A MEASURING SYSTEM FOR MAGNETS WITH CYLINDRICAL SYMMETRY

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Summary

A simple and very precise system for the measurement of magnets with cylindrical symmetry is described.

This system was first used to determine the shims and, subsequently, for the final magnetic measurements of the sextupole and octupole lenses for resonance compensation in the CERN Intersecting Storage Rings (ISR). The method of measurement is based on the use of a coil which rotates stepwise around the axis of symmetry of the magnet. The harmonic coefficients of the magnetic potential are determined by a numerical harmonic analysis of the flux linked by the coil. The high precision is obtained by using a coil which measures only the azimuthal component of the field; the harmonic response of this type of coil is given. The use of compensation coils to suppress the main harmonic and measure only the higher multipole content of the field is also described. The measuring machine is controlled by a 16 K-bit computer, which also performs the harmonic analysis of the measurements on-line. A magnet can be measured in 20 minutes.

1. - Basics

In high energy particle accelerators, the length of the magnets is usually small in comparison with the wavelength of the betatron oscillation. This means that only the integrals along the z-axis (see Figure 1) of the induction B and its derivatives are of interest.

\[ I(r,\theta) \text{ and the integrals } \int_{-\infty}^{\infty} H_{r} \, dz \text{ being given by:} \]

\[ I(r,\theta) = I_0 + \sum_{n=1}^{\infty} \frac{a_n}{n} r^n \sin n\theta + b_n r^n \cos n\theta \]

\[ \int_{-\infty}^{\infty} H_{r} \, dz = \frac{2I}{r} \sum_{n=1}^{\infty} n r^{n-1} \left( a_n \sin n\theta \right) \]

\[ \int_{-\infty}^{\infty} H_{\theta} \, dz = \frac{2I}{r} \sum_{n=1}^{\infty} n r^{n-1} \left( b_n \cos n\theta \right) \]

The harmonic analysis of \( H_r \) or \( H_\theta \) measured with a long coil at \( N \) points at a distance \( R \) from the origin, at steps of \( 2\pi/N \) radians, will give the coefficients \( a_n, b_n \). The principle for this method of measuring magnetic fields has been discussed in several papers(4 to 6).

The flux linked by a coil measuring \( H_\theta \) only, will be:

\[ \Phi(\theta) = \sum_{n=0}^{\infty} K_n \left( a_n \cos n\theta - b_n \sin n\theta \right) \]

where \( K_n \) are coefficients depending on the coil's geometry. By measuring \( \Phi(\theta) \) at \( N \) points, the total flux \( \Phi \) as a function of the angular position of the coil can be determined. The harmonic analysis of this function will have the form:

\[ \Phi(\theta) = \sum_{n=1}^{\infty} A_n \cos n\theta + B_n \sin n\theta \]

where \( A_n, B_n \) are the harmonic coefficients of the measured flux.

From (5) and (6) we get:

\[ a_n = \frac{A_n}{K_n} \quad b_n = \frac{B_n}{K_n} \]

The tilt of the magnet's median plane may also be determined knowing \( a_n, b_n \), as shown in Section 5.5.

2. - Choice of the Coil Geometry

The coil must be designed so as to meet the following requirements:

- high sensitivity
- exploration of the outer boundary of the useful field region, as near as possible to the pole tips
- simplicity in construction, winding, alignment
- low sensitivity to geometrical errors like radial displacement, etc.
- possibility of compensating the main harmonic of the field
- high sensitivity to the higher harmonics.

One can choose between measuring the flux due to the azimuthal or the radial field components (Coils A and B in Figure 2).

\[ \Phi_n = \psi \int_0^{2\pi} \frac{H_n r}{r} \, rd\theta \]

The influence of the coil geometry is given by:
\[ K_n = 2l r_n \sin n \frac{\theta_2 - \theta_1}{2} \]
\[ K_n = l (r_2 - r_1) \]

The term \( n \frac{\theta_2 - \theta_1}{2} \) in the response of the radial flux coil makes it difficult to obtain a good response for every harmonic. This term may be made \( \sim 1 \) for a given \( n \), but will be \( \ll 1 \) for other values of \( n \) and the accuracy of the corresponding harmonics will be poor. In the present case, a compromise may be made by choosing \( \theta_2 - \theta_1 \approx 14^\circ \), but a low sensitivity must then be accepted. An increase in the number of windings will complicate the winding operation and increase the uncertainty in the computation of the harmonic response coefficient \( K_n \).

Because of the above considerations, a coil geometry of type B was chosen, which has the following advantages:
- high sensitivity; since a smaller number of turns is required, the coil's section is smaller; as a result, it can be used more effectively near the pole tips and its response can be calculated more accurately;
- good sensitivity for every harmonic of the field;
- simple construction.

The influence of the coil width on the response coefficients \( K_n \) and on their sensitivity to radial displacements is shown in Figures 3.a and b. It can be seen that, according to the magnet type, a compromise must be found between coil sensitivity and the sensitivity to radial alignment errors.

The use of a compensation coil to avoid the measurement of the main harmonic of the field is described in Section 4.

a) Coil coefficients \( K_n \) versus coil width \( (R_2 = 0.1 \text{ m}) \).
b) Relative error in the coil coefficients for \( dR = 0.5 \text{ mm} \), as a function of \( R_1 \) \( (R_2 = 0.1 \text{ m}) \).
\[ B_y = u_0 \left( H_z \sin \epsilon + H_0 \cos \epsilon \right) \]
\[ B_y = u_0 \sum_{n=1}^{\infty} \frac{a_n \cos(n-1)\epsilon}{n^2} \left\{ \cos n\theta \left[ a_n \sin(n-1)\epsilon \right. \right. \]
\[ + b_n \sin(n-1)\epsilon \left. \left. - \sin n\theta \left[ a_n \sin(n-1)\epsilon \right. \right. \right. \]
\[ + b_n \cos(n-1)\epsilon \left. \right. \right\} \]

The flux linked by the coil is:

\[ \Phi = \lambda k \int_{-a}^{a} \int_{-b}^{b} \int_{-d}^{d} B \, db \, da \, dx \]

where:
- \( \lambda \) = winding density/surface unit
- \( k \) = coil length

Combining (10) and (11) and neglecting the \( \sin(n-1) \) terms for reasons of symmetry,

\[ \Phi = 2u_0 k \int_{-a}^{a} \int_{-b}^{b} \left( \frac{a_n \cos n\theta - b_n \sin n\theta}{n^2} \right) \left( \frac{x^2 + y^2}{1 + \frac{y^2}{a^2}} \right)^{n-1} \cos \left[ (n-1) \arctan \frac{y}{x} \right] \, db \, da \, dx \]

The harmonic coefficients of the coil response are then,

\[ K_n = 2u_0 k \int_{-a}^{a} \int_{-b}^{b} \left( \frac{a_n \cos n\theta - b_n \sin n\theta}{n^2} \right) \cos \left[ (n-1) \arctan \frac{y}{x} \right] \, db \, da \, dx \]

The triple integral can be computed by numerical integration and the term \( 2u_0 k \) is determined from a measurement of the coil's effective surface \( S_e \) by,

\[ 2u_0 k = \frac{S_e}{2AB(R2R1)} \]

4. - Compensation of the Main Harmonic of the Field

For particle accelerators, one is generally interested in having multiple lenses in which only the main field harmonic is present. The magnet's quality is then dependent on the presence of higher or lower components than the main field component. By suitably designing the search coil system, the response for the main field harmonic can be suppressed. The signals arising from the field errors will be small so eliminating linearity problems in the integrator.

Let us consider two single loop coils as shown in Figure 5.

![Figure 5 - Compensation of the Main Field Harmonic.](image)

The flux linked by these coils will be respectively:

\[ \Phi_1 = n \sum_{n=1}^{\infty} \phi_{1,n} = u_0 k \sum_{n=1}^{\infty} \left( \frac{a_n \cos n\theta - b_n \sin n\theta}{n^2} \right) \left( \frac{R2 - r_1}{R1} \right) \]

\[ \Phi_2 = n \sum_{n=1}^{\infty} \phi_{2,n} = u_0 k \sum_{n=1}^{\infty} \left( \frac{a_n \cos n\theta - b_n \sin n\theta}{n^2} \right) \left( \frac{r_4 - r_3}{R1} \right) \]

where:
- \( l_1 = l_2 = \) coil length.

By suitably choosing \( r_1, r_2, r_3 \) and \( r_4 \), we can obtain for a given \( n \),

\[ \phi_{1,n} = \phi_{2,n} \]

By subtracting the signals from these two coils, the nth harmonic is then removed. For practical multi-turn coils, we have the simple relation (with \( k_1 = k_2 \)):

\[ r_2 \approx n \sqrt{\frac{N_1}{N_2} \left( \frac{r_2}{r_1} - \frac{r_3}{r_1} \right) + \frac{r_2}{r_1}} \]

where:
- \( n \) = order of the harmonic to be compensated.
- \( N_1, N_2 \) = number of turns.

A compensation coil also has the advantage of reducing the system's sensitivity to radial displacements for the lower harmonics. This is best if \( r_2 \) is as close as possible to \( r_3 \)
5. - Description of the Measuring Technique

The measuring equipment is automated with a small on-line computer for the control of the system and the data analysis.

The voltage induced in the coil during its rotation is integrated by a specially developed integrator, the design of which is based on the principle of the dual slope integrating voltmeter. To compensate for integrator drift, each rotation step is repeated in the opposite sense and the result of both measurements is averaged by the computer.

After a complete rotation of the search coil, the computer sums the flux values measured at each step to get the total flux as a function of the angular position of the coil.

The system is diagrammatically shown in Figure 6.

![Figure 6 - Schematic Layout of the Measuring System.](image)

5.1 - Coils

The coils were wound on glassfibre-epoxy bars and to reduce the winding operations, a multifilamentary wire was chosen. The filaments were put in series on a small epoxy board fixed on one extremity of the epoxy bar (see Figure 7).

![Figure 7 - Rotating Frame with a Coil](image)

5.2 - Rotating Frame

The finished coils were fixed on a rigid glassfibre-epoxy plate, which was bolted on a rotating frame. This frame is made of two facing flanges, coupled by three tie-rods, equally made of glassfibre-epoxy. It is thus possible to use the same rotating frame with different plates, adapted to the dimensions of the magnet to be measured (see Figure 7). The two bearing flanges are identical, so that the whole frame may be turned round end to end on its support.

By the use of wedges, the coil is aligned on the supporting plate, so that it lies in the median plane of the magnet.

5.3 - The Measuring Bench

The measuring bench is shown in Figure 8. The angular position of the rotating frame, which is driven by a stepping motor, is mechanically located within 0.1 mm by notches cut in the disk visible in Figure 8. A pneumatically-driven shaft locks the angular position of the disk in steps of 3°.

The high-precision bearings ensure that the rotation is circular to within 0.01 mm. The fine adjustment of the starting position of the rotating frame is done by moving the locking shaft tangentially.

5.4 - Computer Software

A Hewlett-Packard 2114 computer with a 16 K-bit memory was used for:

- control of the measuring system
- data analysis.

The routines for the system control (integrator, data input, stepping motor, pneumatic system, interlocks) were written in Assembler language. The data analysis (checks during the measurement, construction of the flux function, harmonic analysis,
field error computations) was carried out by routines in FORTRAN. The master program was also written in FORTRAN, which made the preparation of the measurements fast and added a large degree of flexibility to the system.

3.3 - Alignment

The measuring bench is aligned with respect to external references on the magnet which will be used for its alignment in the machine. The difference between the mechanical axis of the magnet, defined by its external references, and its magnetic axis is found by pulsing the magnet. Between two measurements, the search coil is rotated by 180°, starting from the angular positions where

\[ B_0 = 0 \quad \text{and} \quad \frac{\partial B_0}{\partial \phi} = \text{max}. \]

These points are found by pulsing the magnet and adjusting the angular position until the measured flux is equal to zero.

If the coil's plane is horizontal at the start of the rotation, the tilt of the magnetic median plane with respect to the horizontal plane can be determined from the coefficients of the field expansion as

\[ \theta_t = \arctg \frac{a_n}{b_n}, \]

where \( n \) is the order of the main field harmonic.

\( \theta_t \) may be accurately determined as follows:

The search coil and the magnet are very precisely levelled. Two complete measurements are then made by stepwise rotation of the coil. For the second of these measurements, the rotating frame is turned round end to end.

By averaging the two measurements obtained, the error due to the uncertainty in the position of the coil's median plane is removed (see Fig. 9).

\[ a \]

\[ 1 \]

\[ 2 \]

\[ 1: \text{Magnet median plane} \]

\[ 2: \text{Coil} \]

\[ \theta_t \]

\[ b \]

\[ 1 \]

\[ 2 \]

\[ \text{Figure 9 - Turning the Coil Support by 180° to Compensate Misalignment Errors} \]

1) 1st Measurement

b) 2nd Measurement

6. - System Performance

For the sextupole and octupole lenses for resonance compensation in the ISR, the maximum acceptable value of the error function:

\[ \varepsilon = \left( \left| \frac{\partial N}{\partial X} \right|_{X=0} \right) \left( \left| \frac{\partial N}{\partial X} \right|_{X=0} \right)^{-1} \]

(17)

where:

\( n = 2 \) for the sextupole

\( n = 3 \) for the octupole

\( x_1 = \) limit of the useful region,

was fixed at 3.2. The limit of the useful region was fixed at 76 % and 64 % of the bore radius for the sextupoles and the octupoles, respectively.

For simplicity, although the use of a compensation coil would have given better reproducibility, a measurement with only one search coil, without compensation of the main field harmonic, was found to be sufficient.

The performance of the measuring system will be described using the example of the octupoles. For their measurement, a coil with the following characteristics was wound:

- Winding height 1 mm (2A in Figure 4)
- Winding width 0.5 mm (2B in Figure 4)
- Outer radius 107 mm (R2 in Figure 4)
- Inner radius 62 mm (R1 in Figure 4)

Let us now define the relative field errors as follows:

\[ a_n = \left( \frac{n \times a}{m \times a} \right)^{n-1} \]

\[ b_n = \left( \frac{n \times b}{m \times b} \right)^{m-1} \]

where:

\( R_c = \) outer radius of the search coil

\( m = \) order of the main field harmonic.

6.1 - Systematic Errors

The relative errors in \( a_n \) and \( b_n \) due to an uncertainty of 0.1 mm on the winding height are less than \( 3 \times 10^{-6} \) up to \( n = 20 \).

Figure 10 shows the relative error in \( \% \) in \( a_n \) and \( b_n \) for a radial uncertainty of 0.05 mm.

In formula (17), this radial uncertainty leads to an imprecision of \( 4 \times 10^{-3} \). A reduction of the radial uncertainty to 0.01 mm will reduce these errors by a factor 5 and ensure accurate measurements.

6.2 - Reproducibility of the Measurements

The reproducibility on the field at the outer coil radius was \( 5 \times 10^{-5} \).
Figure 10 - Relative Error in % on the $a_n$ for a Radial Displacement of the Octopole Coil by $0.05 \text{ mm}$.

$$\frac{n \times \Delta(a_n \text{ or } b_n)}{a_n} \leq 2 \times 10^{-5}$$

These values give a reproducibility of $4 \times 10^{-2}$ in function (17).

The use of a compensation coil would improve the measuring reproducibility by a factor between 10 and 100, according to the coil sensitivity and mechanical accuracy.

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References


