The D1-D5 Brane System in Type I String Theory

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Abstract

We construct the supergravity solution for the intersecting D1-D5 brane system in Type I String Theory. The solution encodes the dependence on all the electric charges of the $SO(32)$ gauge group. We discuss the near horizon geometry of the solution and a proposed dual $(0,4)$ superconformal field theory.
1 Introduction

The D1-D5 brane system in Type I string theory may provide us with yet another example of a duality between superstring theory on AdS$_3$ background and two dimensional SCFT. We will consider the system of parallel D1 and D5 branes with charges $Q_e$ and $Q_p$ respectively, where the D5 branes wrap a compact space $M$ which is $T^4$ or $K3$. The purpose of this letter is to construct the corresponding supergravity solution with non-zero $SO(32)$ gauge fields.

The supergravity solution in Type I theory describing D1 and D5 branes intersecting in one spatial dimension has been constructed in [1] and partially in [2, 3]. However, in these solutions the $SO(32)$ gauge group in the open string sector is not turned on. Therefore, they are not complete Type I solutions, as the consistency of the Type I superstring theory without spacetime anomalies requires the existence of the $SO(32)$ gauge group. This is what distinguishes the Type I supergravity solutions from the corresponding solutions in the Type IIB theory. We will therefore construct the supergravity solution for the intersecting D-brane configuration with all the 496 electric charges of the $SO(32)$ gauge group turned on.

The strategy which we will use is to first construct the supergravity solution for intersecting fundamental strings and NS5-branes with non-zero $SO(32)$ gauge group in heterotic theory and then apply the $S$-duality transformation relating Type I and $SO(32)$ heterotic string theories to obtain the required Type I solution.

The paper is organized as follows. In section 2, we briefly review the Type I - $SO(32)$ heterotic duality in ten dimensions. We then discuss the technique for generating the solution. Finally, we present the non-extreme as well the BPS heterotic and Type I solutions. In section 3 we construct the near horizon geometry of the Type I solution and discuss a proposed duality to a $(0,4)$ SCFT in two dimensions.

2 The Supergravity Solution

In this section, we construct the supergravity solution of Type I superstring theory describing intersecting D1 and D5 branes with non-zero $SO(32)$ gauge field. For this purpose, we first construct intersecting fundamental string and NS5-brane solution with non-zero $SO(32)$ gauge group in heterotic theory and then apply the $S$-duality transformation relating Type I and $SO(32)$ heterotic string theories to obtain this Type I theory solution.
2.1 Type I - $SO(32)$ heterotic duality in ten dimensions

In this subsection, we summarize the $S$-duality transformation that relates the Type I superstring to the $SO(32)$ heterotic superstring in ten dimensions for the purpose of fixing notations.

The bosonic part of the low energy effective action for the $SO(32)$ heterotic string theory is described by the massless bosonic string states, which are the graviton $G_{MN}^{(H)} (M, N = 0, 1, \ldots, 9)$, the dilaton $\Phi^{(H)}$ and the two-form field $B_{MN}^{(H)}$ in the NS sector of the closed heterotic string along with gauge fields $A_{M}^{(H)a} (a = 1, \ldots, 496)$ in the adjoint representation of the $SO(32)$ gauge group. The action in the string frame is given by

$$S^{(H)} = \frac{1}{16\pi G_{N}} \int d^{10}x \sqrt{-G^{(H)}} e^{-\Phi^{(H)}} \left[ R_{H} + G_{MN}^{(H)} \partial_{M}^{(H)} \partial_{N}^{(H)} \right]$$

$$- \frac{1}{12} H_{MNP}^{(H)} H_{(H)MNP}^{(H)} - \frac{1}{4} \text{Tr}(F_{MN}^{(H)} F_{(H)MN}^{(H)})$$

where $G_{N}$ is the ten-dimensional Newton’s constant, $R_{H}$ is the Ricci scalar of the metric $G_{MN}^{(H)}$, and field strengths $H_{MNP}^{(H)}$ and $F_{MN}^{(H)}$ of $B_{MN}^{(H)}$ and $A_{M}^{(H)}$ are defined as

$$H_{MNP}^{(H)} = \partial_{M} B_{NP}^{(H)} - \frac{1}{2} \text{Tr} \left( A_{M}^{(H)} F_{NP}^{(H)} - \frac{\sqrt{2}}{3} A_{M}^{(H)} [A_{N}^{(H)}, A_{P}^{(H)}] \right) + \text{cyc. perms. in } M, N, P,$$

$$F_{MN}^{(H)} = \partial_{M} A_{N}^{(H)} - \partial_{N} A_{M}^{(H)} + \sqrt{2} [A_{M}^{(H)}, A_{N}^{(H)}].$$

Here, the trace $\text{Tr}$ is in the vector representation of $SO(32)$.

The Type I superstring theory is defined as the orientifold projection of the Type IIB superstring theory. In the massless bosonic NS sector of the closed string, only the graviton $G_{MN}^{(I)}$ and the dilaton $\Phi^{(I)}$ survive the orientifold projection. In the bosonic RR sector, the only surviving massless mode is the two-form field $B_{MN}^{(I)}$. The bosonic open string sector gives rise to gauge fields $A_{M}^{(I)a} (a = 1, \ldots, 496)$ of the $SO(32)$ gauge group. In the string frame, these massless bosonic modes are described by

$$S^{(I)} = \frac{1}{16\pi G_{N}} \int d^{10}x \sqrt{-G^{(I)}} \left[ e^{-\Phi^{(I)}} \left( R_{I} + G_{MN}^{(I)} \partial_{M}^{(I)} \partial_{N}^{(I)} \right) \right]$$

$$- \frac{1}{12} H_{MNP}^{(I)} H_{(I)MNP}^{(I)} - \frac{1}{4} \text{Tr}(F_{MN}^{(I)} F_{(I)MN}^{(I)})$$

where $R_{I}$ is the Ricci scalar of the metric $G_{MN}^{(I)}$, and field strengths $H_{MNP}^{(I)}$ and $F_{MN}^{(I)}$ of $B_{MN}^{(I)}$ and $A_{M}^{(I)}$ are defined similarly as above.

Note, these two theories have the same field contents and also the same supersymmetry. In fact, the actions (1) and (3) become identical, provided one relates the fields in the two actions in the following way [4]:

$$G_{MN}^{(I)} = e^{-\frac{\Phi^{(I)}}{2}} G_{MN}^{(H)}, \quad \Phi^{(I)} = -\Phi^{(H)}, \quad B_{MN}^{(I)} = B_{MN}^{(H)}, \quad A_{M}^{(I)} = A_{M}^{(H)}.$$
Since $g_s = e^{\phi}$ is defined as the string coupling constant, one sees that the strong coupling limit of one theory is related to the weak coupling limit of the other theory. Under this $S$-duality transformations, the fundamental string [the solitonic NS5-brane] of the heterotic theory and the D-string [the D5-brane] in the Type I theory are related.

### 2.2 Solution generating technique

Fundamental strings and (solitonic) NS5-branes in heterotic string respectively carry electric and magnetic charges of the NS two-form field $B^{(H)}_{MN}$. Additionally, we want the $p$-brane configuration to be charged under the $SO(32)$ gauge group. We are therefore interested in the configuration which carries electric charges of the $SO(32)$ gauge group, i.e. the so-called colored non-Abelian solution $^\ast$. Such solutions are constructed by embedding the $U(1)$ groups in the non-Abelian gauge group as the adjoint representation. Namely, one takes the Ansatz for the non-Abelian $SO(32)$ gauge field $A^{(H)}_M$ of the form $A^{(H)}_M = \beta^a A_M$, where $A_M$ is an $U(1)$ gauge field and the parameters $\beta^a$ satisfy the constraint $\gamma_{ab} \beta^a \beta^b = 1$. Here, $\gamma_{ab}$ is an invariant metric of the $SO(32)$ group. Then, the field strength $F^{(H)}_{MN} (2)$ of the “non-Abelian” $SO(32)$ gauge group reduces to the field strengths $F^{(H)}_{MN} = \beta^a F^0_{MN}$ of 496 $U(1)$ gauge fields $A^{(H)}_M = \beta^a A_M$. So, the effective action (1) becomes that with 496 Abelian $U(1)$ gauge fields. In particular, 496 electric charges $Q^{(H)}_a$ of the $U(1)$ gauge fields form the adjoint representation of the $SO(32)$ gauge group. One can induce these $p$-brane charges and the electric charges of the $SO(32)$ gauge group on the uncharged black $p$-brane solution by first compactifying it down to 3 dimensions and then applying the appropriate boost transformations in the $U$-duality symmetry of the 3-dimensional action.

Since we are interested in constructing the intersecting fundamental string and NS5-brane solution which is localized along the overall transverse directions, only, we start from the following uncharged black fivebrane solution in $D = 10$:

$$G^{(H)}_{MN} dx^M dx^N = -(1 - \frac{2m}{r^2}) dt^2 + dx_1^2 + \cdots + dx_5^2 + (1 - \frac{2m}{r^2})^{-1} dr^2 + r^2 d\Omega_3^2, \quad (5)$$

where $d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi_2^2 + \cos^2 \theta d\phi_2^2$ is the infinitesimal line element on $S^3$. The remaining fields are zero.

One can compactify the solution (5) in the $t$, $x_i$ and $\phi_2$ directions on a torus down to three dimensions, since the solution is independent of these coordinates. The Kaluza-Klein Ansatz of the metric for such compactification is given by

$$G^{(H)}_{MN} = \left( e^{\varphi} h_{\mu\nu} + G_{mn} A^{m}_\mu A^{n}_\nu A^m \right)$$

$$A^m G_{mn}, \quad (6)$$

$^\ast$The other class of solutions, which are regarded as genuine non-Abelian solutions, are neutral under the non-Abelian gauge group, i.e. do not carry “global charges” of the non-Abelian gauge group.
where $\phi \equiv \Phi^{(H)} - \frac{1}{2} \ln \det G_{mn}$ is the 3-dimensional dilaton and the indices run as $\mu, \nu = r, \theta, \phi_1$ and $m, n = t, \phi_2, x_1, \ldots, x_5$. The ten-dimensional heterotic action (1) compactifies to the following three-dimensional form, which we write down only the final form as the details on its derivation and the field definitions can be found elsewhere [5, 6, 7]:

$$
\mathcal{L} = \frac{1}{4} \sqrt{h} \left[ R_h + \frac{1}{8} h^{\mu\nu} \text{Tr} (\partial_\mu M L \partial_\nu M L) \right],
$$

where $R_h$ is the Ricci scalar of the metric $h_{\mu\nu}$. Note, since we keep all the 496 $U(1)$ gauge fields in the adjoint representation of the $SO(32)$ gauge group, the scalar moduli matrix $M$ is now an $O(8, 504)$ matrix and $L$ is an invariant metric of the $O(8, 504)$ group. The action (7) is invariant under the following $O(8, 504)$ duality transformation:

$$
h_{\mu\nu} \rightarrow h_{\mu\nu}, \quad M \rightarrow \Omega M \Omega^T,
$$

where $\Omega \in O(8, 504)$. The $SO(1, 1)$ boost transformations in this $O(8, 504)$ transformation group induce electric and magnetic charges of $p$-branes and $U(1)^{496} \subset SO(32)$ gauge group on the uncharged solution (5).

### 2.3 Non-extreme solutions

#### 2.3.1 Heterotic solution

After applying the $SO(1, 1)$ boost transformations in the $O(8, 504)$ duality group (8) with the boost parameters $\delta_e, \delta_p$ and $\delta_a (a = 1, \ldots, 496)$ on the uncharged black 5-brane solution (5) compactified down to three dimensions, one obtains the following non-extreme supergravity solution describing the intersecting fundamental string and NS5-brane with 496 electric charges of the $SO(32)$ gauge group:

$$
\begin{align*}
G_{MN}^{(H)} dx^M dx^N &= \left[ 1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2} \right]^{-2} \left[ -\left( 1 + \frac{2m \sinh^2 \delta_e}{r^2} \right) \left( 1 - \frac{2m}{r^2} \right) dt^2 \\
&\quad + \frac{2m \sinh \delta_e \cosh \delta_e (\Delta - 1)}{r^2} \left( 1 - \frac{2m}{r^2} \right) dt dx_1 \\
&\quad + \left( 1 + \frac{2m \cosh^2 \delta_e \Delta - 1}{r^2} + \frac{m^2 \cosh^2 \delta_e (\Delta - 1)^2}{r^4} \right) dx_1^2 \\
&\quad + \sum_{i=2}^5 dx_i^2 + \left( 1 + \frac{2m \sinh^2 \delta_p}{r^2} \right) \left( 1 - \frac{2m}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3^2 \right],
\end{align*}
$$

$$
e^{\Phi^{(H)}} = \frac{1 + \frac{2m \sinh^2 \delta_p}{r^2}}{1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}},
$$

$$
B_{tr_1}^{(H)} = \frac{1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}}{1 + \frac{m \sinh \delta_e \cosh \delta_e (\Delta + 1)}{r^2}}.
$$
\[
B_{\phi_1\phi_2}^{(H)} = 2m \sinh \delta_p \cosh \delta_p \sin^2 \theta,
\]
\[
A_t^{(H)} = \frac{\sqrt{2}m \cosh^2 \delta_e \sinh \delta_a \Delta_a}{1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}} \quad (9)
\]
\[
A_{x_1}^{(H)} = \frac{\sqrt{2}m \sinh \delta_e \cosh \delta_e \sinh \delta_a \Delta_a}{1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}},
\]
where
\[
\Delta = \prod_{i=1}^{496} \cosh \delta_i, \quad \Delta_a = \prod_{i=a+1}^{496} \cosh \delta_i \quad (10)
\]

The charges \(Q_e\) and \(Q_p\) of the fundamental string and the solitonic NS5-brane and the electric charges \(Q_a\) of the \(U(1)^{496} \subset SO(32)\) gauge group carried by this brane configuration are given by
\[
Q_e = m \sinh \delta_e \cosh \delta_e (\Delta + 1), \quad Q_p = 2m \sinh \delta_p \cosh \delta_p,
\]
\[
Q_a = \sqrt{2}m \cosh^2 \delta_e \sinh \delta_a \Delta_a. \quad (11)
\]

Note, in the limit where the electric charges \(Q_a\) of the \(SO(32)\) gauge group are zero \((\delta_a = 0)\), the solution (9) reduces to the non-extreme version of the intersecting fundamental string [8] and NS5-brane [9, 10] solution.

### 2.3.2 Type I solution

The following non-extreme supergravity solution describing the intersecting D1 and D5 branes in the Type I string theory is obtained by applying the S-duality transformation (4) on the heterotic solution (9):

\[
G_{MN}^{(I)} dx^M dx^N = \left(1 + \frac{2m \sinh^2 \delta_p}{r^2}\right)^{-\frac{1}{2}} \left[1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}\right]^{-\frac{1}{2}}
\]
\[
\times \left[-(1 + \frac{2m \sinh^2 \delta_e}{r^2})(1 - \frac{2m}{r^2}) dt^2 + \frac{2m \sinh \delta_e \cosh \delta_e (\Delta - 1)}{r^2} (1 - \frac{2m}{r^2}) dtdx_1 + \left(1 + \frac{2m \cosh^2 \delta_e (\Delta - 1)}{r^2} + \frac{m^2 \cosh^2 \delta_e (\Delta - 1)^2}{r^4}\right) dx_1^2\right]
\]
\[
\times \left[1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2} \right]^{\frac{1}{2}} \left(\sum_{i=2}^{5} dx_i^2\right) + \frac{2m \sinh \delta_p}{r^2}\right)^{\frac{1}{2}} \left[1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}\right]^{\frac{1}{2}} \left(\frac{dr^2}{1 - \frac{2m}{r^2}} + r^2 d\Omega_3^2\right),
\]
\[ e^{\Phi^{(I)}} = \frac{1 + \frac{m \cosh^2 \delta_e (\Delta + 1) - 2m}{r^2}}{1 + \frac{2m \sinh^2 \delta_p}{r^2}}, \quad (12) \]

where the remaining fields have the same forms as the heterotic solutions (9).

The D1 brane charge \( Q_e \), the D5 brane charge \( Q_p \) and electric charges \( Q_a \) of the \( U(1)^{496} \subset SO(32) \) gauge group are given as in (11).

In the limit that the electric charges \( Q_a \) of the \( SO(32) \) gauge group are zero \( (\delta_a = 0) \), the solution (12) reduces to the non-extreme generalization of the intersecting D1 and D5 brane solution constructed in [1].

### 2.4 The BPS solutions

The BPS limit of the above solutions (9) and (12) is defined as the limit in which the non-extremality parameter \( m \) goes to zero while keeping \( 2m \sinh \delta_p \cosh \delta_e = Q \) and \( 2m \sinh \delta_p \cosh \delta_p = Q_p \) as finite non-zero constants. In the following, we write down the \( p \)-brane solutions (9) and (12) in the BPS limits.

#### 2.4.1 Heterotic solution

The BPS intersecting fundamental string and NS5-brane solution with the electric charges of the \( SO(32) \) gauge group is given by:

\[
G_{MN}^{(H)} dx^M dx^N = \left[ 1 + \frac{Q(\Delta + 1)}{2r^2} \right]^{-2} \left[ -(1 + \frac{Q}{2r^2}) dt^2 \\
+ \frac{Q(\Delta - 1)}{r^2} dtdx_1 + (1 + \frac{Q\Delta}{r^2}) dx_1^2 \\
+ \sum_{i=2}^5 dx_i^2 + (1 + \frac{Q_p}{r^2})(dr^2 + r^2 d\Omega_3^2) \right] \\
+ \frac{5}{2} dx_1^2 + (1 + \frac{Q_p}{r^2})(dr^2 + r^2 d\Omega_3^2),
\]

\[
e^{\Phi^{(H)}} = (1 + \frac{Q_p}{r^2})(1 + \frac{Q(\Delta + 1)}{2r^2})^{-1},
\]

\[
B_{i_1x_1}^{(H)} = \frac{Q(\Delta + 1)}{2r^2} \left( 1 + \frac{Q(\Delta + 1)}{2r^2} \right)^{-1}, \quad B_{\phi_1\phi_2}^{(H)} = Q_p \sin^2 \theta,
\]

\[
A^{(H)}_a = \frac{Q_a}{r^2} \left( 1 + \frac{Q(\Delta + 1)}{2r^2} \right)^{-1} = A_{x_1}^{(H)} a.
\]

Note, the charge \( Q_e \) of the fundamental string is defined as \( Q_e = Q(\Delta + 1) \).
2.4.2 Type I solution

The BPS intersecting D1 and D5 brane with electric charges of the $SO(32)$ gauge group in the open string sector is as follows:

\[
G^{(l)}_{MN} dx^M dx^N = (1 + \frac{Q_e}{r^2})^{-\frac{\Delta}{2}} (1 + \frac{Q(\Delta + 1)}{2r^2})^{-\frac{\Delta}{2}} \left[ -(1 + \frac{Q}{r^2}) dt^2 
+ Q(\Delta - 1) \frac{r^2}{dt} dx_1 + (1 + \frac{Q}{r^2} d^2 x_1^2) \right]
+ (1 + \frac{Q(\Delta + 1)}{2r^2})^\frac{\Delta}{2} (1 + \frac{Q}{r^2})^{-\frac{\Delta}{2}} \sum_{i=2}^{5} dx_i^2
+ (1 + \frac{Q}{r^2})^\frac{\Delta}{2} (1 + \frac{Q(\Delta + 1)}{2r^2})^\frac{\Delta}{2} (dr^2 + r^2 d\Omega_3^2),
\]

\[e^{\Phi(l)} = (1 + \frac{Q(\Delta + 1)}{2r^2}) (1 + \frac{Q}{r^2})^{-1}. \quad (14)\]

where the D1 brane charge $Q_e$ is given by $Q_e = Q(\Delta + 1)$.

3 The Near-Horizon Geometry and $\theta$ SCFT

The decoupling limit of the worldvolume theory of the intersecting $D$-brane configuration corresponds to the near-horizon limit of the corresponding supergravity solution. The near-horizon geometry of the supergravity solution (14) is obtained by keeping only the $1/r^2$ terms in the harmonic functions in the solution. The resulting metric has the following form:

\[
G^{(l)}_{MN} dx^M dx^N = \left(\frac{2}{\Delta + 1}\right)^\frac{\Delta}{2} \frac{r^2}{\sqrt{QQ_p}} \left[ -dt^2 + (\Delta - 1) dt dx_1 + \Delta dx_1^2 \right]
+ \sqrt{\Delta + 1} \frac{1}{2} \sqrt{QQ_p} \left( dx_2 + \cdots + dx_7^2 \right)
+ \sqrt{\Delta + 1} \frac{1}{2} \sqrt{QQ_p} \left( \frac{dr^2}{r^2} + d\Omega_3^2 \right). \quad (15)\]

If one redefines the coordinates $t$ and $x_1$ in the following way:

\[
t \rightarrow t' = \sqrt{\frac{2}{\Delta + 1}} t, \quad x_1 \rightarrow x'_1 = \sqrt{\frac{2}{\Delta + 1}} x_1, \\
t' \rightarrow t'' = t' - \frac{\Delta - 1}{2} x'_1, \quad x'_1 \rightarrow x''_1 = \frac{\Delta + 1}{2} x'_1. \quad (16)\]
then the metric (15) takes the recognizable form (in the following double primes are suppressed):

\[ G^{(f)}_{MN} dx^M dx^N = \frac{r^2}{Q_e Q_p} (-dt^2 + dx_1^2) + \sqrt{Q_e Q_p} r^2 dr^2 + \sqrt{\frac{Q_e}{Q_p}} \sum_{i=2}^{5} dx_i^2 + \sqrt{Q_e Q_p} d\Omega_3^2. \]  

(17)

This corresponds to the metric describing AdS$_3 \times M \times S^3$ with the radii of AdS$_3$ and $S^3$ being $R_{AdS}^2 = R_{S^3}^2 = \sqrt{Q_e Q_p}$ and the volume of $M$ being $v_M = Q_e/Q_p$.

The D1-D5 brane configuration in Type I string theory is expected to encode the information on the moduli space of instantons on $M$. The worldvolume theory of $k$ flat D5-branes in Type I theory has as its Higgs branch the moduli space of $SO(32)$ $k$-instantons on $R^4$ [11]. Using a D1 brane probe extending parallel to the D5-branes, one gets the ADHM data encoded in the Yukawa couplings of the $(0,4)$ sigma model of D1 brane worldvolume theory [12, 13]. More precisely, the condition for $(0,4)$ supersymmetry of the sigma model requires that the couplings satisfy the ADHM equations. For every instanton, one gets a $(0,4)$ sigma model that flows in the infrared to a solution of string theory for the corresponding instanton. This generalizes to other non-compact manifolds such as ALE spaces where a similar ADHM construction exists.

When the D5 branes wrap a compact manifold $M$, there should presumably be a similar relation between the $(0,4)$ theory on the D1 brane worldvolume to the instanton moduli space. This is not yet known. In the spirit of other examples of the AdS/SCFT correspondence [14, 15, 16] one is led to conjecture that the $(0,4)$ SCFT in the infrared limit of the D1 brane worldvolume theory is dual to Type I string theory on the AdS$_3 \times M \times S^3$ background (17). The supergroup $SU(1,1/2) \times SL(2,R) \times SU(2)$ of the Type I compactification is mapped under the conjectured duality to the symmetry supergroup of the $(0,4)$ SCFT.

For charges $Q_e, Q_p >> 1$, one can use the supergravity approximation. The supergravity theory can be reduced first on $M$ from ten to six dimensions. One can then study the spectrum of the Kaluza-Klein excitations of $N = 1$ supergravity on AdS$_3 \times S^3$ as in [17]. It will be interesting to construct the string theory picture as done for the D1-D5 system in Type IIB string theory [18].

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References