Neutralino Annihilation into Two Photons

Zvi Bern\textsuperscript{1}

Department of Physics
University of California at Los Angeles
Los Angeles, CA 90095-1547

Paolo Gondolo\textsuperscript{2}

Max Planck Institut für Physik
Föhringer Ring 6, 80805 München, Germany

and

Maxim Perelstein\textsuperscript{3}

Department of Physics
University of California at Los Angeles
Los Angeles, CA 90095-1547

Abstract

We compute the annihilation cross-section of two neutralinos at rest into two photons, which is of importance for the indirect detection of neutralino dark matter in the galactic halo through a quasi-monochromatic gamma-ray line. We include all diagrams to one-loop level in the minimal supersymmetric extension of the standard model. We use the helicity formalism, the background-field gauge, and an efficient loop-integral reduction method. We confirm the result recently obtained by Bergström and Ullio in a different gauge, which disagrees with other published calculations.

\textsuperscript{1}bern@physics.ucla.edu
\textsuperscript{2}gondolo@mppmu.mpg.de
\textsuperscript{3}maxim@physics.ucla.edu
The composition of dark matter is one of the major issues in astrophysics. A popular candidate for non-baryonic dark matter is a neutral stable Majorana fermion, the lightest neutralino, appearing in a large class of supersymmetric extensions of the Standard Model. In a wide range of supersymmetric parameter space, relic neutralinos from the Big Bang are in principle abundant enough to account for the dark matter in our galactic halo. (See ref. [1] for a thorough analysis and ref. [2] for a comprehensive review.)

Several methods have been proposed to detect galactic neutralinos: elastic scattering in a low-background terrestrial detector, energetic neutrino fluxes from neutralino annihilations in the core of the Sun or of the Earth, gamma-rays and cosmic ray antiprotons or positrons from neutralino annihilation in the halo. (For a review see ref. [2].)

In particular, the annihilation into two photons gives a distinct signature against backgrounds from known astrophysical sources – a narrow line in the gamma-ray spectrum at the energy equal to the neutralino mass [3]. Observation of such a line would provide evidence for the existence of supersymmetric dark matter, while non-observation can be used to put constraints on models of the galactic dark matter. A number of new experiments with reasonable energy resolutions are being planned [4] which would be sensitive to photon production in the galactic halo.

There exist several previous calculations of the neutralino annihilation into two photons, all but one incomplete. The first calculation was carried out by Bergström and Snellman [3] for neutralinos which are pure photinos. Rudaz [5] corrected their results and considered also pure higgsinos. Giudice and Griest [6] removed the restriction on the neutralino state mixture, and confirmed Rudaz’s work. In all these early calculations the sfermion and the \(W\)-boson masses were assumed to be much larger than any other mass. This assumption was relaxed by Bergström [7], who computed the fermion-sfermion loop contributions for arbitrary squark masses, but only for a pure photino. Bergström and Kaplan [8] evaluated the contribution from virtual \(W\)’s in a leading-logarithm approximation. Jungman and Kamionkowski [9] improved on the leading-logarithm approximation, but did not include all contributing diagrams (our \(A_{W\psi}^b\) below was missing). Only very recently, Bergström and Ullio [10] (hereafter BU) presented a full one-loop calculation. Their results, however, differ from some previous partial calculations when corresponding expressions are compared.

We provide an independent complete one-loop calculation of the neutralino annihilation into two photons. We confirm the BU result using a different calculational procedure. To perform the calculation we take advantage of some recent improvements in the calculational techniques for one-loop amplitudes [11]. In particular, we use a helicity basis [12, 13] for the photons, background field Feynman-'t Hooft gauge [14, 15] and an efficient integral reduction method [16].

As in previous calculations, we focus only on the first term in the relativistic expansion of \(\sigma v\), where \(\sigma\) is the annihilation cross-section and \(v\) the relative neutralino-neutralino velocity. This kinematic limit is appropriate for neutralino annihilation in the galactic halo, where the neutralino velocities are of the order of the galactic rotation speed, \(v/c \simeq 10^{-3}\). This kinematic configuration considerably simplifies the form of the amplitudes. Our evaluation methods are however general and
do not depend on the special kinematics.

2 Evaluation of the Amplitudes

In this section we outline the computation of the amplitudes for the annihilation of two neutral massive Majorana fermions at rest into two photons via vector boson, scalar and fermion loops. The Feynman diagrams for the process under consideration are given in fig. 1. Since photons do not change the identity of the particle they couple to, the masses in the fermion and scalar lines within the loops are uniform.

![Feynman diagrams for neutralino annihilation into two photons.](image)

Figure 1: Feynman diagrams for neutralino annihilation into two photons.

2.1 Feynman Rules and Diagrams

We use background field Feynman–’t Hooft gauge \[14, 15\] because of its technical advantages. In this gauge the gauge fixing part of the Lagrangian is

\[
\mathcal{L}_{gf} = -\frac{1}{2} \left( \partial_{\mu} W^i{}^{\mu} + g e^{ijk} \tilde{W}_{j\mu} W_k{}^{\mu} + \frac{ig}{2} \sum (\phi_k^i T^j \phi_{0k} - \phi_{0k}^i T^j \phi_k^i) \right)^2 - \frac{1}{2} \left( \partial_{\mu} B^{\mu} + \frac{ig'}{2} \sum (\phi_k^i \phi_{0k} - \phi_{0k}^i \phi_k^i) \right)^2 ,
\]

where \( \tilde{W} \) and \( \tilde{B} \) are respectively the \( SU(2) \) and \( U(1) \) external background fields, \( W \) and \( B \) are the corresponding quantum fields appearing in the loops, and \( \phi_{0k} \) is the VEV of the \( k \)-th Higgs field of the model: \( \phi_k = \phi_{0k} + \phi_k' \). The Fadeev-Popov ghosts are rather simple and at one-loop have the same coupling as scalars; only the overall sign differs because of Fermi statistics. The Feynman rules
Figure 2: Feynman rules in the background field gauge; overall factors of $i$ and coupling constants have been removed.

for performing this calculation may be found in ref. [15] and are summarized in fig. 2. In this figure $\gamma$ and $Z$ are background fields.

An important technical advantage of this gauge is that only one of the three terms in the $WW\gamma$ vertex contains loop momentum. This decreases the number of the integrals with high powers of loop momentum in the numerator that have to be evaluated. Typically, such integrals are the most difficult to compute. Furthermore, the gauge fixing terms have been chosen so that they cancel the $WH\gamma$ coupling from the Lagrangian, which reduces the number of diagrams to be evaluated. (This is similar to non-linear $R_\xi$ gauges [17, 8, 9].) This cancellation works for any number of Higgs fields assuming that they are doublets of the Standard Model $SU(2)$. For the massive four-point amplitudes computed in this paper, the background field method captures the main technical advantages of using the gauge invariant cutting techniques reviewed in ref. [11].

2.2 Helicity and Kinematics

We use a helicity representation for the final-state photons, since this generally leads to compact expressions for amplitudes. In the spinor helicity formalism [12, 13] the photon polarization vectors are expressed in terms of massless Weyl spinors $|k^\pm\rangle = \frac{1}{2}(1 \pm \gamma_5)|k\rangle$,

$$
\epsilon^+_{\mu}(k; q) = \frac{\langle q^-| \gamma_\mu |k^-\rangle}{\sqrt{2}\langle qk\rangle}, \quad \epsilon^-_{\mu}(k; q) = \frac{\langle q^+| \gamma_\mu |k^+\rangle}{\sqrt{2}\langle kq\rangle},
$$

(2)

where $k$ is the photon momentum, $q$ is an arbitrary null ‘reference momentum’ which drops out of final gauge-invariant amplitudes and

$$
\langle i,j \rangle \equiv \langle k_i^-|k_j^+\rangle, \quad [i,j] \equiv \langle k_i^+|k_j^-\rangle,
$$

(3)

which satisfy the normalization condition $\langle i,j \rangle [j,i] = 2k_i \cdot k_j$. The plus and minus labels on the polarization vectors refer to the outgoing photon helicities. We denote the massive Majorana spinors of the initial-state neutralinos by $|1\rangle$ and $|2\rangle$. (For the massive spinors we choose the normalizations of ref.[18]; our massless spinors follow the conventions of ref. [13].)
Since the two annihilating particles are identical Majorana fermions, the initial-state wavefunction must be antisymmetric. Therefore the initial spin state must be the singlet with angular momentum zero; since it is conserved, the final-state photons must also have vanishing angular momentum and be of the same helicity. This means that there are only two helicity cases that are required, namely where both photons are of positive or both are of negative helicity. These two helicity configurations are, however, simply related as can easily be seen by making the reference momentum choice
\[ q_3 = k_1, \quad q_4 = k_3, \quad (4) \]
where \( q_3 \) and \( q_4 \) are the reference momenta of the two photons. With this choice of reference momenta
\[ \epsilon_4^- = \epsilon_{34}^+ \epsilon_{3}^+, \quad \epsilon_3^- = \epsilon_{34}^+ \epsilon_{4}^+ \quad (5) \]
Since the amplitudes are symmetric under the interchange of photons 3 ↔ 4, we have
\[ A(1\tilde{\chi},2\tilde{\chi},3\chi,4\chi) = \frac{\langle 34 \rangle^2}{\langle 34 \rangle^2} A(1\tilde{\chi},2\tilde{\chi},3\chi,4\chi), \quad (6) \]
where legs 1 and 2 are the initial state neutralinos and the ± labels on legs 3 and 4 refer to the photon helicities. Thus, the amplitudes for either helicity choice are identical up to an overall phase factor and we need only compute the annihilation into two positive helicity photons to obtain the entire result.

A number of important simplifications occur because of the special kinematics. For annihilation at rest, we take the kinematic configuration to be of the form
\[ k_1 = (m_{\tilde{\chi}},0,0), \quad k_2 = (m_{\tilde{\chi}},0,0), \]
\[ k_3 = (-m_{\tilde{\chi}},0,0), \quad k_4 = (m_{\tilde{\chi}},0,0), \quad (7) \]
where \( m_{\tilde{\chi}} \) is the neutralino mass, \( k_1 \) and \( k_2 \) are the neutralino four momenta and \( k_3 \) and \( k_4 \) the photon momenta. The Mandelstam variables are then given by
\[ s \equiv (k_1 + k_2)^2 = 4m_{\tilde{\chi}}^2, \quad t \equiv (k_1 + k_4)^2 = -m_{\tilde{\chi}}^2, \quad u \equiv (k_1 + k_3)^2 = -m_{\tilde{\chi}}^2 \quad (8) \]

The four-point kinematics allows us to reduce any spinor structure that appears in the diagrams (before we symmetrize in the photons) to a linear combination of \( \langle 1|2 \rangle, \langle 1|\gamma_5|2 \rangle, \langle 1|\not{k}_4|2 \rangle \), and \( \langle 1|\gamma_5\not{k}_4|2 \rangle \). The permutation 3 ↔ 4 then corresponds to \( k_3 \leftrightarrow k_4, \ t \leftrightarrow u \). (Note that the reference momentum choice (4) respects the exchange symmetry.) But in our kinematics \( t = u \), so we only need to symmetrize the spinors. Using \( \langle 1|\not{k}_3 + \not{k}_4|2 \rangle = 0, \langle 1|\gamma_5(\not{k}_3 + \not{k}_4)|2 \rangle = 2m_{\tilde{\chi}} \langle 1|\gamma_5|2 \rangle \), we see that the answer after symmetrizing in the photons contains only \( \langle 1|2 \rangle \) and \( \langle 1|\gamma_5|2 \rangle \). At the kinematic point (7) we have \( \langle 1|2 \rangle = 0 \), so all amplitudes are proportional to \( \langle 1|\gamma_5|2 \rangle \).

Another simplification due to this kinematics is that we need only consider pseudo-scalar Higgs particles and Z bosons in the s-channel, the other neutralino currents vanish. This means that in fact we do not need to calculate diagrams \( A_{\phi W}^4 \), and in the diagrams \( A_{\phi \psi}^4 \) we need to consider only pseudo-scalar intermediate Higgs. Moreover, diagrams \( A_{\phi \phi}^4 \) do not contribute because CP conservation (which we impose) forbids the \( H^+H^-H_3^0 \) and \( H^+H^-G^0 \) couplings.
2.3 Integral Reductions

In evaluating the diagrams in fig. 1, one encounters loop integrals of the form,

$$I_n[P(\ell^\mu)] \equiv -i(4\pi)^{2-\epsilon} \int \frac{d^{4-2\epsilon} \ell}{(2\pi)^{4-2\epsilon}} \frac{P(\ell^\mu)}{L_1^2 L_2^2 ... L_n^2},$$

where

$$L_i^2 = \ell_i^2 - m_i^2.$$  \hspace{1cm} (10)

Here the $\ell_i$ and $m_i$ are the momentum and the mass flowing through the $i$th loop propagator, and $P(\ell^\mu)$ is a polynomial in the loop momenta. For the box (four-point) integrals,

$$\ell_1 = \ell, \quad \ell_2 = \ell - k_1, \quad \ell_3 = \ell - k_1 - k_2, \quad \ell_4 = \ell + k_4.$$  \hspace{1cm} (11)

(Our sign conventions for the integrals $I_n$ are adjusted to agree with Oldenborgh’s FF program, and differ from those of refs. [19, 20, 11] for the three-point integrals.) Although, the amplitudes are ultraviolet finite we include dimensional regularization because a few of the integrals encountered in the calculation are divergent. (We comment that we use the ‘four-dimensional helicity scheme’ [21] to maintain compatibility with the helicity formalism; however, since the amplitudes are ultraviolet finite the specific scheme choice is unimportant.)

We now outline the procedure used to reduce the tensor integrals (i.e. those with powers of loop momentum in the numerator of eq. (9)) to linear combinations of scalar integral functions used to express the amplitudes. This procedure has been used previously in refs. [16, 22, 11]. The basic technique is to extract as many inverse propagators as possible from the spinor inner products appearing in the numerators of the integrands. In this way we can simplify the tensor box integrals with two or three powers of loop momenta, which are by far the most complicated of the integrals occurring in the calculation.

As an example, consider the box diagram in fig. 1(a) with a scalar $\phi$ and a charged fermion $\psi$ in the loop, given by

$$\int \frac{d^4 \ell \cdot d^{-2\epsilon} \mu}{(2\pi)^4 (2\pi)^{-2\epsilon}} \frac{\langle 1 | f_2 + m_2 + \mu | 2 \rangle \epsilon^+_3 \cdot (\ell_4 + \ell_3) \epsilon^+_4 \cdot (\ell_1 + \ell_4)}{(\ell_1 - \mu^2 - m_3^2)(\ell_2 - \mu^2 - m_4^2)(\ell_3 - \mu^2 - m_3^2)(\ell_4 - \mu^2 - m_4^2)}.$$  \hspace{1cm} (12)

As usual when using four-dimensional helicities [23, 22, 11], we have broken up the loop integral into a four-dimensional part, $\ell$, and a $(-2\epsilon)$-dimensional part, $\mu$. See, for example, appendices A.2 and C of ref. [22] for further details.

Using the polarization vectors (2), with the choice of reference momentum (4) the dot products of the polarization vectors and loop momenta may be re-expressed as

$$\epsilon^+_3 \cdot (\ell_4 + \ell_3) \epsilon^+_4 \cdot (\ell_1 + \ell_4) = -2 \frac{\langle 3^+ | f_4 \rangle \langle 4^+ | f_4 \rangle \langle 4^+ | f_4 \rangle | 3^+ \rangle}{\langle 3^+ \rangle^2}.$$  \hspace{1cm} (13)

We may then extract inverse propagators from the spinor products using,

$$\langle 3^+ | f_4 \rangle \langle 4^+ | f_4 \rangle | 3^+ \rangle \hspace{1cm} (14)$$
where the combinations appearing in the parentheses are inverses of propagators appearing in the loop integral (12). By canceling propagators we can reduce the tensor box integral (12) to a linear combination of box integrals with at most one power of loop momentum and triangle integrals with at most two powers of loop momentum. In some cases we can reduce the tensor triangle integrals by again extracting inverse propagators; however, this strategy fails for some terms because one cannot form an inverse propagator appropriate for the triangle integral. For example, in eq. (14), the term
\[(\ell_2^2 - m_\phi^2 - \mu^2)2 \ell_4 \cdot k_3 = (\ell_4^2 - m_\phi^2 - \mu^2) [(\ell_3^2 - m_\phi^2 - \mu^2) - (\ell_4^2 - m_\phi^2 - \mu^2)], \tag{15}\]
contains two factors of \((\ell_2^2 - m_\phi^2 - \mu^2)\) which cannot be completely canceled against a single propagator. For such terms one can use, for example, the integration formulas of refs. [24, 19, 20] to directly evaluate the tensor integrals in terms of scalar integrals; the expressions are relatively simple since these are triangle integrals.

The integrals with a single power of \(\mu\) in the numerator vanish, while the ones containing a \(\mu^2\) are proportional to \(\epsilon\) times a higher-dimensional integral [22, 11],
\[
\int \frac{d^4 \ell}{(2\pi)^4} \frac{d^{-2\epsilon} \mu}{(2\pi)^{-2\epsilon}} \mu^2 f(\mu^2, \ell^\prime) = -4\pi \epsilon \int \frac{d^6 \ell}{(2\pi)^6} \frac{d^{-2\epsilon} \mu}{(2\pi)^{-2\epsilon}} f(\mu^2, \ell^\prime). \tag{16}\]
The higher dimensional integral may be re-expressed in terms of \(D = 4 - 2\epsilon\) integrals using the integral relations appearing in refs. [19, 20]. Observe that because of the explicit \(\epsilon\), we need only evaluate the parts of the higher dimensional integrals that are singular in \(\epsilon\). In some cases, such as for box integrals containing a single power of \(\mu^2\), the integrals are finite and may therefore be dropped immediately since they are suppressed by an overall power of \(\epsilon\). It turns out that for the amplitudes presented in this paper, all such higher dimension integrals cancel.

The above procedure allows us to reduce all four-point diagrams in fig. 1 to a linear combination of scalar box, triangle and bubble integrals.

2.4 The Cross-Section

We calculate the cross-section in the framework of the minimal supersymmetric extension of the Standard Model. (For a comprehensive discussion, see ref. [25].) We use the following notation for the MSSM interaction terms that contribute to the diagrams in fig. 1:
\[
\mathcal{L}_{\text{int}} = W_\mu \bar{\chi} \gamma^\mu (g^V_{W\chi'\psi} + g^A_{W\chi'\psi})\psi + \phi^* \bar{\chi} (g^S_{\phi\chi'\psi} + g^P_{\phi\chi'\psi})\psi + h.c. \\
+ g^A_{\psi\psi} Z_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi + g^P_{\phi\psi\psi} \phi^* \bar{\psi} \gamma_5 \psi + h.c. \\
+ \frac{1}{2} g^A_{Z\chi'\chi'} Z_\mu \bar{\chi} \gamma^\mu \gamma_5 \chi + \frac{1}{2} g^P_{\phi\chi'\chi'} \phi^* \bar{\chi} \gamma^\mu \gamma_5 \chi + h.c. \tag{17}\]
Here \(\chi, \chi'\) are Majorana fermions, \(\psi, \psi'\) are Dirac fermions, and \(\phi\) can be a neutral \((\phi^* = \phi)\) or a charged scalar. Explicit expressions for the coupling constants \(g_{ijk}\) can be obtained in refs. [25, 26].

We write the amplitude as
\[
\mathcal{A}(1, 2, 3^+, 4^+) = \frac{4i \alpha m_\chi^2}{\pi (3\tilde{4})^2} \frac{\langle 1 | \gamma_5 | 2 \rangle}{\tilde{4}}, \tag{18}\]
where \( \alpha \) is the fine structure constant. With this notation, \( \tilde{A} \) matches the corresponding expression in refs. [9, 10].

The cross-section may be directly obtained from the amplitudes listed in eq. (22) below using the formula

\[
\sigma_v = \frac{\alpha^2 m_v^2}{16\pi^3} |\tilde{A}|^2 .
\]

We have averaged over the four possible initial states, summed over the two possible photon helicity configurations and inserted the symmetry factor of \( \frac{1}{2} \) for identical final state photons. In evaluating the squared matrix elements we have used

\[
|\langle 34 \rangle | = 2m_\tilde{\chi}, \quad |\langle 1\gamma_5 |2 \rangle | = 2\sqrt{2}m_\tilde{\chi} ,
\]

valid for the kinematics (7).

We find the following non-vanishing contributions to the amplitude:

\[
\tilde{A} = \sum_{\psi = \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} \tilde{A}^{a+b+c}_{W\psi} + \sum_{\phi = H_1^\pm, G_1^\pm} \tilde{A}^{a+b+c}_{\phi\psi} + \sum_{\phi = H_0^0, G_0^0} \tilde{A}^{a+b+c}_{\phi\psi} + \sum_{\psi = f, \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} \tilde{A}^d_{\phi\psi} + \sum_{\psi = f, \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} \tilde{A}^e_{Z\psi}
\]

where the sums are over all the Standard Model fermions \( f \) (quarks and leptons), the corresponding sfermions \( \tilde{f} \), the charginos \( \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm \), and the unphysical Higgs bosons \( G^0, G^\pm \) with masses \( m_{G^0} = m_Z \) and \( m_{G^\pm} = m_W \), which appear in 't Hooft-Feynman gauge. The individual amplitudes are

\[
\tilde{A}^{a+b+c}_{W\psi} = -2 S_{W\tilde{\chi}} \left[ 2I_3^{[1]}(m_W) + 2(m_W^2 + m_W^2 - m_\tilde{\chi}^2)I_4^{[a]} + 2m_W^2 I_4^{[b]} + 3m_\tilde{\chi}^2 I_4^{[c]} + I_3^{[2]}(m_W, m_\psi) \right] + 8 D_{W\tilde{\chi}} m_\psi m_\tilde{\chi}(I_4^{[b]} + I_4^{[c]}),
\]

\[
\tilde{A}^{a+b+c}_{\phi\psi} = c_\psi e_\psi^2 S_{\phi\tilde{\chi}} \left[ 2m_\psi^2 I_4^{[b]} + m_\psi^2 I_4^{[c]} + I_3^{[2]}(m_\phi, m_\psi) \right] + 2c_\psi e_\psi^2 D_{\phi\tilde{\chi}} m_\psi m_\tilde{\chi} I_4^{[b]},
\]

\[
\tilde{A}^d_{\phi\psi} = c_\psi e_\psi^2 S_{\phi\tilde{\chi}} \left[ g^P_{\phi\tilde{\chi}} g^P_{\phi\psi} \frac{m_\psi}{m_\tilde{\chi}} \frac{4m_\tilde{\chi}^2}{m_\phi^2 - 4m_\tilde{\chi}^2} I_3^{[1]}(m_\psi) \right],
\]

\[
\tilde{A}^e_{Z\psi} = c_\psi e_\psi^2 S^{A}_{Z\tilde{\chi}} \left[ g^A_{Z\tilde{\chi}} g^A_{Z\psi} \frac{m_\psi}{m_\tilde{\chi}} \frac{4m_\tilde{\chi}^2}{4m_\tilde{\chi}^2 - m_Z^2} I_3^{[1]}(m_\psi) \right],
\]

where \( e_\psi \) is the electric charge of particle \( \psi \) in units of the positron charge, \( c_\psi \) is a color factor (3 for quarks; 1 for leptons and charginos),

\[
S_{Wjk} = |g^V_{ijk}|^2 + |g^A_{ijk}|^2, \quad D_{Wjk} = |g^V_{ijk}|^2 - |g^A_{ijk}|^2, \quad S_{\phijk} = |g^S_{ijk}|^2 + |g^P_{ijk}|^2, \quad D_{\phijk} = |g^S_{ijk}|^2 - |g^P_{ijk}|^2.
\]

The labels for amplitudes follow the labels appearing in fig. 1. For example, \( \tilde{A}^{a+b+c}_{W\psi} \) is the gauge invariant combination of diagrams appearing in fig. 1(a), (b) and (c) which have a W boson in the loop. Similarly, \( \tilde{A}^{a+b+c}_{\phi\psi} \) is the same combination of diagrams, but with the W in the loop replaced by a scalar.

For the triangle diagrams, \( \tilde{A}^d_{\phi W} \) and \( \tilde{A}^d_{\phi \phi} \) do not contribute because of the special kinematics (see discussion in sect. 2.2). \( \tilde{A}^e_{ZW}, \tilde{A}^e_{Z\phi} \) and the part of \( \tilde{A}^e_{Z\psi} \) with the vector coupling between \( Z \)
and the fermion loop vanish by Furry’s theorem. The term $A_{Z \psi}^c$ requires special attention because of the anomaly, which arises when the $Z$ coupling with the fermion loop is axial. We have dropped the anomalous contributions in eqs. (22) since they add to zero when summing over all fermions in the loop. The remaining contribution of this diagram is non-zero due to the large mass splitting between the two quarks of the third generation and between the charginos.

Individual Feynman diagrams are, of course, not gauge-invariant. However, the combinations appearing in eq. (22) are gauge invariant. We have verified that when the photon polarizations are between the two quarks of the third generation and between the charginos.

Substituting the at rest kinematics (7) into the general integral identity given in eq. (18) of ref. [19], we can reduce the box integrals appearing in $\tilde{A}_{W \psi}^{a+b+c}$ to triangle integrals:

$$
I_4^{[a]} = \frac{I_3^{[2]}(m_W, m_\psi) - I_3^{[1]}(m_W)}{m_\chi^2 + m_\psi^2 - m_W^2}, \\
I_4^{[b]} = \frac{I_3^{[2]}(m_\psi, m_W) - I_3^{[1]}(m_\psi)}{m_\chi^2 + m_W^2 - m_\psi^2}, \\
I_4^{[c]} = \frac{I_3^{[2]}(m_\psi, m_W) - I_3^{[2]}(m_\psi, m_\phi)}{m_\psi^2 - m_W^2}.
$$

The integral functions in $\tilde{A}_{\phi \psi}^{a+b+c}$ are obtained from eqs. (24) by replacing $m_W$ with $m_\phi$.

We use the standard methods of ref. [27] to express the triangle integrals in terms of logarithms and dilogarithms [28],

$$
I_3^{[1]}(m) = \begin{cases} 
\frac{1}{8m_\chi^2} \left[ \log^2 \left( \frac{1+x}{1-x} \right) - \pi^2 - 2i\pi \log \left( \frac{1+x}{1-x} \right) \right], & m \leq m_\chi; \\
-\frac{1}{2m_\chi^2} \left( \arctan \frac{1}{\sqrt{m^2/m_\chi^2 - 1}} \right)^2, & m > m_\chi;
\end{cases}
$$

$$
I_3^{[2]}(m_1, m_2) = \frac{1}{2m_\chi^2} \left[ \text{Li}_2 \left( \frac{m_1^2 - m_\chi^2 - m_2^2 - \sqrt{\Delta_1}}{2m_1^2} \right) + \text{Li}_2 \left( \frac{m_1^2 - m_\chi^2 - m_2^2 + \sqrt{\Delta_1}}{2m_1^2} \right) \\
- \text{Li}_2 \left( \frac{m_1^2 + m_\chi^2 - m_2^2 - \sqrt{\Delta_2}}{2m_1^2} \right) - \text{Li}_2 \left( \frac{m_1^2 + m_\chi^2 - m_2^2 + \sqrt{\Delta_2}}{2m_1^2} \right) \right],
$$

where

$$
x = \sqrt{1 - m^2/m_\chi^2}, \\
\Delta_1 = (m_1^2 + m_\chi^2 - m_2^2)^2 + 4m_\chi^2m_2^2, \\
\Delta_2 = (m_1^2 - m_\chi^2 - m_2^2)^2 - 4m_\chi^2m_2^2.
$$

One convenient way to obtain the values of the integrals is from Oldenborgh’s program FF [29]. In the notation used by FF the triangle integrals are

$$
I_3^{[1]}(m) = I_3(m^2, m^2, m^2; 4m_\chi^2, 0, 0), \\
I_3^{[2]}(m_1, m_2) = I_3(m_2^2, m_1^2, m_1^2; -m_\chi^2, 0, m_\chi^2).
$$
We have compared the results in eq. (22) to those of BU [10] and we find complete agreement. We also find disagreement between our results and those of ref. [9]. After using eq. (24) to express the amplitudes in terms of triangle integrals, the comparison is simple. For example, for the integrals of class 1 and 2 of BU the translation between the real parts of our integrals and those of BU is, $\text{Re} \ I_1[a,b]/4m^2_\chi \leftrightarrow -I_3[a,b]/2m^2_\chi$ and $\text{Re} \ I_3[a,b]/m^2_\chi$. (Notice that $I_3[a/b,1/b] = I_2[a,b]$). Similar formulas hold for the class 3 and 4 integrals. The notation for the coupling constants is related by $g_{L,P} = g_{V,A} + g_{S,P} \gamma_5$ (for vector bosons; $g_{S,P} \gamma_5$ for scalar bosons) with $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$. (Note that BU use unitary gauge for the $Z$ in the diagrams in fig. 1(e), while we use Feynman-'t Hooft gauge.)

3 Discussion

3.1 Large mass behavior of amplitudes

An interesting feature of our result is that in the limit when the neutralino is a pure higgsino with a mass much greater than $m_W$ the cross-section does not fall off when $m_\chi$ is increased, but remains constant. This behavior, which was numerically observed by BU, is caused by the diagrams in fig. 1(b). In the pure higgsino limit, $\chi$ is almost degenerate with the lightest chargino and the denominator of $I_{4[b]}$ in eq. (24) becomes small, so this contribution dominates. This allows us to obtain an analytic expression in the limit $m_\chi \simeq m_\chi^\pm \gg m_W$. Using eqs. (24), (25), and the expansion

$$\text{Li}_2(1+i\epsilon) + \text{Li}_2(1-i\epsilon) \simeq \frac{\pi^2}{3} - \epsilon\pi,$$

the asymptotic behavior of $I_{4[b]}$ is

$$I_{4[b]} \simeq \frac{\pi}{2} \frac{1}{m_\chi^3 m_W}.$$

This yields the asymptotic behavior of the cross section,

$$\sigma v \simeq \frac{\alpha^4 \pi}{4m^2_W \sin^2 \theta_W} \approx 10^{-28}\text{cm}^3/\text{s},$$

where we have used $S_{W\chi^+} = D_{W\chi^+} = g^2/4 = \pi\alpha/\sin^2 \theta_W$ and set the other couplings to zero which is appropriate for pure higgsinos.

3.2 Annihilation to gluons

We may also use the above results to obtain the cross section for neutralino annihilation into two gluons. The diagrams for this process are a subset of the ones appearing in the photon calculation. (The diagrams containing three-gluon vertices vanish.) This allows one to obtain the result for gluon annihilation directly from the results for photon annihilation. For the gluon case, the amplitude is given by

$$\tilde{A} = \sum_{\phi=\bar{q}} \tilde{A}_{\phi\psi}^{a+b+c} + \sum_{\phi=H^0,C^0} \tilde{A}_{\phi\psi}^d + \sum_{\psi=\bar{q}} \tilde{A}_{Z\psi}^e,$$
where we set $e_\psi = c_\psi = 1$ in the individual amplitudes and sum over quarks and squarks. The color sum [3] amounts to simply changing $\alpha^2$ to $2\alpha^2$ in (19), so our result agrees with BU. This confirms a small but significant error in previous calculations [30, 2], which generally led to an over-estimation of antiproton fluxes.4

4 Summary and Conclusions

In this paper we presented a complete one-loop calculation for neutralino annihilation into two photons or two gluons within the Minimal Supersymmetric Standard Model. In performing the calculation we assumed that initial-state neutralinos are non-relativistic, which is the relevant case for galactic dark matter. The techniques we used are, however, not limited to this special kinematics.

Our results confirm the very recent calculation of Bergstrøm and Ullio [10] which disagrees with previously published ones, especially in the case where the neutralino is a heavy pure higgsino.

We thank Graciela Gelmini for encouragement and a number of stimulating discussions. This work was supported in part by the DOE under contract DE-FG03-91ER40662 and by the Alfred P. Sloan Foundation under grant BR-3222. P.G. thanks Piero Ullio and Lars Bergstörm for useful discussions and David Cline, Graciela Gelmini and Roberto Peccei for the extended visit at UCLA which made this work possible.

References

M.C. Chantell et al, preprint astro-ph/970437

4M. Kamionkowski has reported to us that the error in the text of ref. [30] did not propagate to the numerical analysis of that paper.
F.A. Berends, R. Kleiss, P. De Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. 103B:124 (1981);


G. ’t Hooft, in Acta Universitatis Wratislaviensis no. 38, 12th Winter School of Theoretical Physics in Karpacz; *Functional and Probabilistic Methods in Quantum Field Theory*, Vol. 1 (1975);
B.S. DeWitt, in Quantum gravity II, eds. C. Isham, R. Penrose and D. Sciama (Oxford, 1981);


K. Fujikawa, Phys. Rev. D7:393 (1973);
M. Base and N.D. Hari Dass, Ann. Phys. 94:349 (1975);


W. Siegel, Phys. Lett. 84B:193 (1979);
D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B167:479 (1980);


L.M. Brown and R.P. Feynman, Phys. Rev. 85:231 (1952);
L.M. Brown, Nuovo Cimento 21:3878 (1961);


J.F. Gunion and H.E. Haber, Nucl. Phys. B272:1 (1986);


L. Lewin, *Dilogarithms and Associated Functions* (Macdonald, 1958)
