DISK INSTABILITY MODELS

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Abstract. We review various aspects of disk instability models. We discuss problems and difficulties and present ways that have been suggested to solve them.

INTRODUCTION

Almost everyone interested in the subject would agree with the statement that the thermal–viscous disk instability model (DIM) describes both dwarf nova (DN) outbursts and soft X-ray transients. If one asked, however, what is meant by the thermal–viscous DIM, the answer would be far from unanimous; and if one asked for examples of systems correctly described by the DIM, there would be no answer at all. One or two examples would we given but a confrontation of the DIM predictions with observations would show them to be too optimistic.

What we usually mean when we say that the DIM is the model describing DN’s and SXT’s is that it is a general idea (we resisted the temptation to call it a ‘paradigm’) according to which both DN outbursts and SXTs are triggered by an accretion disk instability due to an abrupt change in opacities at temperatures at which hydrogen is partially ionized. All versions of the DIM have this ingredient. They differ in assumptions about viscosity, and about what happens at the inner and outer disk radii. In some versions convection is ‘switched off’ in order to make a viscosity ansatz work; in most, the results depend on the number of grid points in the numerical code. There are models in which some terms in the energy equation are dropped even if they are of the same order as the terms left (the authors do not hide this fact). Finally, some versions of the model, when applied to particular classes of systems, require the help of tidal forces and sometimes extremely low values of the parameter describing viscosity. Of course some of these versions are the result of attempts to describe particular systems.

In this respect the success of the DIM is not overwhelming. Attempts to
produce ‘standard’ models of such ‘prototype’ systems as the DN SS Cyg and
the SXT A0620–00 failed to produce results that would reproduce fundamental
properties of the prototype (such as the presence of ‘inside–out’ and ‘outside
in’ outbursts in SS Cyg or the correct rise–time and recurrence time in A0620–
00).

So why does almost everybody (including the present authors) agree that
the DIM is the model describing DN and SXT outbursts? And why is it so
difficult to build a model that would incorporate the general idea about the
DIM?

The answer to the first question is that there is no competition. At the very
beginning, when it was established that a dwarf nova is a binary system in
which a white dwarf accretes, through a disk matter lost by a Roche–lobe fill-
ning secondary (see Warner 1995 and references therein) star, there were some
doubts as to where the outburst takes place. Soon it became clear that the site
of the outburst is the accretion disk (and not one of the stellar components)
and two possibilities were considered. The outburst could be due either to an
instability in the disk or to a (sudden) increase of the mass transfer rate from
the stellar companion of the compact object. At first, the physical reason for
either mechanism was not known, but after it was realized, in the early eight-
ies, that accretion disks in the parameter range corresponding to dwarf nova
properties are thermally and viscously unstable, the disk instability mecha-
nism gained favour in the eyes of most astrophysicists. The trigger for the
supposed mass transfer instability remained unknown and some observations,
such as the behaviour of the disk’s outer radius during outbursts, seemed to
be inconsistent with this mechanism. It was concluded, therefore, that the
dwarf nova phenomenon cannot be due to a sudden increase in mass transfer
from the secondary.

An attempt to revive the mass transfer model [10] in the context of SXTs
was short lived because it was shown [9] that this model, which used the X–ray
irradiation of the secondary, cannot reproduce observed time–scales.

The (partial) answer to the second question is the subject of this review.

THE VISCOSITY PROBLEM

The disk instability model of dwarf–nova outburst was formulated in the
early eighties by several groups of authors (for the history of the early de-
developments see Cannizzo [4], [5]). The various versions of the model used a
fundamental property of accretion discs: in the range of temperatures at which
hydrogen recombines, the resulting dramatic reduction of opacities makes the
disk thermally and viscously unstable. This property is usually shown on the
$\Sigma - \nu\Sigma$ (surface density–(averaged) viscosity) plane: at a given radius, the
disk thermal equilibria form an $S$ - shaped curve, the middle branch of which
represents unstable solutions (see Fig. 1 where $S$ - curves are plotted on the
surface density, effective temperature ($T_{\text{eff}}$) plane). If the rate at which matter is fed into the disk corresponds, at some radius, to a locally unstable state, the disk would have to oscillate between the lower, cold, and the upper, hot, branch of the $S$-curve. The resulting limit cycle is supposed to represent the dwarf nova outburst cycle.

From the very beginning it became clear that the existence of the instability itself, as well as the disk behaviour, depend crucially on properties of viscosity in the accretion flow. The viscosity is described by the usual Shakura–Sunyaev prescription $\nu = \alpha c_s H$ where $H$ is the half-thickness of the disk, $c_s$ the sound speed and $\alpha$ the viscosity parameter, which in the original formulation (Shakura & Sunyaev 1973) was assumed to be a constant $\leq 1$. It was first noted by Smak [37] that if $\alpha$ is kept constant, “only very short-period, very low-amplitude variations could be produced”. To obtain light curves of dwarf novae Smak had to assume that $\alpha$ in a cold state is 4 times lower than $\alpha$ in the hot disk. This jump in $\alpha$ is necessary because the jump in the temperature itself, when moving from the cold to the hot branch is not big enough to increase the viscosity to values that give the right amplitudes and time-scales of dwarf nova outbursts.

As pointed out by Smak [37] one should not expect the simple $\alpha$ parameterization to represent all the complex phenomena due to turbulent viscosity. Rather, once a reliable model is found, observations of dwarf nova out-
bursts might be used to put constraints on viscosity models. For example, it is thought that the ratio $\alpha_{\text{hot}}/\alpha_{\text{cold}}$ must be between 4 and 10 in order to reproduce characteristic time-scales of DN outbursts. In most cases values $\alpha_{\text{cold}} \sim 0.01$ (see e.g. Livio and Spruit 1991) and $\alpha_{\text{hot}} \sim 0.1$ are believed to be required for models to correspond to observations.

On the other hand, observations of quiescent DNs suggest completely different values of $\alpha_{\text{cold}}$. Quiescent DN disks seem to be optically thin [14], [41], whereas the DIM predicts that between outbursts the disks are optically thick. The optical thickness is directly related to the value of $\alpha$ because low $\alpha$'s imply high column densities. In a quiescent disk of the DIM optical depths are high. Observed quiescent disks show the presence of a Balmer jump in emission and brightness and colour temperature distributions that are incompatible with an optically thick disk. Attempts to fit observations with a simple model in which emission is produced by a uniform, isothermal slab give $\alpha_{\text{cold}} \gtrsim 100$, which is unacceptably high. Modeling quiescent disk emission using a radiative transfer code [39] gives $\alpha_{\text{cold}} \approx 0.5$ [16], which is in contradiction with the DIM.

It is not surprising that the main difficulties encountered in the DIM are connected with viscosity. Despite the progress in the understanding of the origin and nature of viscous turbulence in accretion disks owing to the seminal work of Balbus and Hawley, (see Balbus – this volume) we are still forced to use the $\alpha$ prescription. Only very recently have results of numerical simulation been directly applied to disk model (e.g. Abramowicz, Brandenburg & Lasota 1996).

The first application of Balbus & Hawley simulations to the problem of dwarf nova outbursts questioned the validity of DIM itself [8]. Gammie and Menou [8] calculated the magnetic Reynolds number $R_M$ in a quiescent dwarf nova disk. The disk properties were determined using the DIM code of Hameury et al. (1998) (which we will discuss below), where, as usual, $\alpha_{\text{cold}} \sim 0.01$. They obtained $Re_M < 10^4$. According to the simulations with such low values of $Re_M$ the instability creating the turbulence dies away. It follows, if turbulence in accretion disks can result only from an MHD instability [2] (but see e.g. Yoshizawa & Kato 1997), that there would be no transport of angular momentum in quiescence, i.e. no accretion. This would mean that matter can accumulate only at the outer disk rim. Since in quiescence there is no viscosity, the outburst would have to be triggered by a MHD instability and not by a thermal one. The DIM would have to be strongly modified.

It is too early however to try to modify drastically the ‘standard’ version of the DIM. First, observations clearly show that there is accretion in quiescence (see also below). To this objection Gammie and Menou answer that the observed emission could be due to a hot corona above the cold ‘dead’ disk. This a hypothesis that would have to be checked. Second, one observes quite often ‘inside-out’ outbursts which, it seems, it would be difficult to get when matter is allowed to accumulate only in the outer disk regions.
It is interesting to note, however, that if \( \alpha_{\text{cold}} \) were \( \sim 0.5 \) then \( R_M > 10^4 \) and turbulence would be present the quiescent disk. This value of \( \alpha_{\text{cold}} \) is the same as the one required by observed emission from the disk [15]. Maybe this is not a coincidence.

In any case viscosity models are not yet ready to describe the vertical stratification of viscosity or even its dependence on temperature and other variables. It is therefore not surprising that attempts were made to guess or to postulate viscosity’s functional dependence on variables such as temperature and radius. In the framework of the ‘\( \alpha \)–prescription’ one can only play with \( \alpha \). The formula \( \alpha = \alpha_0 (H/R)^n \) is rather popular nowadays. It was first proposed by Meyer & Meyer–Hofmeister (1983) and used by Mineshige & Wheeler (1989) in their model of SXTs and by Cannizzo [7], [6] also in application to SXTs. Ludwig, Meyer–Hofmeister & Ritter (1994) arrived at the conclusion that the \( \alpha_{\text{hot}}/\alpha_{\text{cold}} = 4 – 10 \) prescription cannot produce ‘outside–in’ outbursts and suggest that \( \alpha = \alpha_0 (H/R)^n \) might be necessary to reproduce such types of outbursts. As shown, however, by Hameury et al. (1998) (HMDLH; see also [17]) the lack of ‘outside–in’ outbursts in Ludwig et al. (1994) and similar calculations is mainly due to the outer boundary condition that assumes a fixed outer disk radius. When the radius is allowed to move during and after the outburst (as it has in fact observed to do) there is no problem with obtaining ‘outside–in’ outbursts.

Another argument in favour of \( \alpha = \alpha_0 (H/R)^n \) is based on the shape of light–curves during the decay from outburst [7], [5]. It is claimed that in order to produce strictly exponential light–curves \( \alpha \) has to be of the form (2) with \( n = 1.5 \). The value of \( \alpha_0 \) is supposed to depend on the mass of the accreting body and to be 50 for \( M = 10M_\odot \) (Mineshige & Wheeler favor \( \alpha_0 \sim 10^3 \) in their model of the same systems). There are several problems with this approach. First, if \( \alpha_0 \sim 50 \) then one encounters the same problem as when one assumes \( \alpha_0 = \text{const.} \): the jump in temperature on the \( S \)–curve is not big enough to produce an outburst. Discovering this difficulty, Cannizzo et al. [7] decided to modify the physics leading to the appearance of an \( S \)–curve. Convection, as a cooling mechanism, plays an important role in the accretion disk structure close to the local surface density maximum on this curve. If one artificially, switches off convection the resulting disk models produce the required amplitudes. It is true that models of convection in accretion disks are far from being perfect, but discarding this physical mechanism just to obtain the right shape of part of a light–curve seems a bit far–fetched.

Especially since it is not certain how the form of \( \alpha \) is related to the shape of the observed light–curves. First, it is not clear that they are exactly exponential. According to Cannizzo (1998a) the light curve of the SXT A0620–00 is rigorously exponential whereas light curves of WZ Sge–type dwarf novae are ‘flat top’ (power–like). As shown, however, by Erik Kuulkers (private communication) the \( V \) light curves of A0620–00 and of the WZ Sge–type system AL Com have exactly the same shape. In any case optical emission from SXT
disks in outburst is dominated by radiation due to X–ray reprocessing (Mc Clintock & van Paradijs 1995). The shape of the light curve during the decay from outburst may be only very indirectly related to the α prescription (King & Ritter 1997).

Finally, if \( \alpha_0 = 50 \) then the rise to outburst is much too slow to agree with observations (Cannizzo 1998b). The rise to outburst is due to the propagation of the heat front through the disk. This front brings the disk from the quiescent cold state to the hot outburst configuration. It propagates with the speed \( v_{fr} \approx \alpha c_s \) In the hot state \( H/R \approx 0.01 \) [6] so that (with \( \alpha_0 = 50 \)) according to \( \alpha \approx 0.05 \) which is roughly ten times lower than the value of \( \alpha \) necessary to obtain the correct (i.e. observed) rise-times to outburst. It is not surprising that Cannizzo (1998b) gets rise-times 10 times too long.

As a conclusion one can state that arguments in favour of the \( \alpha = \alpha_0 (H/R)^n \) formula are rather weak.

**NUMERICAL CODES**

Equations describing time–dependent accretion disks in the Keplerian, geometrically thin \((H/R \ll 1)\) approximation are equivalent to a system of 4 non–linear, first order partial differential equations. The rise to outburst is the result of the propagation through the disk of a heating front; the decay from the maximum is, in general, when illumination effects are neglected, the result of the propagation through the disk of a cooling wave. The problem is not mathematically very complicated but if one wishes to resolve the fronts and describe their propagation at scales differing by up to six orders of magnitude, the required computer time may quickly become prohibitive. At first, the number of (fixed) grid points in the DIM codes was less than 50, and only recently (except for Mineshige [29]) Cannizzo [7] [6] increased this number to 1000 (see HMDLH for discussion).

Recently a systematic study of the DIM numerical problem was undertaken in several important articles. Cannizzo [4], in a pioneering article, showed that light curves (i.e. rise–times, amplitudes, number of outbursts and recurrence times) strongly depend on the number of grid points. Unfortunately the number of points chosen by Cannizzo himself (200 to 400) is, in general, not sufficient to give physically reliable results (HMDLH). Moreover the character of the outbursts (‘inside-out’) and the shape of the light-curves in his models [4] are the result of keeping the outer disk radius fixed. The same assumption was made in the important investigations of Ludwig et al. (1994) and Ludwig and Meyer [24]. In this last article it was shown that non–Keplerian effects arising from steep gradients in the fronts can be neglected.

All versions of the DIM codes suffered from various drawbacks such as insufficient resolution of the grid, prohibitive computing time (for explicit codes) and/or ill–adapted boundary conditions. A new version of the DIM code [13]
FIGURE 2. V magnitude light curves calculated using a fixed grid at various resolutions: (N=100, 400, 1600), adaptive grid with fixed (N=400) and variable $R_{\text{out}}$ (N=1600).

which is free of all these inconveniences has been constructed by HMDLH. This implicit code uses an adaptive grid and the size of the disk is allowed to vary. The number of grid points can be as high as 4000 but this is not necessary. HMDLH find that global properties of transient discs can be addressed by codes using a high, but reasonable, number of fixed grid points. However, the study of the detailed physical properties of the transition fronts generally requires resolutions which are out of reach of fixed grid codes. An example in which fixed grid and adaptive grid calculations are compared is shown on Figure 2. The HMDLH code can be considered as a continuation of Smak’s (1984) code and it has a lot in common with Ishikawa & Osaki (1992).

This new code will allow a systematic study of the DIM in the context of both the dwarf–nova and X–ray transient outbursts. It has already been used to model outbursts of WZ Sge and the rise to outburst of GRO J1655-40 (see next Section).

ADVECTION DOMINATED ACCRETION FLOWS

In the ‘standard’ version of the DIM one assumes that the disk extends all the way down to the central accreting object (or to the last stable orbit in the case of a central black hole) and that mass transfer from the secondary is constant. Both these assumptions are difficult to maintain when confronted
FIGURE 3. Quiescent luminosities of black-hole and neutron star soft X–ray transients versus orbital period. Squares represent neutron star SXTs and diamonds SXTs with confirmed ($M_{BH} > 3M_{\odot}$) black holes. (Frank & Lasota, 1997 unpublished)

with observations.

The DIM requires all quiescent disks to be in the cold state (‘on the lower branch of the S–curve’). This implies that the accretion rate throughout the disk must be lower than the critical value corresponding to the local maximum surface density (the quiescent disk is obviously not in viscous equilibrium; if it were there would be no outbursts). Since this critical accretion rate varies like $R^{2.7}$ (see e.g. HMDLH) the resulting accretion rate onto the central body could be ridiculously small; in any case it is quite often several orders of magnitude lower than the value deduced from observations (see e.g. Lasota 1996). In the case of dwarf–novae whose quiescent hard X–ray and UV emission is in contradiction with the DIM, Meyer and Meyer–Hofmeister [28] found that the inner part of a quiescent dwarf nova disk will evaporate and form a hot, tenuous accretion flow which can emit the X–rays and UV required by the observations.

In the case of quiescent black hole SXTs (BHSXTs) where the observed quiescent X–ray flux cannot be explained in the framework of a ‘standard’ disk model, Narayan et al. [31, 33] showed that an inner advection dominated accretion flow (ADAF) (see Menou, Quataert & Narayan 1998 for a recent review) provides a natural explanation of the observational data. Quiescent spectra of the three SXTs in which X–rays in quiescence were detected are very well reproduced by an ADAF model [31, 33, 12].

The inner ADAF extends up to about $10^4$ Schwarzschild radii; the outer
accretion flow forms a dwarf–nova type disk subject to the thermal–viscous instability which triggers the transient event. This model was confirmed by the observation [34] of the rise to outburst of GRO J1655-40 [12].

In ADAFs most of the heat released by accretion is not radiated away in the flow itself but advected to the central body. If this is a black hole energy is lost forever below the event horizon; but if accretion is onto a neutron star (or a white dwarf) the energy will eventually be re-radiated from its surface. Therefore BHSX Ts in quiescence should be fainter than quiescent systems containing neutron stars. Of course this statement is true if we compare systems with the same mass transfer rate. Narayan, Garcia & McClintock (1997) see also McClintock – these proceedings compare outburst amplitudes versus maximum luminosities of BHSX Ts and neutron star systems. As predicted by the ADAF model there is a clear difference between the two classes of objects. The same clear difference is seen if one compares histograms showing the distribution of the outburst amplitudes for the two classes (Cannizzo [5] sees no difference because he includes in the histogram lower limits of BHSXT amplitudes).

Finally, one can (see Chen et al. these proceedings) try to compare actual luminosities and not their ratios. The problem is to be able to determine the mass transfer rate. If one assumes that it is related to the orbital period one obtains the result shown on Fig. 3. It is clear that the present data is not in contradiction with the ADAF model but that much more observations are needed. One should also add that quiescent neutron star SXTs could be dimmer than expected because of the action of the propeller effect [42].

**ENHANCED MASS TRANSFER**

As shown by Lasota, Narayan & Yi (1996) in the ADAF + cold disk model the outer disk can be (marginally) stable with respect to the thermal instability. In this case the outburst would have to be triggered by an enhanced mass transfer which would bring the outer disk into the unstable regime.

In any case it is observed that the mass transfer rate from the secondary varies on various time–scales (see e.g. Warner 1995). There is also evidence that the mass transfer rate increases prior to superoutbursts of SU UMa dwarf–novae and during some ‘normal’ outbursts (Smak 1996). This enhanced mass transfer could be due to irradiation of the secondary. It has been shown [21], [11] that if one wishes to avoid using extremely low values ($\lesssim 10^{-4}$) of the viscosity parameter required by the standard DIM one should consider a model in which the inner part is evaporated and mass–transfer is increased (by irradiation) during the outburst.

Despite growing evidence to the contrary, it is still often asserted that dwarf nova outbursts can be explained by assuming that the mass transfer rate is constant prior and during the outburst (see e.g. Cannizzo 1993a; Osaki 1996).
The confrontation of such models with observations does not, however, give satisfactory results. Clearly the role of variations of the mass transfer rate in dwarf nova systems requires more study.

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