Imprint of the Global Hubble Flow on Galactic Rotation Curves∗

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Abstract
We identify an apparent imprint of the global Hubble flow on the systematics of galactic rotation curves, and discuss its possible implications for gravitational theory.

Even though the astrophysics community is currently engaged in an extremely vigorous debate as to the explicit nature (macho or wimp) of the dark matter which is widely believed to pervade and even dominate the universe, it is nonetheless instructive (even if only to sharpen the dark matter debate itself) to consider the possibility that dark matter may not in fact actually exist (or at least not in such copious amounts), and that the apparent need for it actually heralds the explicit failure of the standard Newton-Einstein gravitational theory on galactic and larger distance scales, viz. precisely on distance scales which are altogether larger than the solar system one on which the standard theory was first established. In order to address this issue it is thus of value to try to identify phenomenological regularities in the data, regularities which may then possibly point in the direction of some other candidate gravitational theory, or which at the very minimum would serve to constrain dark matter dynamics.

Perhaps the most startling aspect of dark matter (other than its very existence) is that none is actually needed in the solar system at all, and nor (as

noted in various presentations in these proceedings) is any apparently needed in any sub-galactic system such as a globular cluster or a molecular cloud, with the standard gravity of the known luminous objects in such systems providing a completely adequate accounting of their dynamics.\footnote{Moreover, the very success of standard gravity in its precision fitting of a binary pulsar, a typical sub-galactic system, even constrains the density of the nearby, tidally disrupting, galactic dark matter which is thought to be transiting the Milky Way with the halo virial velocity. As regards this halo, we note that while the MACHO project is finding evidence for halo lenses, it is quite perplexing that their inferred masses would need to be above the hydrogen burning threshold. Since the luminosity of the LMC is much larger than that of the resolved source stars being monitored, some detected events could be due to the brightening of otherwise unseen LMC source stars as they are lensed, to then possibly reduce the true microlensing optical depth.} However, once one goes to galactic systems not only do the Newton-Einstein gravitational contributions of the known luminous constituents underestimate the available data, the associated shortfalls are found to be even bigger on even larger distance scales such as those associated with systems which contain large numbers of galaxies.\footnote{In fact, this very need for dark matter could itself be regarded as being merely a parameterization of the detected luminous Newtonian shortfalls, with the needed amounts of dark matter being determined (to be large or small) only after rather than before the fact.} From this trend it would thus appear that there is some intrinsic scale associated with the systematics of luminous Newtonian shortfalls, and thus we shall seek to extract an explicit one out from galactic data, to find below that not only is there evidence of such a new scale, but that it intriguingly turns out to be one which is associated with the global cosmological Hubble flow.

Given the above trend in the growth of the luminous Newtonian shortfall with distance as we go from galaxies to clusters of galaxies and then beyond, it is thus suggestive to look at the dependence on (radial) distance of the analogous shortfalls within individual galaxies themselves. Thus we shall specifically analyze HI spiral galactic rotation curves, these being the curves which go out the furthest beyond the optical disk in distance and which precisely provide the primary evidence for the need for galactic dark matter in the first place. Now while the flatness of many of these rotation curves is their immediately most striking and celebrated feature, it is more instructive to look not at the total observed rotation velocities, but rather to look at the discrepancies, i.e. to look at the luminous Newtonian shortfalls themselves.
And, indeed, in cases where the total velocities are in fact flat, since the luminous Newtonian contribution undergoes a Keplerian falloff, it thus follows that the discrepancies themselves must in fact actually be growing with distance in such cases. Now out of the currently available set of 33 HI spiral galaxies K. G. Begeman, A. H. Broeils and R. H. Sanders (MNRAS 249, 523 (1991)) identified a subset of 11 of them as being particularly reliable, a subset which contains dwarfs (whose rotation curves are typically actually rising rather than being flat), intermediate spirals (with flat rotation curves) and bright spirals (with curves which typically slightly decline), this being a subset which contains galaxies which vary in luminosity by a factor of more than 1000. Now it was noted by P. D. Mannheim (ApJ 479, 659 (1997); Found. Phys. 26, 1683 (1996)) that even the non-flat rotation curves of the dwarfs and the bright spirals have discrepancies that likewise grow with distance out to the largest distances, to thus make growing discrepancies the most comprehensive qualitative feature of rotation curve systematics. Moreover, on noting that the centripetal accelerations \(\frac{v^2}{c^2 R}\) at the last available data points in each of these 11 galaxies are all found to lie in the surprisingly narrow range of \(1.51 \times 10^{-30}\) to \(7.25 \times 10^{-30}\) cm\(^{-1}\) (i.e. surprisingly close given the huge range in luminosity), Mannheim then found that all of these accelerations could be parameterized according to the universal three-component relation \(\frac{v^2}{c^2 R}\) = \(\gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2\) where the two new universal constants \(\gamma_0\) and \(\gamma^*\) take the numerical values \(3.06 \times 10^{-30}\) cm\(^{-1}\) and \(5.42 \times 10^{-41}\) cm\(^{-1}\) respectively, where \(\beta^* = 1.48 \times 10^5\) cm, and where \(N^*\) is the total amount of visible stellar (and gaseous) material in solar mass units in each galaxy. In addition we now note, that while not being quite as definitive as the preferred 11 galaxies in the 33 galaxy sample, the other 22 galaxies (listed for instance in R. H. Sanders, ApJ 473, 117 (1996)) have rotation curves which in general again all show a trend of increasing discrepancy with distance. Moreover, for this larger group the centripetal accelerations at the last detected points are all found to fall in the range \(1.32 \times 10^{-30}\) to \(5.38 \times 10^{-30}\) cm\(^{-1}\), i.e. just the same narrow range found for the other 11 galaxies.

Since there is nothing significant about the last detected data points (they are fixed by the sensitivity of the 21 cm line detectors and not by any dynamics within the galaxies themselves), the existence of this empirically found structure for \(\frac{v^2}{c^2 R}\) should thus be taken as being phenomenologically significant, and it should thus serve as a constraint on all theories of galactic
rotation curves. Moreover, since the numerical value extracted out for the parameter $\gamma_0$ turns out to be of order the inverse of the Hubble radius, the rotation curves would thus appear to be endowed with a cosmological imprint. Further, on recognizing that the $\frac{\gamma_0}{2} + \gamma^*N^*/2$ term is serving as a linear potential, we thus see that it its inferred numerical value immediately implies that it would be negligible for sub-galactic distances where the luminous Newtonian $\beta^*N^*/R^2$ term would then dominate, while becoming ever more important on galactic and larger distances scales. The scale associated with this linear potential thus precisely characterizes when there should or should not be any luminous Newtonian shortfall.

Now that we have identified this pattern it is immediately natural to look for a dynamics which might produce it, and indeed Mannheim has noted (as discussed in his above papers which give detailed related references) that its emergence is actually quite natural in fourth order conformal gravity, a fully covariant, pure metric based candidate alternative to standard Einstein gravity. Indeed, what is found there is that the familiar standard gravity Schwarzschild metric exterior to a star of Schwarzschild radius $2\beta^*$ is generalized in conformal gravity to $-g_{00} = 1/g_{rr} = 1 - 2\beta^*/r + \gamma^*r$, to thus yield, for a system of $N^*$ stars, the asymptotic $(v^2/c^2R)_{last} = \gamma^*N^*/2 + \beta^*N^*/R^2$ on large enough distance scales. We thus see that the departure from standard gravity is described by none other than a linear potential. However, unlike Newtonian potentials, linear potentials are not asymptotically negligible, and thus, again unlike Newtonian gravity, it is no longer possible to neglect the gravitational effects due to matter exterior to any system of interest. Thus we need to determine what effect the rest of the matter in the universe might have on motions within individual galaxies. And, quite remarkably, it was further shown that in conformal gravity the entire Hubble flow is found to act on such galaxies just like a universal linear potential term (generated by the scalar curvature $k = -\gamma^2_0/4$ of a necessarily topologically open cosmology), to give the additional $(v^2/c^2R)_{last} = \gamma_0/2$ term just as desired. With these two linear potential terms, conformal gravity was then able to yield parameter free fits to the all of the rotation curve data points of the selected 11 galaxies (i.e. those at all radial distances and not just the furthest ones) without any need for dark matter.³ We thus identify an imprint of cosmology on galactic

³In these proceedings J. F. Navarro reports on work with C. S. Frenk and S. D. M. White in which they derived a cosmology based dark matter model with generic density
rotation curves, and suggest that it is its neglect which has generated the conventional appeal to dark matter. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

\[ \rho(r) = \frac{\rho_0}{r(1 + r/r_s)^2}. \] It is thus of interest to note that in the limit of large \( r_s \), a galactic halo with such a density would act precisely the same way as a linear potential. At the present time their model is not yet detailed enough to determine exactly what specific luminous matter distribution (i.e. what particular luminosity and optical disk scale length) would be trapped in any particular halo with specific values for \( \rho_0 \) and \( r_s \), and so they currently do standard dark matter fitting to rotation curves by adjusting dark to luminous matter distributions galaxy by galaxy. It might thus be quite instructive to see whether cosmological dark matter dynamics could reproduce the universal structure found for \( (v^2/c^2R)_{\text{last}} \) with the same facility as conformal gravity.