Phase Transitions in Neutron Stars

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Abstract
Phase transitions in neutron stars due to formation of quark matter, kaon condensates, etc. are discussed with particular attention to the order of these transitions. Observational consequences of phase transitions in pulsar angular velocities are examined.

1 Introduction
The physical state of matter in the interiors of neutron stars at densities above a few times normal nuclear matter densities is essentially unknown. Interesting phase transitions in nuclear matter to quark matter [1, 2], kaon [3, 4] or pion condensates [5, 6], neutron and proton superfluidity [7], hyperonic matter, crystalline nuclear matter [5], magnetized matter, etc., have been considered. We discuss how these phase transitions may exist in a mixed phase, the structures formed and in particular the order of the transition. Observational consequences are discussed.

2 Mixed Phases and Order of Transitions
The mixed phase in the inner crust of neutron stars consists of nuclear matter and a neutron gas in $\beta$-equilibrium with a background of electrons such that the matter is overall electrically neutral [8]. Likewise, quark and nuclear matter can have a mixed phase [1] and possible also nuclear matter with and without condensate of any negatively charged particles such as $K^-$ [9], $\pi^-$, $\Sigma^-$, etc. The quarks are confined in droplet, rod- and plate-like structures [2] analogous to the nuclear matter and neutron gas structures in the inner crust of neutron stars [8]. Depending on the equation of state, normal nuclear matter exists only at moderate densities, $\rho \sim 1 - 2\rho_0$. With increasing density, droplets of quark matter form in nuclear matter and may merge into rod- and later plate-like structures. At even higher densities the structures invert forming plates, rods and droplets of nuclear matter in quark matter. Finally pure quark matter is
formed at very high densities unless the star already has exceeded its maximum mass.

A necessary condition for forming these structures and the mixed phase is that the additional surface and Coulomb energies of these structures are sufficiently small. Excluding them makes the mixed phase energetically favored [1]. That is also the case when surface energies are small (see [2] for a quantitative condition). If they are too large the neutron star will have a core of pure quark matter with a mantle of nuclear matter surrounding and the two phases are coexisting by an ordinary first order phase transition.

The quark and nuclear matter mixed phase has continuous pressures and densities [1] when surface and Coulomb energies are excluded. There are at most two second order phase transitions. Namely, at a lower density, where quark matter first appears in nuclear matter, and at a very high density, where all nucleons are finally dissolved into quark matter, if the star is gravitationally stable at such high central densities. However, due to the finite Coulomb and surface energies associated with forming these structures, the transitions change from second to first order at each topological change in structure [2]. If the surface and Coulomb energies are very small the transitions will be only weakly first order but there may be several of them.

Fig. 1. Nuclear and quark matter structures in a $\sim 1.4M_\odot$ neutron star. Typical sizes of structures are $\sim 10^{-14}m$ but have been scaled up to be seen.
3 Rotation

As rotating neutron stars slow down, the pressure and the density in the core region increase due to the decreasing centrifugal forces and phase transitions may occur in the center. At the critical angular velocity $\Omega_0$, where the phase transitions occur in the center of a neutron star, we find that the moment of inertia, angular velocity, braking index etc. change in a characteristic way.

The general relativistic equations for slowly rotating stars were described by Hartle [10]. Hartle’s equations are quite elaborate to solve as they consist of six coupled differential equations as compared to the single TOV equation in the non-rotating case. However, Hartle’s equations cannot be used in our case because the first order phase transition causes discontinuities in densities so that changes are not small locally. This shows up, for example, in the divergent thermodynamic derive $d\rho/dP$. From Einstein’s field equations for the metric we obtain from the $l = 0$ part the generalized rotating version of the TOV equation [11]

$$\frac{1}{\rho + P} \frac{dP}{da} = -G \frac{m + 4\pi a^3 P}{a^2(1 - 2Gm/a)} + \frac{2}{3} \Omega^2 a,$$

where $m(a) = 4\pi \int_0^a \rho(a')a'^2 da'$. In the centrifugal force term we have ignored frame dragging and other corrections of order $\Omega^2 GM/R \sim 0.1\Omega^2$ for simplicity and since they have only minor effects in our case.

The rotating version of the TOV equation (1) can now be solved for rotating neutron stars of a given mass and equation of state with a phase transition. If a first order phase transition occur in the center at $\Omega_0$ the moment of inertia is generally [11]

$$I(\Omega) = I_0 \left( 1 + \frac{1}{2} c_1 \frac{\Omega^2}{\Omega_0^2} - \frac{2}{3} c_2 (1 - \frac{\Omega^2}{\Omega_0^2})^{3/2} + \ldots \right),$$

for $\Omega \leq \Omega_0$. Here, $c_2 \sim \Delta \rho \Omega_0^3$ is a small number proportional to the density difference between the two phases and to the critical angular velocity to 3rd power. For $\Omega > \Omega_0$ the $c_2$ term vanishes.

The pulsars slow down at a rate given by the loss of rotational energy which commonly is assumed proportional to the rotational angular velocity to some power (for dipole radiation $n = 3$)

$$\frac{d}{dt} \left( \frac{1}{2} I(\Omega) \Omega^2 \right) = -C\Omega^{n+1}.$$
Consequently, as the braking index depends on the second derivative $I'' = dI/d^2\Omega$ of the moment of inertia and thus diverges as $\Omega$ approaches $\Omega_0$ from below

$$n(\Omega) \equiv \frac{\dot{\Omega}_\Omega}{\Omega^2} = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega} \simeq n - 2c_1\frac{\Omega^2}{\Omega_0^2} + c_2\frac{\Omega^4/\Omega_0^4}{\sqrt{1 - \Omega^2/\Omega_0^2}}. \quad (4)$$

For $\Omega \geq \Omega_0$ the term with $c_2$ is absent.

4 Cooling

If the matter undergoes a phase transition at a critical temperature but is supercooled as the star cools down by neutrino emission, a large glitch will occur when the matter transforms to its equilibrium state. If the cooling is continuous the temperature will decrease with star radius and time and the phase transition boundary will move inwards. The two phases could, e.g., be quark-gluon/nuclear matter or a melted/solid phase. In the latter case the size of the hot (melted) matter in the core is slowly reduced as the temperature drops freezing the fluid. Melting temperatures have been estimated in [8] for the crust and in [2] for the quark matter mixed phase. Depending on whether the matter contracts as it freezes as most terrestrial metals or expands as ice, the cooling will separate the matter in a liquid core of lower or higher density respectively and a solid mantle around. When the very core freezes we have a similar situation as when the star slows down to the critical angular velocity, i.e., a first order phase transition occurs right at the center. Consequently, a similar behavior for the moment of inertia, angular velocities, braking index may occur by replacing $\Omega(t)$ with $T(t)$ in Eqs. (2-4).

5 Glitches

The glitches observed in the Crab, Vela, and a few other pulsars are probably due to quakes occurring in solid structures such as the crust, superfluid vortices or possibly the quark matter lattice in the core [2]. These glitches are very small $\Delta\Omega/\Omega \sim 10^{-8}$ and have a characteristic healing time.

In [12] a drastic softening of the equation of state by a phase transition to quark matter leads to a sudden contraction of the neutron star at a critical angular velocity and shows up in a backbending moment of inertia as function of frequency. As a result, the star will become unstable as it slows down, will suddenly decrease its moment of inertia and create a large glitch. If the matter supercools and makes a sudden transition to its stable phase the densities may
also change and the star will have to contract or expand. Consequently, a large glitch will be observed.

6 Summary

We have discussed various possible phase transitions in neutron stars and have argued that we expect several first order order phase transitions to occur when the topological structure of the mixed phase change in the inner crust, nuclear and quark matter mixed phase or Kaon condensates. If a first order phase transitions is present at central densities of neutron stars, it will show up in moment of inertia and consequently also in angular velocities in a characteristic way. For example, the braking index diverges as \( n(\Omega) \sim c_2/\sqrt{1 - \Omega^2/\Omega_0^2} \).

References