + i\infty} f(s) e^{st} ds
\]  

(10)

then the equation (8) becomes, (taking \( X \) sufficiently far to the right so that all poles are included)

\[
\lambda s^2 \ddot{v}(s) + s \ddot{v}(s) = s f(s) + \frac{1}{\lambda} f(s)
\]

\[
v(s) = \frac{(s + \frac{1}{\lambda}) f(s)}{\lambda s(s + \frac{1}{\lambda})}
\]

\[
= \frac{f(s)}{s\lambda}
\]  

(11)

This solution is stable, if the poles of \( f(s) \) are at real negative \( s \), or imaginary \( s \).
Now \( f(s) = \int_0^\infty F(t) e^{-st} \, dt \)

and \( F(t) = \frac{\lambda}{C} I_0 \)

Assume \( I_0(t) = I_0 \left\{ 1 + \sum_{n=1}^{\infty} a_n e^{\pm i\omega_n t} \right\} \frac{-2\pi}{\omega_n} t + \frac{2\pi}{\omega_n} \) (12)

and \( F(t) = \frac{\lambda}{C} I_0 \left\{ 1 + \sum_{n=1}^{\infty} a_n e^{\pm i\omega_n t} \right\} \)

Thus \( f(s) = \int_0^\infty dt \left\{ \frac{\lambda}{C} I_0 \left\{ e^{-st} + \sum_{n=1}^{\infty} a_n e^{(\pm i\omega_n - s)t} \right\} \right\} \)

\[ = \frac{\lambda}{C} I_0 \left[ \frac{e^{-st}}{s} + \sum_{n=1}^{\infty} \frac{a_n e^{(\pm i\omega_n - s)t}}{1 + \frac{i\omega_n - s}{s}} \right] \]

\[ = \frac{\lambda}{C} I_0 \left[ \frac{1}{s} + \sum_{n=1}^{\infty} \frac{a_n}{s + i\omega_n} \right] \]

since in \( e^{i\omega_n t} \), \( t \) lies between \( \pm \frac{2\pi}{\omega_n} \), then \( \sum a_n e^{i\omega_n t} e^{-st} \to 0 \) as \( t \to \infty \)

Therefore \( v(s) = \frac{I_0}{C} \left[ \frac{1}{s^2} + \sum_{n=1}^{\infty} \frac{a_n}{s(s + i\omega_n)} \right] \) (13)

Thus the solution is stable for \( \omega_n \) real. If \( \omega_n \) is complex and given by

\[ \omega_n = a_n \pm i\beta_n \]
then the solution will be of the form

\[ e^{\pm \beta n t} + \ldots \ldots \ldots \]

However, for most purposes \( \omega_n \) can be taken to be a real number. If it is not real, it means that \( I_0 \) is probably decreasing continuously i.e. something wrong with the Van de Graaff which the control circuit is not able to cope with. The solution is then

\[ V_T = \frac{I_0}{C} t - Qt + \bar{V}_T + \frac{I_0}{C} \sum_{-\infty}^{\infty} \frac{a_n}{\omega_n} \sin \omega_n t \]  

(14)

The Van de Graaff is charged by means of current passing up a belt and it is more than likely that periodic variations of \( I_0 \) are slow (i.e. \( \frac{1}{\omega_n} \leq 10^{-3} \text{ secs} \)) and that \( a_n I_0 \ll I_0 \) (since the belt is equipped with a current stabilizer). Thus the magnitude of variations in the top terminal voltage \( V_T \) is governed mostly by fast transients e.g. due to electrical breakdown of top terminal or short time perturbations of \( I_0 \).

In this case equation (6) may be written, neglecting \( I_0 \), as

\[ \lambda \dot{V}_T + \dot{V}_T + \frac{L}{C} = 0 \]  

(15)

and the transients examined by imposing initial delta function like disturbances on \( V_T \).

This equation has the solution

\[ V_T = -\lambda \left( \frac{d\bar{V}_T}{dt} + \frac{L}{C} \right) \left( e^{-t/\lambda} - 1 \right) - \frac{L}{C} t + \bar{V}_T \]  

(16)

where \( V_T = \bar{V}_T \) at \( t = 0 \)

and \( \frac{dV_T}{dt} = \frac{d\bar{V}_T}{dt} \) at \( t = 0 \)
and \[ \dot{V} = \left\{ \frac{dV}{dt} + \frac{L}{C} \right\} e^{-t/\lambda} - \frac{L}{C} \] (16a)

Let us now take the case where \( V_T \) is increasing after suffering from a large negative transient, and has just become larger than \( V_R \) so that at \( t = 0 \), \( \dot{V}_T = V_R \), \( \frac{dV_T}{dt} = \frac{1}{C} (I_o - I_{b\,\text{min}}) \) and, of course, \( L \) has just become \( (I_{b\,\text{max}} - I_o) \).

Thus \[ V_T = -\lambda \left\{ \frac{1}{C} (I_o - I_{b\,\text{min}}) + \frac{1}{C} (I_{b\,\text{max}} - I_o) \right\} \left\{ e^{-t/\lambda} - 1 \right\} \]

\[ - \frac{(I_{b\,\text{max}} - I_o)}{C} t + V_R \]

\[ = -\frac{\lambda}{C} I_{b\,\text{max}} - I_{b\,\text{min}} (e^{-t/\lambda} - 1) - \frac{(I_{b\,\text{max}} - I_o)}{C} t + V_R \]

which for \( I_{b\,\text{max}} = 2I_o \gg I_{b\,\text{min}} \) yields

\[ V_T = +\frac{\lambda}{C} 2I_o (1 - e^{-t/\lambda}) - \frac{I_o}{C} t + V_R \]

Now \( V_T \) is given by \( \dot{V}_T = 0 \)

i.e. by \( e^{-t/\lambda} = 1/2 \) or \( t = \lambda \log_e 2 \approx 0.7\lambda \)

i.e. \[ V_{T\,\text{max}} = \frac{\lambda}{C} 2I_o (1 - 1/2) - \frac{I_o}{C} \lambda \log_e 2 + V_R \]

\[ V_{T\,\text{max}} - V_R = \frac{I_o}{C} \lambda (1 - \log_e 2) = \frac{I_o}{C} \cdot 3.3 \lambda \]

Then for \( V_{T\,\text{max}} - V_R = 100 \) volts, \( I_{b\,\text{max}} = 1 \) mA, \( C = 10^{-10} \) F

we find that

\[ \lambda \approx 70 \mu\text{secs.} \]
The equations also show that $V_T$ returns to $V_R$ in $t \sim \lambda$ at which time the volts are falling at the rate

$$\frac{dV_T}{dt} \frac{\Omega}{-1/3} \frac{I_o}{C} \text{ for } V_T \geq V_R = 0$$

and then $V_T$ is found by using $L = I_b \min - I_o \Omega - I_o$ in equation (15)

i.e.

$$V_T = \frac{\lambda}{C} \frac{4}{3} I_o \left\{ 1 - e^{-t/\lambda} \right\} + \frac{I_o}{C} t + V_R$$

where

$$V_T = -\frac{\lambda}{C} \frac{4}{3} \frac{e^{-t/\lambda}}{\lambda} + \frac{I_o}{C}$$

and $V_{T \max}$ occurs at

$$e^{-t/\lambda} = \frac{3}{4}$$

$$e^{t/\lambda} = \frac{4}{3}$$

$$t = \lambda \log_e \frac{4}{3}$$

$$= 0.3 \lambda$$

and is given by

$$V_{T \max} - V_R = -\frac{\lambda}{C} \frac{4}{3} I_o \left\{ 1 - e^{-0.3} \right\} + \frac{I_o}{C} \frac{0.3\lambda}{C}$$

$$= -\frac{1}{9} \frac{\lambda}{C} I_o$$

Taking same values as before and $\lambda = 70 \mu \text{sec.}$

$$V_{T \max} - V_R = -40 \text{ volts}$$
\( V_T \) returns to \( V_R \) again when

\[
- \frac{4}{3} \frac{\lambda}{C} I_o \left\{ 1 - e^{-t/\lambda} \right\} + \frac{I_o}{C} t = 0
\]

i.e. when \( t = \lambda \)

at which time the new

\[
\frac{d\bar{V}_T}{dt} = - \frac{4}{3} \frac{I_o}{C} \cdot \frac{1}{3} + \frac{I_o}{C} = \frac{I_o}{2C}
\]

Then the top terminal voltage will have the following form after suffering a negative voltage transient

Also since \( I_{b\max} = 2I_o \), the inverse of this curve is produced after a positive voltage transient.

It must be remembered that in all the above, we have assumed an extremely thin beam i.e. \( K \sim 1 \) in the expression for \( V_p \) – this means a beam of width of about 4 equivalent volts. In practice the beam width is more likely about 1/2 mm in diam i.e. 400 equivalent volts wide, and \( K \sim 1/100th \).

In this case the equation is

\[
\frac{1}{2} (I_{b\max} - I_{b\min}) \left\{ 1 + \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-x^2} dx \right\} = \int_0^\infty K(V_T-V_R) e^{-x^2} dx
\]

\[
= I_o - I_{b\min} - CV_T + \lambda I_o - \lambda CV_T
\]

(17)

PS/2658
and we are interested only in the region (call it region (2))

\[-2 < K(V_T - V_R) < 2\]

because outside this region the solution to the equation is that given above for \( K \sim 1 \). In other words the voltage \( V_T \) of the top terminal will always damp down to the region (2).

In region (2) the response of the probe is described as

\[
V_P = V_o \left\{ 1 + \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-x^2} dx \right\}
\]

This expression is most inconvenient when it comes to solving equation (17). However, in region (2) we may assume a linear dependence of \( V_P \) on \( (V_T - V_R) \) without detracting from the accuracy of the analysis, i.e.

\[
V_P = \frac{2V_o}{2V_B} \left\{ V_T - (V_R - V_B) \right\} \text{ for } V_R - V_B < V_T < V_R + V_B
\]

FS/2658
\[
\frac{1}{2V_B} \left\{ I_{b \ max} - I_{b \ min} \right\} \left\{ V_T - V_R + V_B \right\} = I_0 - I_{b \ min} - CV_T + \frac{1}{2V_B} \left\{ I_{b \ max} - I_{b \ min} \right\} \left\{ V_B - V_R \right\} + \lambda I_0 - \lambda CV_T \\
\]

or \[
\lambda V_T + V_T + Q + N + MV_T = F(t) + \frac{1}{\lambda} F(t)
\]

where \[
Q = \frac{I_{b \ min}}{C} ; \quad N = \frac{1}{2V_B C} \left\{ I_{b \ max} - I_{b \ min} \right\} \left\{ V_B - V_R \right\}
\]

and \[
M = \frac{1}{2V_B C} \left\{ I_{b \ max} - I_{b \ min} \right\}
\]

and \[
\frac{1}{C} I_0 = F(t)
\]

Thus applying the Laplace inverse transformation, we have \[
\lambda s^2 v(s) + sv(s) + Mv(s) = sf(s) + \frac{1}{\lambda} f(s)
\]

i.e. \[
v(s) = \frac{(s + \frac{1}{\lambda}) f(s)}{\lambda s^2 + s + N}
\]

Since \( \lambda \) and \( N \) are real positive constants, the solutions can again be shown to the stable by same methods as before (i.e. when \( \omega_n \) real where

\[
I_0(t) = I_0 \left[ 1 + \sum_{n=1}^{\infty} a_n e^{\pm i\omega_n t} \right]
\]

PS/2658
As before we can assume \( I_o(t) = 0 \) i.e. \( I_o = \text{constant since} \ \omega_n \text{ is likely to be less than} \ 10^3. \))

Thus equation may be written

\[
\dddot{V}_T + \frac{1}{2C} \ddot{V}_T + M \dot{V}_T + Z = 0
\]

(18)

where

\[
Z = Q + N - \frac{I_o}{C}
\]

\[
= \frac{(I_{b\text{ min}} - I_o)}{C} + \frac{(I_{b\text{ max}} - I_{b\text{ min}})}{C} \frac{(V_B - V_R)}{2V_B}
\]

\[
M = \frac{(I_{b\text{ max}} - I_{b\text{ min}})}{C} \cdot \frac{1}{2V_B}
\]

Equation (15) has the solution

\[
V_T = \left( \bar{V}_T + \frac{Z}{M} \right) e^{-t/2\lambda} \cos \sqrt{\frac{2M}{\lambda} + \frac{1}{4\lambda^2}} \cdot t - \frac{Z}{M}
\]

where \( V_T = \bar{V}_T \) at \( t = 0 \)

Now

\[
\frac{Z}{M} = (V_B - V_R) + \frac{(I_{b\text{ min}} - I_o)}{(I_{b\text{ max}} - I_{b\text{ min}})} 2V_B
\]

Thus putting \( I_{b\text{ max}} = 2I_o - I_{b\text{ min}} \) where \( \frac{Z}{M} = -V_R \) and

\[
V_T = (\bar{V}_T - V_R) e^{-t/2\lambda} \cos \omega t + V_R
\]

where

\[
\omega = \sqrt{\frac{-M}{\lambda} + \frac{1}{4\lambda^2}}
\]

\[
\ddot{V}_T = -\frac{1}{2\lambda} (\bar{V}_T - V_R) e^{-t/2\lambda} \cos \omega t - \frac{1}{\omega} (\bar{V}_T - V_R) e^{-t/2\lambda} \sin \omega t
\]
At \( t = 0 \)

\[
\frac{\dot{V}_T}{V_T} = \frac{(\bar{V}_T - V_R)}{2\lambda}
\]

or

\[
\lambda = \frac{(\bar{V}_T - V_R)}{2\dot{V}_T}
\]

Since at \( t = 0 \), we have \( \dot{V}_T > I_o \frac{C}{T} \), then for \( (\bar{V}_T - V_R) > 200 \text{ volts} \), so that the amplitude of the \( V_T \) oscillations are within \( \pm 100 \text{ volts} \) of \( V_R \) after a time \( t \approx \lambda \), we must have

\[
\lambda > \frac{(\bar{V}_T - V_R)}{2I_o \frac{C}{\Omega}} \approx \frac{200}{10} \times 10^{-6}
\]

i.e. \( \lambda > 20 \text{ \mu} \text{sec} \).

In the above if \( I_{b \text{ max}} = 2I_o - I_{b \text{ min}} \) the top terminal voltage becomes independent of beam width as long as

\[
N \ll \frac{1}{4\lambda}
\]

The preceding analysis deals with mean values of the current so that in practice the mean repetition rate of the pulsed current should be appreciably larger than \( \frac{1}{\lambda} \). This is to ensure that the mean currents are relatively well defined. When the beam current is being pulsed at the minimum repetition rate, the mean current is then virtually the average belt charging current which must, of course, have a well defined mean value.

One may conclude from the preceding analysis that the electronic circuitry should be designed in such a manner that \( \lambda \) may be varied from 10 to 100 \( \mu \text{sec} \) and that both \( I_{b \text{ min}} \) and \( I_{b \text{ max}} \) may be varied by a factor of about two while their mean value should be in the range

\[
I_{b \text{ max}} = 2I_o - I_{b \text{ min}} \text{ and } I_{b \text{ min}} \sim \frac{1}{10} I_o \text{ to } \frac{1}{5} I_o.
\]
Further with $I_0$ of the order of $1/2$ mA it seems that a minimum repetition rate of 1 pulse per 20 μsec and maximum of 1 pulse per μsec is adequate for controlling the top terminal voltage to within ± 1 part in $10^4$.

Finally, the operation of the stabilizer can be made independent of the beam width if $I_{b \text{ max}} = 2I_0 - I_{b \text{ min}}$. Clearly, too large a beam width at the probe can mean erratic stabilization if the electron density is not uniform over the cross section of the beam.

Again for the same reason, it cannot be made too small in width because in this case the beam may be entirely lost to the collimating slits before it enters the spectrometer.

Under these circumstances a beam diameter of about $\frac{1}{8}$ to $\frac{1}{2}$ mm would be reasonable (i.e. 150 to 450 equivalent volts) consisting of about a few hundred electrons per pulse.
APPENDIX II

The electronics necessary to control the stabilizer must consist of a probe, filter and pulse generator external to the Van de Graaff, together with the circuitry necessary to transmit and receive between the control box and the top terminal. This breaks down into four discrete parts each situated in a chain in the one servo loop.

1. Control Unit
   A. Filter

   ![Circuit Diagram]

   $C_1$ charges through $D_1$ to the peak value of the pulses and can leak down in about 25 μsec more or less linearly through $R_1$. If no pulses arrive the potential of $C_1$ is caught at zero by $D_2$. $R_2$ and $C_2$ serve to introduce some desired time constant ($\lambda$) and $C_2$ can be switched to various values. This circuit has a slightly asymmetric $\lambda$ and this could be remedied by applying positive clearing pulses at $A$ immediately prior to the arrival of the input pulses but this is not felt to be necessary.

B. Pulse Generator

   ![Circuit Diagram]

   $V_2$ 180 F  $V_3$ 88 CC  $V_4$
$P_1$ and $P_2$ limit the voltage excursion of $E$ and hence fix the $I_{b\ max}$ and $I_{b\ min}$. $R_3$ changes the gain of $V_2$ and hence controls the loop gain. $V_2$ needs a response time of about $5\ \mu$sec and hence $R_3\ max = 100\ K$ with $50\ pF$. $V_3$ is a multivibrator whose rate depends on the voltage at $B$, and we can easily arrange its component values for a rate variation from $5 \times 10^4$ p.p.s. to $1 \times 10^6$ p.p.s. $V_4$ will pass positive output pulses to the light source at the bottom end of the Van de Graaff.

2. **Light Source**

This will be a small C.R.T. using RCA Phosphor type P16 (which has a decay time of well under $1\ \mu$sec) with about $1\ KV$ on its anode. By pulsing the grid positive we get our light pulses which travel up a plexiglass (polymethyl methacrylate) tube to the top terminal of the Van de Graaff. This tube is $2$ metres long and absorbs about $40\ o/o$ of the light.

3. **Light Receiver (Top Terminal)**

A photomultiplier, R.C.A. type 931 A in the top terminal receives the light pulses. This photomultiplier has round ends and is good for withstanding the high pressure (30 atm. or 280 p.s.i.) in the top terminal. It is powered by $1\ KV$. A three valve pulse amplifier generating a standard pulse will serve to drive the electron gun (in the top terminal) into bursts of current such as to control the top terminal voltage.

4. **Probe**

The Photomultiplier which views the plastic scintillator has its dynode time constants less than $1\ \mu S$ so that changes of pulsing rate from $1\ mc/s$ to $50\ kc/s$ do not affect its gain.
APPENDIX III

Parameters Governing Probe Design

The vacuum in the region of the Van de Graaff is to be as near as possible to $10^{-9}$ mm Hg. Thus at the probe, which is expected to be some 8 metres from the Van de Graaff, pressures of the order of $10^{-7}$ mm Hg should be adequate. It should be possible to obtain such a pressure at the probe without heating as long as there is not much outgassing in this region. In the present design, the crystal detector is placed outside the vacuum system and only a very small number of electrons pass through the window of the aluminium flange. Thus the outgassing rate ought to be quite low - if it is not the design is such that the crystal and photomultiplier may be separated from the flange and so allowing the system to be heated. A diagram of the design is included at the end of this report.

The material for the flange was chosen to be aluminium for the following reasons: it is easily obtainable, not difficult to machine, light in weight and so allows passage of the electrons through a "window" of reasonable thickness ($\lesssim 0.5$ mm); data exist on electron transmission and scattering in aluminium. This last reason is an important one in that without this information the optimum thickness of the window cannot be calculated. The data used in obtaining an optimum of 0.3 to 0.5 mm thickness is given at the end of this report.

The resolution of the probe is to be investigated by using a collimating system which is to be accurate to 0.01 mm (see for example P. Kirstein's movable slit systems) and uses a $\beta^-$ emitter (probably Ytrium) as a source of electrons. The results of such an investigation will be described in a following report.
Vacuum Side

Photo Multiplier Side

Aluminium Window for Probe Window 0.3 mm
Angular distribution of 9 MeV electrons scattered by aluminium. The three curves show:

1. 0.1 mm
2. 0.2 mm
3. 0.5 mm thickness respectively.

Thickness of aluminium vs. intensity of electrons of energy 1 MeV. (J. M. Cork, "Radioactivity and Nuclear Physics", p 130, Van Nostrand 1947)