Abstract
In these lectures I give a short review of the main theoretical ideas underlying the extensions of the Standard Model of elementary particle interactions.

1. INTRODUCTION

Studying physics beyond the Standard Model means looking for the conditions of the Universe in the first billionth of a second, when its temperature was above $10^{14}$ K. This clearly requires a gigantic intellectual leap in the investigation. It is even more striking that modern accelerators can reproduce particle collisions similar to those that continually occurred in the thermal bath in the very first instants of our Universe. We are now entering the age in which, with the joint effort of experiments and theory, we are likely to unravel the mystery of the fundamental principles of particle interactions lying beyond the Standard Model.

The Standard Model describes the interactions of three generations of quarks and leptons defined by a non-Abelian gauge theory based on the group $SU_3 \times SU_2 \times U_1$. The precision measurements at LEP have given an extraordinary confirmation of the validity of the Standard Model up to the electroweak energy scale (for reviews, see ref. [2]), and we have no firm experimental indications for failures of this theory at higher energies. Our belief that the Standard Model is a low-energy approximation of a new and fundamental theory is based only on theoretical, but well-motivated, arguments.

First of all, the electroweak symmetry breaking sector is not on firm experimental ground. The Higgs mechanism, which is invoked by the Standard Model to generate the $Z^0$ and $W^{\pm}$ masses, predicts the existence of a new scalar particle, still to be discovered. From the theoretical point of view, the Higgs mechanism suffers from the so-called “hierarchy” or “naturalness” problem which, as discussed in sect. 5, leads us to believe that new physics must take place at the TeV energy scale.

Furthermore, the complexity of the fermionic and gauge structures makes the Standard Model look like an improbable fundamental theory. To put it in a less qualitative way, the Standard Model contains many free parameters \textit{e.g.} the three gauge coupling constants, the nine fermion masses and the four Cabibbo–Kobayashi–Maskawa mixing
parameters); these correspond to important physical quantities, but cannot be computed in the context of the model. Simplifying the Standard Model structure and predicting its free parameters are therefore basic tasks of a successful theory.

In these lectures I review the main current ideas about theories beyond the Standard Model, keeping the discussion at a qualitative level and making no use of advanced mathematics. More comprehensive reviews can be found in refs. [3] (for GUTs), [4] (for supersymmetry) and [5] (for technicolour).

2. **GUT $SU_5$**

The first attempts to extend the structure of the Standard Model have led to the construction of Grand Unified Theories (GUTs) [6]. The basic idea is that gauge interactions are described by a single simple gauge group, which contains the Standard Model $SU_3 \times SU_2 \times U_1$ as a subgroup and as a low-energy manifestation. At first this may seem impossible, since a simple gauge group contains a single coupling constant $g_X$ and the strong, weak and electromagnetic couplings have different numerical values. However it should be remembered that, in a quantum field theory, the coupling constants depend on the energy scale at which they are probed, as a consequence of the exchange of virtual particles surrounding the charge. The evolution of the gauge coupling constants as a function of the energy scale can be computed using renormalization group techniques and perturbation theory, and the relevant equations are described in sect. 7. There, we will also find that, as we include the quantum effects of all Standard Model particles, the three gauge coupling constants approach one another as the energy scale is raised. For the moment, let us assume that the three gauge couplings meet at a single value for a specific energy scale ($M_X$) and study possible GUT candidates describing the physics above $M_X$ with a single gauge coupling constant $g_X$.

The simplest example of a GUT is based on the group $SU_5$. Each fermion family is contained in a $10 + \bar{5}$ representation of $SU_5$. This can be understood from the decomposition in terms of the Standard Model group:

\[
SU_5 \rightarrow SU_3 \times SU_2 \times U_1
\]

\[
10 \rightarrow (\bar{3}, 1, -\frac{2}{3})_{\nu_R} + (3, 2, \frac{1}{6})_{q_L} + (1, 1, 1)_{e_R}
\]

\[
\bar{5} \rightarrow (\bar{3}, 1, \frac{1}{3})_{d_R} + (1, 2, -\frac{1}{3})_{\ell_L}.
\]

Equation (1) shows that the degrees of freedom for all the (left-handed) fields in one Standard Model family are described by the two $SU_5$ fields $10$ and $\bar{5}$. In GUTs not only is the gauge group unified, going from $SU_3 \times SU_2 \times U_1$ to $SU_5$ in this specific example, but also the fermionic spectrum is simplified. As quarks in QCD come with different colours, in GUTs different quarks and leptons are just different aspects of the same particle. This also explains the simple integer relations among the electric charges of different quarks and leptons.

3. **EXPERIMENTAL TESTS FOR GUTS**

Theoretical elegance is of course not a sufficient argument to convince us that GUTs have anything to do with Nature. We need to establish GUTs predictions which can be confronted with experimental data. The basic idea of GUTs, gauge coupling unification,
provides such a prediction. Indeed at the GUT scale $M_X$ we can compute the weak mixing angle:

$$\sin^2 \theta_W = \frac{e^2}{g^2} = \frac{\text{Tr}(T_3^2)}{\text{Tr}(Q^2)} = \frac{3}{8}. \quad (2)$$

Here $T_3$ is the third isospin-component and $Q$ is the electric charge. The trace in eq. (2), taken over any $SU_5$ representation, follows from a correct normalization of the GUT generators. Before comparing eq. (2) with experiment, one has to rescale it to the low energies where coupling constants are measured. We will do this in sect. 7, and show that eq. (2) gives a successful prediction for a class of theories which we have not yet introduced, supersymmetric GUTs. We anticipate here that, if gauge coupling unification has any chance to succeed, the unification scale $M_X$ must be extremely large, of the order of $10^{15} - 10^{16}$ GeV, which, in the thermal history of our Universe, brings us to consider events occurring in the first $10^{-35} - 10^{-38}$ s.

Since we have promoted the gauge group to $SU_5$, we expect new gauge bosons and therefore new forces which may have experimental consequences. The decomposition of the $SU_5$ gauge bosons in terms of Standard Model ones is:

$$SU_5 \rightarrow SU_3 \times SU_2 \times U_1; \quad 24 \rightarrow (8, 1, 0)_g + (1, 3, 0)_W + (1, 1, 0)_B + (3, 2, -\frac{5}{6})_X + (\bar{3}, 2, \frac{5}{6})_{\bar{X}}. \quad (3)$$

Together with the familiar degrees of freedom for the gluons ($g$) and the electroweak gauge bosons ($W^\pm, W^0, B$), we find new particles ($X$ and $\bar{X}$) which carry both colour and weak quantum numbers. The gauge bosons $X$ and $\bar{X}$ affect weak interactions, but modify standard processes only by an amount $(M_W/M_X)^2$, a fantastically small number, whose effect is completely undetectable even in the most precise measurements. Nevertheless, the $X$-mediated interactions may not be so invisible. Let us inspect the interactions between $X$, $\bar{X}$ and the fermionic currents, which are dictated by $SU_5$ gauge invariance:

$$L = \frac{gX}{\sqrt{2}} \left\{ X^\mu \left[ \bar{d}_R \gamma^\mu e_R^c + \bar{d}_L \gamma^\mu \epsilon _L^c + \epsilon_{\alpha \beta \gamma} \bar{u}_L^c \gamma^\mu u_L^\beta \right] + \right.$$  

$$+ \bar{X}^\mu \left[ -\bar{d}_R \gamma^\mu e_R^c - \bar{u}_L \gamma^\mu \epsilon _L^c + \epsilon_{\alpha \beta \gamma} \bar{u}_L^c \gamma^\mu u_L^\beta \right] + \text{h.c.} \right\}. \quad (4)$$

Notice that one cannot assign a conserved baryon ($B$) and lepton ($L$) quantum number to $X$ and $\bar{X}$; the new interactions violate both $B$ and $L$. In the Standard Model $B$ and $L$ are accidental global symmetries, in the sense that they are just a consequence of gauge invariance and renormalizability. It is not surprising that $B$ and $L$ are then violated in extensions of the Standard Model, in particular in GUTs where quarks and leptons are different aspects of the same particle.

The experimental discovery of processes that violate $B$ and $L$ would be clear evidence for physics beyond the Standard Model. One of the most important of such processes is proton decay, which has the dramatic consequence that ordinary matter is not stable. It is easy to see from eq. (4) that the $X$ boson mediates the transition $uu \rightarrow e^+ \bar{d}$. When dressed between physical hadronic states, this transition is converted into the proton decay modes $p \rightarrow e^+ \pi^0, e^+ \rho^0, e^+ \eta, e^+ \pi^+ \pi^-$, and so on. The calculation of the proton lifetime yields

$$\tau_p = (0.2 - 8.0) \times 10^{31} \left( \frac{M_X}{10^{15} \text{ GeV}} \right)^4 \text{ yr}. \quad (5)$$
The uncertainties in the numerical coefficient in eq. (5) come mainly from the difficulty in estimating the matrix elements relating quarks to hadrons. For reasonable GUT masses, \( M_X \simeq 10^{15} - 10^{16} \text{ GeV} \), eq. (5) predicts a proton lifetime \( 10^{21} - 10^{25} \) times larger than the age of the Universe. It is fascinating that experiments can probe such slow processes by studying very large samples of matter. The present experimental bound on the lifetime of the decay mode \( p \to e^+ \pi^0 \), the dominant proton decay channel in \( SU_5 \), is [7]

\[
\tau(p \to e^+ \pi^0) > 5.5 \times 10^{32} \text{ yr}.
\] (6)

This bound already sets important constraints on possible GUT models.

GUTs also provide a framework in which the creation of a primordial baryon asymmetry can be understood and computed. Although this is not an experimental test, it is clearly a very attractive theoretical feature. Observations tell us that the present ratio of baryons to photons in the Universe is a very small number, \( n_B/n_\gamma = 4-7 \times 10^{-10} \). If \( n_B/n_\gamma \) is then extrapolated back in time following the thermal history of the Universe, one finds that the excess of baryons over antibaryons at the time of the big bang must have been \( \Delta B \equiv (n_B - n_B)/n_B \sim 3 \times 10^{-8} \). We find it disturbing to consider that the present observed Universe is determined by a peculiar initial condition prescribing that for each three hundred million baryons there are three hundred million minus one antibaryons.

The hypothesis of baryogenesis is that \( \Delta B = 0 \) at the time of the big bang and that the small cosmic baryon asymmetry was dynamically created during the evolution of the Universe. The physics responsible for the creation of \( \Delta B \) must necessarily involve interactions which violate \( B \). GUTs are therefore a natural framework for baryogenesis and it has been proved [8] that they have all the necessary ingredients to generate the observed value of the present baryon density.

4. \( SO_{10} \) AND NEUTRINO MASSES

I have presented \( SU_5 \) as the simplest GUT, but models based on larger groups can also be constructed. Probably the most interesting of them [9] is based on the orthogonal group \( SO_{10} \), which contains \( SU_5 \) as a subgroup. The 16-dimensional spinorial representation of \( SO_{10} \) decomposes into \( 10 + \overline{5} + 1 \) under \( SU_5 \). We recognize the fermion content of one Standard Model family. It is quite satisfactory that quarks and leptons with their different quantum number assignments can be described by a single \( SO_{10} \) particle, for each generation.

In addition to the ordinary quarks and leptons contained in the \( 10 + \overline{5} \) of \( SU_5 \), the spinorial representation of \( SO_{10} \) contains also a gauge singlet. This can be interpreted as the right-handed component of the neutrino, allowing the possibility of Dirac neutrino masses. The neutrino mass term can now be written in the form

\[
(\bar{\nu}_L \nu^c_L) \mathcal{M} (\nu^c_R \nu^R) + \text{h.c.},
\] (7)

where, for simplicity, we are considering only the one-generation case. The different entries of the neutrino mass matrix \( \mathcal{M} \)

\[
\mathcal{M} = \begin{pmatrix} T & D \\ D^T & S \end{pmatrix}
\] (8)

can be understood in terms of symmetry principles. The term \( S \) transforms as a singlet under the Standard Model gauge group and therefore is naturally generated at the scale
where the $SO_{10}$ symmetry is broken, $S \sim M_X$. The other two terms, $T$ and $D$, transform respectively as a triplet and a doublet under the weak group $SU_2$; therefore they can be generated only after the Standard Model gauge group is broken. However, vacuum expectation values of triplet fields lead to an incorrect relation between the strengths of neutral and charged weak currents. We conclude therefore that $T \simeq 0$ and $D \simeq m_f$, where $m_f$ is a typical fermion (quark or charged lepton) mass. After diagonalization of the matrix in eq. (8), we find one heavy eigenstate with mass of order $M_X$ and one (mainly left-handed) eigenstate with mass [10]:

$$m_{\nu} \simeq \frac{m_f^2}{M_X} = 10^{-6} \text{eV} \left( \frac{m_f}{\text{GeV}} \right)^2 \left( \frac{10^{15} \text{GeV}}{M_X} \right).$$ (9)

In the context of the $SO_{10}$ GUT, not only do we expect neutrinos to be massive, but we also understand in terms of symmetries why their masses must be much smaller than the typical scale of the other fermion masses.

5. THE HIERARCHY PROBLEM

The hierarchy (or naturalness) problem [11] is considered to be one of the most serious theoretical drawbacks of the Standard Model and most of the attempts to build theories beyond the Standard Model have concentrated on its solution. It springs from the difficulty in field theory in keeping fundamental scalar particles much lighter than $\Lambda_{\text{max}}$, the maximum energy scale up to which the theory remains valid.

It is intuitive to require that if a particle mass is much smaller than $\Lambda_{\text{max}}$ there should exist a (possibly approximate) symmetry under which the mass term is forbidden. We know an example of such a symmetry for spin-one particles. The photon is, theoretically speaking, naturally massless since the gauge symmetry $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ forbids the occurrence of the photon mass term $m^2 A_\mu A^\mu$. Similarly, we can identify a symmetry which protects the mass of a fermionic particle. A chiral symmetry, under which the left-handed and right-handed fermionic components transform differently $\psi_L \rightarrow e^{i\alpha} \psi_L, \psi_R \rightarrow e^{i\beta} \psi_R, \alpha \neq \beta$, forbids the mass term $m \psi_L \psi_R + \text{h.c.}$ Scalar particles can be naturally light if they are Goldstone bosons of some broken global symmetry since their non-linear transformation property $\varphi \rightarrow \varphi + a$ forbids the mass term $m^2 \varphi^2$.

In the case of the Higgs particle, required in the Standard Model by the electroweak symmetry breaking mechanism, we cannot rely on any of the above-mentioned symmetries. In the absence of any symmetry principle, we expect the Higgs potential mass parameter $m_H^2$ to be of the order of $\Lambda_{\text{max}}^2$. Even if we artificially set the classical value of $m_H^2$ to zero, it will be generated by quadratically divergent quantum corrections:

$$m_H^2 = \frac{\alpha}{\pi} \Lambda_{\text{max}}^2,$$ (10)

where $\alpha$ measures the effect of a typical coupling constant.

One may argue that in a renormalizable theory, the bare value of any parameter is an infinite (or, in other words, cut-off dependent) quantity, without a precise physical meaning. Since all divergences can be reabsorbed, one can just choose the renormalized quantity to be equal to any appropriate value. However, we believe that a complete description of particle interactions in a final theory will be free from divergences. From this point of view, the cancellation between a bare value and quadratically divergent quantum
corrections looks like a conspiracy between the infra-red (below $\Lambda_{\text{max}}$) and the ultraviolet (above $\Lambda_{\text{max}}$) components of the theory. We do not accept such a conspiracy, but, on the other hand, we know that the parameter $m_W^2$ sets the scale for electroweak symmetry breaking and it is therefore directly related to $m_W^2$. We thus require that the quantum corrections in eq. (10) do not exceed $m_W^2$. This implies an upper bound on $\Lambda_{\text{max}}$:

$$\Lambda_{\text{max}} \lesssim \sqrt{\frac{\pi}{\alpha}} M_W \simeq \text{TeV}.$$  \hspace{1cm} (11)

We can conclude that the Standard Model has a natural upper bound at the TeV scale, where new physics should appear and modify the ultraviolet behaviour of the theory.

The hierarchy problem becomes most apparent when one considers GUTs. Here the Higgs potential of the model contains two different mass parameters: one is of order $M_X$ and sets the scale for the breaking of the unified group; the other is of order $M_W$ and sets the scale for the ordinary electroweak breaking. By explicit calculation, one can show [12] that these parameters mix at the quantum level and the hierarchy of the two mass scales can be maintained only at the price of fine-tuning the parameters by an amount $(M_X/M_W)^2$.

6. \textsc{Supersymmetry}

Supersymmetry [13], contrary to all other ordinary symmetries in field theory, transforms bosons to fermions and vice versa. This means that bosons and fermions sit in the same supersymmetric multiplet. In the simplest version of supersymmetry (the so-called $N = 1$ supersymmetry), each complex scalar has a Weyl fermion companion and each massless gauge boson also has a Weyl fermion companion; similarly the spin-2 graviton has a spin-3/2 companion, the gravitino. Invariance under supersymmetry implies that particles inside a supermultiplet are degenerate in mass. It is therefore evident that, in a supersymmetric theory, if a chiral symmetry forbids a fermion mass term, it forbids also the appearance of a scalar mass term, such as the notorious Higgs mass parameter. The hierarchy problem discussed in the previous section can now be solved. Indeed, it has been proved that a supersymmetric theory is free from quadratic divergences [14]. The contribution to $m_H^2$ proportional to $\Lambda_{\text{max}}^2$ in eq. (10) coming from a bosonic loop is exactly cancelled by a loop involving fermionic particles. Since the dependence on $\Lambda_{\text{max}}^2$ has now disappeared, we can extend the scale of validity of the theory without provoking any hierarchy problem.

It should also be mentioned that when supersymmetry is promoted to a local symmetry, which means that the transformation parameter depends on space-time, then the theory automatically includes gravity and is called supergravity. Because of this characteristic, supersymmetry is believed to be a necessary ingredient for the complete unification of forces.

Here we are interested in the minimal extension of the Standard Model compatible with supersymmetry. Each Standard Model particle is accompanied by a supersymmetric partner: scalar particles (squarks and sleptons) are the partners of quarks and leptons, and fermion particles (e.g. gluinos) are the partners of the Standard Model bosons (e.g. gluons). Supersymmetry also requires two Higgs doublets, as opposed to the single Higgs doublet of the Standard Model, and their fermionic partners mix with the fermionic
partners of the electroweak gauge bosons to produce particles with one unit of electric charge (charginos) or no electric charge (neutralinos).

Supersymmetry ensures that the couplings of all these new particles are strictly related to ordinary couplings. For instance, the couplings of squarks to one or two gluons, of gluinos to gluons, of squarks and gluinos to quarks are solely determined by $\alpha_s$, the QCD gauge coupling constant.

The supersymmetric generalization of the Standard Model is therefore a well-defined theory where all new interactions are described by the mathematical properties of the supersymmetric transformation. As such, however, the theory is not acceptable since it predicts a mass degeneracy between the ordinary and the supersymmetric particles; in Nature, therefore, supersymmetry is not an exact symmetry. In order to preserve the solution of the hierarchy problem we need to break supersymmetry while maintaining the good ultraviolet behaviour of the theory. It has been shown [15] that if only a certain set of supersymmetry-breaking terms with dimensionful couplings are introduced, then the quadratic divergences still cancel, but the mass degeneracy is removed. Let us generically call $m_S$ the mass that sets the scale for the dimensionful couplings which softly break supersymmetry. This scale has a definite physical meaning, since all new supersymmetric particles acquire masses of order $m_S$. It is the energy scale at which supersymmetry has to be looked for in experiments.

By explicit calculation one finds that, in a softly broken supersymmetric theory, quadratic divergences cancel, but some finite terms of the kind $(\alpha/\pi)m_S^2$ remain. From eq. (10) we recognize that $m_S$ behaves as the cut-off of quadratic divergences in the Standard Model. This is not entirely surprising since, in the limit $m_S \rightarrow \infty$, all supersymmetric particles decouple and one should recover the ultraviolet behaviour of the Standard Model. Therefore we conclude that, in a softly broken supersymmetric theory, the cut-off of quadratic divergences has a physical meaning since it is related to $m_S$, the mass scale of the new particles. Moreover, following the same argument that led us to eq. (11) we find that these new particles cannot be much heavier than the TeV scale, if supersymmetry solves the hierarchy problem. In sect. 8, I will make this argument more quantitative.

Although technically successful, it may appear that the introduction of the soft supersymmetry-breaking terms is too arbitrary to be entirely satisfactory. But, on the contrary, it has a very appealing explanation [16]. Let us first promote supersymmetry to supergravity, possibly a necessary step towards complete unification of forces. Then assume that supergravity is either spontaneously or dynamically broken in a sector of the theory that does not directly couple to ordinary particles. In this case, gravity communicates the supersymmetry breaking, and the low-energy effective theory of the supersymmetric Standard Model contains exactly all the terms which break supersymmetry without introducing quadratic divergences.

From this point of view, the appearance of the soft-breaking terms can be understood in terms of well-defined dynamics. However, we do not yet know which mechanism breaks supersymmetry and therefore we are not able to compute the soft-breaking terms. This is unfortunate because these define the mass spectrum of the new particles. All we can do now is to keep them as free parameters and hope they will be determined by experimental measurements or calculated, if theoretical progress is made. In the minimal version of the theory, there are only four such parameters but, if some assumptions are
relaxed, the number of free parameters can grow enormously.

7. **SUPERSYMMETRIC UNIFICATION**

In the previous section, we have extended the Standard Model to include supersymmetry in order to solve the hierarchy problem. We can now incorporate within this model the ideas of grand unification, and construct a supersymmetric GUT [17].

![Figure 1: Gauge coupling constant unification predictions obtained by varying $\log_{10} M_X$ (as shown by the numbers inside the bands) for the Standard and supersymmetric GUT theories. The bands represent the uncertainties in the prediction. Also shown are the present experimental data and the data available in 1981. (Courtesy S. Dimopoulos).](image)

As discussed in sect. 3, the first test of a GUT is gauge coupling unification. At the one-loop approximation the evolution of the $SU_3 \times SU_2 \times U_1$ gauge coupling constants with the energy scale $Q^2$ is given by

$$\frac{d\alpha_i}{dt} = -\frac{b_i}{4\pi} \alpha_i^2 \quad \Rightarrow \quad \alpha_i(t) = \frac{\alpha_i(0)}{1 + \frac{b_i}{4\pi} \alpha_i(0)t}, \quad i = 1, 2, 3, \quad (12)$$

where $t = \log(M_X^2/Q^2)$. The coefficients $b_i$ take into account the numbers of degrees of freedom and the gauge quantum numbers of all particles involved in virtual exchanges. For the Standard Model, we find

$$b_3 = -7 + \frac{4}{3}(N_g - 3), \quad b_2 = -\frac{19}{6} + \frac{4}{3}(N_g - 3), \quad b_1 = \frac{41}{6} + \frac{20}{9}(N_g - 3), \quad (13)$$

where $N_g$ is the number of generations. In the supersymmetric case all new particles influence the running of the gauge coupling constants and modify the $b_i$ parameters,

$$b_3 = -3 + 2(N_g - 3), \quad b_2 = 1 + 2(N_g - 3), \quad b_1 = 11 + \frac{10}{3}(N_g - 3). \quad (14)$$

Assuming $N_g = 3$ and gauge coupling unification, i.e. $\alpha_3(0) = \alpha_2(0) = 5/3\alpha_1(0)$, we can compute the QCD coupling $\alpha_s(M_Z)(\equiv \alpha_3(M_Z))$ and $\sin^2 \theta_W(\equiv [1 + \alpha_2(M_Z)/\alpha_1(M_Z)]^{-1})$.
as a function of $M_X$, taking $\alpha^{-1}(M_Z) = 127.9 \pm 0.1$ ($\alpha^{-1} \equiv \alpha^{-1}_1 + \alpha^{-1}_2$). The results of the theoretical calculations in the Standard and supersymmetric models are shown in fig. 1, together with the experimental result [7]. Unification of couplings is clearly inconsistent with the Standard Model evolution for any value of $M_X$. This rules out any simple GUT which breaks directly into $SU_3 \times SU_2 \times U_1$, with only ordinary matter content. Inclusion of additional light particles or intermediate steps of gauge symmetry breaking may reconcile the Standard Model with the idea of unification. Of course, in this case, any prediction from gauge coupling unification is necessarily lost. More interesting is the supersymmetric case in which unification is achieved in the minimal version of the model, with $M_X \simeq 10^{16} \text{ GeV}$. From the historical point of view, it is amusing to notice that in 1981, when supersymmetric GUTs were first proposed, the experimental data [18] were compatible with standard GUTs, but disfavoured supersymmetric unification; see fig. 1.

8. ELECTROWEAK SYMMETRY BREAKING

As a realistic theory of particle interactions, the supersymmetric model should describe the correct pattern of electroweak symmetry breaking. This is obtained by the Higgs mechanism. As already mentioned in sect. 6, supersymmetry requires two Higgs doublets, as opposed to the single one of the Standard Model. Along the neutral components of the two Higgs fields, the scalar potential is:

$$V(H^0_1, H^0_2) = m_1^2 |H^0_1|^2 + m_2^2 |H^0_2|^2 - m_3^2 (H^0_1 H^0_2 + \text{h.c.}) + \frac{g^2 + g'}{8} \left(|H^0_1|^2 - |H^0_2|^2\right)^2$$

where $g$, $g'$ are respectively the $SU_2$ and $U_1$ gauge coupling constants. The mass parameters $m_1^2, m_2^2$ and $m_3^2$ originate from soft-breaking terms and are therefore of the order of $m_S$, the mass scale introduced in sect. 6. The stability of the potential for large values of fields along the direction $H^0_1 = H^0_2$ requires

$$m_1^2 + m_2^2 > 2|m_3^2|.$$  \hspace{1cm} (16)

Since electroweak symmetry is broken, the origin $H^0_1 = H^0_2 = 0$ must correspond to an unstable configuration, which implies:

$$m_2^2 < m_3^4.$$  \hspace{1cm} (17)

It is often assumed that the soft-breaking terms satisfy some universality conditions around $M_X$. Notice that, should for instance $m_1^2 = m_2^2$, eqs. (16) and (17) cannot be simultaneously satisfied and electroweak symmetry remains unbroken. Nevertheless, before drawing any conclusion, we have to include the renormalization effects of changing the scale from $M_X$ to the electroweak scale $M_W$. These effects are important as they are proportional to a large logarithm, $\log(M_X^2/M_W^2)$, and they have been systematically computed up to two loops [19]. Generically, the effect of gauge interactions is to increase the masses as we evolve from $M_X$ to $M_W$. Therefore, if all masses are equal at $M_X$, we expect gluinos to be heavier than charginos and neutralinos, and similarly squarks to be heavier than sleptons, because of the dominant QCD effects. On the other hand, Yukawa interactions decrease the masses in the renormalization from high to low energies. Therefore, the stops will be the lightest among squarks, since the top quark coupling gives the dominant Yukawa effect.
Let us now consider the evolution of the Higgs mass parameters. As they do not feel QCD forces at one loop, their gauge renormalization is not very significant. The Yukawa coupling effect is important for $m_1^2$, because $H_2$ is the Higgs field responsible for the top quark mass, but not for $m_2^2$. Therefore, as an effect of the heavy top quark, $m_2^2$ decreases and it is likely to be driven negative around the weak scale, while $m_1^2$ remains positive. For $m_1^2 > 0$ and $m_2^2 < 0$, eqs. (16) and (17) can be easily satisfied and electroweak symmetry is broken [20].

In conclusion, the supersymmetric model is consistent with electroweak symmetry breaking and the mechanism involved is appealing in several ways. First of all, the breaking is driven by purely quantum effects, a theoretically attractive feature. Then it needs a heavy top quark, which agrees with the Tevatron discovery. Finally, we have found that the dynamics itself chooses to break down $SU_2$. In a supersymmetric theory, colour $SU_3$ could spontaneously break if squarks get a vacuum expectation value, but this does not happen since squark masses squared receive large positive radiative corrections.

The minimization of the Higgs potential in eq. (15) gives:

$$\frac{M_Z^2}{2} = \frac{g^2 + g'}{8} v^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1},$$

$$\sin^2 \beta = \frac{2m_2^2}{m_1^2 + m_2^2},$$

where

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}} \cos \beta, \quad \langle H_2^0 \rangle = \frac{v}{\sqrt{2}} \sin \beta.$$

Equation (18) can be interpreted as a prediction of $M_Z$ in terms of the soft supersymmetry-breaking parameters $(a_i)$ which determine $m_1^2, m_2^2,$ and $m_3^2$. Unfortunately, we are not able to compute supersymmetry breaking, and therefore we can only use eq. (18) as a constraint which fixes one of the parameters $a_i$ in terms of the others.

We can also use eq. (18) to define a quantitative criterion for obtaining upper bounds on supersymmetric particle masses from the naturalness requirement [21]. It is intuitive that, as the supersymmetry-breaking scale $m_S$ grows, eq. (18) can hold only with an increasingly precise cancellation among the different terms. We therefore require, for each parameter $a_i$:

$$\left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right| < \Delta,$$

where $M_Z^2$ is given by eq. (18) and $\Delta$ is the degree of fine tuning. Equation (21) can now be translated into upper bounds on the supersymmetric particle masses. Independently of specific universality assumptions on supersymmetry-breaking terms, we find [22], for instance, that the chargino and the gluino are respectively lighter than 120 and 500 GeV, if fine tunings no greater than 10% ($\Delta = 10$) are required.

9. **HIGGS SECTOR**

Supersymmetry requires two Higgs doublets and therefore an extended spectrum of physical Higgs particles. Out of the eight degrees of freedom of the two complex doublets, three are eaten in the Higgs mechanism and five correspond to physical particles. These form two real CP-even scalars $(h, H)$, one real CP-odd scalar $(A)$, and one complex
The Higgs potential contains three parameters \(m_1^2, m_2^2, m_3^2\) and one of them is fixed by the electroweak symmetry-breaking condition, eq. (18). Therefore, all tree-level masses and gauge couplings of the five Higgs particles are completely described by only two free parameters.

Another important feature of the supersymmetric Higgs potential is that the quartic coupling is given in terms of gauge couplings, see eq. (15). In the Standard Model case, the quartic Higgs coupling measures the Higgs mass. Therefore, it is not surprising to find that in supersymmetry the mass of the lightest Higgs is bounded from above:

\[
m_h < M_Z |\cos 2\beta|.
\]

Supersymmetry does not only provide a solution to the hierarchy problem by stabilizing the Higgs mass parameter, but also predicts the existence of a Higgs boson lighter than the \(Z^0\).

Note that eq. (22) holds only at the classical level. There are important radiative corrections to the lightest Higgs mass proportional to \(m_t^4\) [23]:

\[
\delta m_h^2 \simeq \frac{3}{\pi^2} \frac{m_t^4}{v^2} \log \frac{m_S}{v}.
\]

The upper bound given in eq. (22) is then modified, and the result is shown in fig. 2 [25]. For extreme values of the parameters, \(m_h\) can be as heavy as 150 GeV, but it is generally much lighter.

This is an excellent opportunity for LEP2, where the Standard Model Higgs boson can be discovered via the process \(e^+e^- \rightarrow hZ^0\) in essentially the entire kinematical range \(m_h < \sqrt{s} - M_Z\). In the supersymmetric case, the search is more involved, because of the extended Higgs sector. For \(\tan \beta\) close to 1, the supersymmetric Higgs boson resembles the Standard Model counterpart and the LEP2 search is unchanged. For large values of \(\tan \beta\), the cross-section for \(e^+e^- \rightarrow hZ^0\) is reduced and can become unobservable at LEP2. However, at the same time, the CP-odd Higgs boson \(A\) becomes light and the cross-section for the process \(e^+e^- \rightarrow hA\) is then sizeable. The two different Higgs production mechanisms are therefore complementary and allow the search for the supersymmetric Higgs boson at LEP2 for most of the parameters. Nevertheless, a complete exploration of the whole supersymmetric parameter space will be possible only at the LHC, at the beginning of the next millenium.

The discovery of a light Higgs boson is certainly not a proof of the existence of supersymmetry at low energies. However, in the Standard Model, vacuum stability imposes a lower bound on the Higgs mass as a function of the top quark mass [24]. This is shown in fig. 2 [25], where the validity of the Standard Model is assumed up to the Planck mass. For comparison, the upper bound on the supersymmetric Higgs mass is also shown in fig. 2. Notice that, for \(m_t < 175\) GeV, the Higgs discovery can discriminate between the supersymmetric model and the Standard Model with \(\Lambda_{\text{max}} = M_{Pl}\). Although this is not strictly true for \(m_t > 175\) GeV, it is clear that the Higgs search can in general give good indications about the scale of new physics.

10. **SUPERSYMMETRY AND EXPERIMENTS**

If the Higgs search is certainly an important experimental test, evidence for low-energy supersymmetry will come only from the discovery of the partners of ordinary particles.
Figure 2: Left: Lower bound on the Standard Model Higgs boson mass (thick lines) from metastability requirements, as a function of the top quark mass, for a cut-off scale $\Lambda = 10^{19}$ GeV. Upper bound on the supersymmetric Higgs boson mass (thin lines) for $m_S < 1$ TeV. The dashed lines show the uncertainties in the bounds. Right: Lower bounds on the Standard Model Higgs boson mass from the metastability requirements, as a function of the cut-off scale $\Lambda$ for $m_t = 170, 180, 190, 200$ GeV (lines from bottom to top). (Courtesy M. Quiros)

The most important feature of supersymmetry phenomenology is the existence of a discrete symmetry, called $R$-parity, which distinguishes ordinary particles from their partners. This is not an accidental symmetry, in the sense that it is not an automatic consequence of supersymmetry and gauge invariance. Nevertheless, it is usually assumed, or else dangerous $B$- or $L$-violating interactions are introduced. It can be understood as a consequence of gauge symmetry in GUT models which contain left-right symmetric groups. If $R$-parity is indeed conserved only an even number of supersymmetric partners can appear in each interaction. As a consequence, supersymmetric particles are produced in pairs and the lightest supersymmetric particle is stable.

In most of the models, this stable particle turns out to be the lightest neutralino ($\chi^0$). This is fortunate for the model, since the present density of electric- or colour-charged heavy particles is very strongly limited by searches for exotic atoms [26]. A stable neutral particle is not only allowed by present searches but also welcome since it can explain the presence of dark matter in the Universe (see ref. [27]). From the point of view of collider experiments, $\chi^0$ will behave as a heavy neutrino which escapes the detector, leaving an unbalanced momentum and missing energy in the observed event. The distinguishing signature of supersymmetry is therefore an excess of missing energy and momentum. For example, in $e^+e^-$ colliders, charginos and sleptons are pair-produced with typical electroweak cross-sections and then decay, giving rise to events such as:

\[
e^+e^- \rightarrow \chi^+\chi^- \rightarrow \text{isolated leptons and/or jets} + \not{E}_T ,
\]

\[
e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^- \rightarrow \text{isolated leptons} + \not{E}_T .
\]

Using these processes, LEP1, working at the $Z^0$ peak, was able to rule out the existence of
these particles with masses less than $M_Z/2$ [7]. LEP2 should cover most of the kinematical range, and discover or exclude $\chi^\pm$ and $\ell^\pm$ with masses almost up to $\sqrt{s}/2$. This is certainly going to be a very critical region since, as we have seen in sect. 8, the 10% fine-tuning limits place the weakly-interacting supersymmetric particles at the border of the LEP2 discovery reach.

Strongly-interacting particles, such as squarks and gluinos, can be best studied at hadron colliders where they are produced with large cross-sections. The signature is again missing transverse energy carried by the neutralinos produced in the decays of squarks and gluinos. Tevatron experiments have set limits on the masses of these particles of about 150–200 GeV, depending on the particular model assumptions. At the LHC squarks and gluinos can be searched even for masses of several TeV, well above the 10% fine-tuning limits.

It is worth pointing out that although $e^+e^-$ colliders are the ideal machines for a systematic search of new weakly-interacting particles, charginos and neutralinos may also be discovered at hadron colliders, for instance in the process:

$$p\bar{p} \rightarrow \chi_1^0\chi_2^0, \quad \chi_1^\pm \rightarrow \ell^\pm \nu\chi_1^0, \quad \chi_2^0 \rightarrow \ell^+\ell^-\chi_1^0.$$  \hspace{1cm} (25)

The signal of three leptons and missing transverse energy in the final state has almost no Standard Model background, when sufficient lepton isolation requirements are imposed. However, it is difficult to obtain lower bounds on the new particle masses, because the leptonic branching ratios of charginos and neutralinos depend strongly on the model parameters.

In conclusion, this generation of colliders is testing the theoretically best-motivated region of parameters in the supersymmetric model. We can be confident that, after the LHC has run, either low-energy supersymmetry will have been discovered or it must be discarded, since its main motivation is no longer valid.

11. THE FLAVOUR PROBLEM

The Standard Model Lagrangian for gauge interactions is invariant under a global $U_3^g$ symmetry, with each $U_i$ acting on the generation indices of the five irreducible fermionic representations of the gauge group $(q_L, u_R^i, d_R^i, \ell_L^i, e_R^i)$, $i = 1, 2, 3$. This symmetry, called flavour (or generation) symmetry, implies that gauge interactions do not distinguish among the three generations of quarks and leptons. In the real world, this symmetry must be broken, as quarks and leptons of different generations have different masses. However, the breaking must be such as to maintain an approximate cancellation of Flavour-Changing Neutral Currents (FCNC). This is called the flavour problem.

In the Standard Model the flavour problem is solved in a simple and rather elegant way. The flavour symmetry is broken only by the Yukawa interactions between the Higgs field and the fermions. After electroweak symmetry breaking, these interactions give rise to the various masses of the three generations of quarks and leptons. The attractive feature of this mechanism is that all FCNC exactly vanish at tree level [28]. This is a specific property of the Standard Model with minimal Higgs structure and it is not automatic in models with an enlarged Higgs sector. Small contributions to FCNC are generated at loop level and generally agree with experimental observations. Although this mechanism provides a great success of the Standard Model, it prevents us from computing any of the quark or lepton masses, as these are introduced in terms of some free parameters.
In supersymmetry, the solution of the flavour problem is more arduous. Most of the soft-breaking terms introduced in sect. 6 generally violate the flavour symmetry and give too large contributions to the FCNC. This can be understood by recalling that, in a softly-broken supersymmetric theory, the mass matrices for quarks and squarks are independent and therefore cannot be simultaneously diagonalized by an equal rotation of the quark and squark fields. Thus neutral currents involving gluino-quark-squark vertices can mediate significant transitions among the different generations. Only if squarks and gluinos were heavier than 10–100 TeV could generic soft-breaking terms be consistent with observations of FCNC processes. Since, as discussed in sect. 8, the very motivation for low-energy supersymmetry implies that squarks and gluinos must be lighter than 500–1000 GeV, we have to postulate that the supersymmetry-breaking terms have some specific property.

The first possibility is that the supersymmetry-breaking terms respect the flavour symmetry in the limit of vanishing Yukawa couplings. This possibility is often advocated in models based on supergravity, on the basis of the hypothesis that all gravitationally-induced interactions are flavour-invariant. However, this hypothesis has been shown to be incorrect both in supergravity models with generic Kähler metrics [29] and in models derived from superstrings [30]. Nevertheless, this is an interesting possibility, since it significantly reduces the number of free parameters in the supersymmetry-breaking terms and allows sharp predictions testable at future colliders.

The other possibility is that the supersymmetry-breaking terms violate the flavour symmetry but are approximately aligned with the corresponding flavour violation in the fermionic sector (e.g. with the Yukawa couplings). This can be the result of some new symmetry [31] or some dynamical mechanism [32].

It is likely that the solution of the flavour problem is linked with the mechanism of supersymmetry breaking and therefore it will only be unravelled after significant theoretical developments. Now we can only speculate that an understanding of the flavour problem may help us to calculate the amount of flavour breaking and ultimately all quark and lepton masses.

12. TECHNICOLOUR

We have seen how supersymmetry can cure the hierarchy problem of the Standard Model by stabilizing the mass scale in the Higgs potential. Technicolour [33] offers a different solution to the hierarchy problem, based on the idea of removing all fundamental scalar particles from the theory. The mass scale which sets the electroweak breaking is dynamically determined in a strongly interacting gauge theory with purely fermionic matter.

The presence of light scalars (mesons) in the hadronic spectrum does not pose a problem of hierarchy. The description of mesons as fundamental particles is valid only up to about $\Lambda_{QCD}$. Above this scale, physics is described in terms of quarks and gluons, and hadrons have to be interpreted as composite particles. Technicolour aims to describe the Higgs boson as a composite particle, similarly to the case of mesons in QCD.

In order to illustrate the main idea of technicolour, let us consider as a toy model QCD with only two massless flavours ($m_u = m_d = 0$). In this limit, the theory has a chiral $SU(2)_L \times SU(2)_R$ invariance, in which the left-handed and right-handed components of the up and down quarks are rotated independently. As QCD becomes strongly-interacting
at $Q^2 \lesssim \Lambda^2_{QCD}$, the quark condensates are formed:

$$\langle u\bar{u} \rangle = \langle d\bar{d} \rangle = \mathcal{O}(\Lambda^3_{QCD}) .$$  \hspace{1cm} (26)

If the two condensates are equal, the chiral symmetry is broken to the vectorial part $SU(2)_{L+R}$. Goldstone's theorem ensures the existence of three massless scalar particles in the spectrum, the pions $\pi^0, \pi^\pm$. In the real world, quark masses explicitly break chiral symmetry and give small masses to the pions. Also, if the strange quark is included, the chiral symmetry $SU(3)_L \times SU(3)_R$ is broken to $SU(3)_{L+R}$, giving rise to the meson octet as approximate Goldstone bosons.

Let us turn on weak interactions in our toy model. Since the $W$ boson couples to quarks, it also interacts with the pions. This coupling can be obtained from PCAC, which determines the matrix element of the broken current ($j^a_\mu$) in terms of the pion decay constant $f_\pi$:

$$\langle 0 | j^a_\mu | \pi^b \rangle = f_\pi q_\mu \delta^{ab} .$$  \hspace{1cm} (27)

Here $a, b$ are $SU(2)$ indices and $q_\mu$ is the pion four-momentum. From eq. (27) and the coupling of the $W$ boson to the weak current, we obtain the coupling between $W^a_\mu$ and $\pi^b$:

$$\frac{g}{2} f_\pi q_\mu \delta^{ab} .$$  \hspace{1cm} (28)

Consider now the correction of one-pion exchange in the $W$ propagator:

$$\frac{1}{q^2} + \frac{1}{q^2} \left( \frac{g}{2} f_\pi q^a \right) \frac{1}{q^2} \left( \frac{g}{2} f_\pi q_\mu \right) \frac{1}{q^2} .$$  \hspace{1cm} (29)

The first term corresponds to the uncorrected massless $W$ propagator and the second term corresponds to the exchange of a massless pion between two $W$ propagators with the coupling given in eq. (28). We can insert an infinite number of pion exchanges, but it is not difficult to sum the whole series:

$$\frac{1}{q^2} \sum_{n=0}^{\infty} \left[ \left( \frac{g}{2} f_\pi \right)^n \right] = \frac{1}{q^2 - \left( \frac{g}{2} f_\pi \right)^2} .$$  \hspace{1cm} (30)

Equation (30) shows that the effect of the pion exchange is to shift the pole value of the $W$ propagator to

$$M_W = \frac{g}{2} f_\pi .$$  \hspace{1cm} (31)

The $W$ boson has acquired mass, which is not a surprising result if we think that we have promoted a global broken symmetry to a local invariance. The value for the $W$ mass given by eq. (31) is about 30 MeV, certainly too small to explain the experimental data.

We can use the result of this toy model and explain the physical value of $M_W$, if we introduce a new force, called technicolour. Technicolour behaves in a similar fashion to the ordinary colour forces but it becomes strong at a much larger scale $\Lambda_{TC} \simeq 500$ GeV. The simplest technicolour model is very easy to construct. Take a doublet of fermions with the same electroweak quantum numbers as the up and down quarks, assign to them a technicolour charge and call them techniquarks $U$ and $D$. The condensates

$$\langle \bar{U} U \rangle = \langle \bar{D} D \rangle = \mathcal{O}(\Lambda^3_{TC})$$  \hspace{1cm} (32)
generate three composite Goldstone modes, which become the longitudinal degrees of freedom of the $W$ and $Z$ gauge bosons. We have then built a model of electroweak symmetry breaking with no fundamental Higgs boson. The experimental signature is the presence of strongly interacting dynamics at the TeV scale, which produces new resonances similar to those found in the hadronic spectrum at the GeV scale.

Although the mechanism for generating electroweak breaking in technicolour is very elegant, several difficulties have prevented the construction of a fully realistic model. The first problem is the communication of electroweak breaking to the quark and leptonic sectors of the theory. This can be done via new interactions, called extended technicolour (ETC) forces [34], which couple quarks to techniquarks. If the ETC symmetry is broken (possibly by some dynamical mechanism) at a scale $M_{ETC}$ larger than $\Lambda_{TC}$, quarks and leptons receive masses of the order of

$$m_f \sim \frac{\langle F \bar{F} \rangle}{M_{ETC}^2} \sim \frac{\Lambda_{TC}^3}{M_{ETC}^2},$$

where $\langle F \bar{F} \rangle$ is the corresponding technifermionic condensate. The trouble is that measurements of FCNC processes generally impose stringent lower bounds on $M_{ETC}$, of the order of 100 TeV. This means that the ETC mechanism can generate the masses for the first generation of fermions, but has difficulties to explain the larger masses of the second and third generations. The task is particularly arduous for the top quark, since a dynamical mechanism which explains the large isospin breaking in the difference between $m_t$ and $m_b$ generally leads to large corrections to the $\rho$ parameter, the ratio between the strengths of the neutral and charged weak currents. Finally, the effect of the strong technicolour dynamics always gives sizeable corrections to the electroweak precision data in LEP1, which have been shown to agree with the Standard Model with great accuracy [2].

The hope is that these problems can be cured in technicolour theories with dynamics substantially different from a scaled-up QCD. There has been some effort in this direction, trying to construct theories in which the ultraviolet behaviour of the technifermion self-energy enhances the quark mass contribution, while the infra-red behaviour determines the $W$ mass. This may occur in theories with slowly running coupling constants (the so-called walking technicolour [35]) or in fixed-point gauge theories [36], although the non-perturbative nature of the problem prevents us from making reliable calculations.

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