**D-term Inflation in Superstring Theories**

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**Abstract**

An inflationary stage dominated by a $D$-term avoids the slow-roll problem of inflation in supergravity and may emerge in theories with a non-anomalous or anomalous $U(1)$ gauge symmetry. The most intriguing and commonly invoked possibility is that the Fayet-Iliopoulos $D$-term triggering inflation is the one emerging in superstring theories. We discuss the complications one has to face when trying to build up a successful $D$-term inflationary scenario in superstring models. In particular, we show that the “vacuum shifting” phenomenon of string theories is usually very efficient even in the early Universe, thus preventing inflation from taking place. On the other hand, when $D$-term inflation is free to occur, the presence of a plethora of fields and several non-anomalous additional abelian symmetries in string theories may help in reconciling the value of the Fayet-Iliopoulos $D$-term required by the COBE normalization with the value predicted by string theories. We also show that in superstring $D$-term inflation gravitinos are likely to pose no cosmological problem.

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The flatness and the horizon problems of the standard big bang cosmology are elegantly solved if during the evolution of the early Universe the energy density happens to be dominated by some form of vacuum energy and comoving scales grow quasi-exponentially [1]. An inflationary stage is also required to dilute any undesirable topological defects left as remnants after some phase transition taking place at early epochs. The vacuum energy driving inflation is generally assumed to be associated with some scalar field \( \phi \), the inflaton, which is initially displaced from the minimum of its potential. As a by-product, quantum fluctuations of the inflaton field may be the seeds for the generation of structure. The level of density and temperature fluctuations observed in the present Universe, \( \delta \rho / \rho \sim 10^{-5} \), require the inflaton potential to be extremely flat. This means that the couplings of the inflaton field to the other degrees of freedom cannot be too large: large couplings induce large loop corrections to the inflaton potential, spoiling its flatness. This is the main reason why inflation is more natural in the context of supersymmetric theories. Introducing very small parameters to ensure the extreme flatness of the inflaton potential seems very fine-tuned in most non-supersymmetric theories, while this naturalness is achieved in supersymmetric models. The nonrenormalization theorems in exact global supersymmetry guarantee that we can fine-tune any parameter at the tree-level and this fine-tuning will not be destabilized by radiative corrections at any order in perturbation theory [2].

There is, however, a severe problem one has to face when dealing with supersymmetric inflation model building in the context of supersymmetric theories. The generalization of supersymmetry from a global to a local symmetry automatically incorporates gravity and, therefore, inflation model building must be considered in the framework of supergravity theories. The supergravity potential \( V \) consists of two pieces, the so-called \( D \)-term and \( F \)-term. For models where the \( D \)-term vanishes, the slow-roll parameter \( \eta = M_{Pl}^2 V'' / V \), where \( M_{Pl} \simeq 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck scale, generically receives various contributions of order \( \pm 1 \) [3]. This is the so-called \( \eta \)-problem of supergravity theories: there are contributions of order \( \pm H^2 \) to the mass-squared of every scalar field and the troublesome contributions to \( \eta \) may be regarded as contributions to the coefficient \( m^2 \) in the expansion of the inflaton potential. Therefore, it is very difficult naturally to implement a slow-roll inflation in the context of supergravity. The problem basically arises since inflation, by definition, breaks global supersymmetry because of a nonvanishing cosmological constant (the false vacuum energy density of the inflaton). In supergravity theories, supersymmetry breaking is transmitted by gravity interactions and the squared mass of the inflaton becomes
naturally of order of $V/M^2_P \sim H^2$. The perturbative renormalization of the Kähler potential is therefore crucial for the inflationary dynamics due to a non-zero energy density which breaks supersymmetry spontaneously during inflation. How severe the problem is depends on the magnitude of $\eta$. If $\eta$ is not too small then its smallness could be due to accidental cancellations. Having $\eta$ not too small requires that the spectral index $n = 1 - 6\epsilon + 2\eta$ [$\epsilon = \frac{1}{2}M^2_P(V'/V)^2$ is another slow-roll parameter] be not too small, so the observational bound $|n - 1| < 0.3$ is already beginning to make an accident look unlikely.

2. Solutions to the $\eta$-problem already exist in the literature [4]. Among them, $D$-term inflation seems to be particularly promising [5–7]. It is based on the observation that, when the vacuum energy density is dominated by nonzero $D$-terms and supersymmetry breaking is of the $D$-type, scalars get supersymmetry soft breaking masses which depend only on their gauge charges. Scalars charged under the corresponding gauge symmetry obtain a mass much larger than $H$, while gauge singlet fields can only get masses from loop gauge interactions. In particular, if the inflaton field is identified with a gauge singlet, its potential may be flat up to loop corrections and supergravity corrections to $\eta$ from the $F$-terms are not present since the latter vanish during inflation. The toy model adopted in Refs. [6,7] contains three chiral superfields $S$, $\Phi_+$ and $\Phi_-$ with charges equal to 0, +1 and −1 respectively under a $U(1)$ gauge symmetry. The superpotential has the form

$$W = \lambda S\Phi_+\Phi_-.$$  

The scalar potential in the global supersymmetry limit reads

$$V = \lambda^2 |S|^2 \left( |\phi_-|^2 + |\phi_+|^2 \right) + \lambda^2 |\phi_+\phi_-|^2 + \frac{g^2}{2} \left( |\phi_+|^2 - |\phi_-|^2 + \xi \right)^2$$

where $\phi_\pm$ are the scalar fields of the supermultiplets $\Phi_\pm$, $g$ is the gauge coupling and what is crucial is the presence of the Fayet-Iliopoulos (FI) $D$-term $\xi > 0$. The global minimum is supersymmetry conserving, but the gauge group $U(1)$ is spontaneously broken

$$\langle S \rangle = \langle \phi_+ \rangle = 0, \quad \langle \phi_- \rangle = \sqrt{\xi}.$$  

*It is interesting to notice that $D$-term inflation may provide a solution to the moduli problem too [8].

†See, however, Ref. [9] for further comments.
However, if we minimize the potential, for fixed values of $S$, with respect to other fields, we find that for $S > S_c = g\sqrt{\xi}/\lambda$, the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $S > S_c$ and $\phi_+ = \phi_- = 0$ the tree level potential has a vanishing curvature in the $S$ direction and large positive curvature in the remaining two directions $m^2_{\pm} = \lambda^2|S|^2 \mp g^2\xi$. For arbitrarily large $S$ the tree level value of the potential remains constant, $V = g^2\xi^2/2$, and $S$ plays the role of the inflaton. As stated above, the charged fields get very large masses due to the $D$-term supersymmetry breaking, whereas the gauge singlet field is massless at the tree-level.

Along the inflationary trajectory $\phi_\pm = 0$, $S \gg S_c$, all the $F$-terms vanish and large supergravity corrections to the $\eta$-parameter do not appear. Therefore, we do not need to make any assumption about the structure of the Kähler potential for the $S$-field: minimal $S^*S$ and non-minimal quartic terms in the Kähler potential $(S^*S)^2$ (or even higher orders) do not contribute in the curvature, since $F_S$ is vanishing during inflation.

Since the energy density is dominated by the $D$-term, supersymmetry is broken and this amounts to splitting the masses of the scalar and fermionic components of $\Phi_\pm$. Such splitting results in the one-loop effective potential for the inflaton field

$$V_{\text{1-loop}} = \frac{g^2}{2} \xi^2 \left( 1 + \frac{g^2}{8\pi^2} \ln \frac{\lambda^2|S|^2}{\mu^2} \right),$$

where $\mu^2$ is the renormalisation scale. Equivalently,

$$V_{\text{1-loop}} = \frac{g^2}{2} (\mu^2 = \lambda^2|S|^2) \xi^2,$$

where the loop-correction is absorbed in the gauge coupling running with the supersymmetric one-loop RG

$$\mu \frac{dg^2}{d\mu} = \frac{1}{16\pi^2} g^4 \sum_i Q_i^2,$$

with the sum extending to all the fields in the model. Note that scale invariance of the effective potential $dV/d\mu = 0$ can be used to evaluate the coefficient of the one-loop logarithmic contribution directly.

The end of inflation is determined either by the failure of the slow-roll conditions or when $S$ approaches $S_c$. COBE [10] imposes the following normalization

$$5.3 \times 10^{-4} = \left( \frac{V^{3/2}}{V^* M_{Pl}^3} \right),$$

where the $*$ denotes this is to be evaluated at the scale at which the relevant fluctuations leave the horizon. This can be written in the equivalent form
\[
\left( \frac{V^{1/4}}{e^{1/4}} \right)_o = 8 \times 10^{16} \text{ GeV}.
\] (8)

More or less independently of the value of \( |S| \) at the end of inflation, this gives with the above potential

\[
\sqrt{\xi_{\text{COBE}}} = 6.6 \times 10^{15} \text{ GeV}.
\] (9)

This normalization is independent of the gauge coupling constant \( g \) in the toy model, but if we write the potential during inflation as

\[
V = V_0 (1 + c \ln |S|),
\] (10)

it depends on the numerical coefficient \( c \) in the one-loop potential and it scales like \( e^{1/4} \). This, in turn, depends upon the particle content of the specific model under consideration through the RG beta function of the gauge coupling, as explained above.

Notice that, if the theory contains an abelian \( U(1) \) gauge symmetry (anomalous or not), the FI \( D \)-term

\[
\xi \int d^4 \theta \ V = \xi D
\] (11)

is gauge invariant and therefore allowed by the symmetries and it may lead to \( D \)-type supersymmetry breaking. It is important to notice that this term may be present in the underlying theory from the very beginning. Successful \( D \)-term inflation models based on this observation have been constructed in the framework of Grand Unified Theories (GUT's) [11].

In the rest of the paper, however, we would like to focus on a more intriguing possibility that is usually invoked to motivate \( D \)-term inflation: in string theories, there can be a \( U(1)_A \) gauge symmetries which is anomalous. This means that \( \text{Tr} Q_A \neq 0 \) where the trace is evaluated over the massless string states. Indeed, string theory provides a different mechanism, the Green-Schwarz mechanism [12], by which the anomaly may be cancelled even though the trace is nonvanishing. Such a nonvanishing trace leads to the appearance of a one-loop FI \( D \)-term of the form [13]

\[
\xi = \frac{g^2}{192 \pi^2} \text{Tr} Q_A M_{Pl}^2.
\] (12)

Then \( \sqrt{\xi} \) is expected to be of the order of the string scale, \((10^{17} - 10^{18}) \text{ GeV} \) or so.
3. There are still many open questions related to $D$-term inflation in the framework of superstring theories:

-**The Fayet-Iliopoulos $D$-term from string theories faces the COBE normalization**-

Comparing the value of the FI $D$-term normalized according to COBE to the value predicted by string theories, it is clear that the $\xi_{\text{COBE}}$ looks too small to be consistent with the value arising in many compactifications of the heterotic string. Notice that this problem may be exacerbated if cosmic strings are formed at the end of inflation [14]. Some level of flexibility may be allowed in the case in which, in the strong coupling limit, the ten-dimensional $E_8 \otimes E_8$ heterotic string can be described as the compactification of an eleven-dimensional theory known as M-theory [15]. When the ten-dimensional heterotic coupling is large, the fundamental eleven-dimensional mass parameter $M_{11}$ becomes of the order of the unification scale and it might be that the value of the FI $D$-term may be reduced to a value close to the one required by COBE. However, to our knowledge no explicit example has been constructed so far. It is also clear that the whole issue is strictly related to another unsolved problem in heterotic string theories, the stabilisation of the dilaton field. Since the dilaton potential most likely is strongly influenced by the inflationary dynamics, the actual value of $\xi$ at the moment when observationally interesting scales crossed the horizon during inflation might be quite different from the one "observed" today [16–18].

-**Constructing a viable model**-

The presence of the FI $D$-term (12) leads to the breaking of supersymmetry at the one-loop order at very high scale, an option which is not phenomenologically viable. The standard solution to this puzzle is to give a nonvanishing vacuum expectation values (VEV's) to some of the scalar fields which are present in the string model and are charged under the anomalous $U(1)_A$. In such a way, the FI $D$-term is cancelled and supersymmetry is preserved. In the context of string theory, this procedure is called "vacuum shifting" since it amounts to moving to a point where the string ground state is stable. While maintaining the $D$- and $F$-flatness of the effective field theory, such vacuum shifting may have important consequences for the phenomenology of the string theory. Indeed, the vacuum shifting not only breaks the $U(1)_A$, but may also break some other gauge symmetries under which the fields which acquire a VEV are charged. This is because the anomalous $U(1)_A$ is usually accompanied by a plethora of nonanomalous abelian symmetries.

The vacuum shifting can generate effective superpotential mass terms for vector-like states that would otherwise remain massless or may even be responsible for the soft mass
terms of squarks and sleptons at the TeV scale [19].

What is relevant for our considerations is that in string theories the protection of supersymmetry against the effects of the anomalous $U(1)_A$ is extremely efficient. If we now apply a sort of “minimal principle” [20,17] requiring that a successful scenario of $D$-term inflation should arise from a “realistic” string model leading to the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge structure at low energies, the requirement of cancellation of the FI $D$-term by the vacuum shifting mechanism may (and usually does) represent a serious problem. In other words, how can we guarantee that during inflation the FI $D$-term is not cancelled by one of the many scalar fields which are charged under the anomalous $U(1)_A$ and are not coupled to the inflaton? Does a successful $D$-term inflationary scenario in string theory require many inflatons to render the vacuum shifting mechanism inoperative? Is it possible that the presence of several fields and non-anomalous $U(1)$’s may solve the problem of the mismatch between the value of the Fayet-Iliopoulos $D$-term required by the COBE normalization and the value predicted by string theories? Do gravitinos or other dangerous relics pose any cosmological problem? It is clear that only a systematic analysis of specific models can answer these and similar questions. This requires the identification of possible inflatons and $D$- and $F$-flat directions for a large class of perturbative string vacua. This classification [21] is a prerequisite to address systematically the issue of $D$-term inflation in string theories as well as the phenomenological issues at low energy [21–23].

4. As an illustrative example of the possible complications one has to face in building up a successful model of $D$-term inflation in the framework of 4D string models, we consider the massless spectrum of a compactification on a Calabi-Yau manifold with Hodge numbers $h_{1,1}, h_{2,1}$, etc. The four-dimensional gauge group is $SO(26) \times U(1)$. There are then $h_{1,1}$ left-handed chiral supermultiplets transforming as $(26, \sqrt{\frac{1}{3}}) \oplus (1, -2\sqrt{\frac{1}{3}})$ and $h_{2,1}$ supermultiplets transforming as $(26, -\sqrt{\frac{1}{3}}) \oplus (1, 2\sqrt{\frac{1}{3}})$. In this case the $U(1)$ is clearly anomalous. The one-loop $D$-term is given by

$$
\xi = \frac{g^2 M_P^2}{192 \pi^2} \sum_i n_i q_i h_i = \frac{g^2 M_P^2}{192 \pi^2} \cdot 2 \cdot \frac{24}{\sqrt{3}} (h_{1,1} - h_{2,1})
$$

where the sum is over the (positive helicity) states with $U(1)$ charge $q_i$ and multiplicity $n_i$. It is positive if the Euler characteristic $\chi = 2(h_{1,1} - h_{2,1}) > 0$, the sign being fixed by the dominant contribution of the $SO(26)$ non-singlet fields. In addition we suppose the model has a gauge singlet field $S$ which will play the role of the inflaton. Further we assume that there is a discrete $R$-symmetry that ensures $S$-flatness. These assumptions are quite ad hoc
and in a realistic model we would have to demonstrate the existence of such a field, but we use this simple example to illustrate another problem that must be overcome if one is to obtain a realistic string model of $D$-term inflation.

With this field we may try to construct an inflationary potential along the lines of Eq (2). In particular one may generate masses for the $h_{2,1}$ vectorlike combinations of the $SO(26)$ singlet and non-singlet fields via the couplings in the superpotential of the form

$$W = \lambda S \left[ (26, \sqrt{1/3}) \cdot (26, -\sqrt{1/3}) + (1, -2\sqrt{1/3}) \cdot (1, 2\sqrt{1/3}) \right].$$

(14)

This leaves light $(h_{1,1} - h_{2,1})$ fields transforming as $(26, \sqrt{1/3}) \oplus (1, -2\sqrt{1/3})$ and leaves unchanged Eq. 13. Only the $SO(26)$ singlet fields, $\phi_i$, are now available to cancel the anomalous $D$-term and indeed their tree-level couplings to $D$ are negative $:\Sigma_i Q_i |\phi_i|^2 < 0$, as is expected if supersymmetry is not to be broken by the FI $D$-term. However this prevents us from implementing $D$-term inflation because the scalar potential dependence on the $\phi_i$ fields arises only through the anomalous $D$-term of the form

$$\frac{g^2}{2} \left( \xi + \sum_i Q_i |\phi_i|^2 \right)^2.$$

(15)

The vacuum expectation values of the fields $\phi_i$ will rapidly flow to cancel the $D$-term preventing inflation from occurring.

This example illustrates the problem in implementing $D$-term inflation in a string theory. It arises because the minimum of the potential should not break supersymmetry through the anomalous $D$-term and so there must be light fields (here the $\phi_i$) with the appropriate $U(1)$ charge to cancel it. To implement $D$-term inflation these fields must acquire a mass for large values of $S$ but this was not possible in this example because the $\phi_i$ were protected by chirality from acquiring mass by coupling to the $S$ field.

Thus we conclude that it is crucial to consider all fields with non-trivial $U(1)$ quantum numbers when discussing the possible inflationary potential in the framework of superstring theories.

5. We will consider now further examples to capture other possible aspects of $D$-term inflation in superstring theories. For illustrative purposes, we will use the specific string models, discussed in [2,24] whose space of flat directions was recently analyzed in [21]. The emphasis will be on exploring the different possibilities that may be realized rather than proposing a working model of inflation. In so doing we will often restrict the analysis to some subset of the fields present in the model and ignore the rest. In view of what we concluded
above, this is not consistent, but the examples that follow should only be considered as toy models attempting to capture some of the stringy characteristics one should expect when trying to construct a fully realistic model of $D$-term inflation in superstring inspired scenarios.

The presence of several (non-anomalous) additional $U(1)$ factors is a generic property of string models. For the discussion of $D$-term inflation, the relevant objects are thus no longer single elementary fields but rather multiple-field directions in field space along which the $D$-term potential of the non-anomalous $U(1)$’s vanishes [22]. These directions would be truly flat if an anomalous $U(1)_A$ (or some $F$-terms) were not present. To study whether a given direction remains flat in the presence of the anomalous $U(1)_A$, the important quantity is the anomalous charge along the direction. If the sign of this charge is opposite to that of the Fayet-Iliopoulos term, VEVs along the flat direction will adjust themselves to cancel the FI term and give a zero potential. If the charge has the same sign of the FI term, the potential along that direction rises steeply with increasing values of the field. The interesting case corresponds to zero anomalous charge, in which case the potential along the given direction is flat and equals, at tree level, $g_A^2 \xi^2/2$. The parallelism with the toy model of section 2 is evident.

The condition $Q_A = 0$ ensuring tree-level flatness is not by itself sufficient. We must also require that the direction is stable for large values of the field, that is, all masses deep in the inflaton direction must be positive (or zero). However the presence of the FI term in the scalar potential can induce negative masses for those fields which have a negative anomalous charge (recall we are taking $\xi > 0$):

$$\delta m_i^2 = g_A^2 Q_A^i \xi.$$  

(16)

To ensure that masses are positive in the end one can use $F$-term contributions (to balance the negative FI-induced masses) coming from superpotential terms of the generic form

$$\delta W = \lambda \langle I' \rangle \Phi_+ \Phi_-,$$

(17)

where $I'$ stands for some product of fields that enter the inflaton direction while $\Phi_\pm$ do not. Fields of type $\Phi_+$ and $\Phi_-$ which couple to the inflaton direction in the superpotential terms get a large $F$-term mass, $\lambda \langle I' \rangle$.

Consider the simplest example, a toy model with two chiral fields $S_1$ and $S_2$ of opposite $U(1)$ charges, so that the direction $|S| = |S_1| = |S_2|$ can play the role of the inflaton. Assume
that deep in this direction ($S \gg \sqrt{\xi}$) the masses of all fields are positive (or zero) and thus no other VEVs are triggered. Then we can minimize the $D$-term scalar potential

\[ V_D = \frac{1}{2} g_A^2 \left[ Q_1^4 \left( |S_1|^2 - |S_2|^2 \right) + \sum Q_i^4 |\phi_i|^2 + \xi \right]^2 + \frac{1}{2} \sum g_{\alpha}^2 \left[ Q_1^4 \left( |S_1|^2 - |S_2|^2 \right) + \sum Q_i^4 |\phi_i|^2 \right]^2, \]  

(18)

[where $\alpha = 1, \ldots, n$ counts the additional $D$-term contributions of the non-anomalous $U(1)$'s] for $S_1$ and $S_2$ only.

If $\xi = 0$, $|S_1| = |S_2|$ is flat and necessarily stable, as $V = 0$. For $\xi > 0$ however, the flat direction is slightly displaced and lies at

\[ \delta S^2 \equiv |S_1|^2 - |S_2|^2 = -\frac{g_A^2}{G_{11}^2} Q_1^4 \xi, \]  

(19)

where $G_{ij}^2 = g_A^2 Q_i^4 Q_j^4 + \sum g_{\alpha}^2 Q_i^\alpha Q_j^\alpha$. This displacement is the result of the destabilization effect of $\xi$ referred to above and occurs when the fields in the inflaton direction carry anomalous charge: as the inflaton direction must have zero anomalous charge, the fields forming it have anomalous charges of opposite signs and one of them will get a negative mass of the form (16). Notice that a term like (17) but with say $\Phi_-$ belonging to the inflaton direction cannot be used to stabilize inflaton fields, because it would spoil the $F$-flatness of the inflaton direction.

Taking into account this displacement, the value of the potential along the inflaton direction is, at tree level

\[ V_0 = \frac{1}{2} \frac{g_A^2}{G_{11}^2} \xi^2 \sum g_{\alpha}^2 (Q_1^\alpha)^2 \leq \frac{1}{2} g_A^2 \xi^2 \leq \frac{1}{2} g_A^2 \xi^2. \]  

(20)

A few comments on this result are in order. We first realize that $\xi_{eff}$ entering the estimates for a successful inflation can be smaller than the naive $\xi$. We will discuss later whether this can improve the COBE constraint (9). Note also that, if $Q_1^4 = 0$ (i.e. if the fields forming the inflaton do not carry anomalous charge) then $\xi_{eff} = \xi$. If, on the other hand, $Q_i^\alpha = 0$ [i.e. if these fields do not carry charges under the additional $U(1)$'s], then $\xi_{eff} \to 0$, and we recover the vacuum shifting phenomenon described in the previous section. Thus we

\[ ^* \]

\[ ^4 \]

In writing this potential we are assuming for simplicity that kinetic mixing of different $U(1)$'s is absent. For this to be a consistent assumption the vanishing of $\text{Tr}(Q_A Q_\alpha)$ and $\text{Tr}(Q_\alpha Q_\beta)$ is a necessary condition.

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see how the presence of additional, non-anomalous $U(1)$'s can play a very significant role in preventing the relaxation of the potential to zero in these inflationary models.

For a viable inflationary model we should ensure that the one-loop potential is appropriate to give a slow roll along the inflaton direction. Thus, we must consider the one-loop corrections proportional to the Yukawa couplings introduced in the terms of eq. (17). The field-dependent masses for the scalar components of the chiral fields $\Phi_{\pm}$ along the inflaton direction are

$$m_{\pm}^2 = \lambda^2 \langle P \rangle^2 + g_A^2 Q_{\pm}^A (Q_{\pm}^A \delta S^2 + \xi) + \sum_{\alpha} g_{\alpha}^2 Q_{\pm}^\alpha \delta = \lambda^2 \langle P \rangle^2 + g_A^2 a_{\pm} \xi,$$

while the fermionic partners have masses-squared equal to $\lambda^2 \langle P \rangle^2$. For large values of the field $\langle P \rangle$, the one-loop potential takes the form

$$32 \pi^2 \delta V_1 = 2 g_A^2 (a_+ + a_-) \lambda^2 \langle P \rangle^2 \xi \left( \log \frac{\lambda^2 \langle P \rangle^2}{Q^2} - 1 \right) + g_A^4 (a_+^2 + a_-^2) \xi^2 \log \frac{\lambda^2 \langle P \rangle^2}{Q^2}. \quad (22)$$

In this more complicated model the scalar direction transverse to the inflaton gains a very large mass deep in the inflaton direction. In addition, the gauge boson corresponding to the broken $U(1)$ symmetry and one neutralino also become massive. These fields arrange themselves in a massive vector supermultiplet, degenerate even if $\xi \neq 0$, and their contribution to the one-loop potential along the inflaton direction cancel exactly. The potential of Eq. (22) can be also rewritten as a RG-improved\(^5\) tree-level potential with gauge couplings evaluated at the scale $\lambda \langle P \rangle$.

The term quadratic in $\lambda \langle P \rangle$ would spoil the slow-roll condition necessary for a successful inflation, but it drops out because

$$g_A^2 (a_+ + a_-) = (G_{1+}^2 + G_{1-}^2) \delta S^2 + g_A^2 (Q_{1+}^A + Q_{1-}^A) \xi$$

$$= -G_{1P}^2 \delta S^2 - g_A^2 Q_{1P}^A \xi \propto G_{1P}^2 \delta S^2 + g_A^2 Q_{1P}^A \xi = 0,$$ \quad (23)

where we have made use of the $U(1)$ invariance of $P \Phi_{\pm} \Phi_{\mp}$ to write the third expression which vanishes by Eq. (19).

\(^5\)In doing so, a careful treatment of the possibility of kinetic mixing of different $U(1)$'s is required. The details of our analysis are modified in the presence of such mixing but the generic results are not changed.
The results just described for the simplest inflaton direction containing more than one field are generalizable to more complicated inflatons. One could have inflatons containing more than two elementary fields while still having only a one-dimensional flat direction. Another possibility is that the flat direction has more than one free VEV (multidimensional inflatons). It is straightforward to verify that the results obtained above for two mirror fields are generic provided the inflaton does not contain some subdirection capable of compensating the FI term.

As we have noticed above, the fact that the vacuum energy driving inflation is proportional to an effective FI $D$-term $\xi_{eff}$ may help in solving the problem of the mismatch between the value of the Fayet-Iliopoulos $D$-term required by the COBE normalization and the value predicted by string theories. Using the notation of Eq. (10), it is easy to show that the COBE normalization imposes the generic constraint $V_0^{1/4} = 1.6 \times 10^{16}$ GeV. This means that for the model under consideration we can obtain the following lower bound on the COBE normalized value of the FI $D$-term

$$\sqrt{\xi}_{\text{COBE}} = 1.6 \left( \frac{a_x^2 + a_z^2}{4\pi^2} \right)^{1/4} \times 10^{16} \text{ GeV}. \quad (24)$$

If we now presume that $g_\alpha \gg g_A$ and $|Q_\alpha| \sim |Q_i^A|$ with $\beta \gg 1$, the COBE normalized value of the FI $D$-term becomes enhanced by a factor $\beta^{1/2}$. Whether the enhancement factor is large enough to reconcile the value of the Fayet-Iliopoulos $D$-term required by the COBE normalization with the value of string theories is very model-dependent and should be checked case by case. We feel encouraged, though, by the fact that the presence of a plethora of fields and several non-anomalous additional abelian symmetries in string theories may help in solving the mismatch problem.

As the next step in complexity we now examine the case in which, besides the inflaton VEVs $|S_1|$ and $|S_2|$, some other field $\varphi_i$ is forced to take a VEV (this can be triggered by $\xi$ in the anomalous $D$-term of the potential or by $S^2$ in any $D$-term). In general, the new VEV can induce further VEVs too. For simplicity, we assume that this chain of destabilizations ends with $\langle \varphi_i \rangle$. By minimizing the $D$-term potential, all VEVs are determined to be

$$\delta S^2 = |S_1|^2 - |S_2|^2 = -\frac{g_A^2}{\det G^2} (C_{\alpha}^2 Q_\alpha^A - C_{\alpha}^2 Q_i^A) \xi$$

$$\langle \varphi_i^2 \rangle = -\frac{g_A^2}{\det G^2} (-C_{\alpha}^2 Q_\alpha^A + C_{\alpha}^2 Q_i^A) \xi,$$  \quad (25)

with $\det G^2 = G_{\alpha}^2 G_{\alpha}^2 - G_{\alpha}^2$. The tree level potential along this direction is
\[ V_0 = \frac{1}{2} g^2 \frac{\xi^2}{\det G^2} \sum_{\alpha, \beta} g^2_{\alpha \beta} Q^\alpha Q^\beta (Q^\alpha Q^\beta - Q^\alpha Q^\beta) \leq \frac{1}{2} g^2 \xi^2. \]  

In this background, the masses of the scalar components of \( \Phi_\pm \) appearing in the superpotential (17) are

\[ m^2 = \lambda^2 \langle I \rangle^2 + g^2_A Q^A (D_A) + \sum_a g^2_{\alpha a} Q^\alpha (D_a) = \lambda^2 \langle I \rangle^2 + g^2_A a \pm \xi, \]  

and again, one finds \( a_+ + a_- = 0 \).

To illustrate the above discussion, consider the following example of a string model [24] that satisfies the conditions required for \( D \)-term inflation, at least when we restrict the analysis to a subset of the fields. The \( U(1) \) charges of these fields are listed in Table I (we follow the notation of ref. [21] with charges rescaled). For every listed field \( S_i \), a ”mirror” field \( \overline{S}_i \) exists with opposite charges. At trilinear order the superpotential is

\[ W = \overline{S}_{11} (S_5 \overline{S}_8 + S_6 \overline{S}_9 + S_7 \overline{S}_{10} + S_{12} S_{13}) + S_{11} (\overline{S}_5 S_8 + \overline{S}_6 S_9 + \overline{S}_7 S_{10} + \overline{S}_{12} S_{13}). \]

<table>
<thead>
<tr>
<th>Field</th>
<th>( Q_A )</th>
<th>( Q_3 )</th>
<th>( Q_4 )</th>
<th>( Q_5 )</th>
<th>( Q_6 )</th>
<th>( Q_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_5 )</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>( S_6 )</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( S_7 )</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( S_8 )</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>( S_9 )</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( S_{10} )</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( S_{11} )</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( S_{12} )</td>
<td>0</td>
<td>1</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( S_{13} )</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I: List of non-Abelian singlet fields with their charges under the \( U(1) \) gauge groups. The charges of these fields under \( U(1)_{1,2,8,9} \) are zero and not listed.

The role of the inflaton direction can be played by \( \langle S_{11} \overline{S}_{11} \rangle \), formed by fields with zero anomalous charge. However for this to be viable there should be no higher order terms in the superpotential involving just the inflaton directions fields (or terms involving just a single non-inflaton direction field) for these will spoil the \( F \)-flatness of the inflaton direction \( \langle S_{11} \overline{S}_{11} \rangle \) must be invariant under continuous gauge symmetries and so the only symmetry

**Superpotential terms with dimension strictly greater than three terms in the superpotential are suppressed by inverse powers of the Planck mass and so may not be so large as to prevent inflation. However, given the requirement that the inflaton must have a very large VEV \( \mathcal{O}(\sqrt{\xi}) \), only very high dimension terms will be acceptable.**

\[ \text{12} \]
capable of ensuring such $F$-flatness is a discrete R-symmetry. Unfortunately we do not know whether the models considered have such a discrete R-symmetry and thus they may allow the dangerous terms. Henceforth we will ignore this problem and assume the dangerous terms are absent.

The rest of the fields in the subset of Table 1 acquire large positive masses deep in the inflaton direction due to the Yukawa couplings in (29), guaranteeing the stability of the inflaton direction $S = S_{11} = \overline{S}_{11}$. One-loop corrections to the inflaton potential proportional to $S^2$ are absent and only the $\sim \xi^2 \log S^2$ dependence remains, providing the slow-roll condition. However, the end of inflation poses a problem for the present example: no set of VEVs for the selected fields can give zero potential. As is well known, a flat direction ($V = 0$) is always associated with an holomorphic, gauge invariant monomial built of the chiral fields. To compensate the FI-term and give $V = 0$, this monomial should have negative anomalous charge. However, in the considered subset $Q_A = Q_7/2$ and all holomorphic, gauge invariant monomials must have then $Q_A = 0$. To circumvent this problem we enlarge the field subset by adding an extra field, $S_1$ with \( \overline{Q}(S_i) = (Q_A; Q_\alpha) = (-4; 0, 1, 0, 0, -2) \). It is easy to see that, for example, the flat direction \( \langle 1^3, 5, 6, 10, 13 \rangle \) can cancel the FI-term and give $V = 0$. Other flat directions exist, but clearly all of them involve $S_1$. However, the superpotential (29) does not provide a large mass for $S_1$ when we are deep in the flat direction. Unless higher order terms in (29) provide a positive mass for $S_1$, the FI-term induces a destabilization of the inflaton direction and $S_1$ is forced to take a VEV:

\[
\langle S_1^2 \rangle = -\frac{g_A^2}{G_{11}^2} Q_1^4 \xi, \tag{30}
\]

where we use the definition $G_{ij}^2 = g_A^2 Q_i^4 Q_j^4 + \sum_\alpha g_\alpha^2 Q_i^4 Q_\alpha^2$. This is not a problem in itself because the rest of the fields are forced to have zero VEVs and so the potential cannot relax to zero. The presence of additional $U(1)$ factors prevents the vacuum shift that was problematic for the example of section 4. The value of the potential in the presence of a VEV for $S_1$ is

\[
V = \frac{1}{2} g_A^2 \xi_{eff}^2, \tag{31}
\]

with

\[
\xi_{eff}^2 = \frac{\sum_\alpha g_\alpha^2 (Q_\alpha)^2}{G_{11}^2} \xi^2. \tag{32}
\]

The masses of the rest of the fields are also affected and read:
\[ m_i^2 = \lambda_i^2 (f_i')^2 + \frac{g_A^2}{G_{11}^2} (Q_i^A C_{11}^2 - Q_i^A G_{11}) \xi, \]  

where \( \lambda_i \) are some of the Yukawa couplings in (29).

In general, when all the fields in the model are included, the presence of the FI term will induce VEVs for the fields with negative anomalous charge which are not forced to have zero VEV by F-term contributions. These non-zero VEVs will in turn induce, through other D-terms, non-zero VEVs for other fields, even if they have positive anomalous charge. Finding all the VEVs requires the minimization of a complicated multifield potential that includes both F and D contributions. It is intriguing, though, that the interplay of all the various fields in the game may help in reducing the FI D-term. As we have noted, this suggests a way of reconciling the COBE normalized value of \( \sqrt{\xi} \) with the one suggested by string theory.

In many cases, as in the example of section 3, the field VEVs adjust themselves to give \( V = 0 \) and no D-term inflation is possible. In other cases however, especially in the presence of additional \( U(1) \) factors, there is a limited number of fields that must necessarily take a VEV to cancel the FI term. If the inflaton direction provides a large F-term mass for them, cancelation of the FI-term is prevented. Even if many other fields are forced to take VEVs, no configuration exists giving \( V = 0 \) and D-term inflation can take place in principle. To determine if that is the case, one should minimize the effective potential for large values of the inflaton field and determine all the additional VEVs triggered by the FI-term. These VEVs, of order \( \xi \) will affect the details of the potential along the inflaton direction, both at tree level (offering the possibility of reducing the effective value of \( \xi \)) and at one-loop, via their influence on the field-dependent masses of other fields.

6. Let us now discuss the post-inflationary phenomenology of reheating in D-term inflation in superstring inspired models. D-term inflation is characterized by the problem of maintaining the reheating temperature \( T_R \) small enough not to overproduce dangerous relics such as gravitinos [25]. In fact, this problem is common to any supersymmetric hybrid model of inflation—including the ones where inflation is driven by some F-term [26,27]—with the COBE normalized value of the vacuum energy \( V^{1/4} \) close to the GUT scale [4]. Indeed, when inflation ends, some heavy field that during inflation is located at the origin rolls down to the true minimum and promptly releases the vacuum energy. The reheating temperature is therefore quite large, \( T_R \sim V^{1/4} \sim 10^{15} \) GeV. On the other hand, for unstable gravitinos in the mass range 100 GeV to 1 TeV, one has to require \( T_R \lesssim (10^7 - 10^9) \) GeV [25].
We now argue that the gravitino bound may be naturally satisfied in $D$-term inflation inspired by superstring theories. As we have shown, it is very likely—if not unavoidable—that the vacuum energy density during superstring $D$-term inflation is carried by a combination of fields. After slow roll, these fields begin to oscillate about their minima of the potential, and the vacuum energy that drives inflation is converted into coherent scalar field oscillations corresponding to a condensate of nonrelativistic particles. Reheating takes place when these particles decay into light fields, which through their decays and interactions, eventually produce a thermal bath of radiation. During the epoch of coherent oscillations the Universe is matter dominated and the energy trapped in the condensates decreases as the cube of the scale factor. The reheating temperature is determined by the decay time of the scalar field oscillations, which is given by the inverse of the decay width $\Gamma$ of the condensates. If $\Gamma$ is smaller than the Hubble parameter $H$, the coherent oscillations phase is relatively long and the reheating temperature $T_R \simeq \sqrt{\Gamma M_{Pl}}$. On the other hand, if $\Gamma \gtrsim H$, oscillations decay rapidly, and $T_R \sim V^{1/4}$, corresponding to 100% conversion of the vacuum energy into radiation. Since in $D$-term inflation the decay rate of all the condensate oscillations is of order of $\sqrt{\xi}$—much larger than the Hubble rate $H \sim \xi/M_{Pl}$—the oscillation energy is promptly released into radiation with $T_R \sim \sqrt{\xi}$. Notice that, at this stage, there is no sequence of separate reheating processes from the different condensates since their decay rates are all larger than $H$.

One should not admit defeat too soon, though. It has been shown that it is quite natural to have a late stage of “thermal” inflation [28] which releases a large but controlled amount of entropy at the electroweak scale which solves this problem. However in the present case there is another natural way to avoid this cosmological disaster which has the merit that it does not invoke a different sector of the theory. Suppose that the vacuum manifold is characterized by some global accidental symmetry. This occurs, for instance, if in the true vacuum with unbroken supersymmetry the FI $D$-term is cancelled by two fields $\phi_1$ and $\phi_2$ with equal $U(1)_A$ charge $q$, $|\phi_1|^2 + |\phi_2|^2 = \xi/q$. In such a case the vacuum manifold is a circle in the $(\phi_1, \phi_2)$ plane and the accidental symmetry is an abelian global symmetry. The presence of a plethora of fields and several non-anomalous additional $U(1)$'s in string models makes the possibility of having accidental symmetries very likely [29,14]. This symmetry of the vacuum manifold is only accidental, in the sense that it is not respected by the Yukawa interactions in the superpotential. Therefore, once supersymmetry is broken, the vacuum manifold is tilted and the Goldstone mode $\theta$ associated to the accidental symmetry gets
a mass $\overline{m}$—it becomes a pseudo-Goldstone boson. The value of its mass depends upon the details of supersymmetry breaking. In scenarios in which supersymmetry is broken in some hidden sector and is transmitted to the other fields only gravitationally, once expects that $\overline{m} \sim 1$ TeV. However, if supersymmetry is broken dynamically at some scale $\Lambda$ and transmitted to the other fields by gauge interactions, the mass of the Pseudo-Goldstone boson may be much larger than 1 TeV if, for instance, it communicates with the hidden sector via some $U(1)$ gauge interaction, $\overline{m} \sim \alpha \Lambda$ with $\alpha$ the gauge coupling constant of the messenger $U(1)$.

When the Hubble parameter becomes of the order of $\overline{m}$—much later than the end of inflation—coherent oscillations of the Pseudo-Goldstone mode start. The Universe remains cold until $H$ drops below the decay width $\Gamma_\theta \sim \overline{m}^3/\xi$. At this point the condensate oscillations decay and the decay products start thermalizing the Universe again, reheating it up to a temperature $T_R \simeq \overline{m}^{3/2} \sqrt{M_{Pl}/\xi}$. This reheating temperature is large enough to permit successful nucleosynthesis and may be even larger than the weak scale, which will allow for electroweak baryogenesis. What is relevant for us, however, is that the release of a huge amount of entropy at late epochs will dilute any products of the previous stage of reheating, including harmful gravitinos. Thus, in $D$-term inflation inspired by superstring theories, gravitinos seem to pose no cosmological problem††.

7. In conclusion, we have illustrated through different examples the main complications one has to face when trying to build up a successful $D$-term inflationary scenario out of superstring models. The latter are usually characterized by a great number of true flat directions along which the FI term can be cancelled. This makes difficult to implement the necessary conditions for $D$-term inflation. The requirement one should impose in such a case is that the theory possesses the necessary couplings between some inflaton (usually to be identified with some flat direction) and the true flat directions, in such a way that large values of the inflaton make massive at least one field along every flat direction. However, this is not always possible because flat directions may be protected by chirality from acquiring mass by coupling to the inflaton field. This is equivalent to say that in the context of string theory, the “vacuum shifting” phenomenon is operative even in the early Universe, thus preventing

††We observe that this way of solving the gravitino problem via the decay of Pseudo-Goldstone modes at late epochs applies to any model of supersymmetric hybrid inflation, provided the vacuum manifold after inflation possesses accidental symmetries.
inflation from taking place. In those cases in which the “vacuum shifting” is not so efficient, 
$D$-term inflation may take place with an effective FI $D$-term whose value generally depends 
upon the details of the minimization of a complicated multifield potential. We have also 
shown that the presence of a plethora of fields and several non-anomalous additional abelian 
symmetries in string theories may solve the problem of the mismatch between the value of 
the Fayet-Iliopoulos $D$-term required by the COBE normalization and the value predicted 
by string theories. Primordial gravitinos may be efficiently diluted by the large amount of 
entropy released at late epochs when a pseudo-Goldstone boson parametrizing almost flat 
direction of the vacuum manifold decays into light states.

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