ABOUT TRANSIENT BEAM LOADING AND ENERGY CONSERVATION

by

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Geneva, June 1976
1. Introduction

In large electron storage rings the energy extracted by one bunch at every passage through the RF cavities has become a concern. The following formulae contain, in substance, no new information and are based on nothing but energy conservation. They do, however, express the situation in terms of few, generally known quantities and show how little one has to choose.

2. Required energy extraction

The definition of the quality factor, $Q$, of a cavity resonator is given by

$$ Q = \frac{W \omega}{P_k} $$  \hspace{1cm} (1)

where $W$ is the total stored electromagnetic energy, $\omega/2\pi$ the cavity frequency and $P_k$ the power loss to the cavity walls. If, on the other hand, $P_b$ is the average power to be delivered to the beam in order to make up for synchrotron radiation and all higher-mode losses, the energy, $U$, to be delivered to each bunch at each passage through the cavities is given by

$$ U = \frac{P_b}{k_f f_{\text{rev}}} \hspace{1cm} (2) $$

where $k_b$ is the number of bunches and $f_{\text{rev}}$ the revolution frequency. Introducing the harmonic number

$$ h = \frac{\omega}{2\pi f_{\text{rev}}} $$

one obtains, from (1) and (2)

$$ \eta = \frac{U}{W} = \frac{P_b}{\frac{P_k}{k_b} k_f Q} \hspace{1cm} (3) $$

for the fraction of stored energy that must be extracted at each passage (cf. ref. 1).

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If $W$ is interpreted as the stored energy at the arrival of a bunch then, clearly, $\eta$ cannot be larger than one. But then $P_L$ is the power loss at that moment, not necessarily the average loss.

3. Maximum possible energy extraction

The energy conservation arguments presented above only give an upper bound for the energy gain of a bunch. In the following, we shall calculate a tighter upper bound by considering the beam-cavity interaction in a little more detail.

A particularly simple model applies to the case where the bunch spacing is large enough that all the higher modes excited by the passage of the bunches decay between successive bunch passages. In this case, we only have to consider two types of field, the field in the lowest cavity mode driven by the RF transmitter and left behind by previous bunch passages, and the field induced by the passage of the bunch to which all modes contribute. The energy gained by a bunch on passing through an RF cavity can then be written as follows\(^2\):

\[ U = 2(W_{oi} W_{or})^{\frac{1}{2}} \sin \phi - \sum_{\lambda=0}^{\infty} W_{\lambda r} \tag{4} \]

Here $W_{oi}$ is the stored energy in the lowest cavity mode just before the passage of the bunch, and the $W_{\lambda r}$ are the energies radiated into the $\lambda$-th mode. In particular, $W_{or}$ is the energy which the bunch would radiate into the lowest cavity mode if the cavity was not driven by the RF transmitter. $\phi$ is the phase angle of the RF measured from zero crossing. Again, (4) expresses nothing but energy conservation. If we introduce

\[ W_{or} = n W_{oi} \tag{5} \]

and

\[ \sum_{\lambda=0}^{\infty} W_{\lambda r} = B W_{or} \tag{6} \]

\[ \]

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we can put (4) into the following form:

\[
\frac{U}{W_{oi}} = 2n \sin \phi - Bn
\]  

(7)

Here, (5) says only that \( W \) is some unspecified multiple or fraction of \( W_{oi} \). B describes the effect of the higher-order modes excited by the bunch passage. It has been called the beam loading enhancement factor \(^1\).

We may look for the maximum of (7) with respect to \( n \). It occurs at

\[
n_{\text{opt}} = \left( \frac{\sin \phi}{B} \right)^2
\]  

(8)

and has a value

\[
\left. \frac{U}{W_{oi}} \right|_{\text{opt}} = \frac{\sin^2 \phi}{B}
\]  

(9)

It is quite clear that \( W \) and hence \( n \) is proportional to the square of the bunch charge. \( B \) is only a function of the bunch and cavity geometry and is always bigger than one. It can be obtained by standard energy loss calculations \(^2\). \(^3\). Therefore, there is just one bunch charge which fulfills (8) and hence is able to extract the fraction (9) of the stored energy from the cavity. For all other bunch charges, the energy extraction by the bunch will be smaller.

For \( e^+ e^- \) storage rings up to energies of about 15 GeV, such as PEP or PETRA, \( \sin \phi \ll 1 \), and hence only a small fraction of the stored energy can be extracted. In higher energy machines, such as LEP, \( B \) is more important in limiting the energy extracted, since \( \sin \phi \approx 1 \).

4. **Numerical examples**

The table below shows the parameters of several machines and the energy extraction efficiency \( n \) calculated by using (3).

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<table>
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<th>PEP 4)</th>
<th>PETRA 5)</th>
<th>LEP I 6)</th>
<th>LEP II 7)</th>
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<td>0.20</td>
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</table>

It may be seen from this table that in all machines a very substantial fraction of the stored energy must be extracted by the beam in order to make up for the synchrotron radiation losses. In LEP I, the fraction is actually greater than one. This was the reason for abandoning this machine in favour of LEP II.

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5) Updated version of the PETRA proposal, Hamburg 1976.
6) E. Keil, CERN/ISR-LTD/76-17.
7) E. Keil, CERN/ISR-LTD/76-17 Rev.