MAGNETIC HORN AND NEUTRINO FLUX CALCULATIONS

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Summary:

The focusing behaviour of the magnetic horn, some points of its construction, as well as method and results of neutrino flux calculations are described.
1. Introduction

In view of enhancing the fluxes of high energy neutrinos required for experiments as those described in the following papers, it is useful to focus towards the neutrino detector the π's and K's produced by the protons interacting in the target. The statistically determined change in direction of the neutrino with respect to its parent pion or kaon will set a limit to the precision of focusing that is useful.

It can be shown that the chance of a neutrino hitting the detector is in first approximation proportional to

\[
\frac{1}{(1 + \gamma^2 \phi^2)^2}
\]

where \(\gamma\) refers to the pion (or kaon) and \(\phi\) is the decay angle required in order that the neutrino hits the detector (Fig. 1).

Obviously, by focusing, i.e. by reducing \(\phi\), the situation may be improved, until \(\gamma \phi \ll 1\), in which case further reduction is useless. On the other hand, for \(\gamma \phi > 1\) the chance that the neutrino will go through the detector decreases rapidly. For instance, for \(\gamma \phi = 2\) the chance is reduced by a factor 25, as compared with \(\gamma \phi = 0\). A typical average value of \(\gamma \phi\) for unfocused particles is 3 for pions, 1.2 for kaons \(\text{\(^\text{x}\)}\).

\(\text{\(^\text{x}\)}\) The required decay angle is on the average about 1.3 times the emission angle (see Fig. 1).

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The neutrino momentum also depends on the decay angle:

\[ P \gamma \approx \frac{A}{1 + \gamma^2 \varphi^2} \]

with \( A = 0.42 \ p_{\pi} \) for \( \pi \) decay
\( A = 0.95 \ p_{K} \) for \( K \) decay

Again, \( \gamma \varphi \) should be made small if high energy neutrinos are required.

The gain to be obtained in principle from focusing does not depend much on the particle momentum. This is because average emission angles are inversely proportional with \( \gamma \) (constant average transverse momentum), so that \( \gamma \varphi \) does not depend much on momentum for an unfocused beam. At very high momentum, where \( \varphi \) becomes of the same order as the angle subtended by the detector at the decay point, the gain by focusing decreases. With the PS experimental set-up this effect is not yet very important even at the highest neutrino energies that are considered.

2. Focusing properties of the magnetic horn

The magnetic horn that focuses the pions and kaons was described in earlier publications \((1,2)\). The shape of its inner conductor has been modified since, in order to obtain more high energy neutrinos than with the old designs. It consists now of a cylindrical part and two conical parts with different opening angles, as shown in Fig. 2. The 25 cm long copper target is PS/4076
located in the cylindrical part. The cones are shaped so that most particles with a momentum of 6 GeV/c are made about parallel with the axis, independent of their emission angle (curves 1 and 2, Fig. 2). Particles of higher or lower momentum will be slightly divergent or convergent, respectively, after leaving the horn. The dependence on momentum is not very strong, however, since e.g. particles with higher momentum are less deflected, but will therefore stay longer inside the magnetic field region.

At still lower momentum (\(< 3 \text{ GeV/c}\)) the trajectories will enter the magnetic field twice (curve 3, Fig. 2).

The shape of the trajectories depends on the exact location of their starting point in the target. Since the most effective part of the target is the upstream one, the horn was accordingly designed. Particles emitted from the downstream part are deflected somewhat less, so that their decay angles must be larger if their neutrinos are to pass through the detector. Consequently, the downstream part of the target has a relatively better efficiency for the low than for the high energy part of the spectrum.

Fig. 3 shows an example of the focusing behaviour of the horn. This diagram is valid for a point 6 cm from the upstream end of the target. It shows the angle after the horn as a function of emission angle and particle momentum. The horizontal lines at the bottom (emission angles below 1.7°) represent particles that do not traverse the inner conductor. The near vertical lines in the centre correspond to particles that traverse the cylindrical part.
of the inner conductor, are deflected by the field, and leave the
horn through the long conical part of the inner conductor. The hori-
zontal lines at the left represent particles that pass twice through
the deflecting field, crossing the axis once. Finally, the near-
hyperbolic curves in the top are due to particles that leave the
horn through its end plate or through the outer conductor.

A similar diagram for the particles of opposite sign, which
are defocused, is shown in Fig. 4. The discontinuities in the
curves are caused by discontinuities in the shape of the outer
conductor.

The data for constructing the diagrams of Fig. 3 and Fig. 4
were obtained by electronic computation. Unfortunately, the
differential equations describing the particle motion in the field
of an axial conductor cannot be solved analytically, so that hand
computation of the trajectories is time-consuming.

3. Construction and operation of the horn

Fig. 5 shows a section of the upstream part of the horn,
in which the inner and outer conductor and the target rod (4 mm \( \Phi \))
are visible. The current pulse (max. 300 kA) is supplied from a
12 kV, 150 kJ capacitor bank through 16 coaxial low-inductance
cables whose attachments can be seen (top, left). Each cable can
be fixed in two different positions as shown. In this way the po-
larity can be changed, so that either positive or negative par-
ticles can be focused. Either neutrinos or antineutrinos will then

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be predominant in the detectors.

The current pulse has the shape of a strongly damped sine wave with a half-period of about 180 μsec. It is synchronised with the ejection system so that the 2 μsec proton pulse arrives when the current is at its maximum. At that moment the current has only penetrated about 1.5 mm into the Al inner conductor. The total wall thickness of this conductor is 4 mm in the narrow cylindrical part and 3 mm at the wide end. This thickness is necessary for mechanical reasons: the inward pressure of the magnetic field tries to buckle the cone and also has a strong axial component pulling on the cylindrical part.

The inner conductor is cooled by spraying distilled water on it between pulses.

During tests some inner conductors have broken in various places due to mechanical fatigue. Insulators have been damaged by radiation. Manufacturing procedures were improved, and the present horn is thought to be fairly reliable.

The current is switched with 8 ignitrons of the type BK 178, made by A.E.I., Rugby (Gr. Br.). These are specially made for this kind of service, and it was found that their average life is of the order of $5 \times 10^5$ pulses. In total, over a million pulses at 300 kA were made.

Fig. 6 shows horn and capacitor bank in the laboratory during tests.
4. Calculation of neutrino flux

The basis of all neutrino flux calculations is formed by
data about production of pions and kaons in the target as a function
of momentum and emission angle. For the results presented here the
most recent measurements at CERN (3) were used. These measurements
were done for $\pi^+$, $\pi^-$, $K^+$ and $K^-$ at emission angles of 0 and 100 mr
only; estimates for other angles were made by interpolation and
extrapolation, the general shape of intensity vs. angle curves
being taken from older measurements (4, 5).

Spectrum measurements have mostly been done with beryllium
and lead targets. For the copper target used in most of the neutrino
runs it was assumed that the production per interaction is the
geometric mean of the Be and Pb figures. This assumption may be
quite wrong. There seem, however, to be no experimental data on
this point.

According to (3), the production per interaction differs by
a factor of 1.5 or 2 between Be and Pb. The experimental fact that
a copper target in the horn instead of a tungsten one approximately
doubled the neutrino interaction rate, is due in part to the effect
described in par. 2. With a copper target, more secondaries come
from the downstream end and less from the upstream end. This re-
results in a higher low-energy neutrino flux, and most of the inter-
actions come from the low-energy part of the spectrum.

Starting from the secondary particle flux at the target,
the neutrino flux was computed in two stages:
a) calculation of pion and kaon flux vs. angle and momentum behind the horn;

b) calculation of neutrino flux vs. momentum in the detectors.

a) for computing the pion (or kaon) flux behind the horn, a sampling scheme was used. One momentum value was chosen and particle trajectories were traced through the horn for this momentum, starting from 5 different points in the target, and with different emission angles (intervals of 0.25°). For each trajectory so computed, a figure representing the production intensity at the momentum, emission angle and target position involved was multiplied with a correcting factor for absorption each time the trajectory was found to pass through part of the target or through the inner or outer conductor of the horn. The final intensity figure thus obtained for each trajectory, combined with its angle with the axis after the horn, was added into a table containing one storage point for each interval of 0.1°. After the results for all trajectories were added (interpolation between trajectories being used for better precision), this table gave the angular distribution after the horn.

The whole procedure was repeated for many momentum values, with intervals of 0.5 GeV/c up to max. 12 GeV/c for pions and 14 GeV/c for kaons.
A correction was made for the lateral displacement of each trajectory after the horn. In fact, the distance from the axis at the point of intersection with the detector plane was computed rather than the angle with the axis.

b) The neutrino spectrum was computed, starting from the angular spectrum after the horn, in the following way.

The decay tunnel was divided into 5 "decay sections". For each decay section and for each angle of the angular distribution table, the corresponding decay angle was determined, i.e. the decay angle required for the neutrino to hit the detector.

The probability of decay in the section considered, with this decay angle, and inside a solid angle corresponding to a unit area at the detector, was computed and multiplied with the intensity figure in the angular distribution table. The figure so obtained was added into a "neutrino momentum table", containing storage points for each interval of 0.2 GeV/c neutrino momentum. The neutrino momentum of each contribution to this table was found from the decay angle and the pion (or kaon) momentum.

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*) This was done for different parts of the detector section consecutively, so taking its finite area into account.
Fig. 3

$\theta = 300 kA$

focused particles

angle at target

angle after horn

90% of all pions are emitted at angles below this line.
Fig. 4

c = 300 K/A
defocused particles

angle after horn

angle at target
increase of \( \nu \) flux by horn focusing

\[ p_{\nu} \]

Fig. 8