FREQUENCY AND MOMENTUM TOLERANCES AT INJECTION.

Abstract: The necessary tolerances in frequency and momentum at the time the RF is switched on is discussed. The main result is that the relative frequency or momentum error that may be tolerated is inversely proportional to the square root of the multiple of the rotational frequency on which the RF is working, which may prove to be one of the upper limitations on it.

It should also be noted that the results obtained here are valid for sudden frequency changes later in the accelerating cycle too.

We assume the machine to be perfect, except that the frequency and momentum may be wrong at injection. At the moment the RF is switched on there are two orbits of interest, neglecting the rapid betatron oscillations. One is the orbit in which the particles are actually moving, given by the momentum and the field. The other one is the orbit along which the centre of the 'phase diagram' is moving. This is determined by the frequency and the magnetic field.

We now assume that the two orbits coincide when the frequency is $f_0$ and the momentum is $p_0$, and that this also coincides with the line along which the field is $B_0$ in all magnet sectors.

If the momentum deviates from $p_0$ by $\Delta p$, the particle orbit is shifted by an amount $x_{\text{mean}}$, given by (16) in CMN-P3/K11. Also taking into account (19) in the same report, we may write for $x_{\text{mean}}$

$$x_{\text{mean}} = r_0 \left( \frac{\beta c^2}{\gamma} \right)^2 \Delta p / p_0$$

(1)

where $E_{tr}$ is the transition energy (rest energy included). If the frequency is shifted by an amount $\Delta f$, the 'phase' orbit is shifted by $x_f$, given by
\[
x_f/r_n = (E_0/\gamma_{tr})^2/(1-(E_0/\gamma_{tr})^2) \Delta f/f_0 \\
\]

where \( E_0 \) is the total energy of the particle with momentum \( p_0 \).
(This formula is equivalent to (12) in Courant, "Livingston and Snyder's paper in Phys. Rev., 22, 1190/6.)

In order to trap particles we must have

\[
x_f - x_{mean} < \Delta r_{max} \tag{3}
\]

where \( \Delta r_{max} \) is given by (22) in Courant, "JMB.

Substituting for \( x_f \), \( x_{mean} \) and \( \Delta r_{max} \), we get after some calculation

\[
\frac{(E_0/mc^2)^2}{1-(E_0/mc^2)^2} \frac{\Delta f}{f_0} - \sqrt{1-(E_0/\gamma_{tr})^2} \frac{A_n}{P_0} < \sqrt{\frac{2}{3} \frac{\gamma_{tr}}{\gamma_{tr} - 1} \sqrt{\frac{c}{nc^2}}} \frac{r_0 r_0 B_0 (-\cot \psi_0 - (\psi_0 - \sqrt{2}))}{N} \tag{4}
\]

This result is valid for any particle energy and transition energy. However, the expression may be simplified considerably if \( E_0/mc^2 \approx 1 \), which is usually true near injection, and \( (E_0/\gamma_{tr})^2 \ll 1 \). The latter condition will, of course, be satisfied only at injection and shortly afterwards, and then only for large \( n \)-values.

If these two conditions are fulfilled, we get the simplified expression

\[
\frac{\Delta f}{f_0} - \frac{A_n}{P_0} < 2\sqrt{\frac{c}{nc^2}} \frac{r_0 r_0 B_0 (-\cot \psi_0 - (\psi_0 - \sqrt{2}))}{N} \tag{5}
\]

In order to trap nearly the maximum possible number of particles, the right side of (5) should be divided by a safety factor 3, which should be large.

Example:

Let \( \psi_0 = 15^\circ \), injection energy 50 MeV, \( r_0 = 170 \text{m}, r_n = 150 \text{m} \text{ and } B_0 = 5000 \text{ Gauss/sec}. \) As long as \( r_{tr} \) is above
about 5 GeV, the result is independent of \( B \), and this example, therefore, covers all alternatives I to IV in CERN-8/317 if 50 GeV is used for injection.

The example gives

\[ 1.11 \Delta f/f_0 - \Delta p/p_0 < 2^{-1/2} 1.54 \times 10^{-2} \]

If we assume the necessary safety factor to be about 4 and to work with \( \beta = 100 \), we get

\[ 1.11 \Delta f/f_0 - \Delta p/p_0 < 0.4 \times 10^{-3} \]

These are close tolerances. However, it should be noticed that it seems to be a one-sided tolerance. If, for instance, the left side of (\( \beta \)) is negative, but larger in magnitude than the right side, the particles will certainly not be trapped at once as the \( B^2 \) is switched on, but their orbit will move inwards, as there is no energy gain and the field increases. Therefore, they will in that case 'fall' into the trapping region and be trapped.

It may be very difficult to get a sufficiently strong beam with momentum tolerances satisfying (\( \beta \)), and we may have to inject at a slightly too large energy and rely upon the last process mentioned above.

Bergen, 27th April, 1952.

Kjell Johnsen.
Supplement to CERN-63/61-6.

At the end of my report CERN-63/61-6 I have written: 'However, it should be noted .......... and rely upon the last process mentioned above.'

Goward has now pointed out to me that this is not correct. Particles with too large energy and radius to be trapped 'pass through the synchronous radius via one of the gaps between the stable regions' and get lost to the inner wall.

Consequently, it is as bad to inject with too large energy as with too small energy and the left side of eq. (4) should therefore read

\[
\left| \frac{\sqrt{\frac{(E_0/c^2)^2 \Delta f}{1-(E_0/E_{tr})^2}}}{f_0} - \sqrt{1-(E_0/E_{tr})^2} \frac{\Delta p}{p_0} \right| < \text{ ..........} \quad (4)
\]

and the left side of (5)

\[
\left| \frac{\Delta f}{f_0} - \frac{\Delta n}{n_0} \right| < \text{ ..........} \quad (5)
\]

This makes the difficulties with the tolerances at injection more serious and may put a restriction on our choice of \( n \).

Bergen, 4th May, 1953.

Kjell Johnsen.