THE EFFECT OF SPACE CHARGE IN BEAM TRANSPORT LINES

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Introduction

Space-charge effects can be large in transport lines. For the 50 MeV proton lines at Brookhaven or CERN, an average current of 100 mA corresponds to a peak current of 2.5 A due to the short bunch length (~2 cm). As a result, the space-charge defocusing force is an appreciable fraction of the average external focusing force (1/3 for the CERN line), which leads to significant departures from the beam envelope calculated for zero current, i.e. waists as moved, aperture requirements are increased, and the beam is mismatched for the following synchronrotron. In addition, longitudinal space-charge forces cause an increase in energy spread, typically an increase of ± 200 or ± 300 keV for the CERN line. Finally, non-linear components of the space-charge force twist and filament the distribution leading to an increase in emittance.

Two computer programs, LINEAR and BEAM, have been used to examine these effects. Both trace the evolution of particle distributions in six-dimensional phase space. The first includes only the linear part of the space-charge force and has been incorporated into the program TRANS2: this modified CERN version can be used to trace and optimize beam envelopes with space charge for any combination of drift spaces, quadrupoles, bending magnets and solenoids. The second program is similar to the linac codes of Chasman and Martini and includes non-linear effects. It computes trajectories for 650 super-particles; at each step, the space-charge force on a given particle is the resultant of the Coulomb forces of the remaining N-1 particles. Both programs neglect image forces and the forces due to adjacent bunches. LINEAR requires about 5 seconds of CDC 6600 computer time for 100 meters of transfer line while BEAM requires 40 minutes.

LINEAR

The method follows that of TRANS2. The beam is specified by its 6×6 correlation matrix C which is related to the RMS envelope by \( \chi \propto 1 \chi \propto 1 \), where the 6 components of \( \chi \) are the particle coordinates from bunch center. Since only linear forces are included, \( \eta_0 \) at distance \( \xi \) along the line is related to its initial value \( \eta_0 \) by \( \eta_0 = \eta_0N \) where \( N \) is the 6×6 transfer matrix from \( \eta_0 \) to \( \eta_1 \), \( \eta_1 = \chi \eta_0 \). Our procedure is to transform \( C \) through the system in a series of small steps with the zero-current transfer matrices; after each step a space-charge correction is applied as an impulse. In effect, a series of thin lenses are added to the line to include the defocusing action of space charge. The space-charge force is determined by the 3×3 correlation matrix in \( x,y,z \) space, i.e. the nine components \( \kappa_{xy}, \kappa_{yz}, \kappa_{zx} \) of \( C \) without velocities. If correlations \( \kappa_{xy}, \kappa_{yz}, \kappa_{zx} \) are present (\( \eta_0 \neq 0 \)), this matrix is rotated into normal form.3 Then assuming that the charge density \( \rho \) is uniform (uniform ellipse-oid model), the beam boundary is \( x = \psi_0 x \), and the electric field is \( E = kx \).

\[
\chi = \int_0^{\infty} XYZ \frac{dt}{(X^2+Y^2+Z^2)^{3/2}} \left( \frac{2}{X^2+Y^2+Z^2} \right)^{1/2} \left( \frac{2}{X^2+Y^2+Z^2} \right)^{1/2}
\]

References


*Paper submitted to publication in the Proceedings of the 1971 National Particle Accelerator Conference, Chicago, U.S.A.

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3. Further information is obtainable from T.R. Sherwood, CERN, Geneva, Switzerland.


7. F.J. Sacherer. RMS Envelope Equations with Space Charge. This Conference.

8. P. Tétu, L. Bernard. CERN Internal Note, MPS/LIN - Note 70-7/Nr. 3.

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**Fig. 1** Envelopes (2 standard deviations) for 0, 100 and 200 mA calculated with LINEAR are shown for a transfer line that is matched for zero current. The results from the non-linear program BEAM are also shown (dots) for comparison. The useful aperture is indicated. Quadrupole magnets [], RF cavities []

**Fig. 2** Final design of FES injection line showing 100 mA envelopes obtained from LINEAR and BEAM.

**Fig. 3** Comparison of measured and calculated envelopes for existing 50 MeV Linac-PS injection line.

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