SYNCHROTRON RADIATION FROM A
LARGE ELECTRON-POSITRON STORAGE RING

by

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Abstract

General expressions are compiled for the synchrotron radiation produced in bending magnets and quadrupoles. Numerical examples are given for an $e^+-e^-$ storage ring at 100 GeV.

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1. **INTRODUCTION**

The well known formulae for synchrotron radiation\textsuperscript{1,2} are first compiled for the case of bending magnets. They are then generalized to the case of synchrotron radiation produced in quadrupoles. Finally, numerical examples are shown for the case of an e\textsuperscript{+}-e\textsuperscript{-} storage ring at an energy of 100 GeV.

2. **SYNCHROTRON RADIATION FROM BENDING MAGNETS**

In an isomagnetic machine where the bending radius $\rho$ is constant or zero, the energy loss per turn is given by

$$U_0 = \frac{C_\gamma E^4}{\rho}$$

(1)

Here $E$ is the electron energy, and

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{E_0^3}$$

(2)

where $r_e$ is the classical electron radius and $E_0$ is the electron rest energy. Combining (1) and (2) yields

$$U_0 = \frac{4\pi}{3} \frac{r_e E_0 \gamma^4}{\rho}$$

(3)

The total power loss of a circulating electron current $I$ in a machine with bending radius $\rho$ becomes

$$P_\gamma = \frac{4\pi}{3} \frac{r_e E_0 \gamma^4 I}{\epsilon \rho}$$

(4)

The critical energy $\epsilon_\text{c}$ is defined as follows

$$\epsilon_\text{c} = \frac{3}{2} \hbar c \gamma^2/\rho$$

(5)

The power spectrum $\partial P_\gamma/\partial \epsilon$ is given by

$$\frac{\partial P_\gamma}{\partial \epsilon} = \frac{P_\gamma}{\epsilon_\text{c}} S_\text{b}(\epsilon/\epsilon_\text{c})$$

(6)

where $S_\text{b}(x)$ is the universal function shown in Fig. 1.
\begin{equation}
S_B(x) = \frac{9\sqrt{3}}{8\pi} x \int_{-\infty}^{\infty} K_{\gamma/\epsilon} (s) \, ds
\tag{7}
\end{equation}

\[ S_B(x) \] is identical to the spectral function \( S(\xi) \) defined on p. 115 of ref. 1.

The quantum spectrum \( \frac{\partial N_Y}{\partial \epsilon} \) is

\[ \frac{\partial N_Y}{\partial \epsilon} = \frac{P_Y}{\epsilon_c^2} N_B(\xi/\epsilon_c) \tag{8} \]

where \( N_B(x) \) is another universal function defined by

\[ N_B(x) = x^{-1} S_B(x) \tag{9} \]

We also define the quantities (4), (6) and (8) per unit length of bending magnet

\[ \frac{\partial P_Y}{\partial s} = \frac{2}{3} \frac{\epsilon e \eta_0}{e^2} \gamma^4 \, I \tag{10} \]

\[ \frac{\partial^2 P_Y}{\partial \epsilon \partial s} = \frac{P_Y}{2 \pi \rho \epsilon_c} \, S_B(\xi/\epsilon_c) = \frac{\partial P_Y}{\partial s} \frac{S_B(\xi/\epsilon_c)}{\epsilon_c} \tag{11} \]

\[ \frac{\partial^2 N_Y}{\partial \epsilon \partial s} = \frac{P_Y}{2 \pi \rho \epsilon_c^2} \, N_B(\xi/\epsilon_c) \tag{12} \]

3. \textbf{SYNCHROTRON RADIATION FROM QUADRUPOLES}

A quadrupole differs from a bending magnet by the fact that the bending radius is a function of the electron excursion from the axis of the quadrupole. In order to obtain the synchrotron radiation formulae, the contributions of all electrons have to be superimposed.

Let the distribution of the excursion \( x \) be normalized such that

\[ I = \int_{-\infty}^{+\infty} i(x) \, dx \tag{13} \]

and let the distribution \( i(x) \) be a Gaussian of rms width \( \sigma \). Then \( i(x) \) is given by the following expression
\[ i(x) = \frac{I}{\sigma \sqrt{2\pi}} e^{-x^2/2\sigma^2} \quad (14) \]

It is known that in a quadrupole the bending radius \( \rho(x) \) varies as follows

\[ \rho(x) = \rho(\sigma) \frac{\sigma}{x} = \rho_0 \frac{\sigma}{x} \quad (15) \]

where \( \rho_0 \) is the bending radius at one standard deviation \( \sigma \). In the following, the variation of the excursion \( x \) inside a quadrupole is neglected, i.e. \( x \) is considered constant through the quadrupole.

When studying the synchrotron radiation from quadrupoles it is best to consider only quantities per unit length of quadrupole. Generalizing (10), using (14) and (15) yields

\[ \frac{\partial P_y}{\partial s} = \frac{2}{3} \frac{r_e E_0}{e} \gamma^4 \frac{I}{\sigma \sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2/2\sigma^2} \frac{x^2}{\rho_0^2} \frac{dx}{\sigma^2} \quad (16) \]

The integration is straightforward

\[ \frac{\partial P_y}{\partial s} = \frac{2}{3} \frac{r_e E_0}{e} \gamma^4 \frac{I}{\rho_0^2} \quad (17) \]

It may be seen that the correct result for a quadrupole is obtained by evaluating the bending magnet formula (10) at a distance \( \sigma \) from the quadrupole axis. (This result was quoted to me by A. Hofmann.)

The power spectrum per unit length is obtained by generalizing (11)

\[ \frac{\partial^2 P_y}{\partial \varepsilon \partial s} = \frac{I}{\sigma \sqrt{2\pi}} \int_{-\infty}^{+\infty} P_y(x) e^{-x^2/2\sigma^2} \frac{dx}{2\pi \rho(x) \varepsilon_c(x)} S_Q(\varepsilon/\varepsilon_c(x)) \quad (18) \]

By making the appropriate substitutions for \( P_y(x) \), \( \rho(x) \), \( \varepsilon_c(x) \), and some arithmetic, (18) may be written in the following form

\[ \frac{\partial^2 P_y}{\partial \varepsilon \partial s} = 2 \frac{r_e E_0}{e \rho_0^2 \varepsilon_c(0)} \frac{\gamma^4}{\varepsilon_c(0)} \frac{I}{\varepsilon_c(0)} S_Q(\varepsilon/\varepsilon_c(0)) \quad (19) \]

Here \( \varepsilon_{c0} \) is the critical energy at a distance \( \sigma \) from the quadrupole axis, and \( S_Q(x) \) is a new universal function, also shown in Fig. 1.
\[ S_q(x) = \frac{9\sqrt{3}}{8\pi} \int_0^\infty K_{\frac{3}{2}}(s) \left[ 1 - \text{erf}(x/s\sqrt{2}) \right] ds \]  \hspace{1cm} (20)

Apart from the spectral function \( S_q \), (20) is identical to (11) if the quantities involved are all evaluated at a distance \( \sigma \) from the quadrupole axis.

The quantum spectrum from a quadrupole is given by

\[ \frac{\partial^2 N_Y}{\partial \varepsilon \partial s} = \frac{1}{\varepsilon} \frac{\partial^2 P_Y}{\partial \varepsilon \partial s} = \frac{P_Y}{\partial s} \left| \frac{N_q(\varepsilon/\varepsilon_0)}{\varepsilon_0^2} \right| \]  \hspace{1cm} (21)

where the function \( N_q(x) \) is defined by

\[ N_q(x) = x^{-1} S_q(x) \]  \hspace{1cm} (22)

A comparison of the spectral functions \( S_B(x) \) and \( S_q(x) \) shown in Fig. 1 shows that quadrupoles produce more synchrotron radiation at energies above the critical energy than bending magnets.

4. SYNCHROTRON RADIATION SPECTRA IN LEP

As a practical application of the preceding analysis we calculate below the synchrotron radiation spectra for LEP. We consider two machines

- the 4 bunch machine summarized in ref. 3;
- the 32 bunch machine presently considered as an alternative.

For each machine, photon spectra are shown produced by:

- the normal cell bending magnets;
- special bending magnets with 10\% of the nominal field \(^4\);
- the horizontally focusing quadrupole \( Q2 \). (Since the beam size is largest in this quadrupole, it is likely to be the dominant source of quadrupole synchrotron radiation.)

The parameters entering the calculation are shown in the two tables below.

<table>
<thead>
<tr>
<th>Table I: Bending magnet parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 bunches</td>
</tr>
<tr>
<td>( P_Y ) (2 beams)</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
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</tbody>
</table>
Table II: Quadrupole parameters

<table>
<thead>
<tr>
<th></th>
<th>4 bunches</th>
<th>32 bunches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength of $Q^2$</td>
<td>$K$</td>
<td>$0.02525$</td>
</tr>
<tr>
<td>rms beam radius</td>
<td>$\sigma$</td>
<td>$6.2229$</td>
</tr>
<tr>
<td>Bending radius</td>
<td>$\rho$</td>
<td>$6364$</td>
</tr>
<tr>
<td>Quadrupole length $k_Q$</td>
<td>$3.5$</td>
<td>$3$</td>
</tr>
<tr>
<td>Current $I$</td>
<td></td>
<td>$6.96$</td>
</tr>
<tr>
<td>$(\beta y/\beta s)_0$</td>
<td></td>
<td>$1.512 \times 10^{21}$</td>
</tr>
<tr>
<td>$E_{0}$</td>
<td></td>
<td>$349$</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

Formulæ have been derived for various quantities related to synchrotron radiation, both for bending magnets and quadrupoles. The spectral distribution of synchrotron radiation from quadrupoles has been shown to contain more power above the critical energy than the spectrum from bending magnets. The photon flux at about one MeV from bending magnets can be reduced by installing reduced field bending magnets near the crossing points. When this has been done, most of the synchrotron radiation in that energy range comes from the interaction region quadrupoles.

REFERENCES

1) M. Sands; SLAC-121 (1970).
2) R.A. Mack; CEAL-1027 (1966).
   This paper uses a different normalization of the spectral function $S_B(x)$.
3) E. Keil; CERN/ISR-LTD/76-17 (1976).
Synchrotron radiation from bending magnets

4 bunch machine

- 100% field
- 10% field
Synchrotron radiation from O2 quadrupole:

- 4 bunch machine
- 32 bunch machine

\( \frac{\partial N_\gamma}{\partial \varepsilon} \)

\( \times 10^4 \)
Synchrotron radiation from bending magnets
32 bunch machine

- 100% field
- 10% field