SHELL EFFECTS IN THE SYMMETRIC-MODAL FISSION OF PRE-ACTINIDE NUCLEI

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Mass distributions of fragments in the low-energy fission of nuclei from $^{187}$Ir to $^{211}$At have been analysed. This analysis has shown that shell effects in symmetric mode fragment mass yields from the fission of pre-actinide nuclei could be described if one assumes the existence of two strongly deformed neutron shells in the arising fragments with neutron numbers $N_1 \approx 52$ and $N_2 \approx 68$. A new method has been proposed for quantitatively describing the mass distributions of the symmetric fission mode for pre-actinides with $A \approx 180 \div 220$.

Key words: Nuclear fission; Fission-fragment mass distributions; Semiempirical fission model; Shell effects; Symmetric fission mode; Pre-actinides ($A \approx 180 \div 220$)

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1. INTRODUCTION

On the initiative of Prof. G.N. Smirenkin a cycle of experimental investigations of fragment spectra in the low-energy fission of pre-actinide nuclei with atomic numbers $A = 187 - 213$ was carried out during the last decade [1-6]. In these works it was shown that smooth liquid-drop mass and energy distributions (MED) of fragments from the fission of nuclei lighter than Ra are modulated by sharply expressed irregularities conditioned by the influence of shell structure on the process of fragment formation. The most bright manifestation of these effects takes place in two parts of the MED: at large mass-asymmetric deformations of fissioning nuclei (for heavy fragments with masses $m > 125$), and in a range of the symmetric mode (masses close to $A/2$).

A combined analysis of fragment-mass dependencies of relative yields $Y(m)$, total kinetic energies $E_k(m)$ and energy dispersions $\sigma_E(m)$ showed that the behaviour of these characteristics could be explained within the framework of the heteromodal approach to describe the MED [4]. According to the approach, irregularities observed in the MED in a range of heavy fragments with $m > 125$ are conditioned by the superposition of the symmetric and two asymmetric fission modes (Standard I and Standard II, in terms of Ref. [7]). In the fission of pre-actinide nuclei, the symmetric mode is predominant. Earlier, its origin was explained by pure liquid-drop properties of the fissioning nuclei. But the above-mentioned experiments proved that the formation of the symmetric fission mode is affected not only by liquid-drop properties of the fissioning nuclei, but also by shell structure in the transition state (at the saddle point). Having applied the transition-state method to the analysis of the MED, the authors of these works succeeded to estimate a value for the shell correction to the fission barriers of pre-actinides ($W_t \approx 1$ MeV) and a damping constant (the constant defining the temperature rearrangement of shells) [4-6]. Nevertheless, these investigations did not answer the question
about the nature of the shell-correction origin, particularly the question about the role played by the numbers of neutrons N and protons Z for the formation of the shell effects in a fissioning nucleus and in the arising fragments, and what "magic" numbers Z and N are responsible for these shell corrections.

In order to clear the question up we have undertaken an attempt to perform a combined analysis of all available experimental data on the MED of pre-actinide nuclei. In the course of this, the following problems were solved simultaneously:

1. The determination of shell-correction characteristics ("magic" values N and Z, typical deformation, and amplitude) which are responsible for the shell effects in the fragment yields of the symmetric fission mode for pre-actinide nuclei.

2. The development of a semiempirical method for describing symmetric-mode mass yields in a range of nuclei with $A = 180 - 220$.

2. QUANTITATIVE CONSIDERATION

As it was outlined in investigations of fission-fragment angular distributions [8], pre-actinides have very large deformations in the transition states. According to calculations within different versions of the liquid-drop model [9-11], at such deformations a nucleus has a dumbbell-like shape with a sharply expressed thin neck connecting two elongated fragments, i.e. the configuration at the saddle is close to that in the scission point. These calculations also show that not only the shapes in saddle and scission points are close to each other, but the potential energies as well. Proceeding from this, we suppose that the influence of dynamic effects on the descent from fission barrier to scission point is comparatively small, and therefore, the formation of the MED is generally defined by the deformation potential-energy landscape in the vicinity of the saddle point. As an evidence in favour of this hypothesis one can point out the
investigations [3,12] of mass-distribution dispersions $\sigma_m^2$ and the successful application of the transition-state method to describe fragment mass yields [2-13].

Thus, we further suppose that the dependencies $Y(m)$, $E_4(m)$ and $\sigma_{E4}^2(m)$ for pre-actinide nuclei directly reflect the dependencies of potential energy and deformation on the mass-asymmetric coordinate of the fissioning nucleus. The manifestation of shell effect in the MED of pre-actinide nuclei is demonstrated in fig. 1, where the distributions $Y(m)$, $E_4(m)$ and $\sigma_{E4}^2(m)$ are shown for nuclei from $^{187}$Ir to $^{213}$At at an excitation energy $U \approx 10$ MeV ($U = E^* - B_f$ — is the excitation energy above the fission barrier $B_f$). This figure shows that the MED for $^{207}$Bi and $^{213}$At at $m \geq 127$ have sharply expressed irregularities. This phenomenon has been studied in detail in Refs. [1-3], where it was demonstrated that for nuclei heavier than $^{207}$Tl the distributions $Y(m)$, $E_4(m)$ and $\sigma_{E4}^2(m)$ consist of the MED of three independent fission modes:

1. The symmetric fission mode due to strongly elongated shapes of the fissioning nucleus throughout the trajectory of descent from fission barrier to scission point and comparatively low values of the fragment kinetic energies.

2. The asymmetric mode (Standard I), conditioned by shells in the heavy fragments with deformations close to spherical ones and with masses close to 132.

3. A second asymmetric mode (Standard II), formed by shell effects in the heavy fragments with comparatively small deformation and with masses close to 138.

In low-energy fission of pre-actinide nuclei, as it becomes clear from fig. 1, the shapes of the $Y(m)$-distribution in the symmetric mode strongly depend on the nucleon composition of the fissioning nuclei and deviate significantly from Gaussian distributions predicted by the liquid-drop model. At the same time, at excitation energies $U \geq 25$ MeV, the mass distribution $Y(m)$ for pre-actinide nuclei like $^{207}$Tl (solid curves) becomes close to the Gaussian shape. Such behaviour of $Y(m)$ could be explained by the contribution of shell corrections to the potential energy of the saddle configurations and their influence on the arising fragments [4].
With increasing energy the influence of shell effects decreases, and $Y(m)$ goes toward the liquid-drop limit.

One should note that, generally speaking, it is necessary to consider the shell structure of a nucleus as a whole. But, the calculations within the framework of the shell-correction method [14,15] and the properties of heavy fragments of transactinide nuclei show that the shell structure of the whole nucleus in pre-scission configurations is essentially determined by shells in the arising fragments.

In contrast to $Y(m)$, the dependencies of $E_h(m)$ and $\sigma_h^2(m)$ in the symmetric mode conserve the smooth liquid-drop behaviour at all excitation energies. Taking into account the direct connection between the energy distributions and the configurations of the fissioning nuclei, one can consider this circumstance as an argument that the deformations corresponding to these shells are close to the large deformations which are optimal for liquid-drop fission. Therefore, the shell effects in the fission of pre-actinide nuclei are conditioned by strongly deformed shells. This conclusion is supported by a good agreement between liquid-drop momenta of inertia and experimental ones [16].

In order to clear up the question about the influence of proton number $Z$ and neutron number $N$ on the shell effects in the symmetric fission of pre-actinides let us look at fig. 2 which presents the data on $Y(m)$ for fissioning nuclei with $Z = 83 \div 85$ and $N = 124 \div 128$ at $U \approx 10$ MeV. One can see that at coinciding $N$ and different $Z$ the shapes of the distributions are practically the same (pairs $^{207}$Bi - $^{208}$Po, $^{212}$Po - $^{213}$At), but at coinciding $Z$ and different $N$ the shapes change very fast ($^{208}$Po, $^{210}$Po, $^{212}$Po). So, we suppose that namely the neutron number $N$ in the arising light and heavy fragments plays a predominant role in the formation of shell effects. An analogous situation arises in the fission of heavier nuclei [17,18], where a strong $N$-dependence of the shapes of the $Y(m)$ distributions is also observed. These experimental data have been explained within the shell-correction method, which demonstrates that for heavy
nuclei the values of neutron shell corrections exceed those of the protons by a factor of two and even more. Taking the above-mentioned properties of the symmetric fission mode as a basis, one can attempt to explain the alteration of \( Y(m) \)-distribution shapes, presented in fig. 1, by the existence of two strongly deformed neutron shells with "magic" values \( N_1 < 110/2 \) and \( N_2 > 128/2 \), considering that \(^{187}\text{Ir}\) has 110 neutrons and \(^{213}\text{At}\) has 128 neutrons. In this case, for \(^{187}\text{Ir}\) at \( m \approx A/2 \), the numbers of neutrons in the light and the heavy fragments are approximately equal (\( N_{1L} \approx N_{1H} = 55 \)), and the \( N_1 \)-shells in the light and the heavy fragments add up. This increases the relative yield of fragments with \( m \approx A/2 \). At the same time, the influence of the \( N_2 \)-shell on \( Y(m) \) is weaker, it just increases the yields in the tails of the distributions for heavy fragments with \( m > 109 \). In the fission of \(^{213}\text{At}\), the situation is reflecting that the \( N_2 \)-shell enhances the yield of fragments with \( m \approx A/2 \), and the \( N_1 \)-shell favours those of light fragments with \( m < 91 \). The appearance of sharply expressed two-humped distribution in the fission of \(^{201}\text{Tl}\) could be explained within this approach in the following way. The \( N_2 \)-shell enhances the yield of heavy fragments in the vicinity of masses \( m_{1H} \approx N_2 A/N \), and the \( N_1 \)-shell supports the yield of complementary light fragments with masses \( m_{1L} = (A - m) \approx N_1 A/N \). i.e. the shells \( N_1 \) and \( N_2 \) are located at approximately equal distances from \( A/2 \). This circumstance, altogether with relative symmetry of \( Y(m) \)-shapes for nuclei around \(^{201}\text{Tl}\) observed in fig. 1, allows to assume that there is a connection between \( N_1 \) and \( N_2 \), namely \( N_1 + N_2 \approx 120 \), where 120 is the total number of neutrons in \(^{201}\text{Tl}\).

The hypothesis about the existence of two strongly deformed neutron shells and the direct connection of the deformation potential energy of nuclei at the saddle point with the \( Y(m) \)-distribution became a basis of the proposed method for describing mass yields in the symmetric-mode fission in the region of pre-actinides.

3. ANALYSIS METHOD
In order to describe the mass distributions we used the method outlined in the works of Moretto and co-workers [13,19] and in our previous publications [2-6]. According to this approach, the yield of fragments with mass $m$ is defined by the probability to overcome the conditional fission barrier $B_f(m)$ corresponding to this mass. Following the Bohr-Wheeler formula [20] and the approximation proposed by Moretto[19], one can evaluate relative fragment yields normalised to 200\% by:

$$
\frac{Y(m)}{200\%} = \frac{\int_0^{E-B_f(m)} \rho_f(U)dU}{\sum_{m=0}^{\Lambda} \int_0^{E-B_f(m)} \rho_f(U)dU} \approx \frac{\theta_m \rho_f[E-B_f(m)]}{\sum_{m=0}^{\Lambda} \theta_m \rho_f[E-B_f(m)]}
$$

Here $\rho_f(U)$ is the level density, $\theta_m = \sqrt{(E-B_f(m))}/a$ - the temperature of the fissioning nucleus, $a$ - the level-density parameter in the transition state.

To describe $\rho_f(U)$ we used the relations of the nuclear superfluid model by semiphenomenologically taking into account the shell and collective effects [21], which has successfully been applied to the analysis of integral fission cross sections $\sigma_f$. Since here we consider fissioning nuclei formed in the reactions with low-energy light ions, an influence of angular momentum on $\rho(U)$ could be neglected, and so, the following expression is applicable [21,22]:

$$
\rho(U) = \frac{K_{rot} \exp(S_{f,})}{2\sqrt{2\pi}(J_\perp \theta J_\| \theta \det)^{1/2}} = C(\theta) \exp(S_{f,}).
$$

Here $S_f = 2\sqrt{aU} = 2a\theta$ is the entropy of the fissioning nucleus; $J_\perp, J_\|$ are the perpendicular and parallel momenta of inertia; $\det = 144\pi^2 a^3 \theta^5$; $K_{rot} = J_\perp \theta$ is the rotational enhancement factor of the level density.

At excitation energies above the critical energy of phase transition from the superfluid state to the Fermi-gas $U_{ct} = 0.472a_{ct} \Delta_n^2 - n\Delta_o$, Eq. (2) differs from the generally accepted
Fermi-gas expression by a shift in $U$ by the value of the condensation energy $E_{\text{con}} = 3a_n^2\Delta_n^2/2\pi^2 - n\Delta_n$ ($n = 0, 1, 2$ for even-even, odd-mass and odd-odd nuclei, respectively; $\Delta_n = 14/\sqrt{A}$ MeV — the correlation function of the fissioning nucleus; $a_n$ — the critical value of the level-density parameter).

In this approach, the influence of shell effects on $\rho(U)$ is taken into account by introducing the semiempirical dependence of the parameter $a$ on the effective excitation energy $U' = U - E_{\text{con}}$ and on the value of shell correction to the liquid-drop deformation energy $W_i$, namely [21]:

$$a(U) = \begin{cases} \tilde{a}\{1 + W_i(1 - \exp(-\lambda U'))/U'\}, & \text{at } U \geq U_{cr}, \\ a_{cr} = a(U_{cr}), & \text{at } U < U_{cr}. \end{cases}$$

(3)

where $\tilde{a} = 0.093A$ — the asymptotic value of a level density parameter, $\lambda = 0.064$ is the damping constant.

Below the energy of phase transition, the parameters in Eq. (2) are formulated in the following way [22]:

$$\Phi = (1 - U / U_{cr})^{1/2}; \quad \theta = 2\Phi\theta_{cr}/\ln[(1 + \Phi)/(1 - \Phi)]; \quad \theta_{cr} = 0.567\Delta_n;$$

$$S_i = 2a_n\theta_{cr}^2(1 - \Phi^2)/\theta; \quad \text{det} = 144\pi^{-1}a_n^2\theta_{cr}^4(1 - \Phi^2)(1 + \Phi^2);$$

$$J_4 = J_4^0\theta_{cr}^{-1}(1 - \Phi^2); \quad J_5 = J_5^0/3 + 2J_4^0\theta_{cr}^2(1 - \Phi^2)/3.$$

(4)

Substituting the expression for $\rho(U)$ in Eq. (1) and taking into account that the product $\theta_{cr}C(m)$ has a weak dependence on $m$, one can obtain the approximated expression for $Y(m)$:

$$Y(m)/200\% = \exp[S_i(m)] / \sum_{m=0}^{3}\exp[S_i(m)]$$

(5)

We use this simple expression in our analysis, since, according to test calculations, it is a good enough approximation for the Eq. (1).
The height of the conditional fission barrier $B_t(m)$ is defined as the difference between the potential energies of the nucleus at the ground and at the transition state, and within the shell correction method could be evaluated as:

$$B_t(m) = \left[ \tilde{V}_t(m) + W_t(m) \right] - \left[ \tilde{V}_g + W_g \right] = \left[ \tilde{V}_t(m) - \tilde{V}_g \right] - W_g + W_t(m). \quad (6)$$

Here $\tilde{V}_t(m)$ and $\tilde{V}_g$ are the liquid-drop constituents of the deformation potential energy in transition state (t) and ground state (g); $W_t(m)$ and $W_g$ are the shell corrections in these corresponding states.

The dependence of the liquid-drop potential energy of the fissioning nucleus on the mass of the arising fragments was described with a fast-convergent series [23]:

$$\tilde{V}_t(m) = \tilde{V}_t(2A) + \frac{1}{2} \frac{d^2 \tilde{V}_t(m)}{dm^2} \left( m - \frac{A}{2} \right)^2 + \cdots = \tilde{V}_t(2A) + \frac{8q}{A^2} \left( m - \frac{A}{2} \right)^2,$$

where q is the stiffness parameter of the liquid drop with respect to mass-asymmetric variations of the saddle shape.

So, finally for $B_t(m)$ one can write:

$$B_t(m) = (B_t^{1\ell} - W_g) + \frac{8q}{A^2} \left( m - \frac{A}{2} \right)^2 + W_t(m). \quad (7)$$

Liquid-drop fission barriers $B_t^{1\ell}$ and shell-correction values $W_g$ were taken from the calculations of Myers and Swiateski [9]. One should note that the calculated heights of the fission barriers $B_t = B_t^{1\ell} - W_g$ are in a good agreement with the experimental values $B_t$ [8].

In our approach, the shell correction $W_t$ is defined as the sum of shell corrections in the light and the heavy fragments:

$$W_t(m) = W_{t1}(m) + W_t(A - m) = W_{t1}(m \frac{N}{A}) + W_t(A - m \frac{N}{A}) = W_{t1}(N_{1\ell}) + W_t(N_{1t}). \quad (8)$$

The connection between $N_{1\ell}$, $H_{1\ell}$ and “magic” numbers of shells $N_1$, $N_2$ was established as:
\[ W_H(N_H) = \text{MIN}[F_1(N_H, N_1), F_2(N_H, N_2)] \]
\[ W_L(N_L) = \text{MIN}[F_1(N_L, N_1), F_2(N_L, N_2)] \]

(9)

where the function MIN selects the minimum value of \( F_1 \) and \( F_2 \). The functions \( F_i (i = 1, 2) \) which describe the dependencies of correction amplitudes on the number of neutrons in the fragment were parametrised by the simplest parabolic equations:

\[ F_i(N, N_i) = \begin{cases} b_i [1 - (N - N_i)^2/a_i^2] & \\
0, & \text{for } F_i \geq 0 
\end{cases} \]

(10)

So, the Eqs. (3) - (10) show that in order to solve the problem of describing the \( Y(m) \)-distributions, it is necessary to find a method to determine the values of the liquid-drop stiffness parameter \( q \) for a given nucleus and the shell parameters \( N, \alpha, \) and \( \beta, (i = 1, 2) \) which are common for all pre-actinide nuclei.

With this aim we have analysed 30 experimental \( Y(m) \)-distributions for fissioning nuclei from \(^{187}\text{Ir} \) to \(^{213}\text{At} \) in a range of excitation energies \( U = 7 \pm 25 \text{ MeV} \). These data were described with Eqs. (3) - (10), and values of the above-mentioned description parameters were determined within the least squares' method (the \( \chi^2 \) - method) using the code MINUIT [24]. The evaluation of these distributions was constrained so that the parameters \( N, \alpha, \) and \( \beta, \) are common for all distributions, and the parameter \( q \) can change for different nuclei. It should be noted that \( q \) is characteristic for the liquid-drop potential of a given nucleus and does not depend on the excitation energy \( U \). Nevertheless, in our analysis of \( Y(m) \)-distributions measured for the same nucleus at different excitation energies we allowed the parameter \( q \) to change within limits of \( \pm 5\% \). These variations enable to take into account possible errors of the experimental data which, for the values \( \sigma^2_m \sim 1/q \), amount to about 5\% [3,25].

4. RESULTS AND DISCUSSION
The resulting descriptions of the Y(m)-distributions for 9 nuclei at U ≈ 10 MeV are presented in fig. 3 and 4 where calculations are shown by solid curves, and experimental data correspond to open circles. From these figures one can see that the proposed approach allows to reproduce the sharp alterations of the individual shapes of the distributions at different nucleon compositions of the fissioning nuclei. As it was noted above, the deviations of the yields Y(m) observed in the logarithmic scale for nuclei heavier than 201Tl at m > 127 are conditioned by the contribution of the asymmetric modes, whose description is beyond to scope of the present work. And so, in order to exclude the influence of the asymmetric modes on the results of the analysis, this group of fragments was excluded from the parameter determination routine. As for the symmetric mode of fission, its description practically coincides with experimental data, and local distinctions between the experimental results and their description in Y(m) do not exceed two standard deviations. Taking into account the very simplified parametrization of F(N, N), the neglection of the proton shell contributions, and other approximations of the method, one should admit that the quality of the mass-yield description in dependence of Z and N of the fissioning nuclei is astonishing enough.

The description of the U-dependence of mass yields Y(m) is shown in fig. 5 and 6 for the best-studied nuclei 201Tl and 210Po. These figures demonstrate that the achieved description of temperature damping of the shells allows to reproduce the gradual transition of the experimental mass yields Y(m) to Gaussian-like distributions at U > 25 MeV. We should note that in test calculations with λ = 0.08 the agreement with the experiment was slightly better; the other description parameters which we extracted from the data fitting, remain practically the same. Maybe, the increase of the damping constant could be explained within the following assumption. Although real and liquid-drop deformations of fragments are close to each other, the influence of shell effects can cause some small modifications of fragment configurations in comparison with “pure” liquid drop ones. In this case, with growth of U the rearrangement of
shells is accompanied by a change of fragment shapes that can lead to some increase of $\lambda$. But, taking into account all uncertainties mentioned above and the approximations of the proposed description method, we should accept this explanation with care. This is the reason that all results of our work were represented with the recommended value [22] $\lambda = 0.064$.

As it was pointed out above, one of the objectives of our work is to design a method for determining the dependence of the liquid-drop stiffness parameter $q$ on the nucleon composition of a given fissioning nucleus. With this aim, the optimal values $d^2V/d\eta^2$ found in the analysis are presented in fig. 7 (closed circles) in their dependence on the fissility parameter $Z^2/A$. Open circles show the data on $d^2V/d\eta^2$ obtained by another independent method. Namely on the basis of the analysis of mass dispersions $\sigma^2_m$ measured in the high-energy fission induced by light charged particles and heavy ions [25] (in this analysis the definition of the effective excitation energy at the saddle takes into account the emission of some portion of pre-fission neutrons after the nucleus has passed the fission barrier). This figure demonstrates that within the limits of experimental scattering of the data both sets of data on $d^2V/d\eta^2 (Z^2/A)$ coincide. This is a good argument in favour of the method proposed in the present work. So, we recommend to use the smooth description (solid curve in fig. 7) of the dependence $d^2V/d\eta^2 (Z^2/A)$ obtained by Rusanov and co-workers [25].

The shell parameters used for the description of the experimental data of fig. 3 - 6 are represented in the table 1:

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>51.5</th>
<th>$N_2$</th>
<th>68.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>8.1</td>
<td>$\alpha_2$</td>
<td>5.6</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.37</td>
<td>$\beta_2$</td>
<td>-0.19</td>
</tr>
</tbody>
</table>
The fragment-mass dependencies of the liquid-drop (dashed curves) and full (solid curves) deformation potential energies calculated with these parameters for the nuclei $^{205}$TI and $^{210}$Po are shown in parts "C" in fig. 5 and 6. Parts "D" in these figures demonstrate the dependencies $W_1(m)$ and $W_2(m)$ for light (dashed curve) and heavy (dashed-dotted curve) fragments, respectively, and also the dependencies of $W_2(m)$ for a nucleus as a whole (solid curve). One can see that, though the contribution of shell constituents to the full potential energy of a nucleus is comparatively small, it visibly influences the shapes of the $Y(m)$-distributions.

Coming back to table 1, we should especially note that the extracted "magic" neutron numbers $N_1$ and $N_2$ are close to the neutron shells $N \approx 52$ and $N \approx 68$ appearing at very large deformation parameter $\beta \approx 1$ in the calculations of neutron shell corrections performed by Wilkins and co-workers [26]. This fact is a very important argument in favour of our hypothesis about the predominant role of strongly deformed neutron shells in the formation of fission-fragment mass yields for pre-actinides.

An additional evidence for the existence of the shells with $N_1 \approx 52$ and $N_2 \approx 68$ was obtained in describing $Y(m)$ within the least squares' method by means of more flexible poly-parametric functions for $F_i(N, N_i)$ like Sharlie distributions or asymmetric functions mathematically similar to the Woods-Saxon potential, for example. This calculations showed that the parameters $N_1, N_2$ and $q$ extracted by using the above-mentioned functions practically coincide with those found for parabolic dependencies.

At the same time, effective parameters of shell amplitudes and widths (analogous to the parameters $\alpha$ and $\beta$ for a parabola) visibly (up to 20%) differed from each other and from the values given in table 1. These distinctions point out that our analysis does not allow to make decisive conclusions about details of the functions $F_i(N, N_i)$. This limits the predictive power of the proposed approach to the description of $Y(m)$-distributions. Taking this limitation into
consideration, we recommend to use the parameters of table 1 for the description of the symmetric fission mode only for nuclei within the limits $A \approx 180 \div 220$.

5. CONCLUSIONS

Shell effects in the symmetric mode of fragment mass yields from the fission of pre-actinide nuclei from $^{187}$Ir to $^{213}$At could be explained and described if one assumes the existence of two strongly deformed neutron shells in the arising fragments with neutron numbers $N_1 \approx 52$ and $N_2 \approx 68$.

A new method has been proposed for quantitatively describing the mass distributions $Y(m)$ of the symmetric mode in the fission of pre-actinides with $A \approx 180 \div 220$.

The authors are grateful to Dr. Rusanov A. Ya. for helpful discussions at all stages of this work.
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24. CERN Computer 6600 series program Library Long-Write-UP "MINUIT".


List of figure captions.

1. Relative yields $Y$, total kinetic energies $E_k$ and kinetic-energy dispersions $\sigma^2_k$ as a function of fragment mass. The experimental data points (●) correspond to fission at excitation energy $U \approx 10$ MeV; for $^{201}$Tl the lines show in addition the experimental data taken at $U \approx 25$ MeV.

2. Experimental relative yields $Y$ versus fragment mass for nuclei from $^{207}$Bi to $^{213}$At at an excitation energy $U \approx 10$ MeV.

3. Relative yields $Y$ and their description versus fragment mass for nuclei from $^{187}$Ir to $^{213}$At at an excitation energy $U \approx 10$ MeV. The experimental data (○) and the proposed description (—) are shown.

4. Relative yields $Y$ and their description versus fragment mass for nuclei from $^{187}$Ir to $^{213}$At at an excitation energy $U \approx 10$ MeV in a logarithmic scale. The experimental data (○) and the proposed description (—) are shown.

5. Relative yields $Y$, their description, calculated potential energy and its constituents versus fragment mass for $^{201}$Tl at different excitation energies. Part "A": (○) the experimental yields $Y$ and (—) the proposed description in a linear scale; part "B": (○) the experimental yields $Y$ and (—) the proposed description in a logarithmic scale; part "C": full (—) and liquid-drop (− −) potential energies; part "D": calculated shell corrections in the light fragment group $W_L$ (− −), in the heavy fragment group $W_H$ (− ⋅ −), and for the whole nucleus $W$ (− −).

6. Relative yields $Y$, their description, calculated potential energy and its constituents versus fragment mass for $^{210}$Po at different excitation energies. Part "A": (○) the experimental yields $Y$ and (—) the proposed description in a linear scale; part "B": (○) the experimental yields $Y$ and (—) the proposed description in a logarithmic scale; part "C": full (—) and
liquid-drop (---) potential energies; part "D": calculated shell corrections in the light fragment group $W_l$ (---), in the heavy fragment group $W_h$ (-----), and for the whole nucleus $W_f$ (---).

7. The liquid-drop stiffness with respect to mass-asymmetric deformations at the saddle point $d^2V/d\eta^2$ in the dependence on the fissility parameter $Z^2/A$. (•) The optimal values found in our analysis for nuclei from 187Ir to 213At at excitation energies $U \approx 7 - 25$ MeV, (o) experimental values and (---) their description from Ref. [25].
Fig. 4
Figure 5
Fig 6