COUPLERS FOR CAVITIES

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Abstract
The first part of this chapter is on higher-order-mode couplers. To introduce the subject a coupling approach is examined which makes use of the beam tubes as wave guides with internal vacuum compatible RF loads. Then it is shown how coupling can be considerably enhanced if couplers are themselves regarded as resonators. Examples of couplers used in the LEP and HERA storage rings serve to illustrate this approach. In this first part we regard the cavity as a multi-frequency generator furnishing, via the coupler, power to a room temperature load. In the second part the direction of power flow is reversed. Now, at a single frequency, the cavity acts as a low-temperature load to which RF power has to be delivered and which can be considerably higher than the higher-order-mode power.

1. INTRODUCTION

Cavities are the 'motors' of accelerators, passing energy to the charged particle beams. But charges passing through a cavity are 'active devices' and can either receive energy from, or furnish energy to, a cavity. In an accelerator the first process is the required one though the second inevitably takes place and may hamper the efforts to produce a high quality beam. Couplers are needed to replenish the cavity energy which had been lost to the beam. Such main or power couplers may have to handle very large amounts of RF power, up to the MW level.

Couplers are also needed to mitigate the unwanted interactions between particles and cavities. They then are called higher-order-mode (HOM) couplers or dampers. In fact, to be more precise, couplers are RF devices which allow energy to be exchanged with the modes of oscillation of a cavity. So, at the beginning of this coupler chapter a short discussion of modes is needed.

2. MODES AND THEIR CLASSIFICATION

The ideal cavity is a closed volume, completely surrounded by a metallic boundary as indicated in Fig. 1. For infinite conductivity of the metal walls a closed volume can store electromagnetic field energy $U$ for infinite time. Storage is in the form of free oscillations at eigenfrequencies $\omega_n$, the spectrum of which depends on the size and shape of the volume. At each $\omega_n$, field energy $U_n$ changes periodically between its two possible forms, electric and magnetic, and the patterns of the corresponding fields $E_n$ and $H_n$ are characteristic of each oscillation mode. Computer codes are available (e.g. MAFIA) to calculate the eigenfrequencies and fields of such modes but for simple boundary shapes analytical solutions also exist. Two examples are shown in Fig. 2.

For accelerator cavities it is usual to classify the cavity modes into two groups: the first contains only a single member, the mode used for particle acceleration, the second group contains all the others. Since, by cavity design, the accelerating mode usually has the lowest frequency, one calls it the fundamental mode (fm) and all the others the higher-order modes. Another name for the latter is parasitic modes, indicating that they are deleterious and unwanted. I will also talk about longitudinal and dipole modes. Cavities for accelerators have axial symmetry. Then in a cylindrical coordinate system $\phi$, $r$ and $z$, longitudinal modes have fields with no $\phi$ dependence whereas for dipole modes fields vary with $\cos \phi$. 
3. BEAM TUBES AS HOM DAMPERS

To integrate a cavity into an accelerator one has to attach it to the beam tubes. These have little influence on modes with eigenfrequencies lower than the cut off frequency of wave propagation in the beam tubes. Cavity modes with higher frequencies are often said to be propagating. It is assumed that, by exciting waveguide travelling-wave modes, such cavity modes will quickly lose their stored energy i.e. become very effectively damped.

The interesting question is now: Can all HOM of a cavity be made propagating? What beam tube diameter is required to achieve this goal? To get a first idea we do some back of the envelope calculations. They become simple if the cavity has the shape of a pillbox with radius \( a \) and length \( l \). For this simple shape the fundamental TM\(_{010}\) resonance is at the cut off frequency of the TM\(_{01}\) waveguide mode in a tube of radius \( a \). This mode has the propagation constant:

\[
-k^2 = \left(\frac{2\pi}{\Lambda}\right)^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{2.41}{a}\right)^2
\]

At the cut off the propagation constant is zero and hence at the TM\(_{010}\) resonance frequency \( f_0 \):

\[
\frac{\omega_0}{c} = \frac{2.41}{a}
\]

Note that \( f_0 \) is independent of the pillbox length \( l \). Cavities for ultra-relativistic particles have lengths around \( \lambda_0/2 \) where \( \lambda_0 \) is the free space wavelength at \( f_0 \). The HOM next to the fundamental then is the TE\(_{111}\) dipole mode. At its resonance the cavity length is one half of the TE\(_{11}\) mode wavelength. Working again with the propagation constant we can therefore determine its frequency from

\[
-k^2 = \left(\frac{2\pi}{\Lambda}\right)^2 = \left(\frac{\omega_1}{c}\right)^2 - \left(\frac{1.841}{b}\right)^2.
\]

The beam tube radius \( b \), which at \( \omega_1 \) just allows propagation of the TE\(_{11}\) mode can then be calculated since

\[
\frac{\omega_1}{c} = \frac{1.841}{b}.
\]

Writing finally \( l = p\lambda_0/2 \) and rearranging we obtain for the ratio \( a/b \) of cavity and beam tube radius:

\[
\left(\frac{a}{b}\right)^2 = 1 + \left(\frac{1}{p}\right)^2 \left(\frac{2.41}{1.84}\right)^2.
\]

To have, according to this formula, propagation of all HOM, the beam tubes must be much wider than conventionally used and a reasonable value of \( a/b = 2 \) is only obtained if \( p = 0.75 \) i.e. for a cavity shorter than \( \lambda_0/2 \). This prevents us using such a HOM damping technique for multicell \( \pi \)-mode cavities. Even for a single cell the prognostic from this formula is too optimistic. A check with a code like URMEL reveals [1] that attaching wide beam tubes to the
cell lowers its TE\textsubscript{111} frequency, so the mode 'refuses' to propagate however wide the beam tubes are made!

3.1 Confined and trapped modes

Figure 3 shows $E$-field patterns of a wide-beam-tube design [2] as studied at CERN for possible use in LHC. The FM frequency is 400 MHz and the beam tube diameter 30 cm, about one half of the cavity diameter. The TE\textsubscript{11} and TM\textsubscript{01} waveguide cut off frequencies are 586 and 764 MHz respectively and, as we see in comparing frequencies, in addition to the FM also the TE\textsubscript{111} and TM\textsubscript{110} dipole modes remain confined to the cavity. The TM\textsubscript{011} mode (first harmonic of the fundamental) just reaches the region of propagation. But even a frequency above cut off is no guarantee of sufficient mode damping. At about three times the fundamental mode frequency (at 1232 MHz) we see a HOM which excites only a very small field in the beam tubes. For a cell length of $\lambda_0/2$ the excitation would be even weaker. We have here the example of a trapped mode which, though nominally propagating, may remain weakly damped [3]. Trapped modes become a real headache when designing multicell cavities [4]. They also have smaller cell-to-cell coupling than other modes so, with increasing cell numbers, field distributions become very sensitive to perturbations.

![Fig. 3](image)

3.2 Deconfining the TE\textsubscript{111} mode

To escape from the dilemma of the first two dipole modes not propagating, two solutions were proposed and developed. The first, as pursued at KEK in Japan, simply widens the beam tube further, but only behind a coupling aperture to the cell\textsuperscript{1} as sketched in Fig. 4. However, not only the TE\textsubscript{11} cut off is lowered but also, unnecessarily, the TM\textsubscript{01} cut off, so at the RF absorbing material the FM amplitude might become too high. Therefore, at Cornell the beam tube is widened azimuthally only in sectors, which creates the geometry of a ridged waveguide (see Fig. 4) and lowers particularly the waveguide's dipole mode cut off. These techniques are used for one of the beam tubes, the other tube (see Fig. 5) remaining a simple one. The use of

\textsuperscript{1} This concept was already applied by the Wuppertal team in their designs of 3-GHz structures.
different beam tube sections on the two sides of the single cell helps to avoid trapped modes, measurements on a copper model at Cornell demonstrating efficient damping of all significant modes to external $Q$ values smaller than 50.

Fig. 4 Beam tube forms at KEK and Cornell

Fig. 5 KEK B-factory accelerating module

3.3 The beam tube RF load

Beam tube HOM couplers have to be terminated into a RF load. If high HOM powers are expected this load must be at room temperature. It also must consume little space within the beam tube aperture and should allow cooling to extract the developed heat. The solution which imposes itself is to clad a ring portion of the inner beam tube surface with a uhv-compatible absorbing material. Since, adjacent to a metal surface, only the magnetic field has a component parallel to the absorbing surface layer the material must have magnetic RF losses. Ferrites are such materials and special uhv compatible ferrites were available from earlier work at CERN [5] to dampen parasitic resonances in the beam pickups of the Antiproton Accumulator. The problem was now to attach such ferrites with a good mechanical and thermal bond to a metal substrate. A technique using soft soldering [6] has been developed at Cornell while KEK [7,8] has succeeded in brazing ferrites. The main obstacle to bonding of ferrites to a metal is their
difference of thermal expansion. It is important to keep the dimensions of ferrite tiles small and to use a ductile metal like copper as substrate. Brazing then can be done in a vacuum oven using a standard Ag-Cu brazing alloy while pressing both partners together with 4 kg/cm² force.

3.4 General design implications

The concept of using the beam tubes for HOM damping implies an accelerating-module design where each single-cell cavity is housed in a separate cryostat. As illustrated by the KEK design for a B-factory, the large beam tube sections protrude out of the cryostat to allow ferrite loads at room temperature. The large diameter beam tubes are wide open ports for heat to reach the liquid helium vessel by radiation and conduction. Obviously the simplicity of the HOM damping scheme has to be paid for by enhanced refrigeration costs.

3.5 Ferrite beam tube loads for LEP 2

At CERN we prepare ferrite absorbers for mounting them, if required, into the 10-cm diameter beam tube sections between the 4-cavity modules. In LEP 2, once all the copper cavities have been replaced by superconducting ones, the transversal beam impedance will be reduced, so higher bunch charges can be accelerated. In addition, when the particle energy is increased, bunches may become shorter than they are now. As a consequence the HOM power deposition within a module will increase, particularly at high frequencies, where the performance of the HOM couplers is not well known. Figure 6 shows the power spectrum predictions [9] according to the ABCI program. Since this power is deposited into longitudinal modes and since above 2.2 GHz the TM₀₁ waveguide mode starts to propagate in a 10-cm diameter tube we can reasonably hope to intercept the high frequency tail of the spectrum (about 1 kW for 2 x 4 bunches of 1 mA) at room temperature in between the modules.

The ferrite tiles had to be integrated into the pumping boxes between the modules (Fig. 7). The basic idea for their manufacture was first to braze small (3 x 15 x 18 mm³) ferrite tiles onto flat copper strips which then, by electron beam welding, were joined to form a 10-cm diameter absorber tube. In between the strips 2-mm wide slots allow for pumping. The tube is then integrated into the cylindrical pumping box. Water cooling loops at both ends of the absorber tube remove heat.

Fig. 6 LEP2 module HOM power from ABCI calculations for σₓ = 10 mm

Fig. 7 Pumping box with integrated HOM load

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3.6 Concluding remarks on waveguides as HOM couplers and definition of external $Q$

Apart from the technically difficult vacuum RF load, (but which is only needed if the beam tubes themselves are to be used as waveguides), this HOM coupler concept is of great simplicity. We cut an opening into the wall of the cavity and, through this aperture, let RF power radiate into a wave guide which carries the power as a travelling wave to a termination at room temperature. For a mode with angular frequency $\omega_n$, stored energy $U_n$ and radiating the power $P_n$ we may, since $P_n \propto U_n$, characterize the coupling by defining an external $Q$:

$$Q_{ex,n} = \frac{\omega_n U_n}{P_n}$$

Below cut off a guide cannot carry travelling waves. Thus, by a proper choice of the transverse guide dimensions, coupling to the FM is easily suppressed. But guide and cavity dimensions become comparable and integration into a cryostat while keeping heat leaks at an acceptable level is difficult. So coupling to transmission lines has become a more widespread technique. We then need a filter to suppress FM coupling but we can adapt the line cross-section to the HOM power and minimize heat leaks.

4. COUPLING TO TRANSMISSION LINES AND THE EQUIVALENT-GENERATOR APPROACH

Amongst the different forms of transmission lines the coaxial one is best suited for our purpose. For the moment we will assume that the line is terminated by a matched load, $R=Z_w$, where $Z_w$ is the wave impedance, so again energy transport is by a travelling wave.

Let us now examine the interface between cavity and line in more detail. We have two basic choices. As depicted below we can either leave the inner conductor end 'open', forming a probe, or 'shorted', forming a loop. Seen from the cavity the transmission line terminates this probe or loop by a resistor $R = Z_w$.

![Fig. 8 Two possibilities of coupling, probe or loop](image)

4.1 Non-resonant coupling

For a given loop or probe the question is now: Which value of $R$ gives the lowest $Q_{ex}$?

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3 At CEBAF HOM couplers in waveguide technique are used but with loads in the liquid He bath, but this is only possible because the HOM power is very small.
To answer the question let a mode at \( w \) be excited by some auxiliary device to oscillate with constant stored energy \( U \). We then may regard the probe or the loop as the output port of a RF generator with emf \( V_0 \) and internal impedance \( Z_i \). We can determine \( V_0 \) and \( Z_i \) by 'measuring' open circuit voltage \( V_0 \) and short circuit current \( I_0 \) and have for the loop:

\[
I_0 = \frac{\Phi_m}{L_s} = \frac{\mu_0}{L_s} \int \int \mathbf{H} \cdot d\mathbf{s}
\]

\[
V_0 = j\omega \Phi_m = j\omega \mu_0 \int \int \mathbf{H} \cdot d\mathbf{s}
\]

(2)

\( L_s \) is the self-inductance of the loop. \( H \) is the mode’s magnetic field and we integrate over the loop surface. Evidently \( Z_i = j\omega L_s \) and according to Thevenin’s theorem the coupling port can be described by the \textit{loop inductance} in series with the \textit{induced voltage} as in Fig. 9

![Fig. 9 Equivalent generator circuits for probe and loop coupling](image)

A port equipped with a probe has an equivalent circuit dual to that of the loop. \( C_s \) is the fringe field capacitance of the probe tip. \( I_0 \) is a short circuit current and corresponds to the displacement current of that part of the cavity \( E \)-field which ends on the probe surface.

\[
I_0 = j\omega \Phi_e = j\omega \varepsilon_0 \int \int \mathbf{E} \cdot d\mathbf{s}
\]

(3)

Which power is now extracted from the mode? We find for the \textit{loop} \( Y \) is the \textit{admittance} 'seen' by the voltage source \( V_0 \):

\[
P = \frac{1}{2} V_0^2 \text{Re}(Y) = \frac{1}{2} V_0^2 \frac{1}{Z_w} \frac{Z_w^2}{Z_w^2 + (\omega L_s)^2}
\]

(4)

and for the probe with \( Y_w = 1/Z_w \) \( Z \) is the \textit{impedance} 'seen' by the current source \( I_0 \)

\[
P = \frac{1}{2} I_0^2 \text{Re}(Z) = \frac{1}{2} I_0^2 \frac{1}{Y_w} \frac{y_w^2}{y_w^2 + (\omega C_s)^2}
\]

(5)

The presence of \( L_s \) (or \( C_s \)) diminishes the power flow on the transmission line and hence the damping of the mode.

4.1.1 The limit of obtainable damping

Evidently here we have a dilemma. If, to obtain more damping, we increase the surface of a coupling loop, then also \( L_s \) will increase and take away at least part of the potential benefit of the higher induced voltage. And similarly, for a probe a bigger surface will increase \( C_s \).
Let us study this important phenomenon in more detail. We use a geometry of transmission line and loop (see Fig. 10) which hardly would be used in a real construction project but has the advantage that the loop’s self-inductance can be expressed by a simple formula. We use a strip line and as loop a solenoid with only one turn so \((r)\) is the radius of the solenoid and \(l\) its length:

\[
L_s = \mu_0 \pi \frac{r^2}{l}
\]

Figure 11 sketches how such a loop could couple to the TM_{010} mode of a pillbox cavity. The coupler is in the symmetry plane of the cavity and the loop at the equator where the magnetic field \(B\) is maximal and to first order constant over the loop area. Then:

\[
\frac{1}{P} = \frac{2Z_w}{\left(\omega r^2 \pi B\right)^2} \left( \frac{Z_w^2 + \left(\omega \mu_0 \pi r^2 / l\right)^2}{Z_w} \right)
\]

and it follows

\[
Q_{ex} = \frac{\omega U}{P} = 2 \frac{\omega U}{B^2} \left( \frac{Z_w}{\pi^2 \omega^2 r^4} + \frac{\mu_0^2}{l^2 Z_w} \right)
\]

\[
(6)
\]

The factor \(\omega U/B^2\) describes the cavity and loop position, the two additive terms within the curly brackets the coupler itself. The salient point is now that one of the two terms is independent of the loop radius. So it constitutes a lower limit to the obtainable external \(Q\) (an upper limit of damping). It does not pay to increase the loop radius much beyond the value leading to

\[
\frac{Z_w}{\pi^2 \omega^2 r^4} = \frac{\mu_0^2}{l^2 Z_w}
\]

But this expression can be rearranged to give

\[
\omega L_s = Z_w
\]

\[
(7)
\]

and in this form is of general validity. Having chosen a wave impedance \(Z_w\) for the transmission line and a conductor for the loop, Eq. (7) limits the useful loop size. On the other
hand, as the second term in (6) shows, if the loop area linked to flux remains fixed, increasing the conductor cross-section will lead to improved damping.

An example:

To close this section let us now calculate the lower external $Q$ limit for the fundamental mode if we mount this loop coupler on the equator of the 4-cell LEP cavity. For an accelerating voltage of $V = 1.7$ MV (1 MV/m of accelerating gradient) this cavity has at the equator a magnetic induction of $B = 4$ mTesla and in the $\pi$-mode a $(R/Q)$ of 230 $\Omega$. We now can evaluate $\omega U = V^2/(2R/Q)$ and obtain, for a line impedance of 50 $\Omega$ and a loop length of 8 cm, from the second term of Eq. (6):

$$(Q_{ex})_{\text{min}} = 3900$$

For the reasonable loop diameter of 4.2 cm which produces $\omega L_s = 50$ $\Omega$, the external $Q$ will be twice the minimal one and increasing the loop diameter by a factor $\sqrt{2}$ to 6 cm we will approach the minimal value to within 25%:

$$Q_{ex} = 5000$$

However, much lower values of $Q_{ex}$ can be obtained from the same loop at the price of some more sophistication.

4.2 Resonant coupling

In the simple non-resonant approach $Q_{ex}$ has a lower limit, since part of the induced voltage is lost as voltage drop across the internal impedance. But this impedance is not resistive as in ordinary generators. For a coupler it is a pure reactance and, in contrast to a resistance, a reactance can most easily be compensated by an opposite one, at least at certain frequencies.

For a loop coupler the simplest measure is to connect a capacitor in series with the loop [10]. If we do so for the 6-cm diameter loop we need a capacitive reactance of 100$\Omega$ at 352 MHz. From $1/(\omega C) = 100$ $\Omega$ we obtain $C = 4.55$ pF. Such a capacity is small enough to be easily realised in practice. The result of compensation is often spectacular. In our example, adding the capacitor would divide the external $Q$ by 5!

$$Q_{ex} = 1000$$

And lower values are within reach as the following discussion will show.

4.2.1 The damping limit for resonant coupling

With the compensation condenser added we have the equivalent circuit of Fig. 12. Evidently now, at the mode frequency, we can increase the extracted power, and hence the damping, if we reduce $R$ (e.g. by connecting a $\lambda/4$ transformer between condenser and transmission line) and, in reducing $R$ to zero, it appears as if in the limit $Q_{ex} = 0$ could be obtained. On the other hand, with $R = 0$ there are no losses in the system, and $Q_{ex}$ must be infinite! To remove this contradiction we have to realise that, in adding the compensation condenser, we transformed the coupler itself into a resonator with its own quality factor $Q_{co} = \omega L_s/R$ and tuned to the mode frequency. Figure 13 depicts the situation in representing the mode by a parallel $LC$-resonator which couples via the mutual inductance $M = k \sqrt{L_L L_s}$ to the loop. $k$ is the coupling factor and it follows from the definition of $M$ that the flux of cavity field through the loop is

$$\Phi = M I = k \sqrt{L_L L_s} I$$
We will now express the previous formulae in a coupled-resonator notation and this will help to see the limit of their range of validity. Using $0.5 \ I^2 = U$ we get

$$\Phi^2 = k^2 L_s \cdot LI^2 = 2k^2 L_s U \quad (8)$$

With $P = 0.5V_0^2/R$ we have now

$$P = \frac{1}{2} \left( \frac{\omega \Phi}{R} \right)^2 \frac{1}{2} \left( \frac{\omega \Phi}{\omega L_s} Q_{co} \right)$$

and substituting from (8) gives

$$P = k^2 \omega U Q_{co}$$

If now $\omega U/P = Q_{ex}$ then

$$Q_{ex} = \left( \frac{1}{k} \right) \cdot \frac{1}{kQ_{co}} \quad (9)$$

*Equation (9) does not give anything new.* It still predicts infinite damping for infinite coupler $Q$. But it is written in the language of coupled-resonator theory which says that for $R = 0$ the presence of the coupler resonator will simply split the cavity mode into two, with frequency difference $\Delta f = kf$.

On the other hand, modes of a coupled system are sensitive to perturbations. Damping is one of the possible reasons for perturbations and splitting disappears when $kQ_{co} < 1$. Substituting this into (9) will give a reasonable estimate of the minimal external cavity $Q$ that can be produced with this resonant coupler technique.

$$Q_{ex (min)} = \frac{1}{k} \approx \frac{\sqrt{2UL_s}}{\Phi} \quad (10)$$

In our LEP cavity example, with the loop of 6-cm diameter we have $L_s \approx 46 \ \text{nH}$ and, for a stored energy of 2.9 Ws, a magnetic flux of $\approx 11.3 \ 10^{-6} \ \text{V} \cdot \text{s}$ through the loop. Using (8) we calculate a coupling factor of $\approx 2\%$ and

$$(Q_{ex})_{min} = 50$$
The coupler $Q$ must then be 50 too and this requires a resistive termination of $2\Omega$. In designing couplers for HOM damping one hardly would go to such an extreme because the coupler bandwidth becomes too small and about one coupler per mode would be needed!

### 4.2.2 Fundamental mode filter requirements

As mentioned earlier coupler constructions based on transmission line techniques have to use a filter to suppress coupling to the fm. This appears to complicate the design but the filter can always be integrated into a reactance compensation scheme. For instance, if in Fig. 12 an inductance $L = L_s$ is connected in parallel to $C$, then a stop filter for the FM at $\omega_0$ is formed (see Fig. 14). But at higher frequencies the filter reactance becomes capacitive to compensate the loop reactance at $\sqrt{2}\omega_0$. More general for this circuit, if the HOM frequency at which we want compensation is at $\omega_c$ then $L$ and $C$ are given by:

$$\frac{1}{C} = \omega_0^2 L = (\omega_c^2 - \omega_0^2)L_s$$

(11)

Voltage and current in the filter may reach high values. To illustrate this point we do the 'tentative design' of a LEP cavity HOM coupler compensated at 500 MHz and using a loop (as sketched in Fig. 11) with 3-cm diameter and 4-cm length. Then at 352 MHz $\omega L_s = 50\ \Omega$ and, since $500/352 = \sqrt{2}$, $L = L_s$. At an accelerating gradient of 6 MV/m the equatorial magnetic cavity field is 24 mT and induces the voltage $\omega \Phi = 37.5$ kV, which drops across the filter condenser $C$ and drives a current of $I = 750 \text{ A}$ through $L$. If $C$ has a gap of 3 mm, a bigger gap could lead to multipacting, a surface field of 12.5 MV/m results and if the conductor carrying $I$ has a diameter of 3 cm a surface magnetic field of =10 mT will be produced, values comparable to those in the cavity! Construction from superconducting materials using cavity assembly and surface preparation techniques is mandatory.

Two of the four HOM couplers of the CERN 5-cell 500-MHz prototype sc cavity, built for beam tests in PETRA at DESY, were of this type [11]. An outline of their geometry is given below in Fig. 15. With a coupler $Q \approx 2$ they produced external $Q$ values $\approx 40000$ and were operated up to accelerating gradients of 4MV/m (limit due to a cavity quench).

![Fig. 14 Resonant HOM coupler with fundamental mode stop filter](image1)

![Fig. 15 Outline of resonant HOM loop coupler with FM filter](image2)

However, this approach was not further pursued for two reasons. Making an opening at the cavity equator causes local field enhancement just where the FM $B$-field has its maximum, and putting the coupler at the maximum in turn causes very high field values in the coupler's filter. Looking for a coupler position with smaller FM field and above all a more favourable ratio of HOM to FM fields, a position on the beam tubes of the cavity was identified as optimal. The $\text{TM}_{01}$ mode being in cut off, damping in the beam tube then acts as a prefilter for the FM.

### 4.2.3 Couplers on the beam tube with several resonances

Another feature of the last generation of HOM coupler designs is that they are tailored to be resonant at several frequencies. Figure 16 illustrates for the LEP cavity the distribution of...
HOM with significant $R/Q$ values. As we see, high $R/Q$ HOM come in three clusters around 480 MHz, 650 MHz and 1.1 GHz, so at these frequencies should be resonances of the HOM coupler. A heuristic approach leading to a coupler with the required three resonances is the following. First realize that 650 and 1100 MHz are nearly harmonic and that a transmission line resonator of length $\lambda/2$ at 650 MHz with shorts at both ends would have a second resonance at 1300 MHz.

![Fig. 16](image1.png)

Fig. 16 The high $R/Q$ HOM of the LEP cavity

Replacing the shorts by small inductances $L_1$ & $L_2$, (and $L_1$ would be the coupling loop), we can move these resonances nearer to each other. A third lower frequency resonance finally is obtained by connecting the termination $R$ (see Fig. 17) via a capacitor $C$ to $L_2$. $L_2$, $C$ and $R$ then form a second resonator, which shares with the line resonator the common element $L_2$. In other words: the two resonators are coupled, allowing to split the original 650 MHz resonance into two.

![Fig. 17](image2.png)

Fig. 17 Circuit of a transmission line coupler with three resonances

The HOM couplers [12] used at DESY in HERA and more recently in the TESLA [13,14] project, as well as at CERN on the LEP Nb-Cu cavities all use this approach. As shown in Fig. 18 the HERA coupler has a small antenna added to the loop so that coupling to electric cavity fields is enhanced. The FM filter is in the back of the coupler, parallel to $L_2$, forcing a voltage zero across $C$. In CERN (see Fig. 19) the filter has been moved to the front in returning $L_1$ via a capacitor to ground. In this way coupling to FM $E$-fields is eliminated. Also, to avoid magnetic coupling, the loop is perpendicular to the cavity axis. HOM coupling is electric for the longitudinal modes and predominantly magnetic (to their $B_z$ in the beam tube) for the dipole modes. This modification made it possible to have a demounting flange, which has to carry only the HOM power. In fact, for a successful fabrication of sc cavities by sputter-coating [15], demountability of couplers is a precondition.
At SACLAY and CERN it has also been found that letting a loop protrude into the beam tube is another way to get sufficient coupling to the *E-fields* of the longitudinal (TM$_{0nm}$) modes. This is an interesting variant since it is simpler to fabricate and has been chosen for the LEP cavities. A schematic drawing together with a table of external $Q_s$ is given in Appendix A.

### 4.3 Design aids

The equivalent generator approach combined with a circuit model of the coupler lends itself readily to making meaningful estimations of mode damping. One ingredient is a calculation (by a network analysis code) of the real part of the coupler’s input impedance for probes (or admittance for loops) *including* into the network the coupler’s $C_s$ (or $L_s$ respectively).
The second ingredient is an estimation of the probe's short-circuit current $I_0$ (or the loop's open-circuit voltage $V_0$) with the help of a cavity code which gives the stored energy $U$ and the fields $E$ and $H$ at the coupler's location. We assume that these fields are not too much perturbed by the presence of the coupler, and estimating the integrals of (2) and (3) we have all the needed information to calculate $Q_{ex}$.

During the conceptual design phase of a HOM coupler it is most useful to plot the calculated real part of the input impedance (or admittance) against frequency to identify the position of maxima (resonances) and their relative height. After building hardware models such 'sensitivity' curves can be verified in measuring the transfer function between a small loop or probe (which replace the cavity field) and the coupler's load as illustrated in Fig. 18.

More recently a code (HFSS) has become available which, with the exclusion of the interaction region between coupler and cavity fields, allows one to calculate the $s$-parameters of a complete hardware model. Using it greatly facilitates the translation from the circuit model to the final RF structure.

5. THE HIGHER ORDER MODE POWER

Obviously a good knowledge of the HOM power is needed if one has to decide on the size of the couplers and of cables, connectors and vacuum feedthroughs which serve to connect the couplers to room temperature loads.

5.1 Beam with sinusoidally varying linear charge density

To calculate the RF power (Ref. [16] contains a more general treatment of the subject) from a particular mode with frequency $\omega_m$ first imagine a beam with sinusoidally varying linear charge density which passes an AC current $I$ with frequency $\omega$ through a reference plane of the cavity exciting oscillating fields of the same frequency. If $\omega$ is in the vicinity of $\omega_m$ this mode's field will predominate with an amplitude depending on $\omega$ as typical for a resonance, i.e. reaching at $\omega = \omega_m$ a maximum proportional to $I$. We may measure the accelerating effect of the field by test charges (which have the same speed $v \approx c$ as the paticles of the beam) and call the factor of proportionality between the found accelerating voltage $V$ and $I$ (at $\omega = \omega_m$) the effective shunt impedance $R$.

$$V = RI$$

(12)

$R$ in turn is proportional to the loaded $Q$ of the mode and we call the factor of proportionality $(R/Q)$.

$$R = \left(\frac{R}{Q}\right)Q$$

(13)

The method used to define $V$ implies for the power $P_m$ lost by the beam at resonance to the mode's field:

$$P_m = \frac{1}{2} IV$$

using (12) it follows that

$$P_m = \frac{1}{2} \frac{V^2}{R}$$

But conservation of energy demands that $P_m$ is equal to the power dissipated in the cavity walls and in the HOM coupler load.
\[ P_{\text{diss}} = P_m = \frac{1}{2} \frac{V^2}{R} \] (14)

Further by definition

\[ Q = \frac{\omega U}{P_{\text{diss}}} \] (15)

where \( U \) is the stored energy of the mode. Combining (11), (12) and (13) gives:

\[ \begin{pmatrix} R \\ Q \end{pmatrix} = \frac{1}{2} \frac{V^2}{\omega U} \] (16)

Using a cavity code (SUPERFISH, URMEL, MAFIA…) \( (R/Q) \) can be calculated from Eq. 16 for all non-propagating modes and \( P_m \) could be determined for given \( I \) and \( Q \). But it is more important to realise that all these equations map on the circuit equations of a parallel \( LC \)-resonator (see Fig. 20) to which the beam current is connected in the form of a current source and which has \( 1/(\omega_m C) = (R/Q) \).

![Fig. 20 Circuit model of beam-mode interaction](image)

Trusting the equivalence between circuit model and mode we can now determine the response of the mode to other beam current forms.

5.2 Response to a single bunch

For a point charge \( q \) passing the reference plane at \( t = 0 \) the beam current has the form of a single \( \delta \)-current pulse.

\[ I(t) = q \delta(t) \]

Such a pulse charges the condenser discontinuously to the voltage \( q/C \) which, if the condenser had no charge before, is the initial amplitude of a subsequent free oscillation:

\[ V(t) = q \ h(t) \quad \text{with} \quad h(t) = \frac{u(t)}{C} e^{\gamma t} \]

\( u(t) \) is the unit step function.

With the filling time \( T_f \):

\[ \gamma = -\frac{1}{T_f} + j \omega_m = -\frac{\omega_m}{2Q} + j \omega_m \]

Further, for a linear system such as the circuit here and any beam current \( I(t) \)
\[ V(t) = \int_{-\infty}^{\infty} I(\tau) h(t - \tau) d\tau \]

and for our \( h(t) \):
\[ V(t) = \frac{e^{\gamma t}}{C} \int_{-\infty}^{\infty} I(\tau) u(t - \tau) e^{-\gamma \tau} d\tau \] (17)

We now examine a single beam-current pulse \( I(\tau) \) and \( V(t) \) at times \( t \) when the current has returned to zero. During the pulse then \( u(t - \tau) = 1 \). Also, in accelerators the beam current pulses are very short compared to \( 2Q/\omega_m \). So it makes no difference if we neglect the attenuation and replace the integral in (17) by
\[ \int_{-\infty}^{\infty} I(\tau) e^{-j\omega_m \tau} d\tau \]

This is the Fourier transform of the current pulse \( I(t) \) at \( \omega_m \). A single gaussian bunch of charge \( q \) with
\[ I(t) = \frac{q}{\sqrt{2\pi} \sigma} e^{\frac{-t^2}{2\sigma^2}} \]

has the Fourier transform
\[ I(\omega) = qe^{-\frac{\omega^2 \sigma^2}{2}} \]

and after the passage of a single gaussian bunch
\[ V(t) = V_b e^{\gamma t} \] (18)

with
\[ V_b = \omega_m \left( \frac{R}{Q} \right) e^{-\frac{(\omega_m \sigma)}{2} q} \] (19)

5.3 The HOM voltage due to a bunched beam

We may now go a step further and try to determine the response to a beam made up of bunched charges \( q \) which, passing through the reference plane, constitute a pulsed current with period \( T_b \). In a steady state the excited oscillations must have the same period \( T_b \). On the other hand, in between bunch passages the mode oscillations must be free oscillations i.e. representable by a phasor \( V \) turning with angular speed \( \omega_m \).

Formally, we may turn this phasor back or forward to the moment of the last or next bunch passage and in this way construct phasors \( V_+ \) and \( V_- \). Further, to have the required overall periodicity with \( T_b \), at the moment of a bunch passage, the accelerating voltage induced by the bunch must update \( V \) to become \( V_+ \). Central to the derivation is now that, due to the linearity of the circuit, this updating voltage must be \( V_b \). Writing complex quantities with a tilde:
\[ \tilde{V}_+ = \tilde{V}_- + V_b \]
but also
\[ \tilde{V}_+ = \tilde{V}_+ e^{\gamma T_b} \]
\[ \tilde{V}_+ = V_b \frac{1}{1 - e^{\gamma T_b}} \]
and for the time interval of free oscillations:
\[ \tilde{V}(t) = \frac{V_b e^{\gamma t}}{1 - e^{\gamma T_b}} \quad \text{for} \quad 0 < t < T_b \]  \hspace{1cm} (20)

5.4 The HOM power from a bunched beam

\[ \tilde{V}(t) \] drops across the shunt impedance \( R \), causing in average the dissipation
\[ P_{\text{mode}} = \frac{1}{2} \frac{V_b^2}{T_b} \int_0^{T_b} \frac{\tilde{V}(t)^2}{R} dt \]

\( P_{\text{mode}} \) is furnished by the bunched beam. Substituting (20)
\[ P_{\text{mode}} = \frac{V_b^2}{2RT_b} \left[ 1 - \exp(\gamma T_b) \right]^{\frac{T_b}{2t}} \]
\[ \int_0^{2t} e^{-\frac{\tau}{T_b}} d\tau \]

using \( \tau = \frac{T_b}{T_f} = \frac{\omega_m T_b}{2Q} \) and \( \delta = \omega_m T_b \) (modulo \( 2\pi \))
we have
\[ |1 - \exp(\gamma T_b)|^2 = 1 - 2 \cos \delta \exp(-\tau) + \exp(-2\tau) \]

And evaluating the integral we obtain with \( R = Q(R/Q) \) the result:
\[ P_{\text{mode}} = \frac{1}{2} \frac{V_b^2}{R/Q} \frac{1}{\omega_m T_b} F(\tau, \delta) \]  \hspace{1cm} (21)

where
\[ F(\tau, \delta) = \frac{(1 - \exp(-\tau))(1 + \exp(-\tau))}{1 - 2 \cos \delta \exp(-\tau) + \exp(-2\tau)} \]  \hspace{1cm} (22)

For gaussian bunches of charge \( q \) and substituting from (19)
\[ P_{\text{mode}} = \frac{1}{T_b} k_m q^2 F(\tau, \delta) \]  \hspace{1cm} (23)

\( k_m \) is called the loss parameter \[ 16 \] of the mode at \( \omega_m \).
\[ k_m = \frac{1}{2} \omega_m \left( \frac{R}{Q} \right) e^{-\left( \omega_m \sigma \right)^2} \]  \hspace{1cm} (24)
The pulsed current of a bunched beam corresponds to a spectrum of discrete lines at
distance $1/T_b$ which has the Fourier transform of a single pulse as envelope curve. $\delta = 0$
(modulo $2\pi$) means that $\omega_m$ coincides with one of the spectral frequencies. At $\delta = \pi$ the mode is
just in between two spectral lines. Figure 21 shows a plot of $F(\tau, \delta)$ for these two cases. Note
that $F(\tau, \pi) = 1/F(\tau, 0)$. $F$ is plotted against $2/\tau = 4Q/(\omega_m T_b)$.

![Fig. 21](image)

**Fig. 21** $F(\tau, 0)$ and $F(\tau, \pi)$ plotted against $2/\tau = 4Q/(\omega_m T_b)$

5.4.1 The strong damping limit

Whatever the $\delta$, for increasing damping (increasing $\tau$) $F$ approaches one, and $P_{\text{mode}}$
becomes

$$P_{\text{mode}} \approx \frac{1}{T_b} k_m q^2$$  \hspace{1cm} (25)

In the limit each bunch of the beam sees an “empty” cavity and we conclude that the bunches
now loose an energy $k_m q^2$ to each mode. For strong damping of all HOM the expression for
the total HOM power thus must have the simple form

$$P_{\text{HOM}} \approx \frac{1}{T_b} k q^2$$  \hspace{1cm} (26)

with

$$k = \sum k_m$$  \hspace{1cm} (27)

If the beam is kept on the cavity axis where transversal modes have no accelerating field, only
longitudinal modes contribute to $k$. Codes like TBCI and ABCI allow one to calculate the $k_m$
and their sum, even including the frequency range where modes propagate.
5.4.2 The weak damping limit

If \( \tau = 0.5\omega_m T_b/Q \) is small enough to approximate \( \exp(-\tau) \) by \( 1-\tau \) then we can simplify setting:

\[
F(\tau,0) = \frac{2}{\tau} = \frac{4Q}{\omega_m T_b}
\]

Formula (21) now takes the form

\[
P_{\text{mode}} \approx \frac{1}{2} I_{\text{rf}}^2 Q \left( \frac{R}{Q} \right) R
\]

The expression in brackets is the intensity of the beam current’s spectral line at \( \omega_m \). Calling it the beam’s RF current at \( \omega_m \) we have:

\[
P_{\text{mode}} \approx \frac{1}{2} I_{\text{rf}}^2 Q \left( \frac{R}{Q} \right) Q = \frac{1}{2} I_{\text{rf}}^2 R
\]  

(28)

We arrived at a formula rigourously valid for a single harmonic current source and conclude that used for a bunched beam this approximation is better the smaller the mode’s loaded bandwidth compared to \( 1/T_b \).

5.5 Choices of damping

As we have seen, for a bunched beam we can never reduce the HOM power to zero. The lower limit of power given by (26) is the higher the smaller the number of bunches circulating in a machine for a given average DC beam current \( I_0 = q/T_b \). In fact, by substitution of \( I_0 \) into (26):

\[
P_{\text{HOM}} = k I_0^2 T_b
\]

(29)

Small numbers of bunches also produce a dense spectrum of beam lines, so the scatter of HOM frequencies due to dimensional tolerances of cavity production may become comparable to \( 1/T_b \). It is then impossible to avoid resonances between modes and beam lines and filling times \( T_f \) should be made equal or smaller than \( T_b \) at least for longitudinal modes with significant \( (R/Q) \) values.

On the other hand, if high numbers of equally spaced bunches circulate in a machine it may be possible to detune high \( (R/Q) \) modes from beam lines and weak damping gives an advantage, provided beam stability requirements allow high external \( Q \) values. In all these cases the resulting loaded \( Q \) will be many orders of magnitude smaller than the \( Q_0 \) of modes in a sc cavity. So HOM dissipation in the cavity walls will be negligible compared to the dissipation due to the generator driven fundamental mode.

6. THE FUNDAMENTAL MODE POWER COUPLER

6.1 Matching

Power couplers must have specified properties only near to a single frequency. In this respect they differ considerably from HOM couplers, which in general have to cover broader
frequency ranges. Also, at least for the applications developed so far, they must couple only loosely to the fm. If again we measure coupling by an external $Q$, values smaller than $1\times10^5$ will hardly be needed and reactance compensation techniques to enhance coupling are not required, rather 'matching' is required.

The concept of matching comprises measures which, by an appropriate choice of impedances, optimize the generation and the transport of power. A simple high power RF system has three building blocks: the power amplifier, a line (transmission line or wave guide) and the load. We then want to minimize power losses on the line. This implies that power is transported by a pure traveling wave and so the line must see a load equal to its wave impedance. In most cases, to realise this condition, we need to connect special impedance transforming matching devices between the line and the load proper and to produce a 'load match' is one of the functions of the power coupler.

Another matching task is at the output port of the power amplifier which, for best efficiency, must work on a tightly specified resistive termination. If lines of standard wave impedance $Z_0$ are used, the needed transformers form part of the amplifier circuit. The system is now in a state where it works with highest efficiency, but more matching can be done to improve its behaviour in the presence of load matching errors. The problem is that klystron and tetrode power amplifiers have an internal or 'source' resistance, which may be an order of magnitude higher than their optimal load resistance. Hence, looking back to the amplifier, the line is not matched. This has the consequence that reflections at the load are back reflected from the amplifier adding to the forward power which thus depends on the load. Matching at the amplifier side of the line is called 'source matching'. At microwave frequencies a convenient way of source matching is to connect a circulator between amplifier and line. For a fixed drive power to the amplifier the forward power is then independent of load matching errors.

### 6.2 Matching the external $Q$

The load to which a power coupler shall match is a special one: The FM of a sc cavity, which exchanges energy with the bunches of a particle beam. If we represent the mode by a parallel LC resonator (with $\omega_c L = 1/(\omega_c C) = (R/Q)$ ) then the beam is a current source connected to its terminals. In the discussion here, which focuses on matching, it is sufficient to regard the coupler as an impedance transformer for the wave impedance $Z_0$ of the line, so that looking from the resonator to the line we see a transformed wave impedance $Z$ (see Fig. 22).

![Fig. 22 Line with attached equivalent circuit of loss-free cavity and beam](image)

If this line is source matched then, seen from the resonator, its input impedance is also $Z$, producing as external $Q$

$$Q = \frac{\omega_c C}{Z} = (R/Q)^{-1} Z$$

(30)

We will now assume that the resulting bandwidth $\Delta\omega_c = \omega_c/Q$ is small compared to the bunch frequency. With the resonator tuned near to one particular line of the beam current's spectrum the voltage due to all other lines may then be neglected i.e. the current source in Fig. 22 can be thought of as emitting a sinusoidal RF current of frequency $\omega_c$, identical to that of the RF power generator.
In the following derivations we will write complex variables (phasors) with a tilde, their modules (amplitudes) and other real quantities without.

The circuit equations are here:

\[ \tilde{V}_{\text{forw}} + \tilde{V}_{\text{refl}} = \tilde{V} \]

and

\[ \tilde{I}_{\text{forw}} - \tilde{I}_{\text{refl}} = \tilde{I} = \tilde{I}_c + \tilde{I}_b \] (31)

\[ 2\tilde{I}_{\text{forw}} = (\tilde{I}_{\text{forw}} + \tilde{I}_{\text{refl}}) + \tilde{I}_c + \tilde{I}_b \]

but

\[ I_{\text{forw}} + I_{\text{refl}} = (V_{\text{forw}} + V_{\text{refl}}) / Z = V / Z \quad \text{and} \quad \tilde{I}_c = \tilde{V} / \tilde{Z}_c \]

\[ 2\tilde{I}_{\text{forw}} = \tilde{V} \left( \frac{1}{Z} + \frac{1}{\tilde{Z}_c} \right) + \tilde{I}_b \] (32)

where

\[ \frac{1}{\tilde{Z}_c} = j \left( \frac{\omega_g}{\omega_c} - \frac{\omega_c}{\omega_s} \right) \left( \frac{R}{Q} \right)^{-1} \] (33)

Equation (32) is a relation between phasors in a complex current plane. If now we wish to represent (32) by a phasor diagram, we are free to define the time zero and thus to decide which of the phasors shall be collinear with the real axis.

- Accelerator physicists often declare the RF beam current as real. In fact it appears natural to regard bunch charges as real quantities and hence also the resulting beam current.

- Circuit analysis usually takes the generator signal, here the forward current, as reference and regards it as real.

- For people who develop sc cavities, the accelerating voltage is a main concern. Regarding it as the reference has advantages. The cavity current \( I_c \) (since we neglect cavity losses) is now an imaginary quantity for any tuning and the angle of the RF beam current phasor gives directly the synchronous phase \( \Phi \). If we use the convention to count it from the crest of the cavity voltage. But the forward current now is in general complex and the expressions which relate forward and beam power are under general conditions, rather complicated.

In the following we need to find the optimal conditions for power transfer to the beam. We regard an 'accelerating station' i.e. a group of cavities which, from a power divider, all receive the same forward power. Also, the distance of cavities on the beam axis has been chosen to make for all of them the beam current phasor equal in amplitude and phase which, referenced to the forward current, is called 'station phase' \( \phi \). Under these conditions it is most convenient to regard the forward current as a real quantity, as in the phasor diagram of Fig. 23.

Since \( Z \) is real but \( \tilde{Z}_c \) imaginary, the phasors \( \tilde{V} / Z \) and \( \tilde{V} / \tilde{Z}_c \) are at a right angle and meet on a circle which has \( 2I_{\text{forw}} - I_b \) as diameter. Figure 23 represents a non matched situation: both cavity detuning and coupling are incorrect. To match we first retouch the detuning so that \( \tilde{V} \) becomes collinear with \( I_{\text{forw}} \) (the line 'sees' a real impedance). The result is shown in Fig. 24:

\[ I_c = V/Z_c \] now compensates the imaginary component \( I_{bi} \) of the beam current and if we substitute \( Z_c \) from (33) we get the detuning condition
**Fig. 23** Equation (32) in a phasor diagram representation

**Fig. 24** Phasor sum for a correct cavity detuning

\[
I_c = \frac{V}{Z_c} = V \left( \frac{\omega_g - \omega_c}{Q} \right)^{-1} = -I_{bi} = -I_b \sin \Phi 
\]  
(34a)

or

\[
\frac{(\omega_g - \omega_c)}{0.5(\omega_c/Q)} = -Q \left( \frac{R}{Q} \right) \frac{I_b}{V} \sin \Phi 
\]  
(34b)

\(\omega_c/Q\) is equal to the loaded 3dB bandwidth \(\Delta \omega_c\) if the line which feeds power to the cavity is source matched. When (34) is satisfied, station phase \(\phi\) and synchronous phase \(\Phi\) (measured from the crest of the cavity voltage) become equal.

If the detuning condition is met, Eq. (31) reads:

\[
I_{\text{forw}} - I_{\text{refl}} = I_{br} = I_b \cos \Phi
\]

and to obtain zero reflection we have in addition to satisfy \(I_{\text{forw}} = I_{br}\) . But for zero reflection we have also \(V = V_{\text{forw}}\) . Thus (since \(Z_m = (R/Q)Q\)):

\[
V = V_{\text{forw}} = I_{\text{forw}} Z_m = I_{br} \left( \frac{R}{Q} \right) Q_m
\]

Solving for \(Q_m\) and using \(2P_b = V I_{br}\) we obtain the condition in a more practical form:

\[
Q_m = \frac{1}{2} \frac{V^2}{P_b} \left( \frac{R}{Q} \right)^{-1}
\]  
(35)

\(P_b\) is the power transferred to the beam. Note that, for a given cavity voltage amplitude, \(Q_m\) depends only on \(P_b\) and not on the particular combination of beam current and synchronous phase angle which produces \(P_b\) . The phasor diagram for the matched case is shown in Fig. 25 below:

**Fig. 25** Phasor diagram at the match point
Example:

The cavities in LEP2 work at 10.2 MV (6 MV/m) and transfer 120 kW to the beam. With 
\((R/Q) = 230 \, \Omega\) we calculate from (35) \(Q_m = 1.9E6\). The corresponding 3 dB bandwidth is 
187 Hz. The RF beam current is 28 mA and the synchronous phase -32.8°. Measured in units 
of half a bandwidth the required cavity detuning is then 0.64.

### 6.2 Matching error analysis

In the following we will use a more formal way of analysis to discuss also the 
consequences of matching errors. Introducing normalized values of detuning, beam current 
and cavity voltage:

\[
d = Q \left( \frac{\omega_g - \omega_c}{\omega_c} \right) \approx \left( \frac{\omega_g - \omega_c}{0.5 \Delta \omega_c} \right)
\]

\[
i_b = \frac{I_b}{I_{forw}} = i_{br} + j i_{bi}
\]

and solving (32) and (33) for \(\tilde{V}\) we get:

\[
\tilde{V} = \frac{2 - i_{br}}{1 + jd}
\]

and calculate from \(\tilde{V}\) the reflection coefficient \(\tilde{\rho}\).

\[
\tilde{\rho} = \frac{\tilde{V}_{refl}}{\tilde{V}_{forw}} = \frac{\tilde{V} - V_{forw}}{V_{forw}} = \tilde{v} - 1 = \frac{(1 - i_{br}) - j(d + i_{bi})}{1 + jd}
\]

Let the cavity detuning at which \(\tilde{V}\) and \(\tilde{\rho}\) become real be \(d_0\). At \(d_0\) the expressions in the 
denominator and nominator of (36) must have equal phases:

\[
d_0 = \frac{1}{2 - i_{br}} - \frac{i_{bi}}{2 - i_{br}}
\]

and if we substitute this result back:

\[
v = (2 - i_{br})
\]

and

\[
\rho = 1 - i_{br}
\]

Evidently complete matching is obtained if, in addition to \(d = d_0\), also \(i_{br} = 1\).

Finally, to evaluate also for the general non-matched case the power \(P_b\) transferred to the 
beam we determine \(1 - |\rho|^2\). From (37):

\[
|\rho|^2 = \frac{(1 - i_{br})^2 + (d + i_{bi})^2}{1 + d^2}
\]

\[
\frac{P_b}{P} = 1 - |\rho|^2 = \frac{i_{br}(2 - i_{br}) - i_{bi}(2d + i_{bi})}{1 + d^2}
\]

\(P\) is the forward power needed to transfer \(P_b\) to the beam.
6.2.1 Coupling errors

Let us now compare two cavities of the accelerating station. One, the reference cavity, shall be matched i.e. have \( P_m \) is the forward power and \( I_{\text{brm}} \) the real component of the RF beam current phasor:

\[
I_{\text{brm}} = (I_{\text{forw}})_m = \sqrt{2P_m} / Z_m \Rightarrow 2P_m = I_{\text{brm}} Z_m
\]

and

\[
V_m = (V_{\text{forw}})_m = \sqrt{2P_m} Z_m \Rightarrow 2P_m = V_m Z_m
\]

The second cavity shall be correctly detuned \( (d = d_0) \) and sees the same RF beam current. But with \( Z \neq Z_m \) it will receive from its line a different value of forward current and of forward voltage:

\[
I_{\text{forw}} = \sqrt{2P_m} / Z = I_{\text{brm}} Z_m \Rightarrow i_{\text{br}} = I_{\text{brm}} / I_{\text{forw}} = \sqrt{Z / Z_m} = \sqrt{Q / Q_m}
\]

and

\[
V_{\text{forw}} = \sqrt{2P_m} Z = V_m Z / Z_m = V_m \sqrt{Q / Q_m}
\]

Substituting into (39) and (38) we find for the second non-matched cavity:

\[
\rho = 1 - \sqrt{Q / Q_m}
\]

and for the ratio of cavity voltages:

\[
\frac{V}{V_m} = \sqrt{Q / Q_m} \left( 2 - \sqrt{Q / Q_m} \right)
\]

From \( P_b = (1 - |\rho|^2) P_m \) we verify that (43) also describes the ratio of powers transferred to the beam. \( P_b \) does not vary strongly with \( Q \). For \( Q/Q_m = \sqrt{2} \) we find \( P_b/P_m = 0.964 \).

A more serious consequence of a coupling error is increased voltage in the coupler’s transmission line. \( V_{\text{refl}} \) adds to \( V_{\text{forw}} \) to form a standing wave pattern with voltage maxima of \( V_{\text{max}} = (1+|\rho|) V_{\text{forw}} \). Here we must differentiate between two cases: overcoupling, \( Q < Q_m \), and undercoupling, \( Q > Q_m \):

\[
1 + |\rho| = \begin{cases} 
2 - \sqrt{Q / Q_m} & \text{when } Q \leq Q_m \\
\sqrt{Q / Q_m} & \text{when } Q > Q_m
\end{cases}
\]

Present day couplers work at field levels far below those found in cavities. So, why is increased voltage a problem? The answer is that, in contrast to sc cavities, power couplers may be plagued by multipacting discharges within their operating power range and that, in the presence of mismatch, the forward power at which a given multipacting resonance is met, becomes reduced by \( (1 + |\rho|)^2 \).

Example:
If, with the important mismatch of \( Q/Q_m = \sqrt{2} \), one sends 120 kW to the cavity one loses only the moderate reflected power of 4.3 kW. But multipacting levels up to an equivalent power of 170 kW are now within the operating range of the coupler!
6.2.2 Beam current changes

We now change the real component of the RF beam current from $I_{brm}$ to $I_{br}$ and simultaneously, by a servo loop, the forward power from $P_m$ to $P$ so that the voltage of the reference cavity stays constant. Equation (38) then gives the condition for the changed forward current:

$$V = V_{forw}(2 - i_{br}) = I_{forw}Z_m\left(2 - \frac{I_{br}}{I_{forw}}\right) \equiv V_m = I_{brm}Z_m$$

or

$$2I_{forw} = I_{br} + I_{brm}$$

and remembering that $I_{brm} = (I_{forw})_m$ we have

$$I_{forw} = \frac{1}{2}I_{brm}\left(1 + \frac{I_{br}}{I_{brm}}\right) = \frac{1}{2}(I_{forw})_m\left(1 + \frac{I_{br}}{I_{brm}}\right)$$

it follows that

$$\frac{P}{P_m} = \frac{1}{4}\left(1 + \frac{I_{br}}{I_{brm}}\right)^2$$

There is now a standing wave also on the transmission line to the reference cavity. To know its maximal voltage we again calculate $(1+|\rho|)V_{forw}$. From (39) and (46), after some algebra (see Appendix B)

$$V_{max} = (1 + |\rho|) V_{forw} = \begin{cases} (V_{forw})_m & \text{when } I_{br} \leq I_{brm} \\ \frac{I_{br}}{I_{brm}} (V_{forw})_m & \text{when } I_{br} > I_{brm} \end{cases}$$

Evidently, to avoid increased voltage, coupling has to be correct for the highest expected beam current.

6.2.3 Beam current changes and coupling errors combined

Finally, applying $P$ to the second cavity (which has $Q \neq Q_m$) we have the forward current (more details in appendix C):

$$I_{forw} = \frac{\sqrt{2}P}{\sqrt{Z}} = \sqrt{\frac{P_m}{2Z}}\left(1 + \frac{I_{br}}{I_{brm}}\right) = \frac{1}{2}I_{brm}\sqrt{\frac{Z_m}{Z}}\left(1 + \frac{I_{br}}{I_{brm}}\right)$$

and

$$V_{forw} = I_{forw}Z_m\frac{Z}{Z_m} = \frac{1}{2}V_m\sqrt{\frac{Z}{Z_m}}\left(1 + \frac{I_{br}}{I_{brm}}\right)$$

Substituting (49) and (50) into (38) we obtain the generalised form of (43):

$$\frac{V}{V_m} = \sqrt{\frac{Q}{Q_m}}\left(1 + \frac{I_{br}}{I_{brm}}\right) - \frac{I_{br}}{I_{brm}}\sqrt{\frac{Q}{Q_m}}$$
Note, that as a consequence of equation (51), for $I_{br} < I_{brm}$ and undercoupling ($Q > Q_m$) the cavity field becomes higher than the nominal one. This is illustrated in Fig. 26 below, taking the LEP cavity with its nominal gradient of 6 MV/m as example. The used range of $Q/Q_m$ corresponds to what is found in practice for the present fabrication methods of cavities and couplers.

![Fig. 26 Coupling dependence of the cavity field for three different beam currents](image)

6.2.4 Tuning errors

All the cavities in an accelerating station have independent tuning loops so, in steady state conditions, tuning errors are insignificant. The situation is different for fast perturbations. Tuner constructions available for sc cavities, since they change the length of the cavity, are rather limited in speed. Even the fastest designs, using magnetostrictive rods, cannot tune out vibrations or beam current changes with frequencies above 10 Hz. It is then necessary to change the phase and to increase the amplitude of the forward current to keep the cavity voltage-phasor constant which, for a calculation of the now required additional forward power, forms the most convenient reference of the phasor diagram in Fig. 27.

![Fig. 27 Same diagram as in Fig. 25, but here V/Z is the reference](image)

Note that here, since we decompose in the direction of $V$, the values of $I_{br}$ and $I_{bi}$ are different from those in Fig. 25! With $I_q = I_c + I_{bi}$ we now read from the diagram ($I_q$ is a measure of the deviation from correct tuning):
\[(2I_{\text{forw}})^2 = \left(\frac{V}{Z} + I_{\text{br}}\right)^2 + (I_c + I_{\text{bi}})^2 = \left(\frac{V}{Z} + I_{\text{br}}\right)^2 + I_q^2\]

it follows \( P = \frac{1}{2}(I_{\text{forw}})^2 Z = \frac{Z}{8}\left[\left(\frac{V}{Z} + I_{\text{br}}\right)^2 + I_q^2 \right] \) \( (52) \)

Evidently, for given cavity voltage, RF beam current and synchronous phase (i.e. beam power) the required forward power \( P \) depends on \( Z \) (coupling) and \( I_q \) (tuning) and to minimize \( P \) we have to satisfy the two equations:

\[
\frac{\partial}{\partial I_q} P(Z,I_q) = 0 \quad \text{(i)} \quad \text{and} \quad \frac{\partial}{\partial Z} P(Z,I_q) = 0 \quad \text{(ii)}
\]

or \( I_q = I_c + I_{\text{bi}} = 0 \quad \text{(iii)} \quad \text{and} \quad I_q^2 + I_{\text{br}}^2 - \left(\frac{V}{Z}\right)^2 = 0 \quad \text{(iv)}\)

Satisfying (iii) and (iv) we have a match and the minimum minimorum of the forward power. But with a tuning error, \( I_q \neq 0 \), we still can obtain a relative minimum of the forward power by satisfying only (iv). We find the optimal coupling under such conditions to be tighter than the matching one:

\[
Z_{\text{opt}} = \left(\frac{R}{Q}\right)_{\text{opt}} = \sqrt{\frac{V^2}{I_{\text{br}}^2 + I_q^2}} \leq \frac{V}{I_{\text{br}}} = Z_m \quad \text{(53)}
\]

We now can imagine two scenarios. In the first we had not foreseen a tuning error and matched at a nominal beam current \( I_{\text{brm}} \) i.e. we had chosen \( Z = V/I_{\text{brm}} \). Substituting this value of \( Z \) into Eq. (52) we get:

\[
P = \frac{V}{8}\left(I_{\text{brm}} + 2I_{\text{br}} + \frac{I_{\text{br}}^2}{I_{\text{brm}}} + \frac{I_q^2}{I_{\text{brm}}} \right)
\]

and normalising by the beam power in matched conditions, \( P_{\text{brm}} = 0.5VI_{\text{brm}} \), we obtain a more general form of Eq. (47):

\[
\frac{P}{P_{\text{brm}}} = \frac{1}{4}\left(1 + \frac{I_{\text{br}}}{I_{\text{brm}}}\right)^2 + \left(\frac{I_q}{I_{\text{brm}}} \right)^2 \quad \text{(54)}
\]

\( I_q \) is related to the deviation \( df \) from correct detuning. To first order \( I_q/I_{\text{brm}} = 2df/\Delta f \) where \( \Delta f \) is the loaded 3 dB bandwidth.

Example:

Recently it has been realised [17] that operated at 6 MV/m the LEP cavity suffers from a ponderomotive mechanical oscillation [18]. These oscillations may be suppressed by operating the cavity on its resonance frequency i.e. with \( \omega_c = \omega_g \). But then \( I_c = 0 \) i.e. \( I_q = I_{\text{bi}} \) and at the nominal beam current \( I_{\text{brm}} \):

\[
\frac{P}{P_{\text{brm}}} = \frac{1}{4}\left((1+1)^2 + \left(\frac{I_{\text{bim}}}{I_{\text{brm}}} \right)^2 \right) = 1 + \frac{1}{4}\tan^2 \phi \quad \text{(55)}
\]

Now for \( P_{\text{brm}} = 120 \) kW and \( \Phi = -32.8^\circ \) \( P = 132.5 \) kW and \( (1 + |p|)^2P = 225 \) kW.
In the second scenario we know $I_q$ in advance and choose $Z = Z_{\text{opt}}$. For zero detuning we have $Z_{\text{opt}} = V/I_b$ and the phasor diagram forms an isosceles triangle. Below the LEP example ($\Phi = -32.8^\circ$) is illustrated.

![Fig. 28 Phasor sum for zero detuning and optimal coupling](image)

The synchronous phase is now two times the station phase. Substituting $Z_{\text{opt}}$ into (52) we find:

$$P = \frac{1}{4}VI_b(1 + \cos \phi)$$  \hspace{1cm} (56)

6.2.5 Tuning and coupling errors combined

Finally we will try to visualize what consequences coupling errors have if the cavities of an accelerating station are kept at resonance ($\omega_c = \omega_g$). Here right from the start we had to use a direct numerical approach: First for a reference cavity with $Q = Q_m = 1.88E6$ and $\Phi = -32.8^\circ$ the station phase $\varphi$ was determined by solving (36) numerically. Remembering that $\varphi$ and $P$ are the same for all cavities in a station we then can proceed to determine $\Phi$ and $P_b$ for cavities with different external $Q$, using (36) and (41). The results are shown in Fig. 29. In contrast to the case of correct tuning the synchronous phase (the phase angle difference between cavity voltage and RF beam current) now depends on cavity coupling and rises with increasing external $Q$. Undercoupling thus leads to a sharper increase of power reflection and at the upper limit of the external $Q$ range the equivalent travelling wave power $(1+|p|)^2P$ rises to 280 kW.

6.3 Hardware considerations

Power couplers for sc cavities are normal conducting (nc) devices. Hence their designs could take over many features of earlier constructions for nc cavities. Such couplers may be subdivided into three functional units: first the coupling element proper, probe, loop or coupling iris. Second a ceramic window which seals off the cavity vacuum while letting through the RF power. Third a transition piece to the standard wave guide of the power distribution system.

Couplers for sc cavities have one element more to bridge the gap between room and liquid He temperature while keeping, compared to the cavity dissipation, the heat flux into the He bath small. This thermal transition is a length of guide or line where special care has been taken to minimize both the metal cross-section and thermal conductivity by using copper-plated stainless steel. In addition, cooling with cryogenic gases (He or N$_2$) is employed to intercept heat at discrete points or continuously in a counter current-flow fashion, very similar to the current leads for sc magnets.

For powers around 100 kW and frequencies around 400 MHz wave guides have unnecessarily large cross-sections. So the coaxial line geometry is the preferred ending, to avoid heat conduction by the inner conductor, in an open circuit as the probe which couples to the electric cavity field within the cut-off tube near to one of the cavity end cells. Thus from the inner conductor only heat radiation can reach the He bath and its operation even at room-temperature becomes feasible if no cold window is employed.
Use of such a second cold window (at ≈ 70 K) was the rule in earlier coupler constructions [19,20]. It was felt that only in this way the sensitive SC surfaces of the cavity could be protected if, under the action of the RF fields, gas molecules would be desorbed from 6 warm parts of the coupler and especially from its first room-temperature window.

With the argument that the thermal transition piece should act as an efficient baffle, later [21] only the warm window was kept allowing the use of window constructions with a proven performance from copper-cavity work. Figure 30 shows the LEP power coupler as an example of this approach. Window and 'door-knob' transition to a waveguide are taken over from the coupler for the LEP copper cavities.

Fig. 29 Influence of coupling errors for LEP cavities operated at resonance

Fig. 30 A LEP power coupler with 50 Ω coaxial line

6.4 Conditioning

Applying for the first time higher powers to vacuum RF devices is often a very delicate operation. In fact, if the power is raised abruptly one risks provoking violent discharges which may destroy the device. Two phenomena combine to create this danger. The first is the

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4 The developments for the 1.5 GHz TESLA very-high-gradient SC linac, to be operated at 1.8 K, have reintroduced a second 70 K window as heat shield and to seal the cavity at an early stage of manufacture against dust particle intrusion.
presence of adsorbed layers of gas (predominantly H$_2$O) molecules on RF boundary surfaces. The second is the existence of electron orbits which, starting and ending on surfaces, become self replicating at certain field levels. If now at such a kinematic “multipactor” resonance an initial electron has an impact energy high enough to release more than one secondary then an avalanche occurs with a proportional increase of gas desorption. If the process is not controlled the local gas pressure may rise to the level where ordinary gas discharges set in.

But gas desorption is not only a danger. It is also the effect which makes “conditioning” possible. In fact, the secondary emission coefficient of a copper surface (and similarly for other metals) decreases when H$_2$O layers are removed [22], so multipactor discharges have the tendency to eliminate themselves. To make a discharge disappear one only has to “tickle” it long enough, taking care that pressure bursts do not rise into the unsafe region above $10^{-7}$ Torr.

### 6.5 Deconditioning

Couplers for copper cavities are conditioned together with the cavity. Couplers for sc cavities need more elaborate procedures. Their operation is often plagued by the reappearance of multipactor discharges (deconditioning). This is caused by the baffle action of the warm-cold transition [23].

It has been shown that multipactor orbits in coaxial [24] lines with wave impedances bigger than 50 $\Omega$ start and end on the outer conductor alone which, acting as a cold trap, is unfortunately also the zone to where molecules liberated in the warm window area are cryopumped. Multipactoring recreated in this way is very difficult to condition. Probably molecules desorbed by electron impact are only partially pumped away, the remainder is re-adsorbed at cold surfaces nearby.

To alleviate the problem of deconditioning, power couplers for sc cavities are, after a thorough bake out at 200$^\circ$C, preconditioned on a separate room-temperature test stand and only then transferred to the cavity. Since, during this last operation, contact with humid air is unavoidable LEP couplers are subsequently baked a second time in situ. The window is heated to 200$^\circ$C for 24 hours. Only then are the cavities cooled and RF power applied for the final conditioning.

As a further safeguard against the reappearance of multipactoring the LEP coupler has been fitted with condensers [25] which isolate the central conductor of the coaxial line for DC voltages but let the RF power pass. So, if required, a bias voltage can be applied [26]. It has been found that a bias of 2.5 kV suppresses all multipactor resonances up to a travelling-wave power of at least 200 kW.

Finally, since multipactor resonances belong to certain $E$-field levels at the outer conductor, power flow before multipacting sets in will be the higher the smaller the central conductor’s diameter. With respect to multipacting characteristics the conventional choice of a 50 $\Omega$ wave impedance is not optimal. In consequence the LEP power coupler’s antenna diameter recently has been reduced from 45 mm to 30 mm which corresponds to $Z_0 = 75$ $\Omega$. 
REFERENCES


[25] H.P. Kindermann et al., Status of the RF Power Couplers for Superconducting Cavities at CERN, to be Published in Proc. of the 1996 EPAC, Sitges (Barcelona).

APPENDIX A

Fig. 1 Geometry of the HOM coupler used in LEP

Table 1
Damping of significant HOM with two couplers per LEP cavity

<table>
<thead>
<tr>
<th>f/MHz</th>
<th>461</th>
<th>476</th>
<th>506</th>
<th>513</th>
<th>639</th>
<th>688</th>
<th>1006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
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<td>TE_{111}</td>
<td>TM_{110}</td>
<td>TM_{011}</td>
<td>TM_{111}</td>
<td>TM_{111}</td>
<td>TM_{012}</td>
</tr>
<tr>
<td>(R/Q)/Ω</td>
<td>18</td>
<td>15</td>
<td>20</td>
<td>13</td>
<td>56</td>
<td>25</td>
<td>16</td>
</tr>
<tr>
<td>Q_{ex}</td>
<td>17 000</td>
<td>14 000</td>
<td>5 600</td>
<td>5 700</td>
<td>7 000</td>
<td>10 000</td>
<td>20 000</td>
</tr>
</tbody>
</table>

APPENDIX B

From

\[ V = I_{\text{forw}} Z_m \left( 2 - \frac{I_{\text{br}}}{I_{\text{forw}}} \right) \equiv V_m = I_{\text{brm}} Z_m \quad \text{(B1)} \]

we obtain

\[ 2I_{\text{forw}} = I_{\text{brm}} + I_{\text{br}} \quad \text{(B2)} \]

and using Eq. (39) from the main body of this lecture
we have \( \rho = 1 - \frac{I_{br}}{I_{forw}} = \frac{2I_{forw} - 2I_{br}}{2I_{forw}} = \frac{I_{brm} - I_{br}}{I_{brm} + I_{br}} \) \( \text{(B3)} \)

and remembering that \( I_{brm} = (I_{forw})_m \) we get

\[
1 + |\rho| = \begin{cases} 
\frac{2I_{brm}}{I_{brm} + I_{br}} = \frac{(I_{forw})_m}{I_{forw}} = \frac{(V_{forw})_m}{V_{forw}} & \text{when } I_{br} \leq I_{brm} \\
\frac{2I_{br}}{I_{brm} + I_{br}} = \frac{I_{br}}{I_{brm}} \frac{(V_{forw})_m}{V_{forw}} & \text{when } I_{br} > I_{brm}
\end{cases}
\]

\( \text{(B4)} \)

and \( V_{max} = (1 + |\rho|) V_{forw} = \begin{cases} 
(V_{forw})_m & \text{when } I_{br} \leq I_{brm} \\
\frac{I_{br}}{I_{brm}} (V_{forw})_m & \text{when } I_{br} > I_{brm}
\end{cases} \)

\( \text{(B5)} \)

**APPENDIX C**

From the main body of the lecture:

\[
V = V_{forw} \left( 2 - \frac{I_{br}}{I_{forw}} \right)
\]

\( \text{(C1)} \)

and

\[
\sqrt{\frac{P_m}{2Z}} = \sqrt{2P_m} = (I_{forw})_m \sqrt{Z_m} = \frac{1}{2} I_{brm} \sqrt{Z_m}
\]

\( \text{(C2)} \)

and

\[
I_{forw} = \sqrt{\frac{P_m}{2Z}} = \sqrt{\frac{P_m}{Z}} = \frac{1}{2} I_{brm} \left( 1 + \frac{I_{br}}{I_{brm}} \right)
\]

\( \text{(C3)} \)

it follows that

\[
I_{forw} = 2 I_{brm} \sqrt{\frac{Z_m}{Z}} \left( 1 + \frac{I_{br}}{I_{brm}} \right)
\]

\( \text{(C4)} \)

and

\[
V_{forw} = I_{forw} \frac{Z_m}{Z} = \frac{1}{2} V_{m} \sqrt{\frac{Z}{Z_m}} \left( 1 + \frac{I_{br}}{I_{brm}} \right)
\]

\( \text{(C5)} \)

substituting (C4) and (C5) into (C1):

\[
\frac{V}{V_{m}} = \sqrt{\frac{Z}{Z_m}} \left[ \left( 1 + \frac{I_{br}}{I_{brm}} \right) - \frac{I_{br}}{I_{brm}} \sqrt{\frac{Z}{Z_m}} \right]
\]

\( \text{(C6)} \)