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Transverse Impedance of the Circular Beam due to Two-Stream Instability.

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It is assumed to investigate the influence of two-stream interaction on a stability of the beam rotating in circular accelerator by use of new concept «two-stream transverse impedance», depending on the parameters of the second beam consisting from the particles with opposite sign of the electric charge. (these particles appear due to ionisation of the residual gas inside the vacuum chamber). For a case of an interaction between the circulating ions and the stored electrons, it is derived the formula connecting this impedance to the space-charge impedance of ion beam and the neutralisation degree, which is defined by pressure in the vacuum chamber. The theory is applied to an estimation of the neutralisation degree and the threshold value of the ion momentum spread for two facilities: TWAC (ITEP, Moscow, Russia) and SIS (GSI, Darmstadt, Germany).

Fig. - 3, ref. - 5 name.
1. Introduction

The transverse coupling instability of the coasting circular beam due to an interaction with the opposite sign particles stored inside the beam was examined in a lot of papers (refs. 1-4). It was shown that the instability has resonant character and may be suppressed by Landau damping if the both beams have frequency spreads.

In the present note we assume to describe the two-stream instability by introducing a new kind of the transverse impedance connected with the instability. Such impedance may be included in the standard scheme of transverse stability analysis taking into account all kinds of impedance.

Theoretical results are applied to the ion-electron instability which appears due to accumulation of electrons inside the coasting ion beams. A number of stored electrons is estimated with account of their heating due to Coulomb scattering of the electrons on the circulating ions. The thresholds of the instability have been calculated for two ion rings: SIS (GSI, Darmstadt, Germany) and ITEP heavy ion complex (Moscow, Russia).

2. Equations of motion

Equations of the transverse motion for two individual particles of the circular beams may be written as follows:

\[(\partial^2 \theta + \Omega \partial \theta) x + \Omega_0^2 (Q^2 x + Q_1^2 x_c - Q_2^2 x_c) = 0\]

\[\frac{d^2 y}{dt^2} + \Omega_0^2 (q_1^2 y + q_1^2 y_c + q_2^2 y_c) = 0\]  \hspace{1cm} (1)

Here \(x, y\) are transverse coordinates of two particles (\(x\) corresponds to rotating beam, \(y\) - to stored immovable beam), \(x_c\) and \(y_c\) are coordinates of the beam center of gravity, \(\Omega\) and \(\Omega_0\) are the rotation frequencies for the test particle and the central particle of the rotating beam, \(t\) is a time, \(\theta\) is the azimuthal angle, \(Q\) and \(q\) are incoherent betatron frequencies for two test particles, \(Q_1^2, Q_2^2, q_1^2, q_2^2\) are complex parameters which describe proportional to coordinates of the centers of
gravity. We solve eq. (1) by assuming that the coordinates of the test particles and the beam centers oscillate harmonically in time and space:

\[ x = \alpha \exp[i(n\theta - \omega t)], \quad y = b \exp[i(n\theta - \omega t)] \]

\[ x_c = A \exp[i(n\theta - \omega t)], \quad y_c = B \exp[i(n\theta - \omega t)] \]

Substituting these expressions in Eq. (1) and averaging all frequencies we obtain:

\[ A(1 + Q_1^2 R_1) - B Q_1^2 R_1 = 0 \]

\[ -A q_1^2 R_2 + B(1 + q_1^2 R_2) = 0 \]

(2)

Dispersion integrals \( R_{1,2} \) are defined by:

\[ R_1 = \int_{-\infty}^{\infty} \frac{f(u)du}{Q^2(u) - \omega^2/nQ(u)} / \Omega_0^2 \]

\[ R_2 = \int_{-\infty}^{\infty} \frac{f(v)dv}{Q^2(v) - \omega^2/\Omega_0^2} \]

(3)

Here \( f(u) \) and \( f(v) \) are normalised on unity distribution function on \( u \) and \( v \).

Taking into account that a determinant of an uniform linear system should be equal to zero, we can write the dispersion equation for slow wave (with \( \omega = \Omega_0 (n - Q) \)) in the following form:

\[ (1 + \delta Q_1 V_1) (1 + \delta q_1 V_2) - \delta^2 V_1 V_2 = 0 \]

(4)

where \( \delta Q_1 \) and \( \delta q_1 \) are coherent frequency shifts due to species-species forces, \( \delta \) is the increment of the two-stream instability for monochromatic beams in absence of species-species forces (1), which are defined by:

\[ \delta Q_1 = Q_1^2 / 2Q, \]

\[ \delta q_1 = q_1^2 / 2q \]

\[ \delta^2 = Q_1^2 q_1^2 / 4Qq \]

(5)

For the slow waves dispersion integrals \( V_1 \) and \( V_2 \) are:

\[ V_1 = \Omega_0 \int_{-\infty}^{\infty} \frac{f(u)du}{(n - Q(u))\Omega(u) - \omega} \]

\[ V_2 = \Omega_0 \int_{-\infty}^{\infty} \frac{f(v)dv}{q(v)\Omega_0 - \omega} \]

(6)

Let us rewrite our dispersion equation in the following form:

\[ 1 + V_1 (\delta Q_1 + \delta Q_{1,2}) = 0 \]

(7)

where
\[ \delta Q_{i,2} = \delta^2 \frac{1}{\delta q + V_i^{-1}} \quad (8) \]

Parameter \( \delta Q_{i,2} \) is a coherent frequency shift of the coasting beam due to two stream coherent forces. It is well-known that the coherent frequency shift is proportional to the corresponding transverse impedance \( Z_{1i} \). Therefore we can write:

\[ Z_{1i}^{12} = Z_{1i}^{1} \frac{\delta Q_{i,2}}{\delta Q_{i}} \quad (9) \]

Taking into account Eqs. (5), (6) and (8), we obtain:

\[ Z_{1i}^{12} = \frac{Q_i^{2}q_i^{2}}{2q_i^{2}(\delta q_i + V_i^{-1})} \quad (10) \]

### 3. Ion-electron instability

Let us consider in more details the instability due to interaction between the coasting ion beam and the electrons accumulated in the potential well, which is especially dangerous for heavy ion beams with high intensity and high ion charge number (such beams are assumed to use for plasma physics and accelerator experiments). The final aim of these experiments is an advanced studies concerned Heavy Ion Fusion (HIF).

In such rings the «space charge» transverse impedance of the ion beam \( Z_{se} \) is usually much more than the last terms. This impedance is defined by:

\[ Z_{se} = Z_0 R \frac{1}{\beta^2 \gamma^2} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \quad (11) \]

Here \( Z_0 \) is the free space impedance (\( Z_0 = 377 \) Ohm), \( R \) is the ring radius, \( a \) is the beam size and \( b \) is the vacuum chamber radius.

Our two-stream transverse impedance \( Z_{1i}^{12} \) depends on the electron dispersion integral \( V_2 \), i.e. on a form of the electron distribution function \( f(v) \). Estimations have shown that this dependance is not strong for every reasonable distribution function, and therefore we limit ourselves by a case of «hemicircle» distribution:

\[ f(x_0) = 2 \frac{1}{\Delta q_0 \pi} \sqrt{\frac{1}{\Delta q_0^2 - (\omega/\Omega_0 - q_0)^2}} \quad (12) \]

Then we obtain:

\[ \delta Q_{i,2} = \delta^2 \frac{1}{i \sqrt{\Delta q_0^2 - (\omega/\Omega_0 - q_0)^2 + q_0 + \delta q_1 - \omega/\Omega_0}} \quad (13) \]
Here $q_0$ is the mean value of the electron «betatron» frequency. Taking into account, that the most dangerous value of frequency corresponds to the coherent frequency of the ion oscillations in absence of the electrons, we find the following resonant condition:

$$n = Q_0 + q_0 + \delta Q_1 + \delta q_1$$ (14)

This condition is different from the classical one only due to appearance of two coherent frequency shifts due to species-species interaction. If Eq. (14) is satisfied, the two stream frequency shift reaches its maximal value which is equal to

$$\delta Q_{1,2} = -i \delta \frac{1}{\sqrt{\Delta q_0^2 - \delta q_1^2}}$$ (15)

Using these formulae, we obtain:

$$\delta Q_1 = \delta Q_{\text{anc}}(1-a^2/b^2)$$
$$\delta q_1 = q_e \eta \frac{1-a^2/b^2}{2\sqrt{1-\eta}}$$
$$\delta = \eta^2 \delta Q_1 q \frac{1}{2\sqrt{1-\eta}}$$ (16)

Substituting these expressions in Eq. (15) gives:

$$Z_{1,2} = -iZ_{\infty} \Lambda$$ (17)

where

$$\Lambda = \gamma^2 \frac{k}{\sqrt{1-k^2}}$$

$$k = \eta \frac{1-a^2}{2(1-\eta)\Delta q / q}$$ (18)

We see, that our system is stable, only if $k < 1$, i.e. the neutralization degree

$$\eta \leq \frac{2\Delta q / q}{1-a^2/b^2 + \Delta q / q}$$ (19)

This condition is necessary for the stability for our model; however, we should satisfy as well the standard stability kriterium for the coasting beams, taking into account the transverse impedance due to the electrons. We know, that in resonance point this impedance has pure active character. If we neglect all other sources of impedance, we can write the following condition for momentum spread of the ion beam $\Delta p/p$ (here $\Delta p/p$ is a half width at half height):
\[ \Delta p/p \leq (\Delta p/p)_0 F(\Lambda) \]  
\[ (\Delta p/p)_0 = 0.125 I \left( \frac{R|Z_{\perp}\xi|}{U_p} \right) \frac{Z_i}{A} \frac{1}{\beta \gamma Q|\Gamma||S|} \]  

Here \( F(\Lambda) \) describes an influence of two-stream transverse impedance (this function is plotted at Fig.1).

\[ q = \dot{\varphi} \frac{4R'(1-\eta)}{I_0 \beta' a_1(a_1 + a_2)} \]  

Here \( I_0 \) is Alfvén current \((I_0 = 17 \text{ kA})\), \( I \) is the beam current in Amperes. Substituting Eqs. (21,22) in Eq. (20), we obtain the following expression:

\[ \Delta p/p \leq 0.125 \frac{F(\Lambda) I/I_0}{\sqrt{I/I_0 + \alpha}} \left( \frac{R|Z_{\perp}\xi|}{U_p} \right) \frac{Z_i}{A} \frac{I_0}{\beta \gamma Q|\Gamma|q_0} \]  

Here \( I_0 \) is some reference value of the beam current, \( \alpha = \xi/\Gamma q_0 \), \( S_0 = q_0 + \xi/\Gamma \), \( q_0 \) is a value of electron frequency for \( I = I_0 \).
4. Numerical application

Let us apply this theory to two facilities: 1) SIS (GSI, Darmstadt, Germany); 2) TWAC (ITEP, Moscow, Russia). The list of its parameters is given in Table 1.

Table 1.

<table>
<thead>
<tr>
<th>Facility</th>
<th>ITEP</th>
<th>GSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kind of the ions</td>
<td>Co$_{59}^{27}$</td>
<td>U$_{238}^{27}$</td>
</tr>
<tr>
<td>Kinetic energy (MeV/u)</td>
<td>677</td>
<td>11</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.80</td>
<td>0.158</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>1.011</td>
</tr>
<tr>
<td>Ring radius R (m)</td>
<td>40</td>
<td>34</td>
</tr>
<tr>
<td>Betatron frequency Q</td>
<td>9.7</td>
<td>3.4</td>
</tr>
<tr>
<td>Chromaticity</td>
<td>-20</td>
<td>-1.41</td>
</tr>
<tr>
<td>Chamber radius (cm)</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Horizontal beam emittance (mm-mrad)</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Vertical beam emittance (mm-mrad)</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>$Z_{\perp}^e$ (MΩ/m)</td>
<td>21.2</td>
<td>1100</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.335</td>
<td>0.9408</td>
</tr>
<tr>
<td>Horizontal beam size (cm)</td>
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<td>3.2</td>
</tr>
<tr>
<td>Vertical beam size (cm)</td>
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<td>1.6</td>
</tr>
<tr>
<td>$I_0$ (A)</td>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td>$q_0$</td>
<td>282</td>
<td>94.8</td>
</tr>
</tbody>
</table>

The dependence $\Delta p/p$ on $I/I_0$ for $F(A)=1$ is given at Fig.2. The values of $\Delta p/p$ for different values of the neutralisation degree $\eta$ are plotted at Fig.3 (for $I=I_0$ and the electron frequency spread $\Delta q/q=0.1$). We see that two-stream instability significantly increases the necessary momentum spread of the ion beam and may become dangerous for comparatively small neutralisation degrees. The instability is more dangerous for ITEP facility, since for this case a compensation of the Coulomb electric force by the magnetic force is more weak (at a factor $\gamma^2$), and therefore the influence of the electron-ion force is enhanced. To avoid the instability, as it is shown in Appendix 2, we should keep a high vacuum in the accelerator chamber.
5. Discussion

We see that two-stream instability can significantly change the necessary momentum spread due to the appearance of real transverse impedance, whose value is proportional to the imaginary space charge impedance. To avoid the instability we should restrict the neutralisation degree. If the pressure and the electron concentration are too high, it is necessary to use for
the instability suppression the well-known methods, such as shaking (Ref.). Besides, it is very desirable to construct high frequency damping system suppressing the instability.

In conclusion, let us remark, that this theory has semi-qualitative character, since a number of effects is omitted in a frame of accepted simplifying assumptions. For example, we did not take into account an influence of the magnetic field of the accelerator on the electron dynamics. Moreover, the tails in the electron distribution (i.e., the electrons oscillating outside the ion beam) can result in appearance of more strong Landau damping of the electron dipole oscillations than we have assumed in this note.

Acknowledgements

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Appendix I

For space charge dominated beam it is convinient to express all ion parameters through the ion incoherent frequency shift $\delta Q_{inc}$, which is defined by

$$\delta Q_{inc} = \frac{r_p N_i Z_i^2 R}{\pi \beta^2 \gamma^2 A \alpha_i (\alpha_x + \alpha_y)}$$  \hspace{1cm} (A1-1)

Here $Z_i$ and $A$ are the charge and atomic ion number, $x, y$ are transverse coordinates, $\alpha_x$ and $\alpha_y$ are the corresponding beam sizes; parameter $\delta Q_{inc}$ is given for vertical oscillations. Using (Eq.16), we can write:

$$Q_o^2 = Q_{inc}^2 Q_0 (1 - \eta')$$

$$Q_1^2 = 2 \delta Q_{inc} Q_0 (1 - \alpha^2 / b^2)$$

$$Q_2^2 = 2 \delta Q_{inc} Q_0 \eta \eta' (1 - \alpha^2 / b^2)$$  \hspace{1cm} (A1-2)

Here $\eta$ is a neutralization degree, equalled to a relation of the electron number $N_e$ to $N_i$.

All electron parameters it is convinient to express through electron incoherent frequency in the ion focusing field $q_e$, which is defined by

$$q_e^2 = 2 r_p N_i Z_i R / [\pi \beta^2 \alpha_i (\alpha_x + \alpha_y)]$$  \hspace{1cm} (A1-3)

Then

$$q_1^2 = q_e^2 (1 - \eta)$$

$$q_1^2 = q_e^2 (1 - \alpha^2 / b^2)$$

$$q_2^2 = \eta q_e^2 (1 - \alpha^2 / b^2)$$  \hspace{1cm} (A1-4)

Incoherent frequency shift of the ions $\delta Q_{inc}$ and the squared incoherent frequency of the electrons $q_e^2$ are connected by the following expression:

$$q_e^2 = 2 Q \delta Q_{inc} \frac{m_p A \gamma^2}{m_e Z_i}$$  \hspace{1cm} (A1-5)

Sometimes it is more convinient to use the ion current $I$ instead of number of the ions $N_i$, which are connected by:

$$I = Z_i e \beta c N_i$$  \hspace{1cm} (A1-6)
Appendix 2

We see from Eq (22) that threshold of the two-stream instability is determined by frequency spread of the electron beam $\Delta q/q$ and the neutralization degree $\eta$. Frequency spread appears due to azimuthal variations of the ion size and due to nonlinearity of the ion field. Estimations show that the both sources result in the spread of order 10-15%. Now let us consider the neutralization degree. Equation describing the dependence of the electron number $N_e$ on time is:

$$\frac{dN_e}{dt} = \frac{N_e}{\tau_I} + \frac{N_eZ_i}{\tau_{neut}}$$

A stationary (maximal) neutralization degree is defined by

$$\eta = \frac{\tau_I}{\tau_{neut}}$$

Here $\eta$ is the electron life-time, $\tau_{neut}$ is the time of space charge neutralization.

The electron life time is determined by balance between heating and cooling processes. Main source of the electron heating is their Coulomb scattering by rotating ions; the corresponding heating rate is defined by:

$$\frac{dW_e}{dt} = \frac{4\pi n_e Z_i^2 e^4}{m_e \beta c} L$$

The electron will be lost if his energy is more than the depth of the ion beam potential well $W$, where

$$W = \pi n_e Z_i a^2 (1-\eta)$$

From these equations we find:

$$\tau_e = \frac{a^2 \beta}{4 r_e Z_i c L} (1-\eta)$$

From the other side,

$$\tau_{neut} = \frac{Z_i}{d_m \sigma_{ion} \beta c}$$

Here $d_m$ is the density of the residual gas in the vacuum chamber, $\sigma_{ion}$ is the ionization cross-section. Using these formulae, we obtain:

$$\eta = \eta_0 / (1+\eta_0)$$

$$\eta_0 = \frac{a^2 \beta^2 \sigma_{ion} d_m}{4 r_e Z_i^2} \frac{L}{L}$$

(A2-1)
Here (ref.4)

\[ \sigma_{\text{on}} = Z_i^2 K \beta^2 \left[ C + M_i^2 \left( \ln \frac{\beta^2}{1 - \beta^2} - \beta^2 \right) \right] \]

Here \( P_m \) is expressed in \( 10^{10} \) Torr, \( a \) in cm, and function \( \Phi_{(p)} \) \( (\Phi_{(p)} \approx 1) \) is defined by:

\[ \Phi_{(p)} = 1 + \frac{M_i^2}{C} \left( \ln \frac{\beta^2}{1 - \beta^2} - \beta^2 \right) \]

In this equation \( K = 1.87E(-24) \) m\(^2\), coefficients \( C \) and \( M_i^2 \) depend on a kind of residual gas in vacuum chamber (for \( N_2 \) \( C = 34.8 \) and \( M_i^2 = 3.7 \)). Molecular density \( d_m \) is connected with pressure in vacuum chamber by:

\[ d_m = 3.3 \times 10^{12} P_m \]

Substituting all constants (Coulomb logarithm \( L = 20 \)), we find the final simple expression for \( \eta_b \):

\[ \eta_b = 3.3 \times 10^{-3} P_m \alpha^2 C \Phi_{(p)} \]

(A2-2)

References

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