DUALITY AND BACKWARD PEAKS *)

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ABSTRACT

We investigate the experimental and phenomenological implications of exchange degeneracy patterns of baryons. Also the problem of their parity doubling is discussed.


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The recent experiments on $\pi^+p$ and $K^+p$ differential cross-sections at 5 GeV/c \cite{1,2,3}, covering the whole angular range, show with particular clarity the notion of "baryon exchange". Let us consider the particular example of $\pi^+p$ elastic scattering shown in Fig. 1. At small angles $|t| < 2 (\text{GeV/c})^2$, near $180^\circ$ $|t|$ very large or $|u| < 2 (\text{GeV/c})^2$ the so-called "backward peak" is evident. Despite the fact that the backward peak (typically 10 \cite{2}) is several orders of magnitude smaller than the forward one, a long experimental effort has provided us with good data. This is very important as a wealth of information can be extracted from these measurements.

The physical scattering process $\pi^+p \rightarrow \pi^+p$ we associate with the $s$ channel

\begin{equation}
\begin{array}{c}
\pi^+ \\
\downarrow \\
\Rightarrow \text{(s)} \\
\pi^+
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
s \rightarrow \\
\pi^+ t \\
\downarrow \\
\pi^+ \\
\Rightarrow \\
P + P' + g \Rightarrow A^{(s)} \equiv \text{(s)} \\
P
\end{array}
\end{equation}

The simple argument relating the scattering near $0^\circ$ to a dominant exchange in the $t$ channel, involving $P, P'$ and $g$ Regge poles is well known

\begin{equation}
\begin{array}{c}
\pi^+ \\
\downarrow \\
\Rightarrow t \\
\pi^+
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
\pi^+ t \\
\downarrow \\
P + P' + g \\
\Rightarrow A^{(t)} \equiv \text{(t)} \\
P
\end{array}
\end{equation}

A parallel argument in the $180^\circ$ angular region will associate the backward peak with the $u$ channel exchange diagram

\begin{equation}
\begin{array}{c}
\pi^+ \\
\downarrow \\
\Rightarrow u \\
P
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
\pi^+ u \\
\downarrow \\
N, \Delta \\
P + P' + g \\
\Rightarrow A^{(u)} \equiv \text{(u)} \\
P
\end{array}
\end{equation}
Just as it became fashionable to "Reggeize" $A^{(t)}$ when applied to intermediate and high energy scattering (i.e., when resonance structure has disappeared), the same can be done to $A^{(u)}$. We only have to take into account the fact that in (3) half integer and not integer spins are exchanged ($\alpha \rightarrow \alpha - \frac{1}{2}$) and obtain

$$A^{(u)} = \beta \left( \sqrt{s} \right) \frac{1 + \frac{t}{s} e^{-\frac{i\pi}{2} [\alpha(u) - \frac{1}{2} \gamma_2]}}{\sin \pi [\alpha(u) - \frac{1}{2} \gamma_2]}$$

Taking $t$ instead of $s$ as the physical energy, the amplitude (4) will dominate the annihilation process $p\bar{p} \rightarrow \pi^+ \pi^-$ at fixed $u$ and high energy ($t \gg 2m^2$). Recent measurements on annihilation $p\bar{p} \rightarrow \pi^+ \pi^\mp$ and $p\bar{p} \rightarrow K^+ K^-$ make further phenomenological investigation of Reggeized baryon exchange possible.

For forward scattering it is well known how duality leads to exchange degeneracies of Regge trajectories. Apart from particular successes, it imposed some over-all systematics on the jungle of Regge fitting. This is our motivation to start our discussion by deriving the exchange degeneracy patterns of baryons. Qualitative experimental and theoretical implications of the scheme will be discussed. Only at the end we will not resist the temptation to look more quantitatively at some recent data.

1. - DUALITY CONSTRAINTS ON BARYON EXCHANGES

In the context of a factorizable Regge pole model for $PB \rightarrow BP$ *) scattering we study the implications of the following assumptions

*) We adopt conventional notations for $SU(3)$ multiplets; $P$ refers to the octet of pseudoscalar mesons, $V$ to the nonet of vector mesons, $B$ and $D$ to the octet and decuplet of $\frac{3}{2}^+$ and $\frac{3}{2}^+$ baryons respectively.
(a) - crossing;
(b) - SU(3) symmetry;
(c) - absence of exotics, i.e., no resonances or no exchange forces in channels with exotic SU(3) quantum numbers;
(d) - resonance dominance;
(e) - a weak form of duality, usually referred to as finite energy sum rule duality

Assumptions (a), (b), (c) imply

\[ A_i^{(s)} = \sum_{j \neq \bar{10}, 27} X^{su}_{ij} A_j^{(u)} \]  \hspace{1cm} (5)

Again, \( A_i^{(s)} \), \( A_j^{(u)} \) stand for the s and u channel amplitudes; \( X^{su}_{ij} \) is the SU(3) matrix crossing from the s to the u channel; i, j are SU(3) indices. In the particular case of PB scattering

\[ i, j = 4, 8_{as}, 8_{sa}, 8_{ss}, 8_{aa}, 10, \bar{10}, 27 \]

Because of assumption (c) the summation over \( \bar{10} \) and 27 has been suppressed in (5).

Let us study Eq. (5) in the limit

\[ u \text{ fixed, } s \to \infty \]

As discussed in the introduction,

\[ A_j^{(u)} = \beta_j \frac{\lambda + \tau_j e^{-i \pi (\alpha_j - \nu_j)}}{\sin \pi [\alpha_j - \nu_j]} s^{|\alpha_j - \nu_j|} \]  \hspace{1cm} (6)

Resonance (or Breit-Wigner) saturation of the amplitude implies that in exotic channels the amplitude is purely real \[ \text{[assumptions (c), (d)]} \], therefore

\[ \text{Im} \ A_i^{(s)} = 0 \text{ for } i = \bar{10}, 27 \]  \hspace{1cm} (7)
FESR duality [assumption (c)] states that the global sum of Regge exchanges in the crossed channel builds up the direct channel resonances. This allows us to combine (5), (6) and (7) into the set of equations

$$\sum_{j \neq \bar{10}, \bar{17}} X_{ij}^u S_{ij}^u T_j \mathcal{P}_j S_{j-1/2}^u = 0 \text{ for } \nu = \bar{10}, \bar{17} \tag{6}$$

We will refer to this as $s$ channel constraints (on the Regge exchanges).

As pointed out in the introduction, baryon exchange is also a prominent force in the annihilation $\bar{B}B \rightarrow \bar{P}P$, therefore constraints coming from the absence of exotics in the $\bar{7}$ channel have to be considered.

Starting from $t \leftrightarrow u$ crossing

$$A_{k}^{(t)} = \sum_{l \neq \bar{10}, \bar{17}} X_{kl}^{tu} A_{l}^{(u)}$$

a completely parallel calculation, considering the limit

$$u \text{ fixed, } t \rightarrow \infty$$

yields

$$\sum_{l \neq \bar{10}, \bar{17}} X_{kl}^{tu} \mathcal{P}_l \mathcal{T}_l X_{l-1/2}^{tu} = 0 \text{ for } k = 10, \bar{10}, 27 \tag{9}$$

We call this $t$ channel constraints.

We can briefly summarize the argument as follows. Baryon exchange is at fixed $u$ the prominent force in high energy $PB \rightarrow PB$ scattering and $\bar{B}B \rightarrow \bar{P}P$ annihilation. Duality states that the Regge exchanges build up the resonances or the imaginary part of the amplitude. Whenever $PB \rightarrow PB$ or $\bar{B}B \rightarrow \bar{P}P$ has no imaginary part (i.e., is
exotic such as in the reactions $K^+ p \rightarrow K^+ p$ or $\Sigma^- \bar{p} \rightarrow \Lambda \bar{K}^-$. The Regge exchanges have to conspire to build up a purely real amplitude in the direct channel. In SU(3) language this conspiracy is expressed by Eqs. (8), (9). They constrain severely the Regge couplings $\rho(u)$ and the trajectory functions $\alpha(u)$.

After repeating the same arguments for $FB \rightarrow PD$ scattering [see Refs. 6), 7)], one can look into the implications of these constraints.

2. - EXCHANGE DEGENERACIES OF BARYONS

An important piece of information contained in these duality constraints is obvious. Inverting the argument of the preceding paragraph, we can use Eqs. (8), (9) to guess the dominant Regge exchanges required by duality. In other words, which set of Regge poles fulfills the constraints?

As (8), (9) has to be satisfied for a range of $s$ values, it is clear that the leading exchanges appear in sets with degenerate trajectory functions $\alpha(u)$ (exchange degeneracy). This reduces (8), (9) into equations for their couplings

$$\sum_{j \neq 10, 27} X_{ij}^s \tau_j \rho_j = 0 \text{ for } j = 10, 27 \quad (8')$$

$$\sum_{l \neq 10, 27} X_{kl}^t \rho_l = 0 \text{ for } k = 10 + 10, 27 \quad (9')$$
The solutions of these equations have been discussed in detail \(6), 7\). Especially one of them has a certain phenomenological appeal \(8\) and we will discuss it in detail. It contains, for \(\tau P = +1\)

\[
\begin{align*}
\{8\} & \text{ of } J^P = \frac{1}{2}^+, \quad \frac{3}{2}^+, \quad \frac{5}{2}^+ \ldots (\alpha\text{-sequence}) \\
\{1+8+10\} & \text{ of } J^P = \frac{1}{2}^-, \quad \frac{3}{2}^-, \quad \frac{5}{2}^- \ldots (\delta\text{-sequence})
\end{align*}
\]

all octets have \(F^+ = \frac{1}{2}\) \(\)**. For \(\tau P = -1\)

\[
\begin{align*}
\{8+10\} & \text{ of } J^P = \frac{3}{2}^+, \quad \frac{5}{2}^+ \ldots (\delta\text{-sequence}) \\
\{8\} & \text{ of } J^P = \frac{1}{2}^-, \quad \frac{3}{2}^-, \quad \frac{5}{2}^- \ldots (\beta\text{-sequence})
\end{align*}
\]

all octets have \(F^- = -\frac{1}{2}\).

In terms of the non-strange multiplet components we recognize the \(N^\alpha - N^\gamma\) sequence for \(\tau P = +1\), the \(\Delta^\delta - N^\rho\) sequence for \(\tau P = -1\). In the scheme the relative couplings of \(N^\alpha\) and \(N^\gamma\) are fixed, the same is true for \(\Delta^\delta\) and \(N^\rho\).

Let us make some remarks about the Chew-Frautschi plot that is suggested by this Regge pattern.

- The \(N^\alpha\) and \(N^\gamma\) is accompanied by a \(\Delta^\gamma\). An investigation of its couplings shows it decouples from \(\pi N\). Phase shift analysis contains a \(D_{33}(1670)\) with 12% elasticity \(9\), it could be associated with the \(\Delta^\gamma\) trajectory. We will further ignore its contribution for any process, also because solutions without \(\Delta^\gamma\) exist.

**) \(\tau P = \text{signature} \times \text{parity},\) as exchanges with \(\tau P = +1\) and \(-1\) separate asymptotically, their solutions are independent and have to be discussed separately.

\[
F = \frac{F}{F + D} = \frac{(F/D)}{1 + (F/D)}
\]
- In the $N_b$ sequence the lower recurrence of the $5^{-}_L$ resonance is experimentally missing (it would show up as a $\frac{5}{2}^-$ isobar of 700 MeV). A zero in the $\pi F = -1$ residue should kill this pole. This zero therefore appears also in the $\Delta S$ residue at $\alpha = \frac{1}{2}$.

- A $\frac{3}{2}^+$ octet degenerate with the decuplet is required. Its weak coupling to $\pi N$ has often been invoked as an excuse for its experimental absence. This statement is misleading. Indeed, the coupling to $N\bar{K}$ of the $Y^*_1$ in this octet would only be suppressed by a factor 2 relative to the $Y^*(1385)$. As was done for the $N_b(1/2^-)$, one could in principle account for its absence by a zero in the residue. This would imply the existence of its recurrence $Y^*_2$. Again phase shift analysis provides a possible candidate $F_{17}(1990)$.

This zero, however, has to break duality, it cannot be shared by the $\Delta S$ at $\alpha = \frac{1}{2}$!

- This naturally leads us to the broken duality schemes, proposed in Refs. 10) and 11). Both schemes break duality by ignoring $t$ channel constraints. E.g., the assumption of resonance dominance could break down in $B\bar{B} \to F\bar{F}$ scattering because of annihilation effects. The 5 GeV/c data on $p\bar{p} \to \pi^\pm \pi^\mp$ also show clear deviations from what we expect on the basis of Regge pole models 3).

The main conclusions of the two approaches can be summarized as follows.

Caltech broken duality 10): ignoring equations (9'), one solves Eqs. (8') in a straightforward way.

(i) the $\{8, 8^+\}$ is no longer required;
(ii) $F^\pm$ are not fixed;
(iii) the relative coupling of $N_\alpha$ and $N_\gamma$ trajectories becomes free.

Capeka broken duality 11): one solves Eq. (8') and breaks SU(3) in the sense that $\Lambda_\alpha = \Sigma_\alpha, \gamma$ trajectories are no longer degenerate, as suggested by particle masses *). Surprisingly one finds again

*) This doubles Eqs. (8) as two non-degenerate trajectory functions $\alpha_\Lambda, \alpha_\Sigma$ are entering the equation.
\[ F^\pm = \pm 1/\sqrt{2} \]

and the relative \( N_\omega, N_\rho \) coupling, required before by \( t \) channel constraints, is restored.

In view of the generally accepted assumptions that were used to derive this scheme, a further experimental and phenomenological investigation of the different exchange degeneracies is certainly worth the effort. We want to discuss three possibilities, together with already obtained results:

A. Duality sum rules
B. Effective residues
C. Regge fits.

More detailed dynamical assumptions go into the tests from \( A \rightarrow B \rightarrow C \).

3. EXPERIMENTAL AND PHENOMENOLOGICAL TESTS OF EXCHANGE DEGENERACY PATTERNS OF BARYONS

A. Duality sum rules (D.S.R.)

The absence of Regge pole exchange in exotic channels leads to the absence of a backward peak in the following reactions: \( K^- p \rightarrow p K^- \), \( K^- p \rightarrow n K_0^- \), \( K^- n \rightarrow n K^- \), \( K^- p \rightarrow \Delta^+ K^- \), \( K^- n \rightarrow \Delta^0 K^- \). This argument is well known, but can be pushed further. Performing a straightforward SU(3) decomposition of the \( u \) channel exchange amplitudes, we require the absence of exchange forces in \( \overline{10} \) and 27 representations and obtain the following relations among backward peaks \(^{12})^*\):

\[ A(\pi^+ p \rightarrow \Sigma^+ K^+) - A(\bar{K} p \rightarrow \Sigma^- \pi^+) - A(\bar{\pi} p \rightarrow \Sigma^- K^+) = 0 \]

* These are relations among squared amplitudes. We list only equalities of the type \((10), (11)\) as they apply directly to differential cross-sections. More complicated relations can be derived and could be useful to check interference terms in Regge models, e.g.,
\[
\begin{align*}
[\pi^+ p \rightarrow \rho^- \pi^-] & = [\kappa^- p \rightarrow \Sigma^+ \pi^-] \\
[\pi^+ p \rightarrow \Delta^+ \pi^+] & = 2 [\kappa^- p \rightarrow \gamma^{*0} \pi^-] \\
[\pi^+ p \rightarrow \gamma^{*0} K^+] & = [\kappa^- p \rightarrow \Xi^- K^0] \\
[\pi^- p \rightarrow \Delta^+ \pi^-] & = [\kappa^- p \rightarrow \eta^{*0} \pi^-] \\
[\kappa^- p \rightarrow \Xi^{*0} K^0] & = 2 [\pi^- p \rightarrow \gamma^{*0} K^0] \\
[\pi^+ p \rightarrow \Delta^+ \eta] & = 6 [\kappa^- p \rightarrow \gamma^{*0} \eta] \\
[\pi^- p \rightarrow \Delta^0 \pi^0] & = 2 [\kappa^- p \rightarrow \gamma^{*0} \pi^0] \\
\end{align*}
\]
\[(10)\]

As a next step we can impose the exchanged octets to satisfy

\[ F^\pm = \pm \frac{1}{2} \]

Remember that not only duality but also Capp's broken duality required these \( F/D \) ratios. More sum rules are obtained

\[
\begin{align*}
[\pi^+ p \rightarrow \Sigma^+ K^+] & = [\kappa^- p \rightarrow \Xi^- K^+] \\
[\kappa^- p \rightarrow \Sigma^- \pi^+] & = 2 [\pi^- p \rightarrow \Sigma^0 K^0] = \frac{2}{3} [\pi^- p \rightarrow \Lambda K^0] \\
[\kappa^- p \rightarrow \Lambda \eta] & = \frac{1}{3} [\kappa^- p \rightarrow \Lambda \pi^0] \\
\end{align*}
\]
\[(11)\]

Let us clarify D.S.R. (10) and (11) by the following remarks.

- As absence of exotics and \( F^\pm = \pm \frac{1}{2} \) are features of the exchange degeneracy pattern discussed in Section 2, relations (10), (11) have to be satisfied in the \( N^a - N \), \( \Delta^a - N^\rho \) exchange model.
- Equations (10), (11) do not follow from SU(3) symmetry, but can be supplemented with the well-known SU(3) relations that can be derived for backward peaks \( \gamma \Delta \).
- Relations (10), (11) have to be contrasted with sum rules that
neglect the contribution of $I = \frac{3}{2}$ exchange.

Typical sum rules of this type are:

\[
\begin{align*}
[\pi^- p \rightarrow \Lambda \pi^-] &= \frac{1}{6} \left(2F^+ + 4\right)^2 [\pi^+ p \rightarrow p \pi^+] = 6 [\pi^- p \rightarrow \Lambda \eta] \\
[\pi^- p \rightarrow \Lambda \eta] &= \frac{1}{6} \left(4F^+ - 1\right)^2 [\pi^+ p \rightarrow p \pi^+] \\
&= \frac{4}{3} \left(4F^+ - 1\right)^2 [\pi^- p \rightarrow m \pi^0] \\
&= \frac{4F^+ - 1}{2F^+ + 1} [\kappa^- n \rightarrow \Lambda \pi^-] 
\end{align*}
\]

(12)

For the sum rules mentioned above, $F^+ = \frac{1}{2}$ is consistent
with all existing data. These well-known examples make it
clear how to check (10), (11) experimentally. One can compare the
average magnitude of the backward peaks or the differential cross-
sections integrated near 180°.

- The D.S.R. are invariant under the substitution

\[
\begin{align*}
\pi &\rightarrow \varphi \\
\kappa &\rightarrow \kappa^* \\
\eta &\rightarrow \sin \theta \nu \omega + \cos \theta \nu \varphi
\end{align*}
\]

for the final state meson in both the left and right-hand sides.

The $\omega - \varphi$ mixing angle $\theta_\nu$ is determined by the absence of
exotic mesons.

From the first relation under (12) and the decoupling of $\varphi$ from
NN, we obtain in this way

\[
[\kappa^- p \rightarrow \Lambda \omega] = \frac{\left(2F^+ + 1\right)^2}{12} [\pi^+ p \rightarrow p \rho^+] 
\]

(13)

*) This can be a bad approximation, even in processes of the type
$PB \rightarrow PB$. 

\[PB \rightarrow PB.\]
We take $F^+ = \frac{1}{2}$ and extrapolate the data of Ref. 18) on $K^- p \to \Lambda \omega$ to 5 GeV/c (using Regge pole energy dependence). The comparison with recent measurements 19) on $\pi^+ p \to p \phi^+$ at the same energy is excellent. This is in fact surprising because of the more complicated spin structure of the reactions involved in (13).

Throughout the discussion in Section 1, it was indeed understood that duality constraints hold for each helicity amplitude independently.

A complete phenomenological and experimental investigation of D.S.R. would be very interesting 12) as they just test absence of exotics and SU(3) symmetry for the set (10) and presently accepted octet $F/D$ ratios as well for the set (11).

B - Effective residues

Assuming Regge trajectories are linear one can test the exchange degeneracy pattern, presented in Section 2, in further detail. The method of "effective residues" was outlined in detail in Ref. 16), we briefly sketch the argument below.

Asymptotically the backward differential cross-section can be written 13)

$$\frac{d\sigma}{d\omega} = \frac{1}{3} \left\{ |\tilde{F}^+|^2 + |\tilde{F}^-|^2 \right\}$$

(14)

$\tilde{F}^\pm$ are the $u$ channel helicity amplitudes that are eigenstates of parity. They contain the Regge exchange amplitude (4), which can be written as

$$\tilde{F}^+(\nu,\omega) = \beta(\nu,\omega) R(\omega) S^{\alpha(\omega) - 1/2}$$

(15)
for linear trajectories. $R(u)$ is the Regge propagator. For $u < 0$
$\mathcal{P}(\sqrt{u})$ can be derived from differential cross-section data in a simple
way. Using MacDowell symmetry (see Section 4)

$$\tilde{F}^+ (\sqrt{u}) = - \tilde{F}^- (-\sqrt{u})$$

and real analyticity of the residues $^{20)}$

$$\mathfrak{P} (-\sqrt{u}) = \mathfrak{P}^* (\sqrt{u})$$

we can invert (14) and obtain

$$|\mathfrak{P}(\sqrt{u})| = \left[ \frac{\tilde{\omega} \cdot \tilde{\omega} \alpha(u)}{\tilde{\omega} \cdot R(u) \tilde{\omega}} \frac{d\sigma}{du} \right]^\frac{1}{2}$$

By means of (18) we can extract the Regge residue $\mathfrak{P}(\sqrt{u})$ directly
from differential cross-section data, knowing $\alpha(u)$ (e.g., from their
energy dependence).

At $u = m_j^2$ on the other hand the residue function $\mathfrak{P}(\sqrt{u})$
can be related to the coupling of the particles into which the traject-
ory materializes. In the case of $\pi N$ scattering

$$\mathfrak{P}(m_j) \sim \Gamma_{\pi N}^J$$

The derivation of (18) obviously fails when Regge poles with different
trajectory functions $\alpha(u)$ are exchanged such as in $\pi^+ p \rightarrow p \pi^+ (N, \Delta)$. Where can we actually study by means of (18) and (19) the dual exchanges
of Section 2?

For $T P = +1$,

$K^- N \rightarrow \Lambda \pi^-$ is pure $N \rightarrow N \gamma$ exchange,
$K^+ p \rightarrow p K^+$ is pure $\Lambda \rightarrow \Lambda \gamma$ exchange.
In the first case the residue obtained in the scattering region from 
$K^-n \rightarrow \Lambda\pi^-$ data \(^{21}\) can be compared to the elastic couplings of 
the $\Lambda^*$'s lying along the $N^*_\Lambda - N^*\Sigma$ trajectory \(^{9}\). The so-obtained 
residue is shown in Fig. 2. One can do a similar calculation from 
$K^+p \rightarrow p\Lambda^+$ data \(^{22}\) and the $\Lambda\Sigma$ couplings of the $Y^*$'s along the 
$\Lambda^*_\Lambda - \Lambda^*_\Sigma$ trajectory \(^{9}\). Both examples are, however, related by 
SU(3) symmetry. We explicitly divided out the SU(3) Clebsch-Gordan coefficients in the case of strangeness exchange, the same 
residue should be obtained for both calculations. This is indeed 
the case (Fig. 2).

Figure 2 was obtained by means of (18), (19) with

\[
R_{N^*_\Lambda, N^*_\Sigma} = \frac{-i\pi (\alpha_N - \frac{1}{2})}{(1+e^{-\alpha_N}) + e^{\alpha_N} (1-e^{-\alpha_N})} \left(\frac{\pi}{\alpha_N} + \frac{\pi}{\alpha_N} \right) (\frac{\pi}{\alpha_N})
\]

(20)

Here

\[
\alpha_N = \alpha_{N^*_\Lambda} = \alpha_{N^*_\Sigma}
\]

\[
e = \frac{\beta_{N^*_\Lambda}}{\beta_{N^*_\Sigma}} = \frac{1}{3}
\]

as required by duality and $s_0 = 1$. Factors killing parity doublets 
can be taken into account.

For $\pi^+p \rightarrow \Lambda^-p \pi^-$ \(^{23}\) forms the ideal test 
case for $\Delta^*_\Lambda - N^*_\Lambda$ exchange. The propagator equivalent to (20) is

\[
R_{\Delta^*_\Lambda, N^*_\Lambda} = \frac{-i\pi (\alpha_\Delta - \frac{1}{2})}{(1+e^{-\alpha_\Delta}) + e^{\alpha_\Delta} (1-e^{-\alpha_\Delta})} \left(\frac{\pi}{\alpha_\Delta} + \frac{\pi}{\alpha_\Delta} \right) (\frac{\pi}{\alpha_\Delta})
\]

(21)
Duality requires

\[ \alpha_\Delta = \alpha_\Delta^N = \alpha_\pi^p \]

\[ \delta = \frac{B_NP}{B_\Delta^N} = \frac{1}{4} \sqrt{\frac{2}{3}} \left( \omega \frac{1}{4} \text{ for Caltech broken duality} \right) \]

The Regge residue of the \( \Delta_5 \) is shown in Fig. 3.

The duality zero \( (\frac{3}{2} - \alpha_\Delta)^N \), discussed in Section 2, has been taken into account. Its multiplicity \( n \) can in principle be determined from the \( I = \frac{1}{2} \), \( I = \frac{3}{2} \) interference, i.e., from the combination

\[ \frac{d\sigma}{d\mu} (\pi^+ p \rightarrow p\pi^+) - 2 \frac{d\sigma}{d\mu} (\pi^+ p \rightarrow \pi^- \pi^-) + \frac{3}{5} \frac{d\sigma}{d\mu} (\pi^+ p \rightarrow \pi^+ \pi^-) \]

Also high energy polarization experiments are relevant to this problem.

Inconsistencies in \( \pi^+ p \rightarrow p\pi^+ \) data make any definite conclusion impossible \(^{24}\). Therefore, we took \( n = 1 \), this yields the smoothest residue. As is clear from formula (18), the determination of \( \mathcal{P} (\sqrt{s}) \) should be independent of the energy at which it is performed. That this is indeed the case is illustrated for \( \pi^- p \rightarrow p\pi^- \) data in Fig. 4.

What can we conclude from these simple exercises? Fig. 2 shows that \( N_y, N_y \) (or \( \Lambda_y, \Lambda_y \)) Regge poles, when related in the way required by duality (20), indeed share a common residue function. Although less conclusive, the same is suggested for \( \Delta_5, \Delta^N \) by Fig. 3.

Furthermore the main virtue of a good Regge pole model is to incorporate all data (in scattering and particle regions) with simple residue functions. For \( N_y, N_y \ (\Lambda_y, \Lambda_y) \) a constant residue will do (see again Fig. 2) and therefore this exchange can
be incorporated in Veneziano-type models (where \( s_0 \approx 1 \)). For \( \Delta \bar{\rho} \),
the structure of the residue is more complicated (scattering
data suggest exponential behaviour), Fig. 3 clearly exhibits the
failure of simple Veneziano-type residues in \( \pi^- p \rightarrow p \pi^- \) scattering.

Our conclusion? The dual Regge model is definitely a
good qualitative input in detailed Regge fits. Starting with this
model as a first approximation, quantitative fits to all scattering
data of the type \( PB \rightarrow BP \) can be obtained by including the conventional weak absorptive cuts. E.g., excellent fits to \( \pi^+ p \rightarrow p \rho^+ \)
including the dip at \( u = -0.15 \) can be obtained \(^{25}\).

C - Regge fits

The 6 GeV/c \( \pi^+ p \rightarrow p \pi^+ \) polarization data \(^{26}\), presented
at this meeting by Dr. Poulet, contain a warning against detailed Regge
fits with the presently available backward data. However, without
detailed parametrization of residues or complicated least square fits
of parameters, we want to show that photoproduction and vector meson
production data cast serious doubts, not only on the dual model, but
on Regge models in general when applied to these reactions.

Consider the differential cross-section data \(^{27}\) on the
reactions \( \gamma p \rightarrow n \pi^+ \), \( \gamma p \rightarrow p \pi^0 \). For large values of \( |u| \)
\((|u| > 0.4)\) the scaling in energy of the backward peak is completely
different from the scaling of reactions involving hadrons only. Regge-
pole models are restricted a priori to small backward angles
\((|u| < 0.4) \) *.

\(^{*}\) By a simultaneous study of photoproduction and \( \rho \) production data,
\( \Delta \bar{\rho} \) dominance for big \( |u| \) can be ruled out, see Refs. 26), 29).
Two observations.

(i) In this region the energy behaviour of \( \gamma p \rightarrow n \pi^+ \), \( \gamma p \rightarrow p \pi^0 \) shows \( N^\alpha - N^\omega \) exchange is dominating. To explain the absence of a dip in \( \pi^+ p \rightarrow p \pi^+ \) and its presence in \( \pi^+ p \rightarrow p \pi^+ \) a unique relative coupling of \( N^\alpha, N^\omega \) as proposed by duality is ruled out.*

(ii) From forward scattering, where the \( \rho, \omega \) are Reggeized, we learned that their helicity structure is very different. In backward vector meson production (or in photoproduction supplemented with vector dominance) we study the same couplings. Although \( \rho, \omega \) are now external particles and the \( N \) is Reggeized, we expect a similar helicity structure **. Universal F/D ratios for helicity amplitudes are ruled out.

The extra freedom required by (i), (ii) on the successful scheme discussed in the preceding Section is exactly provided by Caltech broken duality (see Section 2). With simple constant residues we have applied this scheme to backward photoproduction. The result is shown in Fig. 5

By vector dominance (V.M.D.) we obtain \( \pi^+ p \rightarrow p \rho^\pm \), \( \pi^- p \rightarrow \rho^0 n \) differential cross-sections 19,30 (Fig. 6) and the \( \rho^+ \) decay matrix elements (Fig. 7) *** 19).

*) The argument is well known, we have checked it is still true in the presence of weak absorptive cuts.

**) It was indeed required by the data in the fits presented in this paper.

***) Figures 6 and 7 are a one-parameter prediction from photoproduction one helicity amplitude of the \( \Delta g \) not being determined by V.M.D. For details, we refer to Ref. 29).
V.M.D. was done in the canonical way

\[ \sigma_n \frac{d\sigma}{du} [\pi N \to N (p + \frac{1}{2}\alpha)] \sim \frac{\gamma_p^2}{4\pi\alpha} \frac{d\sigma}{du} (\gamma N \to NN) \]

As "decisive" tests are in fashion, we want to point out the smooth behaviour of the fit for the decay matrix elements of the \(F^+\) (Fig. 7), to be contrasted with the structure obtained in strong cut models\(^{31},^{32}\). With present data any conclusion would be premature, but better data on the \(F^+\) (or \(F^-\)) decay matrix elements would be interesting.

Figures 5, 6 and 7 might be invoked as a qualitative success of broken duality supplemented with V.M.D. This would be very misleading because of the two following reasons.

(i) In a next step one should split \(N\) and \(N\) trajectories in order to get consistency with the Chew-Frautschi plot in the particle region. Doing this one predicts a dip in photoproduction or \(F^+\) production at \(u \approx -0.15\) at higher energies (typically 20 GeV, for our particular model).

(ii) Recall Figs. 2 and 3. Using

\[ \sigma(m_J) \sim \Gamma_{\pi N}^J \text{ at } u = m_J^2 \]  

(22)

we convinced ourselves that the residues needed to explain backward scattering were roughly what one expects from the particle couplings along the trajectory. I.e., the order of magnitude of the exchange forces is correct. For photoproduction one can make exactly the same estimate using \(\gamma N \to N\pi\) phase shift analysis. In analogy to (22) we have

\[ \sigma(m_J) \sim \left[ \frac{\gamma_{\pi N}}{\gamma_{\pi N}} \right]^{1/2} \]  

(23)
Using Walker's phase-shift analysis \( \text{33) we conclude:} \\

(a) the absolute magnitude of the forces is wrong (e.g., \( \Delta \delta \)); \\

(b) their relative magnitude is wrong (e.g., only the relative contribution of \( N_{\alpha}, N_{\gamma} \) suggested by phase-shift analysis is encouraging for the mechanism explaining the absence of a dip); \\

(c) the helicity structure is wrong (all contributions \( N_{\alpha}, N_{\gamma} \); \( \Delta \delta, N_{\rho} \) are dominant non-flip in the particle region; this makes the dipping at 180° of the differential cross-sections, suggesting a substantial flip contribution, incomprehensible).

On all or some of these points existing backward photo-production models \( \text{34) (including the strong cut models 31}) \) were found in trouble. For details we refer to Ref. 29). This, together with the large \( |u| \) data, strongly suggests fixed singularities. These are not forbidden by \( u \) channel unitarity in the case of photoproduction. Berger and Fox come to a similar conclusion through detailed fitting with fixed pole and fixed cut models \( \text{28) .} \)

4. **ABSENCE OF PARITY DOUBLETs AND THE BEHAVIOUR OF BACKWARD PEAKS AT LARGE \( |u| \)**

Parity doubling of Regge trajectories is a consequence of MacDowell symmetry. Consider the conventional \( \pi N \) amplitudes \( A, B \) or \( \tilde{A}, \tilde{B} \) where \( \tilde{A} = A - mB \). Their relation to the parity conserving helicity amplitudes \( \tilde{F}^+, \tilde{F}^- \) is given by

\[
\tilde{F}^+ = -\tilde{A} + \sqrt{u} B \\
\tilde{F}^- = \tilde{A} + \sqrt{u} B
\]

(24)  

(25)
From the Mandelstam representation we know that $\tilde{A}, B$ are functions of $u$ (and $s$ of course) not of $\sqrt{u}$. Therefore we obtain from (24), (25)

$$\tilde{F}^+(\sqrt{u}) = - \tilde{F}^-(\sqrt{u})$$  \hspace{1cm} (26)

known as MacDowell symmetry.

Consider (15) and let us exhibit the pole structure of the Regge propagator $R(u)$ explicitly

$$\tilde{F}^+(\sqrt{u}) = \frac{f(s, \sqrt{u})}{-\sin \pi [\alpha(u) - \frac{1}{2}]} \hspace{1cm} \text{at } \alpha(u) = J \hspace{1cm} \sim \frac{g^2}{m_J^2 - u}$$

Therefore

$$\tilde{F}^+(\sqrt{u}) \sim \frac{g^2}{(m_J + \sqrt{u})(m_J - \sqrt{u})}$$ \hspace{1cm} (27)

and by using (26)

$$\tilde{F}^-(\sqrt{u}) \sim \frac{-g^2}{(m_J - \sqrt{u})(m_J + \sqrt{u})}$$ \hspace{1cm} (28)

The factor $(m_J + \sqrt{u})$ in the denominator of $\tilde{F}^+(\sqrt{u})$ produces through MacDowell symmetry a pole in $\tilde{F}^-(\sqrt{u})$. Recall (Section 3B) that for a signatured trajectory $\tilde{F}^\pm$ have opposite parity. Therefore, at each $J$, particles appear in parity doublets of identical mass $m_J$, and coupling $g^2$. We know that at least for the nucleon and $N^*$ this is not the case.

*) This statement has to be interpreted with care [Ref. 35].
As the whole trouble comes from the factor \((m_j + \sqrt{u})\) in Eq. (27), let us perform the following replacement (26)

\[
\tilde{F}^+ (\nu \nu) \sim \frac{g^2}{(m_j + \sqrt{u})(m_j - \sqrt{u})} \rightarrow \frac{g^2}{2m_j (m_j - \nu \nu)}
\]

and obviously (??) the problem is solved.

Do we violate the Mandelstam representation? No, one can check using (29) and (24), (25), (26)

\[
\tilde{A} = -\frac{g^2}{2(m_j^2 - u)}
\]

\[
B = \frac{g^2}{2m_j (m_j^2 - u)}
\]

The drastic change introduced by substitution (29) can be noticed by exhibiting the \(J\) plane structure, indeed

\[
\frac{1}{m_j - \sqrt{u}} \Rightarrow \frac{1}{\sqrt{J - \frac{\alpha'}{\alpha} - \sqrt{u}}}
\]

With the pole a fixed square root cut has been introduced in the \(J\) plane. Its contribution has to be taken into account in the Reggeization of (29). This is schematically represented below.
Durand and Lipinski\textsuperscript{37}) showed that the same trick can be performed with a moving cut, by the further generalization of (30) ($\alpha'_{\text{pole}} = 1$)

\[
\frac{1}{\sqrt{1 - \alpha'_c} - \sqrt{u}} \rightarrow \frac{1}{\sqrt{1 - \alpha'_o - \alpha'_c u} - \sqrt{u}}
\]

provided

\[\alpha'_c < \alpha'_{\text{pole}}\]

Reggeizing an infinite sum of poles of the type (29) one can build dual models free of parity doublets\textsuperscript{38}).

The model has been applied to $\pi N$ backward scattering, leading to excellent fits to the data\textsuperscript{39,40}). Especially at large momentum transfers it is phenomenologically very attractive. This property of the fixed cut gave rise to the forward-backward cut interference model\textsuperscript{41}). In Figs. 1, 8 we show its application to $\pi^+ p \rightarrow p \pi^+$\textsuperscript{1}) and the recent $\pi^- p \rightarrow \pi^0 n$ data\textsuperscript{42}). A more detailed discussion of this type of models can be found in the contribution of Krzywicki\textsuperscript{43}) to this meeting.

Unfortunately in these cases the fixed cut violates u channel unitarity. This can be cured, but unfortunately at the expense of its phenomenological appeal. The same problem does not arise in photoproduction and as anticipated in the preceding Section the introduction of a fixed cut provides an excellent explanation of the absence of a dip at $u \approx -0.15$\textsuperscript{*}) and the energy scaling at large $|u|\textsuperscript{28}).

\textsuperscript{*}) Its presence in $\pi^+ p \rightarrow p \pi^+$ had to be explained by a double zero in the pole residue\textsuperscript{39}). This is rather unnatural.
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**FIGURE CAPTIONS**

**Figure 1** $\pi^+p \to \pi^+p$ total angular distribution, the trend of the data at 3 and 4 GeV/c is compared to a fit at 5 GeV/c involving forward and backward pole + cut models. Data from Ref. 1).

**Figure 2** $N^\alpha -N^\gamma$ Regge residue from the data in Refs. 21, 22), compared to the particle couplings.

**Figure 3** Same as Fig. 2 for the $A_8 - N$ residue. Data from Ref. 23).

**Figure 4** $A_8$ residue extracted from $\pi^-p \to p\pi^-$. Data from Ref. 23). The determination is done for different energies.

**Figure 5** Dual Regge fit for $\gamma p \to n\pi^+$ and $\gamma p \to p\pi^0$ differential cross-sections. Data from Ref. 27).

**Figure 6** V.M.D. prediction (vector dominance) of the $\pi^\pm p \to p\rho^\pm$ and $\pi^-p \to n\rho^0$ differential cross-sections. Data from Refs. 19, 30).

**Figure 7** V.M.D. prediction of the $\rho^+$ decay matrix elements in $\pi^-p \to p\rho^+$. Data from Ref. 19), transformed to the s channel helicity frame.

**Figure 8** $\pi^-p \to \pi^0n$ total angular distribution at 4.83 GeV/c, fitted with forward and backward pole + cut models, Ref. 41). The dashed and solid lines give an idea of the instability of the interference due to ambiguities on Regge parameters. Data from Ref. 42).
\( \beta(N_\alpha - N_\gamma) \) from \( K^- n \rightarrow \Lambda \pi^- \)

\( \beta(\Lambda_\alpha - \Lambda_\gamma) \) from \( K^+ p \rightarrow p K^+ \)

**FIG. 2**
$\pi^- p \rightarrow \pi^0 n$

- 6.0 GeVc J. Schneider
- 5.01 GeVc Yvert
- 4.83 GeVc Brockett et al.

$\frac{d\sigma}{dt} \left[ \frac{\mu b}{(\text{GeV}/c)^2} \right]$