The Minimal Supersymmetric Standard Model (MSSM)

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Abstract

The structure of the MSSM is reviewed. We first motivate the particle content of the theory by examining the quantum numbers of the known standard model particles and by the requirement of anomaly cancellation.

Once the particle content is fixed we can write down the most general renormalizable superpotential. However such a superpotential will contain terms breaking lepton and baryon number which leads us to the concept of R-parity conservation.

The question of supersymmetry breaking is discussed next. We list the possible soft breaking terms. However the Lagrangian involving the most general soft breaking terms is phenomenologically intractable because of the appearance of the many new parameters. It also leads to some unacceptable predictions. To reduce the number of parameters we restrict ourselves to the case with universal soft breaking terms at the GUT scale. We motivate the need for universal soft breaking terms by the apparent unification of gauge couplings in the MSSM and by the absence of flavor changing neutral currents. Then we discuss radiative electroweak symmetry breaking. Radiative breaking arises because the one loop corrections involving the large top Yukawa coupling change the sign of the soft breaking mass parameter of the up-type Higgs doublet, this way introducing a nontrivial minimum in the Higgs potential.

Finally we give an overview of the possible mixings in the MSSM and enumerate the physical (mass eigenstate) fields together with the mass matrices.

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1 Introduction

The Standard Model (SM) of particle physics enjoys an unprecedented success: up to now no single experiment has been able to produce results contradicting this model. Particle theorists are nevertheless unhappy with this theory. The most important features of the SM that are technically allowed but nevertheless theoretically unsatisfying are the following:
   a. There are too many free parameters
   b. The SU(2)⊗U(1) group is not asymptotically free
   c. Electric charge is not quantized
   d. The hierarchy problem.

While the first three problems can be taken care by introducing grand unification, the mystery of the hierarchy problem remains unsolved in GUT’s as well. The hierarchy problem is associated with the presence of elementary scalars (Higgs) in the SM. The problem is that in a general QFT containing an elementary scalar the mass of this scalar would be naturally at the scale of the cutoff of the theory (if the SM were the full story then the Higgs mass would be naturally of $O(M_{Pl})$) due to the quadratically divergent loop corrections to the Higgs mass. If one wants to protect the scalar masses from getting these large loop corrections one needs to introduce a new symmetry. The only known such symmetry is supersymmetry (SUSY), which relates fermions and bosons to each other.\footnote{Another way of solving the hierarchy problem is to assume that the Higgs is not an elementary scalar but a bound state of fermions, which idea leads to technicolor theories.}

In this paper we will review the minimal extension of the SM that includes SUSY, the Minimal Supersymmetric Standard Model (MSSM) \([1, 2, 3, 4, 5, 6]\). We will assume that the reader is familiar with both the structure of the SM and with N=1 global SUSY.

The paper is organized as follows: Section 2 discusses the particle content of the MSSM together with the most general renormalizable superpotential. The need for R-parity is also shown here. The possible form of SUSY breaking terms is described in Section 3 together with the phenomenological constraints (and theoretical biases) on these SUSY breaking terms. That section will be closed with a discussion of radiative electroweak symmetry breaking. In Section 4 we present the possible mixings in the MSSM together with the mass matrices. Finally we conclude in Section 5. We did not attempt to give a comprehensive list of references of the subject. More complete references on the MSSM can be found in \([1, 2, 3, 4, 6]\).

2 Particle content and superpotential

The SM is a spontaneously broken SU(3)⊗SU(2)⊗U(1) gauge theory with the matter fields being

\[\text{2}\]
leptons: \( L_i = \left( \begin{array}{c} \nu \\ e \end{array} \right)_{L_i} = (1, 2, -\frac{1}{2}) \)
\( e_{R_i} = (1, 1, -1) \)
quarks: \( Q_i = \left( \begin{array}{c} u \\ d \end{array} \right)_{L_i} = (3, 2, \frac{1}{6}) \)
\( u_{R_i} = (3, 1, \frac{2}{3}) \)
\( d_{R_i} = (3, 1, -\frac{1}{3}) \)
Higgs: \( H = \left( \begin{array}{c} h^+ \\ h^0 \end{array} \right) = (1, 2, \frac{1}{2}) \quad i = 1, 2, 3, \) \quad (2.1)

where \( i \) is the generation index, \( L \) and \( R \) refer to left and right handed components of fermions and the numbers in parenthesis are the SU(3)⊗SU(2)⊗U(1) quantum numbers.

The MSSM is an extension of the SU(3)⊗SU(2)⊗U(1) gauge theory with \( N=1 \) SUSY (which will be broken in a specific way, see Section 3).

The rules of building \( N=1 \) SUSY gauge theories are to assign a vector superfield (VSF) to each gauge field and a chiral superfield \((\chi \text{SF})\) to each matter field. The physical particle content of a VSF is one gauge boson and a Weyl fermion called gaugino, and of the \( \chi \text{SF} \) is one Weyl fermion and one complex scalar \([8, 9]\). The VSF’s transform under the adjoint of the gauge group while the \( \chi \text{SF}’s \) can be in any representation. Since none of the matter fermions of the SM transform under the adjoint of the gauge group we can not identify them with the gauginos. Thus we have to introduce new fermionic SUSY partners to each SM gauge boson.

If we now look at the matter fields of the SM listed above we see that the only possibility to have two SM fields as each others superpartner would be to have \( \tilde{H} = i\tau_2 H^* \) as a superpartner of \( L \). However this is phenomenologically unacceptable since \( L \) carries lepton number 1, while \( H \) lepton number 0, and the superpartners must carry the same gauge and global quantum numbers. Thus we conclude that we have to introduce a new superpartner field to every single field present in the SM: scalar partners to the fermions (called sleptons and squarks), fermionic partners to the Higgs (Higgsino) and gauge bosons (gaugino).

However we can see that we have introduced one extra fermionic SU(2) doublet Higgs with SU(3)⊗SU(2)⊗U(1) quantum numbers \((1, 2, \frac{1}{2})\). This is unacceptable because of the Witten anomaly and because of the U(1) anomaly that it causes. Thus we need to introduce one more SU(2) doublet with opposite U(1) charge. The need for this second Higgs doublet can also be seen in a different way: in the SM one needed only one Higgs doublet to give masses to both up and down type quarks, because one was able to use both \( H \) and \( \tilde{H} \) in the Lagrangian. However in SUSY theories the superpotential (the only source of Yukawa interactions between only matter fields and its partners) must be a holomorphic function of
\[
\begin{array}{cccccc}
\chi & \text{SU(3)} & \text{SU(2)} & \text{U(1)} & B & L \\
L_i & 1 & 2 & -\frac{1}{2} & 0 & 1 \\
\bar{E}_i & 1 & 1 & 1 & 0 & -1 \\
Q_i & 3 & 2 & \frac{1}{3} & \frac{1}{3} & 0 \\
\bar{U}_i & 3 & 1 & -\frac{2}{3} & -\frac{1}{3} & 0 \\
\bar{D}_i & 3 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 \\
H_1 & 1 & 2 & -\frac{1}{2} & 0 & 0 \\
H_2 & 1 & 2 & \frac{1}{2} & 0 & 0 \\
\end{array}
\]

Table 1: The \(\chi\) SF’s of the MSSM with their gauge and global quantum numbers. \(i = 1, 2, 3\).

the fields thus both \(H\) and \(\tilde{H}\) can not appear at the same time in the superpotential [8]. This again calls for the need of two Higgs doublet \(\chi\) SF’s, one with the quantum number of \(H\), the other with the quantum numbers of \(\tilde{H}\). The final resulting \(\chi\) SF content of the MSSM is given in table 1. (We use the conjugate fields \(\bar{U}, \bar{D}, \bar{E}\) because in the superpotential we can not use conjugation anymore.)

Once the particle content is fixed one can try to write down the most general renormalizable Lagrangian for this N=1 SUSY SU(3)⊗SU(2)⊗U(1) theory. It is known from the structure of N=1 SUSY gauge theories that the Lagrangian is completely fixed by gauge invariance and by supersymmetry, except for the choice of the superpotential, which could contain all possible gauge invariant operators of dimensions not greater than 3. In our case this means that

\[
W = (\lambda_{ij}^u Q^i H_2 \bar{U}^j + \lambda_{ij}^d Q^i H_1 \bar{D}^j + \lambda_{ij}^e L^i H_1 \bar{E}^j + \mu H_1 H_2) + \\
(\alpha_{ijk}^Q L^i L^j \bar{D}^k + \alpha_{ijk}^L L^i L^j \bar{E}^k + \alpha^3 L^i H_2 + \alpha_{ijk} \bar{D}^i \bar{D}^j \bar{U}^k). \tag{2.2}
\]

The terms in the first pair of parenthesis correspond to the SUSY extension of the ordinary Yukawa interactions of the SM and an additional term ("\(\mu\)-term") breaking the Peccei-Quinn symmetry of the two doublet model. However the terms is the second pair of parenthesis break baryon and lepton number conservation. Thus as opposed to the SM where the most general renormalizable gauge invariant Lagrangian automatically conserved baryon and lepton number, here one has to require some additional symmetries to get rid of the B and L violating interactions that are phenomenologically unacceptable.

The easiest way to achieve this is to introduce R-parity and require R-parity conservation.\(^3\)

\(^3\)One could forbid the appearance of the B,L breaking terms by imposing different symmetry requirements. For example a \(Z_2\) subgroup of B⊗L known as matter parity could achieve this goal as well. The point is that once those terms are absent R-parity will necessarily be a symmetry of the Lagrangian.
Under R-parity

\[ H_1, H_2 \rightarrow H_1, H_2, \]
\[ Q, U, D, L, \bar{E} \rightarrow -(Q, U, D, L, \bar{E}) \]
\[ \theta \rightarrow -\theta, \]

which means that

(ordinary particle) \(\rightarrow\) (ordinary particle)

(superpartner) \(\rightarrow\) -(superpartner).

Note that this \(Z_2\) group is a subgroup of a \(U(1)_R\) symmetry where the R-charges of the \(\chi\) SF’s are:

\[ R = 1 \text{ for } H_1, H_2 \]
\[ R = \frac{1}{2} \text{ for } L, \bar{E}, Q, U, D. \]

However the imposition of the full \(U(1)_R\) symmetry forbids Majorana masses for the gauginos which are phenomenologically needed. There are two possible solutions to this problem. One could impose only the \(Z_2\) subgroup, R-parity, which forbids the B,L violating terms in the superpotential, but allows for gaugino mass terms.\(^4\) If however one imposes the full \(U(1)_R\) symmetry then this symmetry has to be spontaneously broken to its \(Z_2\) subgroup leading to complications with the resulting Goldstone boson. We will not discuss this possibility further here. In both cases however R-parity is an unbroken symmetry of the theory.

As a consequence of R-parity conservation superpartners can be produced only in pairs, implying that the lightest superpartner (LSP) is stable if R-parity is exact. Most of the experimental detection modes of SUSY are based on this fact [10].

\section{SUSY breaking, radiative breaking of \(SU(2) \otimes U(1)\)}

In the previous section we have seen the particle content and the superpotential of the MSSM. However we know that this can not be the full story for two reasons:

- SUSY is not yet broken
- \(SU(2) \otimes U(1)\) is not yet broken.

First we discuss SUSY breaking. SUSY was invented to solve the hierarchy problem. However SUSY can not be an exact symmetry of nature since in this case many of the superpartners

\(^4\)R-parity as defined in eq. 2.3 is actually \(Z_2\) subgroup of the continous R-symmetry of 2.5 combined with a baryon and lepton number transformation. The value of the R-parity can be given by \(R = (-1)^{3B + L + 2S}\), where B is the baryon number, L the lepton number and S the spin of a given particle.
should have been observed by experiments. One has two possibilities for SUSY breaking, either explicit or spontaneous breaking. While theoretically spontaneous breaking of SUSY is much more appealing, one nevertheless has to rule out this possibility in the context of MSSM. To see the reason behind this we have to examine the scalar quark mass matrix in detail [11]. The most general scalar mass matrix in N=1 SUSY gauge theories is given by

\[ M^2_a = \begin{bmatrix}
\bar{W}^{ab} W_{b c} + \frac{1}{2} D_a^a D_{a c} + \frac{1}{2} D_{c a} D_a \\
\bar{W}_{a b}^{b c} + \frac{1}{2} D_{a a} D_{a c} + \frac{1}{2} D_{a c} D_a \\
W_{a b} \bar{W}'_{b c} + \frac{1}{2} D_{a a} D_{a c} + \frac{1}{2} D_{a c} D_a
\end{bmatrix}, \quad (3.1) \]

where \( W_a = \frac{\partial W}{\partial \phi_a} |_{\phi = \langle \phi \rangle} \), \( W_{ab} = \frac{\partial^2 W}{\partial \phi_a \partial \phi_b} |_{\phi = \langle \phi \rangle} \), etc. and \( D_a = g_a \phi^{\dagger} T_a \phi \), \( D_{a c} = \frac{\partial D_a}{\partial \phi_c} |_{\phi = \langle \phi \rangle} \), etc., \( W \) is the superpotential, the \( \phi_a \)'s are the complex scalars of the \( \chi \) SF's, the \( g_a \)'s are the gauge couplings, and the \( T_a^\alpha \)'s are the generators of the gauge group in the representations of the \( \chi \) SF's.

Specifying this matrix to the squarks we note that since all the squark VEV's must vanish (so as color and electric charge are unbroken symmetries) \( D_a = 0 \) for the squarks. On the other hand quarks get their masses solely from the superpotential thus \( \bar{W}^{ab} W_{b c} \) is nothing but the square of the quark mass matrix \( m^2 \). Since electric charge and color are not broken one needs to have \( D_1 = D_2 = 0 \) (where 1 and 2 here are SU(2) indices) and \( D_i = 0 \) \((i = 1, \ldots, 8\) of SU(3)). Thus the only possible non-vanishing D-terms are \( D_3 \) and \( D_Y \). Therefore the squark mass matrices can be written in the form

\[ M_{2/3}^2 = \begin{bmatrix}
m_{2/3} m_{2/3}^{\dagger} + (\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y)^1 \\
\Delta \end{bmatrix} \quad \frac{\Delta}{m_{2/3} m_{2/3}^{\dagger} - \frac{2}{3} g' D_Y} \] \quad (3.2)

for the charge 2/3 squarks and

\[ M_{1/3}^2 = \begin{bmatrix}
m_{1/3} m_{1/3}^{\dagger} + (-\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y)^1 \\
\Delta' \end{bmatrix} \quad \frac{\Delta'}{m_{1/3} m_{1/3}^{\dagger} + \frac{1}{3} g' D_Y} \] \quad (3.3)

for the charge -1/3 squarks. Here \( m_{1/3} \) and \( m_{2/3} \) are the 3 by 3 quark mass matrices in generation space. \( M_{2/3}^2 \) and \( M_{1/3}^2 \) are thus 6 by 6 matrices for the 3 generations of left and right handed squarks. The exact form of \( \Delta \) and \( \Delta' \) is not important for us. One may notice that (as a consequence of the tracelessness of the group generators) the sum of the D-terms appearing in the two squark matrices is zero. Therefore at least one of the appearing D-terms is non-positive. Assume for example that \( \frac{1}{2} g D_3 + \frac{1}{6} g' D_Y \leq 0 \). But if \( \beta \) is the normalized eigenvector of the quark mass matrix \( m_{2/3} \) corresponding to the smallest eigenvalue \( m_0 \) we get that

\[ (\beta^{\dagger}, 0) M_{2/3}^2 \begin{pmatrix} \beta \\ 0 \end{pmatrix} \leq m_0^2. \quad (3.4) \]
Therefore there must be a charged scalar state with mass less than the mass of either the u or d quark which is experimentally excluded.

Thus we conclude that we need to introduce explicit SUSY breaking terms in order to circumvent the previous argument. However these terms must be such that the solution of the hierarchy problem is not spoiled. Such terms are called soft SUSY breaking terms, and those are the terms that do not reintroduce quadratic divergences into the theory.

The philosophy behind these soft breaking terms is the following: there is a sector of physics that breaks SUSY spontaneously. This is at much higher energy scales than the weak scale. SUSY breaking is communicated in some way (either through gauge interactions or through gravity) to the MSSM fields and as a result the soft breaking terms appear. One popular implementation of this idea is to break SUSY spontaneously in a “hidden sector”, that is in a sector of fields that do not interact with the SM particles (“visible sector”) except through supergravity which will mediate the SUSY breaking terms to the visible sector. This mechanism with minimal supergravity generates universal soft breaking terms for the visible sector fields at the Planck scale.

Thus one has to handle the MSSM as an effective theory, valid below a certain scale (of new physics), and the soft breaking terms will parametrize our ignorance of the details of the physics of the SUSY breaking sector.

The most general soft SUSY breaking terms are [12]

i. gaugino mass terms
ii. scalar mass terms
iii. scalar quadratic and trilinear interaction terms.

Thus if one wants to implement this program consistently one has to add a separate mass term for each scalar and gaugino and add each quadratic and trilinear interaction term appearing in the superpotential with different coefficients to the Lagrangian:

\[-L_{soft} = \sum_{i=\bar{Q},\bar{U},...} m_i^2 |\phi_i|^2 + \left( \sum_{i=1,2,3} M_i \lambda_i \lambda_i - B \mu H_1 H_2 + \right.\]
\[+ \sum_{ij} A_{ij}^u \lambda_u^i Q^j H_2 \bar{U}^j + \sum_{ij} A_{ij}^d \lambda_d^i Q^i H_1 \bar{D}^j + \sum_{ij} A_{ij}^e \lambda_e^i L^i H_1 \bar{E}^j + h.c. \right) \] (3.5)

This would mean that we introduce 17 new real and 31 new complex parameters into the theory. There are two major problems with this:
- not every set of \((m_i, M_i, B, A_{ij})\) parameters is allowed by phenomenology
- there are too many new parameters to handle the phenomenology.

Let’s first see what the requirements for the soft breaking parameters are. The two most serious restrictions come from the requirements that

1. large flavor changing neutral currents (FCNC) and lepton number violations are absent
2. the theory should not yield too large CP violation.

One can easily understand why a general set of soft breaking parameters introduces large FCNC’s. Let’s look at the $K^0 - \bar{K}^0$ mixing. In the SM one gets contributions from the diagrams shown in Fig. 1. However in the MSSM one has additional contributions from the diagrams of Fig. 2, where the intermediate lines are now gauginos and squarks, and the cross denotes the soft breaking squark masses. In Fig. 2, the usual CKM factors appear at the vertices. Thus the leading part of this diagram is proportional to $V^\dagger M^2 V$, where $V$ is the CKM matrix. The successful implementation of the GIM mechanism in the SM in $K^0 - \bar{K}^0$ mixing is based on the fact that the diagrams are proportional to $V^\dagger V = 1$. However if $M^2$ is an arbitrary matrix then $V^\dagger M^2 V \neq 1$. Thus we can see that in order to maintain the successful GIM prediction in the MSSM one has to require that $M^2 \approx m^2_1$, that is squarks must be nearly degenerate.

Very similar arguments hold for the $\mu \to e\gamma$ process which will result in the need of nearly degenerate sleptons.

The second constraint on the soft breaking terms comes from the fact that the SM can account for all the measured CP violation. Thus there is no need for extra sources of CP violation in the MSSM, therefore it is usually assumed that the soft breaking parameters are real.

Thus we have seen what the phenomenological constraints on these soft breaking parameters are. Now we present a set of assumptions that satisfy these constraints and at the same time highly reduce the number of free parameters of the model:

1. Gaugino unification (common mass for the gauginos at the Planck scale)
2. Unification of soft masses (common soft breaking mass terms for the scalars at the Planck scale)
3. Unification of the soft breaking trilinear couplings $A_{ij}^k$ (common trilinear soft breaking...
term for each trilinear term at the Planck scale)

4. All soft breaking parameters are real.

As one can see these assumptions greatly reduce the number of independent free parameters of the theory. However one has to stress that these are just assumptions, with no solid basis of origin. The strongest argument in favor of these assumptions is that if one takes a supergravity theory in which SUSY is broken in a hidden sector and SUSY breaking is communicated to the visible sector by gravity then one gets flavor independent mass terms, real universal $A$-terms at the Planck scale and real gaugino masses, provided one assumes that the Kähler potential of the supergravity theory is minimal.

The argument for gaugino unification is the following. It is experimentally indicated that gauge couplings do unify in the MSSM [13]. However the 1-loop RGE for the gaugino masses is given by [5]

$$\frac{d}{dt} \left( \frac{\alpha_i}{M_i} \right) = 0, \quad i = 1, 2, 3 \quad t = \log \left( \frac{\Lambda}{M_{GUT}} \right). \quad (3.6)$$

Here $\alpha_i = g_i^2/4\pi$, $g_i$ are the gauge couplings and $M_i$ the gaugino masses. The ratios of gauge couplings to gaugino masses are scale invariant. Thus if gauge couplings unify so must the gaugino masses.

If one accepts these arguments then the independent soft breaking terms are $A_0, m_0, B$ and $M_{1/2}$ (at the Planck scale), and the soft breaking Lagrangian at the Planck-scale is given by

$$-\mathcal{L}_{soft}|_{M_P} = m_0^2 \sum_{i=Q, U, \ldots} |\phi_i|^2 + \left[ M_{1/2} \sum_{i=1, 2, 3} \lambda_i \lambda_i - B \mu H_1 H_2 \right]$$
and the Lagrangian at the weak scale can be obtained by running down these universal parameters from the Planck-scale\textsuperscript{5} to the weak scale. This procedure will yield sufficiently degenerate squark and slepton masses, and if the soft breaking terms are real at the Planck scale then they will not obtain imaginary parts at the weak scale either. Therefore the above assumptions do satisfy the phenomenological constraints and at the same time they greatly increase the predictive power of the theory. Often these assumptions about the soft breaking terms are assumed to be part of the definition of the MSSM. However one can not overemphasize the fact that these assumptions are ad hoc and need not necessarily be satisfied.

In the remainder of this Section we will discuss the breaking of SU(2)\(\otimes\)U(1). The Higgs potential without soft breaking terms is given by

\[
V_{SUSY}(H_1, H_2) = \mu^2 (|H_1|^2 + |H_2|^2) + \frac{g^2}{2} (H_1^T \vec{\tau} H_1 + H_2^T \vec{\tau} H_2)^2 + \frac{g'^2}{2} (H_1^T H_1 - H_2^T H_2)^2. \tag{3.8}
\]

The minimum of this potential is at \(\langle H_1 \rangle = \langle H_2 \rangle = 0\), thus we need to incorporate the soft breaking terms to get electroweak breaking. The full Higgs potential at the Planck (GUT) scale is

\[
V(H_1, H_2)|_{\text{GUT}} = (\mu^2 + m_0^2)(|H_1|^2 + |H_2|^2) - B\mu(H_1 H_2 + h.c.) + \frac{g^2}{2} (H_1^T \vec{\tau} H_1 + H_2^T \vec{\tau} H_2)^2 + \frac{g'^2}{2} (H_1^T H_1 - H_2^T H_2)^2. \tag{3.9}
\]

This potential still does not break SU(2)\(\otimes\)U(1). This can be seen in the following way: in order to have a nontrivial minimum of the Higgs potential

\[
m_H^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - m_{12}^2 (H_1 H_2 + h.c.) + \frac{g^2}{2} (H_1^T \vec{\tau} H_1 + H_2^T \vec{\tau} H_2)^2 + \frac{g'^2}{2} (H_1^T H_1 - H_2^T H_2)^2 \tag{3.10}
\]

the quadratic coefficients have to fulfill the following inequalities:

\[
m_{H_1}^2 + m_{H_2}^2 > 2 |m_{12}|
\]

\[
|m_{12}|^2 > m_{H_1}^2 m_{H_2}^2. \tag{3.11}
\]

The first inequality is required so that the potential remains bounded from below for the equal field direction \(H_1 = H_2\), while the second is required so that the quadratic piece has a negative part enabling a nontrivial minimum. We can see that the potential in eq. 3.9

\textsuperscript{5}Usually the Planck-scale and the GUT-scale are not distinguished and it is common to assume that 3.7 is still valid at the GUT-scale \(\approx 10^{16}\) GeV.
can not fulfill both inequalities at the same time thus electroweak symmetry is not broken at the tree level. However radiative corrections can change this situation. To calculate these radiative effects one needs to evaluate the one loop effective potential:

\[ V_{\text{tree}} = V_{\text{tree}}(\Lambda) + \Delta V_1(\Lambda), \]

(3.12)

where \( V_{\text{tree}} \) is the tree level superpotential with running parameters evaluated at a scale \( \Lambda \) and \( \Delta V_1 \) is the contribution of one loop diagrams to the effective potential evaluated by the method of Coleman and Weinberg. The running of the parameters in the tree level potential is generated by the one loop RGE’s. \( V_{\text{tree}} + \Delta V_1 \) is \( \Lambda \) independent up to one loop order. If we choose the scale \( \Lambda \) to be close to the scale of the masses of the particles of the theory (in our case \( \Lambda \approx M_{\text{weak}} \)) \( \Delta V_1 \) will not contain large logarithms, thus the leading one loop effects will arise due to the running of the parameters of the tree level potential between the Planck and the weak scale. To estimate the running effects on the Higgs parameters we neglect all Yukawa couplings with the exception of the top Yukawa coupling (this is the only large Yukawa coupling so it is reasonable to assume that the largest effects will be caused by it). Then the RGE’s for the soft breaking mass terms of the scalars participating in the top Yukawa coupling of the superpotential are [5]:

\[
\begin{align*}
\frac{dm^2_{H_2}}{dt} &= \frac{3}{5} g_1^2 M_1^2 + 3 g_2^2 M_2^2 - 3 \lambda_t^2 (m^2 + A_t^2) \\
\frac{dm^2_t}{dt} &= \frac{16}{15} g_1^2 M_1^2 + \frac{16}{3} g_2^2 M_2^2 - 2 \lambda_t^2 (m^2 + A_t^2) \\
\frac{m^2_{Q_3}}{dt} &= \frac{1}{15} g_1^2 M_1^2 + 3 g_2^2 M_2^2 + \frac{16}{3} g_3^2 M_3^2 - \lambda_t^2 (m^2 + A_t^2),
\end{align*}
\]

(3.13)

where \( m^2 = m^2_{H_2} + m^2_t + m^2_{Q_3} \), \( t = \frac{1}{16 \pi^2} \log \frac{M_{\text{GUT}}^2}{\Lambda^2} \), \( \Lambda \) is the energy scale, \( g_i \) are the gauge couplings, \( A_t \) the soft breaking trilinear parameter corresponding to the top Yukawa coupling, \( M_i \) the gaugino masses and \( \lambda_t \) the top Yukawa coupling.

One can see that the contributions of the gauge and Yukawa loops are independent of each other and the contributions of the gauge loops are independent of the soft breaking masses \( m_t^2 \). Thus one can solve eq. 3.13 by setting the gauge couplings to zero and at the end add the gauge contribution to the resulting solution. Therefore one has to solve the following equation:

\[
\frac{d}{dt} \begin{pmatrix} m_{H_2}^2 \\ m_t^2 \\ m_{Q_3}^2 \end{pmatrix} = -\lambda_t^2 \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} m_{H_2}^2 \\ m_t^2 \\ m_{Q_3}^2 \end{pmatrix} - \lambda_t^2 A_t^2 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.
\]

(3.14)

This differential equation can be solved easily if one neglects the running of of \( \lambda_t \) and \( A_t \). The solution corresponding to the universal boundary condition at \( t = 0 \) (\( \Lambda = M_{\text{GUT}} \))
\[ m_{H_2}^2 = m_t^2 = m_{Q_3}^2 = m_0^2 \] in the limit \( t \to \infty \) is given by

\[ m_{H_2}^2 = -\frac{1}{2}m_0^2, \]

\[ m_t^2 = 0, \]

\[ m_{Q_3}^2 = -\frac{1}{2}m_0^2. \tag{3.15} \]

Thus we can see that the radiative corrections due to the top Yukawa coupling want to reverse the sign of the soft breaking mass parameter of the up-type Higgs, which is enough to satisfy the conditions for electroweak breaking of eq. 3.11 at the weak scale. The gauge loops will yield additional positive contributions proportional to \( M_1^2 / 2 \), and the solution to 3.14 is more complicated if one takes the running of \( \lambda_t \) and \( A_t \) into account. However the most important feature of the solution in 3.15 is unchanged: appropriate choices of the input parameters \( M_{1/2}, m_0, A_0 \) and \( \lambda_t \) will drive the soft breaking mass parameter of the up-type Higgs (and only of the up-type Higgs) negative which will result in the breaking of electroweak symmetry. This mechanism is called radiative electroweak breaking.

Thus as we have seen loop corrections usually modify the Higgs potential such that at the weak scale \( SU(2) \otimes U(1) \) is spontaneously broken. However it is not enough to require that the symmetry is broken, it has to reproduce the correct SM minimum. The Higgs potential at the weak scale can be written as

\[
(m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 - B \mu (H_1 H_2 + \text{h.c.}) + \frac{g^2}{2} (H_1^\dagger \tau H_1 + H_2^\dagger \tau H_2)^2 + \frac{g'^2}{2} (H_1^\dagger H_1 - H_2^\dagger H_2)^2. \tag{3.16}
\]

The VEV’s of the Higgs doublets are

\[
\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \tag{3.17}
\]

and we define \( \tan \beta = v_2 / v_1, \ v^2 = v_1^2 + v_2^2 \). Minimizing the Higgs potential we find that to fix the \( W, Z \) masses at their experimental values it is necessary that \( [5] \)

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2}M_Z^2, \]

\[
B = \frac{(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) \sin 2\beta}{2\mu}, \tag{3.18}
\]

where all parameters are to be evaluated at the weak scale.
With this we are now able to determine the free parameters of the MSSM. In the soft breaking sector we had $m_0, M_{1/2}, A_0$ and $B$. In the Higgs sector we have $\mu$ and $\tan \beta$, and since the top mass is experimentally not well measured and $\lambda_t$ tends to run to an IR fixed point at $M_Z$, $\lambda_t(M_G)$ is basically an unknown parameter of the theory as well. However from eq. 3.18 $\mu^2$ and $B$ are determined (but not the sign of $\mu$). Therefore the MSSM with R-parity, universal soft breaking parameters and radiative electroweak breaking is determined by 5+1 parameters: $m_0, M_{1/2}, A_0, \tan \beta, \lambda_t$ and the sign of $\mu$.

4 Sparticle masses

In this section we present the possible mixings between the superpartner fields and list the tree level mass matrices [1, 5, 14].

4.1 Sfermions

In principle one must diagonalize 6 by 6 matrices corresponding to the mixing of the L and R scalars of the 3 generations. To simplify this we neglect intergenerational mixings and take only L-R mixing into account.

4.1.1 Squarks

The mass matrices in the L-R basis are for each generation of up-type scalars is

$$M_{\tilde{u},L,R}^2 = \begin{pmatrix} m^2_{\tilde{Q}} + m_u^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2 \theta_W\right)D & m_u(A_u - \mu \cot \beta) \\ m_u(A_u - \mu \cot \beta) & m^2_{\tilde{u}} + m_u^2 + \frac{2}{3}\sin^2 \theta_W D \end{pmatrix},$$

(4.1)

where the mass parameters with a tilde refer to the soft breaking squark mass parameters while the mass parameters without tilde are the usual quark masses, $D = M_Z^2 \cos 2\beta$.

The down-type mass matrix is

$$M_{\tilde{d},L,R}^2 = \begin{pmatrix} m^2_{\tilde{Q}} + m_d^2 - \left(-\frac{1}{2} - \frac{1}{3}\sin^2 \theta_W\right)D & m_d(A_d - \mu \tan \beta) \\ m_d(A_d - \mu \tan \beta) & m^2_{\tilde{d}} + m_d^2 - \frac{1}{3}\sin^2 \theta_W D \end{pmatrix}.$$  (4.2)

The only source of intergenerational mixing is the superpotential, thus in the more general case the diagonal elements $m_{\tilde{u}}^2$ and $m_{\tilde{d}}^2$ must be exchanged to $v_{2,1}^2 (\lambda^\dagger_{\tilde{u},d} \lambda_{\tilde{u},d})_{ij}$, where $ij$ are generation indices. However since the CKM mixing is small and the soft breaking mass terms are large compared to the quark masses, these effects are usually negligible.
4.1.2 Sleptons

In the same notation the sneutrino masses are:

\[ M_\nu^2 = M_L^2 + \frac{1}{2} D \]  

(4.3)

while for \( \tilde{e}, \tilde{\mu}, \tilde{\tau} \) the mass matrices are

\[
M_{\tilde{e}_{L,R}}^2 = \begin{pmatrix} m_{\tilde{L}}^2 + m_e^2 - \frac{\sin^2 \theta_W}{2} D & m_e (A_e - \mu \tan \beta) \\ m_e (A_e - \mu \tan \beta) & m_{\tilde{e}}^2 + m_e^2 - \sin^2 \theta_W D \end{pmatrix}.
\]

(4.4)

4.2 The scalar Higgs sector

We use the notation

\[
H_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}.
\]

(4.5)

The tree level masses are calculated from the mass matrices

\[
\frac{1}{2} \frac{\partial^2 V_{\text{tree}}}{\partial (\text{Im} h_i^0) \partial (\text{Im} h_j^0)} = \frac{1}{2} M_A^2 \sin 2\beta \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix}
\]

\[
\frac{1}{2} \frac{\partial^2 V_{\text{tree}}}{\partial (\text{Re} h_i^0) \partial (\text{Re} h_j^0)} = \frac{1}{2} M_A^2 \sin 2\beta \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} + \frac{1}{2} M_Z^2 \sin 2\beta \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix}
\]

\[
\frac{\partial^2 V_{\text{tree}}}{\partial h_i^- \partial h_j^-} = \frac{1}{2} M_{H^\pm} \sin 2\beta \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix}
\]

(4.6)

where \( M_A^2 = m_{H_i}^2 + m_{H_j}^2 + 2\mu^2, M_{H^\pm} = M_{W^\pm}^2 + M_A^2, i, j = 1, 2. \)

The first mass matrix has eigenvalues 0 (GB eaten by the Z) and \( M_A^2 \) (CP odd scalar). The second matrix gives the masses for the light and heavy Higgs bosons:

\[
M_{H,h}^2 = \frac{1}{2} \left[ (M_A^2 + M_Z^2) \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2M_Z^2 \cos^2 2\beta} \right].
\]

(4.7)

The third matrix has eigenvalues 0 (charged GB’s eaten by \( W^\pm \)) and \( M_{H^\pm}^2 \) (charged scalars).

It is important to mention that for some of the Higgs masses the 1-loop corrections can be significant. For example from the above formula one would get that \( m_h \leq M_Z \), while including 1-loop corrections the corresponding bound will be modified to \( m_h \leq 150 \text{ GeV} \) \cite{14}.
4.3 Charginos

Charginos are mixtures of the charged Higgsinos and the charged gauginos ($\tilde{W}_1, \tilde{W}_2$). The mass matrix is given by $(\lambda^\pm = (\tilde{W}_2 \pm i\tilde{W}_1)/\sqrt{2})$:

\[
\begin{pmatrix}
\lambda^+ & \tilde{h}_2^\pm & \lambda^- & \tilde{h}_1^\pm \\
0 & 0 & M_2 & -g_2 v_1 \\
0 & g_2 v_2 & -\mu & 0 \\
M_2 & -g_2 v_2 & 0 & 0 \\
-g_2 v_1 & -\mu & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\lambda^+ \\
\tilde{h}_2^+ \\
\lambda^- \\
\tilde{h}_1^-
\end{pmatrix}.
\]

The eigenvalues are

\[
M_{C_{1,2}} = \frac{1}{2} \left[ (M_2^2 + \mu^2 + 2M_W^2) \pm \sqrt{(M_2^2 + \mu^2 + 2M_W^2)^2 - 4(M_2^2 - M_W^2 \sin 2\beta)^2} \right].
\]

4.4 Neutralinos

Neutralinos are the mixture of neutral Higgsinos and the neutral gauginos ($\tilde{B}, \tilde{W}_3$). The mass matrix is given by

\[
\begin{pmatrix}
i\tilde{B} & i\tilde{W}_3 & i\tilde{h}_1^0 & i\tilde{h}_2^0
\end{pmatrix}
\begin{pmatrix}
-M_1 & 0 & g' v_1/\sqrt{2} & -g' v_2/\sqrt{2} \\
0 & -M_2 & -g_2 v_1/\sqrt{2} & g_2 v_2/\sqrt{2} \\
g' v_1/\sqrt{2} & -g_2 v_1/\sqrt{2} & 0 & \mu \\
-g' v_2/\sqrt{2} & g_2 v_2/\sqrt{2} & \mu & 0
\end{pmatrix}
\begin{pmatrix}
i\tilde{B} \\
i\tilde{W}_3 \\
i\tilde{h}_1^0 \\
i\tilde{h}_2^0
\end{pmatrix}.
\]

5 Conclusions

We have systematically built up the minimal supersymmetric standard model. We have started from the particle content of the theory, discussed the superpotential and R-parity. Then the question of SUSY breaking and electroweak symmetry breaking have been examined. Finally we listed the physical particles of the theory, together with the tree level mass matrices.

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