NOTE ABOUT A COHESENT, RADIAL Q-SHIFT OF A SPACE CHARGE

COMPENSATED PROTON BEAM

by

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It should be considered a circulating proton and a stationary electron beam, both of circular cross-section and uniform and equal density. The axes of the two beams should be parallel, but displaced from each other by a distance $r$. In the overlapping region a homogenous electric field (parallel to the displacement vector $r$) will be produced:

$$ E_r = \frac{en}{2\varepsilon} \cdot r \quad \text{Equ. 1} $$

e = elementary charge
n = charge density
$\varepsilon$ = vacuum di-electric constant.

If there is also a magnetic field $B_z$ present, perpendicular to the electric one, the electrons (of the overlapping region) will describe a circular motion (due to this thermal velocity) around a guiding center which itself moves with a velocity $v = E/B$ perpendicular to both $E$ and $B$ (parallel to the proton beam); or shortly speaking, the electron trajectory is a cycloide. If we assume a time dependant electric field of the form:

$$ E_r = E e^{i\omega t} $$

it is also possible to solve rigorously the motion of the electron. One obtains for the guiding center velocity:

$$ v = \frac{E}{B} e^{i\omega t} $$

The motion in $r = direction$ is given by:

$$ \Delta r = \frac{E}{B} \frac{1}{\Omega} e^{i\omega t} $$

with $\Omega = \frac{e}{m} B$; $m$ : rest-mass of the electron.

(These formulae hold if : $\omega < \Omega$).
For example with

\[ E = 1 \, \text{kV/cm}; \quad B = 10^4 \, \text{G} \]

one obtains:

\[ v = 10^7 \, \text{cm/sec}; \quad \Delta r_0 = 5 \times 10^{-5} \, \text{cm}. \]

One has the surprising result that the electron beam does not follow the movement of the proton beam in the radial direction. It only oscillates in the proton beam direction with an amplitude of a few mm for a radial betatron frequency \( \omega = 1,5 \times 10^7 \, \text{Hz} \).

Now we consider a space charge compensated proton beam in a storage ring. The tolerances on the revolution frequency (for different points of the beam cross-section) should be sufficiently small, so that a coherent betatron oscillation can be established. A radial deviation of the proton beam, out of its stable position, provokes an electrostatic restoring force. The coherent radial oscillation is roughly given by the following equation:

\[ r + q_r \omega_0^2 r + \frac{e}{M} \frac{\partial U}{\partial r} = 0 \]

with: \( M \): rel. proton mass
\( \alpha \): fraction of the orbit, exposed by the magnetic field.

From equation 1 one obtains

\[ \frac{\partial U}{\partial r} = \frac{en}{2\varepsilon} \cdot r \]

And therefore the radial betatron frequency for a coherent movement of the proton beam becomes:

\[ \omega_r^2 = q_r^2 \omega_0^2 + \alpha \frac{e^2 n}{2\varepsilon M} \omega_0^2 + \Delta \omega^2 \]

For \( n = 10^8 / \text{cm}^3 \) (\( \approx 1 \, \text{Amp. in J.S.R.}; \) and 20 GeV)
\[ \Delta \omega^2 \approx 2.7 \cdot 10^{12} \]

or

\[ \frac{\Delta \omega}{\omega} = 5 \cdot 10^{-3}/\text{Amp.} \]

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