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SPECIFICATIONS FOR DERIVING NEUTRON ELECTRIC POLARIZABILITY FROM THE TOTAL CROSS SECTIONS OF $^{208}$Pb

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1. In spite of numerous and long standing attempts to get an experimental value for the neutron electric polarizability coefficient $\alpha_n$, the only meaningful result

$$\alpha_n = (1.20 \pm 0.15 \pm 0.20) \cdot 10^{-3} \text{ fm}^3$$

was communicated in [1]. An analysis and some criticism of [1] was given in [2] about four years ago. Nevertheless, since we regard the measurement of the $^{208}Pb$ neutron scattering cross section $\sigma(k)$ ($k$ is neutron wave number) performed in [1] as the best one so far, it is very important to understand all the details of the discussed $\sigma(k)$ or $\sigma(E)$ dependence ($E$ is the neutron energy), and not only the $k$- or $\sqrt{E}$-component of it. In other words, the $b$ and $c$ coefficients in the expression

$$\sigma(k) = \sigma(0) + bk^2 + ck^4$$

used in [1] must also be physically reasonable. The second motive of the present work is the unexpected results of recent measurements of $\sigma(E)$ for $^{208}Pb$ performed in Dubna [3]. A very large deficiency in the s-wave resonance strength was discovered in [3] in comparison with that which exists for $^{208}Pb$ in literature [4,5]. We have found that only by taking an unusually strong resonance into account can $\alpha_n$ be derived from $\sigma(E)$ positive.

2. If $\sigma(k)$ is a "potential" cross section, i.e., it is obtained by subtracting the contributions of all known resonances, it must have the form

$$\sigma(k) = \frac{4\pi}{k^2} \sin^2(-ka_{pot}) + \frac{12\pi}{k^2} \sin^2 \delta_1,$$

where $-ka_{pot}$ and $a_{pot}$ are the s-wave phase shift and scattering length, respectively. For the last one we should write

$$a_{pot} = R'_{N} + hE + a_pQ,$$

where $R'_N$ is the nuclear part of the so-called scattering radius, $hE$ are the energy dependent "tails" of unknown and distant resonances, and $a_pQ$ is the neutron polarizability contribution; for $^{208}Pb$

$$a_p = -0.0392\alpha_n \text{ fm}, \quad Q = 1 - \frac{5\pi}{18} kR_N + \frac{5}{21} (kR_N)^2 - \frac{2}{243} (kR_N)^4 + \cdots$$

($R_N \approx 7.1 \text{ fm}$, $\alpha_n$ in $10^{-3} \text{ fm}^3$). The $h$ parameter can be expressed to the first power of the $E/E_0$ approximation via the sum

$$h = -2276 \frac{A+1}{A} \sum \frac{2\Gamma_n^{(0)}}{E^2_0} \text{ fm/eV}$$
over all s-wave resonances which were not taken into account in the $\sigma(k)$ calculation. Here $F_n^{(0)}$ and $E_0$ are the reduced neutron width and the energy of one resonance. Note, both positive and negative resonances give negative contributions to $h$. As for the second term in (3), it is the p-wave part of $\sigma(k)$. We use the results of [6] for neutron scattering by natural $Pb$ and the results of following experiment with $^{208}Pb$ to estimate $\delta_1$ as

$$
\delta_1 = -kR + \arctg(kR) + \arcsin\left[\frac{1}{3}k^{2}R_1^2(R - R_1)\right],
$$
(7)

$$
R = 8.0 \text{ fm}, \quad R_1' = 3.0 \text{ fm},
$$
where $R$ is the channel radius and $R_1'$ is the p-wave scattering radius. Finally, it is convenient to join the constant term of $a_{p}Q$ with $R_{N}$ so instead of (4) one gets:

$$
a_{pot} = R' + hE - a_{p}(1 - Q),
$$
(8)

where $R' = R_{N} + a_{p}$ becomes the value usually seen in experiments.

3. Now, if we present (3) together with (8), (5) and (7) in the form of series (2), its coefficients will be:

$$
\sigma(0) = 4\pi R'^2,
$$
(9)

$$
a = 8\pi c_1 R',
$$
(10)

$$
b = 4\pi(c_1^2 + 2c_2 R' - \frac{1}{3}R'^4),
$$
(11)

$$
c = 4\pi(c_2^2 - 2c_1 R'^2 - \frac{4}{3}c_2 R'^3 + \frac{2}{45}R'^6 + \frac{1}{3}R'R'^2),
$$
(12)

where

$$
c_1 = -\frac{5\pi}{18}a_{p}R_{N}, \quad c_2 = 2.09 \cdot 10^5 h + \frac{5}{21}a_{p}R_{N}^2.
$$

In order to verify the validity of approximation (2) for (3), we found decomposition coefficients of (3) $c_3$ at $k^3$, $c_5$ at $k^5$ and $c_6$ at $k^6$ as well:

$$
c_3 = 8\pi c_1 \left( c_2 - \frac{2}{3}R'^2 \right),
$$
(13)

$$
c_5 = 4\pi c_1 R' \left[ \frac{11}{60}R'^4 - 4c_2 R' - \frac{4}{3}c_1^2 \right],
$$
(14)

$$
c_6 = 4\pi \left[ -\frac{1}{3}c_4 - 4c_3 c_2 R' - 2c_2^2 R'^2 + \frac{5}{12}c_2 R'^4 + \frac{11}{60}c_2 R'^5 - \frac{1}{360}R'^8 + 2R'^2R_1' \left( \frac{2}{15} - \frac{R_1'}{3R} \right) \right].
$$
(15)

Using the values

$$
\sigma(0) = (11.508 \pm 0.005) b, \quad a = (0.69 \pm 0.09) b \cdot fm, \quad b = (-448 \pm 3) b \cdot fm^2, \quad c = (9500 \pm 400) b \cdot fm^4
$$
(16)

from [1] and above-mentioned quantities for $a_{p}$ and $R_{N}$, it is not difficult to get from equations (9)–(11)

$$
R' = (9.5696 \pm 0.0021) fm, \quad c_1 = (0.0029 \pm 0.0004) b, \quad c_2 = (-0.402 \pm 0.013) b \cdot fm, \quad a_p = (-0.046 \pm 0.006) fm, \quad \alpha_n = (1.18 \pm 0.15) \cdot 10^{-3} fm^3, \quad h^* = (-19.0 \pm 0.6) \cdot 10^{-7} fm/eV.
$$
(17)

*) This confirms the result of [3] $h < -11 \cdot 10^{-7} fm/eV.$

3
But substitution of quantities (17) into (12) leads to an unexpected result:

\[
\frac{4\pi}{3} R^4 R'^2 = (-896 \pm 443) \text{ b \cdot fm}^4
\]

(19)

instead of

\[
\frac{4\pi}{3} R^4 R'^2 = 1544 \text{ b \cdot fm}^4
\]

(20)

at \( R \) and \( R' \), according to (7). This means there is an absence of p-wave contribution in \( \sigma(k) \) measured in [1]. The deficiency of \( \sigma(k) \) at 40 keV is the difference of (20) and (19) times \( k^4 \), i.e., 8.9 \pm 1.6 \text{ mb}.

Finally, we estimate the contributions which are proportional to \( k^3, k^5 \) and \( k^6 \) and which were ignored in [1]. Using (17) for \( R', c_1, c_2 \) and (7) for \( R, R' \) we have:

\[
c_3 k^3(40 \text{ keV}) = (-3.55 \pm 0.49) \text{ mb}, \quad c_5 k^5(40 \text{ keV}) = 0.09 \text{ mb},
\]

\[
c_6 k^6(40 \text{ keV}) = -0.86 \text{ mb}.
\]

(21)

Thus, if the neglect of the two last values of (21) can be regarded as justified, the neglect of \( c_3 k^3 \), which is \( \sim 12\% \) from \( ak = 30.18 \text{ mb} \) seems rather unjust for a correct \( \alpha_n \) determination.

4. The main purpose of this work is to demonstrate an alternative method of \( \sigma(E) \) analysis. Having no primary data from [1], we are forced to produce them in an artificial way. One hundred values of \( \sigma(E) \) at \( E \) from 0.4 to 40 \text{ keV}, calculated according to (2) and (16), were spread randomly with a standard deviation \( \Delta \sigma \), and 4 coefficients of (2) were fitted to these quasi-experimental points of \( \sigma(E) \) by the least-square method. We stopped at \( \Delta \sigma = 2 \text{ mb} \) and

\[
\begin{align*}
\sigma(0) &= (11.507 \pm 0.001) b, \quad & a &= (0.68 \pm 0.05) b \cdot \text{ fm}, \\
b &= (-446 \pm 1) b \cdot \text{ fm}^2, \quad & c &= (8600 \pm 500) b \cdot \text{ fm}^4,
\end{align*}
\]

(22)

because the coefficient errors increased very abruptly at larger \( \Delta \sigma \).

<table>
<thead>
<tr>
<th>No.</th>
<th>( R' ), \text{ fm}</th>
<th>h \cdot 10^{-7}, \text{ fm}/\text{eV}</th>
<th>\alpha_n \cdot 10^4, \text{ cm}^3</th>
<th>E_0, \text{ MeV}</th>
<th>\Gamma_n^{(0)}, \text{ eV}</th>
<th>\chi^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.66859(6)</td>
<td>-20.4(3)</td>
<td>1.68(19)</td>
<td>-</td>
<td>-</td>
<td>123</td>
</tr>
<tr>
<td>2</td>
<td>6.302(58)</td>
<td>0</td>
<td>1.49(26)</td>
<td>-3.9</td>
<td>5770(101)</td>
<td>176</td>
</tr>
<tr>
<td>3</td>
<td>6.728(41)</td>
<td>0</td>
<td>1.60(20)</td>
<td>-2.5</td>
<td>3217(46)</td>
<td>122</td>
</tr>
<tr>
<td>4</td>
<td>7.048(40)</td>
<td>0</td>
<td>1.66(21)</td>
<td>-1.9</td>
<td>2170(34)</td>
<td>122</td>
</tr>
<tr>
<td>5</td>
<td>7.511(37)</td>
<td>0</td>
<td>1.79(22)</td>
<td>-1.3</td>
<td>1211(22)</td>
<td>132</td>
</tr>
<tr>
<td>6</td>
<td>8.824(17)</td>
<td>0</td>
<td>3.20(27)</td>
<td>-0.3</td>
<td>101(2)</td>
<td>158</td>
</tr>
<tr>
<td>7</td>
<td>7.591(46)</td>
<td>-5</td>
<td>1.63(21)</td>
<td>-1.9</td>
<td>1702(39)</td>
<td>122</td>
</tr>
</tbody>
</table>

So, the obtained set of \( \sigma(E) \) points was described first by formulas (3), (8), (5) and (7) with three varied parameters, \( R', h \) and \( \alpha_n \). The result of fitting is shown in the first line of the Table (the last column is the \( \chi^2 \) value for 100 points; parameter errors are in parenthesis). After that, it seemed interesting to see what resonance, instead of \( h \), could fill the deficiency and whether it really exists. With this purpose, we added the resonance and interference terms:

\[
\frac{\pi g \Gamma_n^{(0)} \sqrt{E}}{k^2} \cdot \frac{\Gamma_n^{(0)} \sqrt{E} + 2(E - E_0) \sin(2k a_{pot}) - 2 \Gamma \sin^2(k a_{pot})}{(E - E_0)^2 + \Gamma^2/2}
\]

(3')
to (3) for one resonance with a given \( E_0 \) and \( \Gamma_n^{(o)} \), and its interference term:

\[
\frac{2\pi}{k^2} \frac{\Gamma_n^{(o)} \Gamma_n^{(e)} E_0^2 (E - E_0) (E - E_{01}) + \Gamma_1}{[(E - E_0)^2 + \Gamma_1^2/4][E - E_{01})^2 + \Gamma_1^2/4]}
\]

(3"

with the strongest resonance having \( E_{01} = 507 \, keV \), \( \Gamma_n^{(o)} = 74 \, eV \), where \( \Gamma = \Gamma_n^{(o)} \sqrt{E} \).

\( \Gamma_1 = \Gamma_n^{(o)} \sqrt{E} \).

Fitting \( R', \alpha_n, E_0, \Gamma_n^{(o)} \) at the fixed \( h = 0 \) showed a strong correlation between \( E_0 \) and \( \Gamma_n^{(o)} \) and no evident minimum of \( \chi^2 \). So we made several fits with different fixed \( E_0 \) displayed in the Table. The lines 2-6 show that any sufficiently distant negative resonance with proper \( \Gamma_n^{(o)} \) provides acceptable description of the calculated \( \sigma(E) \) points with \( \alpha_n \) slightly dependent on \( E_0 \). The interval for permissible \( E_0 > 0 \) is much less and wholly situated at the searched energies [4,5] where the strongest resonance has \( \Gamma_n^{(o)} = 74 \, eV \). Therefore only a negative resonance can resolve the problem.

Fortunately, the compound-nucleus \( ^{209}Pb \) with double magic \( ^{208}Pb \) core has the well-known level scheme up to \( \sim 5.5 \, MeV \) [7,8] in which there is only one \( 1/2^+ \) level below the neutron binding energy \( 3.94 \, MeV \). It is the single-particle level \( 4s_{1/2} \) at the energy \( 2.03 \, MeV \) and corresponds to the negative resonance at \( E_0 = -1.91 \, MeV \). This resonance was used in [5] as "dummy" resonance with \( \Gamma_n^{(o)} \approx 200 \, eV \) assigned to it. However, if a resonance is based on the single-particle level with the spectroscopic factor close to 1 (as in our case according to [7]) it may well be that such a resonance has \( \Gamma_n^{(o)} \) close to the single-particle Wigner limit which is \( \sim 2300 \, eV \) for a nuclear radius of \( 8 \, fm \).

Thus we have new solid grounds to declare we know now the searched resonance and consider line 4 of the Table as the most probable among the rest. As for line 7, it shows the possible influence on derived \( \alpha_n \) of missed resonances which are equivalent to \( h = -5 \cdot 10^{-7} \, fm/eV \) (for comparison: the strongest resonance at \( E_0 = 507 \, keV \) gives \( h = -6.6 \cdot 10^{-7} \, fm/eV \)).

5. Finally, we tried to improve the \( \sigma(E) \) analysis used in [1] by adding the cubic term \( c_3 k^3 \) to (2). Since the fitting of five independent parameters is practically impossible, we got from (10) and (13) the relation

\[
c_3 = a(c_2 - \frac{2}{3} R'^3) / R' \approx -65a.
\]

(23)

where the approximate equality was obtained by the substitution of values from (17) into the exact equality. This made it possible to fit four parameters as before, but also taking into account the cubic term \( -65ak^3 \). The fitted parameters:

\[
\sigma(0) = (11.508 \pm 0.002) \, b, \quad a = (0.54 \pm 0.26) \, b \cdot fm,
\]

\[
b = (-441 \pm 8) \, b \cdot fm^2, \quad c = (8100 \pm 2100) \, b \cdot fm^4
\]

(24)

do not differ much from (16) and (22) and result in

\[
R' = (9.5696 \pm 0.0009) \, fm, \quad h = (-17.7 \pm 1.4) \cdot 10^{-7} \, fm/eV,
\]

\[
\alpha_n = (0.93 \pm 0.40) \cdot 10^{-3} \, fm^3,
\]

(25)

whose accuracy is a marked degree worse than of (17) and (18).

**Conclusion.** Detailed analysis of formula (2) and its coefficient values (16) given in [1] has shown the following.
1. Most probably, the cross section $\sigma(k)$ measured in the work [1] is somewhat wrong. This issue from the fact that coefficients $b$ and $c$ describing $\sigma(k)$ are in a wrong relation: the $c$ value at a given $b$ is deficient not only for the $p$-wave contribution but for the $k^4$ term from $\sin^2(kR')/k^2$, as well (the value of (19) is not zero but negative). The probable reason for this is a distortion due to background difficulties.

2. The systematic error of (1) is obviously underestimated, and for $\alpha_n$ instead of (1) it should be written something like $\alpha_n \sim (0.7 \pm 1.9) \cdot 10^{-3} fm^3$.

3. In order to get the $\alpha_n$ value from a correctly measured $\sigma(k)$ with errors of about $1 - 2 mb$, we propose two approaches. The first one (see point 5 above) consists of simultaneous taking into account the linear and cubic in $k$ terms of $\sigma(k)$. The alternative approach is application of formula (3) with the supplements (3') and (3'') where $E_0 = -1.91 MeV$ and $\Gamma_n^{(0)}$ is to be fitted. Only the $\alpha_n$ value coincided with these two approaches can be regarded as the reliable one.

References.


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Enik T.L. et al.
Specifications for Deriving Neutron Electric Polarizability from the Total Cross Sections of $^{208}$Pb

A detailed analysis was carried out of the method for determining the neutron electric polarizability and of the only meaningful result $\alpha_n = (1.20 \pm 0.15 \pm 0.20) \times 10^{-3}$ fm$^3$. As some inexactitudes were found in obtaining this result, the conclusion was made that its systematic error should be 3—4 times higher. A complex method for determining $\alpha_n$ from the total cross section of neutrons scattered by $^{208}$Pb nuclei is offered. This method includes taking into account of both the $k^3$-term of the cross section ($k$ is neutron wave number) and the negative neutron resonance at $-1.91$ MeV.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.
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