COHERENT MICROWAVE COOLING (CMC)
OF ELECTRON AND ION BEAMS

Hidetsugu Ikegami
Research Center for Nuclear Physics, Osaka University
Mihogaoka 10-1, Ibaraki, Osaka 567, Japan
and
Department of Radiation Sciences, Uppsala University
Box 535, S-751 21 Uppsala, Sweden

ABSTRACT

A new principle "coherent microwave cooling" (CMC) of charged particle beams is described. Particles are forced, by a magnetic field over some length of their trajectory, to move in a helical path and interact with an rf field. The validity of the principle has been examined experimentally. The experimental results provided some inspiring indications of the "cooling" in the transverse energy-time phase space of the electrons. Electrons initially in the range $8 - 12\,\text{keV}$, were all found to accumulate at one discrete energy, $23\,\text{keV}$. Phase bunching, a feature of the cyclotron maser, was observed indirectly in the gyration of the accumulated electrons of $23\,\text{keV}$ energy. The agreement between the prediction and the experimental results implies that in principle any kind of charged particle beam can be cooled in a single pass through the CMC sections regardless of its energy.

1 INTRODUCTION

The invention of beam cooling has opened up new ways of reducing drastically the phase space of circulating particles [1-6]. In electron cooling the ion beam is merged with a parallel beam of mono-energetic electrons in a cooling section. The thermal energies of the ions seen in the rest frame of the electrons, are transferred to the electron beam as a low temperature reservoir through heat conduction. However for the case of ion beams of higher energy, the cooling times are much longer because of the $\gamma^2$ dependence of the relativistic time dilation effect in the interacting two body system where $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic energy factor [1].

In stochastic cooling, some pick-up systems are installed in a particle storage ring to measure the average positions of circulating particles. Corresponding kicker systems are placed downstream from the pick-up systems to correct any beam excursion at the pick-up systems [3]. In this cooling scheme, especially shaped space and time characteristics of the electric pulses are essential in the kicker systems to yield non-linear effects such as the cooling of particle beam.
This cooling scheme is, in the physical sense, analogous to the action of a sheepdog against flocks and thus it is easy to understand that the cooling time is lengthening in proportion to the number of particles in the pulses.

The *radiation cooling* due to the synchrotron radiation is at present the best mastered scheme for the cooling of the longitudinal phase space of electrons and of protons of extremely high energies but at the sacrifice of heating in their transverse phase space [2]. Other cooling schemes such as *laser cooling* [5] and cooling by inelastic scattering [6] are applicable to only partially ionized ions.

Now, it would be of particular importance to note that most cooling schemes of transverse phase space so far developed are based on the mechanism of *spontaneous* transfer of the *thermal* movement of a particle beam to a low temperature reservoir through heat conduction or radiation but none has been developed based on the stimulated cooling mechanism. From this viewpoint, the most crucial low temperature reservoir would be the coherent microwave field. The newly presented principle *coherent microwave cooling* (CMC) dramatically enhances the *thermal* transfer mechanism by the stimulated coherent emission or absorption of photons and removes drawbacks of present cooling schemes such as the slow cooling speed and the limitations on the energy and the intensity of particle beams to be cooled [7]. Of particular interest is that CMC is in principle applicable to any kind of charged particles with arbitrary energy resulting in an *rapid cooling* in the transverse energy-time phase space. CMC may be understood as a *forced radiation cooling* which is a typical non-Liouvillean phenomenon [7]. This talk outlines the basic concept of CMC [7,8] and presents the first experimental results which provide some indications of CMC supporting the prediction of *cooling* in the transverse phase space of electrons [8,9].

## 2 MECHANISM OF CYCLOTRON MASER

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**— OSCILLATOR APPROACH**

The mechanism of CMC has been discussed so far [7,8] on the basis of the quantum mechanical oscillator approach. In this talk, I would like to present the CMC principle again in the simple quantum mechanical approach for the simplicity of the discussions. However, it would be of significance to understand the mechanism also as a classical physics phenomena in the scheme of dynamics of gyrating particles. For this purpose, a simple approach has been developed based on classical particle dynamics (Appendix). Of special interest is that both approaches lead to identical results!

The CMC can be applied to particles circulating in a storage ring as shown in Fig. 1. In the CMC sections a stimulating rf field is sent antiparallel or parallel to a particle beam in the presence of a solenoidal magnetic field $B_0$. A fraction of the longitudinal energy of particles with rest mass $m_0$ and charge $e$ is changed into transverse energy by introducing the particles in the CMC sections at an angle of inclination to the longitudinal axis of the solenoidal magnetic field $B_0$ yielding a gyration of the particles with the cyclotron frequency $\omega_c \equiv eB_0/m_0$. Quantities with an asterisk refer to the particle system with no longitudinal velocity and MKSA units are used throughout this talk. In the presence of an rf field of angular frequency $\omega_r$, electric field amplitude $E_r$ and energy flow density defined by the absolute value of the Poynting vector $I^* (\omega_r)$ of which electric field vectors are perpendicular to the axis of the solenoid, the gyrating particles are governed by the Lorentz force $F_m = e\beta_\perp B_0$ generated by the magnetic field $B_0$ and an electric force $F_e = eE_r \cos(\omega_r t^*)$ in the particle rest frame where their longitudinal momentum is zero on average. Here, $c$ and $\beta_\perp$ denote the speed of light and the gyration speed, respectively.

The particles gyrating in the solenoidal magnetic field may be described as harmonic oscillators with Landau level spacings, $\hbar \omega_L^*$ in the particle rest frame. In the presence of the rf field,
the gyrating particles, described by harmonic dipole oscillators, undergo radiative transitions leading to a change of their transverse energy $\gamma_\perp m_0 c^2$ with time,

$$\frac{d\gamma_\perp}{dt^*} = \frac{1}{2} \left( \frac{e E_r^*}{m_0 c} \right)^2 R^* F(x_0) = \frac{4\pi r_p}{m_0 c} I^*(\omega_r^*) \tau^* F(x_0),$$

where

$$F(x_0) \equiv (1 + x_0^2)^{-1},$$

$$x_0 \equiv 2\tau^*(\omega_c^* - \omega_r^*).$$

Here, $\gamma_\perp = (1 - \beta_\perp)^{-1/2}$ and $r_p(= e^2/4\pi\varepsilon_0 m_0 c^2)$ denote the relativistic energy factor and the classical radius of particles, respectively, and $\tau^*$ is the damping time of the oscillator — hereafter called phase debunching time. The subscript $\perp$ and $\parallel$ denote the transverse and longitudinal directions, respectively. It is seen that Eq. (1) is a well-known formula of a damping dipole oscillator, which can also be derived classically [10]. The function $F(x_0)$ defined by Eq. (2) is the harmonic oscillator response function of Lorenzian shape, which corresponds to the normalized energy gain of gyrating particles.

As a matter of fact, the gyro-frequency of the particles is $\omega_c^*/\gamma_\perp^*$ instead of $\omega_c^*$ due to the relativistic mass effect on the gyration. The gyrating particles should thus be described as unharmonic oscillators with non-equal Landau level spacings $\hbar \omega_c^*/\gamma_\perp^*$, resulting in a shift of resonance frequency. Thus the variable $x_0$ in Eq. (2) has to be replaced by $x$,

$$x \equiv 2\tau^*[\omega_c^*/\gamma_\perp^* - \omega_r^*].$$
However, this replacement of variable is not enough to describe the unharmonic oscillators. The response function $F(z)$ associated with an unharmonic oscillator generally consists of the dipole term $(1 + z^2)^{-1}$ and an additional quadrupole term which is proportional to $(x - x_0)$ and the derivative of the dipole term having the form $(x - x_0) d(1 + z^2)^{-1} / dz$ [8]. We have thus,

$$F(z) = \frac{1}{1 + x^2} + \frac{2az}{(1 + x^2)^2},$$  \hspace{1cm} (5)

$$a \equiv x_0 - x = 2\omega_c^* r^*(1 - \gamma_{\perp}^{-1}) = 2\omega_c r(1 - \gamma_{\perp}^{-1}).$$  \hspace{1cm} (6)

It should be noted that an opposite sign of $a$ in Eq. (5) would result in $F(z) \to F(x_0)$ losing unharmonicity. The first term of $F(z)$ in Eq. (5) describes incoherent absorptive transitions of dipole oscillator with a relativistically shifted resonance frequency $\omega_c^* / \gamma_{\perp}^*$ while the second $a$-dependent term is concerned with the transitions of quadrupole oscillator of the same resonance frequency. The correction of the gyro-frequency is essentially due to the relativistic mass effect on the gyration, which causes the replacement of $m_0$ in Eq. (1) by $\gamma_{\perp} m_0$ and hence we have,

$$\frac{d\gamma_{\perp}}{dt^*} = \frac{1}{2} \left( \frac{eE_d^*}{\gamma_{\perp} m_0 c} \right)^2 \tau^* F(x) = \frac{4\pi r_0}{\gamma_{\perp}^2 m_0 c} \Gamma^*(\omega_c^*) \tau^* F(x).$$  \hspace{1cm} (7)

The transition rates, Eqs. (5-7), are identical with those derived in the Appendix on the basis of classical dynamical approach, in which a damped dipole oscillator term corresponds to the gyration speed $c\beta_{\perp}$ dependent frictional term whereas an unharmonic quadrupole oscillator term corresponds to the gyro-frequency $\omega_c^* / \gamma_{\perp}$ dependent frictional term. As seen in Eq. (5), the unharmonic oscillation due to the relativistic effect on the gyro-frequency causes a drastic rate in the change of energy proportional to $(\tau^*)^3$ because both $a$ and $x$ are proportional to $\tau^*$. On the contrary, the rate of damped dipole oscillation varies only linearly with the phase debunching time $\tau^*$, which has no coherency of gyration due to the lack of phase bunching effect as explained in the classical dynamical approach (Appendix).

As seen in Fig. 2, the unharmonic quadrupole transition rate is remarkably enhanced with increase of $a$—hereafter called coherency factor. In fact, this mechanism is explained as an effect of the stimulated radiation caused by the coherent gyration, i.e. the bunched particle gyration with respect to the rf field as described below. The unharmonic oscillation may be understood as a super damped oscillation which is a consequence of a large flow of energy from the system of gyrating particles and the rf field to the outside, and vice versa. This phenomenon has been known as the cyclotron maser mechanism and utilized extensively in cyclotron maser amplifiers and/or oscillators to extract radiation of high power from gyrating electrons [11].

To understand the phase bunching mechanism, consider the case where particles of the same transverse energy $\gamma_{\perp}^*$ are initially randomly distributed in phase and are gyrating at the same frequency as the rf field. Some of the particles will gain energy and hence increase their mass. The phase of these particles will slip behind the rf wave since their gyro-frequency decreases. The particles which are initially in the opposite phase and lose energy will similarly advance in phase. After a certain number of gyrations depending on the initial conditions, most particles gyrate at similar phases. After some time, the phase debunching time $\tau^*$, the bunching action is interrupted for example when particles enter the space without rf field. This consideration can be generalized to the case where the particles have initially different transverse energies. More detailed explanations on the phase bunching mechanism are developed in the classical dynamical approach presented in the Appendix.
Figure 2: The response function $F(x)$ gives the normalized energy gain of a gyrating charged particle. As seen in the response function $F(x)$ for $a \gg 1$, the particles with the high energy, i.e. $x < 0$ or $\gamma_\perp > \omega_c^* / \omega_r^*$, emit coherently radiation while the particles with the low energy, i.e. $x > 0$ or $\gamma_\perp < \omega_c^* / \omega_r^*$, absorb coherently radiation through the stimulation of the rf field resulting in the CMC.

3 CMC IN THE TRANSVERSE ENERGY-TIME PHASE SPACE

Since the derivation so far is based on quantum mechanics, the behaviour of the phases of the particles can not be treated explicitly. Classically, the trapping of a DC beam can, however, be discussed by means of the synchrotron phase space. When the phase angles $\phi(t^*)$ of the gyrating particles are distributed uniformly with respect to the rf field and when the mismatching frequency $\Omega^* \equiv (\omega_c^* / \gamma_\perp) - \omega_r^*$ is smaller than the region of stability of synchrotron phase space, a significant portion of the particles will be trapped. For this case, the range of phase angles will be reduced by an amount which depends on the so called maser instability [12].

We have shown that stimulated coherent transitions are caused by the effect of the relativistic mass increase on the gyro-frequency leading to a bunching of the phases of the gyrating particles. Whenever the coherency condition,

$$a \gg 1,$$

is satisfied, the unharmonic oscillator term in Eq. (5) becomes predominant. The unharmonic oscillator term predicts the coherent absorptions for $x > 0$, i.e. $(\omega_c^* / \gamma_\perp^*) > \omega_r^*$, which is interpreted as that the coherently gyrating particles absorb the rf power to increase their mass so as to yield the resonance $\omega_c^* / \gamma_\perp^* = \omega_r^*$. While the coherent emissions are favored to yield the
resonance in the case $x < 0$. This feature is seen in the $F(x)$ curves with $a \gg 1$ in Fig. 2. This finding indicates the possibility of CMC, since the particles with the higher transverse energy i.e. $\gamma_\perp^* > \omega_c^* / \omega_{rf}^*$ undergo coherent emissions while the particles with the lower energy i.e. $\gamma_\perp^* < \omega_c^* / \omega_{rf}^*$ undergo coherent absorptions under the stimulation of the rf field.

Though the principle of CMC is based on the cyclotron maser mechanism, the kernel of the CMC problem is that how control the dynamics of gyrating particles but not to extract extensively radiations from the particles on the contrary to the cyclotron maser developments devoted so far. On the basis of this viewpoint, the following basic conditions are introduced for CMC.

1) At the CMC section, the solenoidal magnetic field is so uniform that its fractional non-uniformity is negligibly small compared to $2\omega_c \tau$ over the field space.

2) The rf cavity is uniformly long so that the unharmonicity of the rf field due to the edge effect can be neglected.

3) The power of stimulating rf field fed to the cavity is much stronger than that radiated or absorbed by the gyrating particles and thus its disturbance or heating can be completely disregarded.

Under these conditions, the CMC mechanism in the transverse energy can be investigated through a simple analytical treatment based on the transition rate formulae Eqs. (5) and (7).

In an experimental situation, the phase debunching time $\tau^*$ is not necessarily constant and determined generally by the gyration state of particles which is influenced by the intrabeam scattering, the irregularities of electromagnetic fields and so on. However, in the case of single pass cooling, the phase debunching time $\tau^*$ is basically the flight time $t^*$ in the rf field. Eqs. (4) and (6) are, as a consequence, replaced by the following equations:

$$a \equiv 2\omega_c t^*(1 - \gamma_\perp^{-1}) , \quad (9)$$

$$x \equiv 2t^* [(\omega_c^*/\gamma_\perp) - \omega_{rf}^*] . \quad (10)$$

The basic discussions of CMC are developed here mostly along the treatments in the previous CMC papers [7,8] which provide a very simple estimate of CMC speed. The rate formula, can be approximated as

$$\frac{d\gamma_\perp}{dt^*} = \frac{4\pi r_F^* (\omega_{rf}^*)^2}{\gamma_\perp^2 m_0 c^2} 2ax , \quad (11)$$

under the coherency condition, $a \gg 1$, and further the initial transverse energy distribution limited between the emission and the absorption maxima i.e. $-3^{-1/2} < x < 3^{-1/2}$ neglecting the second and higher order terms of $x$. The cooling time $\tau_\perp$ for the transverse energy is now defined in the laboratory frame by the logarithmic decrement as,

$$\tau_\perp \equiv \frac{t}{\ln [(\Delta \gamma_\perp)_0 / \Delta \gamma_\perp]} , \quad (12)$$

where $(\Delta \gamma_\perp)_0 m_0 c^2$ and $(\Delta \gamma_\perp)_0 m_0 c^2$ are the half width of transverse energy spread at the half maximum and its initial value, respectively. The value of $\tau_\perp$ is obtained by specifying the tuning conditions which are required for CMC. In the region, $-3^{-1/2} < x < 3^{-1/2}$, the mean value of the distribution shall be such that the gyro-frequency of particles of mean transverse energy $\gamma_\perp m_0 c^2$ is in coincidence with the rf frequency and the energy spread such that the corresponding $x$-values of the particles are kept in the range between the peak values of $F(x)$ in Eq. (5). We have thus $\omega_\perp^*/\sqrt{\gamma_\perp} = \omega_{rf}^*$, i.e.

$$\omega_\perp^*/\sqrt{\gamma_\perp} = (1 - \beta_k \beta_{rf}) \omega_{rf} , \quad (13)$$

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recalling $\omega_c = \omega_c \gamma_t$ and $\omega_r = \omega_r (1 - \beta_t \beta_r) \gamma_t$. Here, $c \beta_t$ denotes the propagation velocity of the rf field along the particle travelling axis. Also we have

$$\Delta \gamma_\perp / \tilde{\gamma}_\perp \sim \Delta \gamma_\perp / 2 \omega_c t.$$  \hfill (14)

Eq. (14) provides an estimate of coolable energy range. Around the resonance $x = 0$, $x$ may be approximated as

$$x \sim -(2 \omega_c t / \tilde{\gamma}_\perp)(\Delta \gamma_\perp / \tilde{\gamma}_\perp).$$  \hfill (15)

Using Eqs. (11), (14) and (15), the transition rate formula is now rewritten in the laboratory frame as,

$$\frac{d}{dt}(\Delta \gamma_\perp) = -\frac{4 \pi \tau_p I(\omega_r)}{\epsilon c B_0} \left( \frac{2 \omega_c t}{\tilde{\gamma}_\perp} \right)^3 \frac{(1 - \beta_t \beta_r)(\tilde{\gamma}_\perp - 1)}{\tilde{\gamma}_\perp^2} \Delta \gamma_\perp.$$  \hfill (16)

and we obtain,

$$\Delta \gamma_\perp = (\Delta \gamma_\perp)_0 \exp \left[-\frac{4 \pi \tau_p I(\omega_r)}{\epsilon c B_0} \left( \frac{2 \omega_c}{\tilde{\gamma}_\perp} \right)^3 \frac{(1 - \beta_t \beta_r)(\tilde{\gamma}_\perp - 1)}{\tilde{\gamma}_\perp^2} t^4 \right].$$  \hfill (17)

The cooling time $\tau_\perp$ defined in Eq. (12) becomes,

$$\tau_\perp \sim \frac{\epsilon c B_0}{\pi \tau_p I(\omega_r)} \left( \frac{\tilde{\gamma}_\perp}{2 \omega_c t} \right)^3 \frac{\tilde{\gamma}_\perp^2}{(1 - \beta_t \beta_r)(\tilde{\gamma}_\perp - 1)}.$$  \hfill (18)

It may be of particular importance to note that the cooling time $\tau_\perp$ is not affected by the relativistic time dilation, that is, independent of the longitudinal velocity of particles, except for the Doppler shift factor $(1 - \beta_t \beta_r)$. However, this factor will shorten the cooling time when the backward travelling rf wave is used, which may be the natural choice in high energy particle accelerators and storage rings since the frequency in the laboratory frame becomes lower and high power levels may more easily be achieved.

4 CMC IN THE LONGITUDINAL ENERGY

As seen in Fig. 1, a fraction of the longitudinal energy is changed to the transverse energy by introducing the particles in the CMC section at a small angle $\theta$ to the longitudinal axis of the section (s-axis in Fig. 1). Further, particle orbits of equimomentum are adjusted to be nearly parallel. Then the particles gyrate keeping a mean value of $\gamma_\perp$, where

$$\gamma_\perp = \gamma_\perp = \gamma - \gamma_\parallel + 1 \sim 1 + (1/2\gamma_\parallel) \left[ (\beta_t \gamma)^2 + (p_\perp/m_0c)^2 \right],$$  \hfill (19)

with $\gamma^2 = 1 + \beta^2 \gamma^2$, $\gamma_\perp^2 = 1 + \beta_t^2 \gamma^2$. Here $p_\perp$ denotes the intrinsic transverse momentum of the particles, that is, the transverse momentum outside the CMC section. The mean width of $\gamma_\perp$ is given by

$$\Delta \tilde{\gamma}_\perp = \left[ D' + (\theta/2)(1 + \gamma_\parallel^2) \right] \theta \Delta \gamma_\parallel + (1/2\gamma_\parallel)(p_\perp/m_0c)^2,$$  \hfill (20)

recalling that $\Delta \theta = D'(\beta_t \gamma)/\beta_\parallel \gamma$ and $\beta_\parallel \gamma \Delta(\beta_t \gamma) = \gamma_\parallel \Delta \gamma_\parallel$. Here $D'$ is the derivative of momentum dispersion of the ring or accelerator lattice at the entrance of the section. The positive sign of $D'$ is defined along the deflection of ange $\theta$. The peak value and the width of the transverse energy distribution can therefore be controlled practically by tuning the lattice elements.

In the practical cases $\gamma_\perp \gg \Delta \gamma_\perp$, the substantial part of $\gamma_\perp$ is transformed from $\gamma_\parallel$ implying that the first term of $\Delta \gamma_\parallel$ in $\Delta \gamma_\perp$ is much larger than the second term of $p_\perp$ in
Eq. (20). The change in the transverse energy, \( d\gamma_\perp /dt \), therefore reflects to the change in the longitudinal energy, \( d\gamma_\parallel /dt \), resulting in the cooling or heating in the longitudinal energy. Besides this an additional change in the longitudinal energy is resulted from the momentum conservation between the particle and the rf field along the longitudinal axis of the rf cavity,

\[
\frac{d(\beta_\parallel \gamma)}{d\gamma} = \beta_{rf} .
\]  

(21)

The momentum conservation, Eq. (21) leads to

\[
\frac{d\gamma_\parallel}{dt} = \frac{\beta_\parallel \beta_{rf}}{(\gamma_\parallel /\gamma) - \beta_\parallel \beta_{rf}} \frac{d\gamma_\perp}{dt} ,
\]  

(22)

recalling again \( \beta_\parallel \gamma \Delta(\beta_\parallel \gamma) = \gamma_\parallel \Delta \gamma_\parallel \) and \( \Delta \gamma = \Delta \gamma_\parallel + \Delta \gamma_\perp \). Summing the change in Eq. (22) and \( d\gamma_\perp /dt \) as a function of \( \gamma_\parallel \) in Eq. (19), we obtain

\[
\frac{d\gamma_\parallel}{dt} = \frac{(\gamma_\parallel /\gamma)}{(\gamma_\parallel /\gamma) - \beta_\parallel \beta_{rf}} \frac{d\gamma_\perp}{dt} .
\]  

(23)

If we replace the above \( d\gamma_\perp /dt \) by the right-hand side of Eq. (11) in the laboratory frame and use Eq. (20) taking out the \( \Delta \gamma_\parallel \) independent term, we have the associated cooling time for the longitudinal phase space,

\[
\tau_\parallel = \frac{(\gamma_\parallel /\gamma) - \beta_\parallel \beta_{rf}}{[D' + (\theta/2)(1 + \gamma_\parallel^{-2})] \theta_\gamma_\parallel /\gamma} \tau_\perp .
\]  

(24)

It is seen in Eq. (24) that the cooling in the transverse energy results in also the cooling in the longitudinal energy i.e. \( \tau_\parallel > 0 \) whenever

\[ |D'| < |\theta/2|(1 + \gamma_\parallel^{-2}) \quad \text{or} \quad D'\theta > 0 . \]  

(25)

5 PHASE LOCKING OF GYRATION

As seen in the previous sections, CMC depends critically on the phase debunching time \( \tau \) which affects the CMC speed and determines together with \( \omega_c \) and \( \gamma_\perp \), the coherency condition \( a \gg 1 \). High energy particles pass through a single CMC section of length \( L_0 \) in a short time \( \tau = L_0 / \beta_\parallel c \) which limits the number of gyrations and hence the coherent emission or absorption as compared to that of cyclotron maser devices such as a gyrotron in which electrons usually gyrate some 10 times. As an example, 2 GeV electrons traverse a CMC section of \( L_0 = 6.0 \) m and \( B_0 = 3 \) T in a time \( \tau = 20 \) ns with a gyro-frequency \( \omega_c /2\pi = 1.3 \times 10^8 \) and the value of \( 2\omega_c \tau = 5 \). This gives \( a < 2 \) for \( \gamma_\perp < 1.4 \) and hence the coherency condition \( a \gg 1 \) is not fulfilled. For ion beams, the situation is evidently worse and \( a = 0 \).

However, since a circulating particle passes the CMC section many times, the effective length over which CMC may act is much greater than \( L_0 \). Furthermore subdivided multiple CMC sections, in which gyration angles of the particles are very small, would be greatly useful in the practical cases in order to make the electrode gap of rf cavities very narrow. It is therefore essential to keep the bunched phases of gyrations unchanged in order to have an additive coherent emission or absorption of radiations during multiple traversals of the particles through the multiple CMC sections. The phase of the gyration can be controlled within a certain range, if the stimulating rf field is synchronized with the phase of the accelerating rf field which determines the synchrotron phase stability of the circulating particles. The condition of phase
locking of gyration will therefore be introduced and this concept is especially essential to the cooling of ion and high energy electron beams.

The condition of phase locking is derived by considering the change of the angle $\phi_0$ of the gyration during one particle revolution in the synchrotron or ring:

$$\phi_0 = \frac{1}{\beta_\parallel \gamma_\parallel c} \int \omega^*(r, s) ds = \frac{\omega_c^*}{\beta_\parallel \gamma_\parallel \gamma_\perp c B_0} \int B_\parallel(r, s) ds .$$

(26)

Here, $\omega^*(r, s)$ is the angular frequency of the gyration due to the longitudinal component of the magnetic field $B_\parallel(r, s)$. The integrals are the closed line integral along the reference particle orbit. If $\phi_0 = 2n\pi$ ($n = 0, 1, 2, \ldots$) is independent of orbits, the particle would recirculate in the ring without any change of gyration phase. The relativistic phase bunching due to $\gamma_\perp$ could therefore act in a repetitive fashion and yield the coherent emission or absorption of radiations in the CMC sections thereby affecting remarkably the transverse kinetic energy.

If the phase angle shifts by an amount $\Delta \phi_0$ per turn, the number of phase bunched recirculations is limited within $\pi/\Delta \phi_0$ resulting in an effective length for CMC of about $\pi L_0/\Delta \phi_0$. This determines the phase debunching time $\tau = \pi L_0/\beta_\parallel c \Delta \phi_0$ and hence $\gamma_\perp/2\omega_c \tau$ as

$$\frac{\gamma_\perp}{2\omega_c \tau} \sim \frac{\beta_\parallel \gamma_\parallel \gamma_\perp}{2 B_0 L_0} \frac{1}{\beta_\parallel \gamma_\parallel \gamma_\perp} \int B_\parallel(r, s) ds \Delta \left[ \frac{1}{\beta_\parallel \gamma_\parallel \gamma_\perp} \right] + \int \Delta B_\parallel(r, s) ds .$$

(27)

The first of the two terms in the large parentheses can be made to vanish independently of $\gamma_\parallel$ and $\gamma_\perp$ under the condition of phase locking:

$$\int B_\parallel(r, s) ds = 0 .$$

(28)

This can be achieved by introducing CMC sections consisting of a pair of solenoidal magnets with antisymmetric configuration and corresponding rf cavities as shown in Fig. 1. The second term in Eq. (27) takes into account variations in the fields and consists of those of solenoidal magnets

$$\int \Delta B_\parallel(r, s) ds = -\frac{1}{4} \frac{d}{ds} B_\parallel(0, s) r^2 + \ldots ,$$

(29)

and of lattice magnets,

$$\int \Delta B_\parallel(r, s) ds = B_\parallel(0, s) r + \frac{1}{6} \frac{d^2}{ds^2} B_\parallel(0, s) r^3 + \ldots .$$

(30)

However, these field integrals vanish for any symmetric equilibrium orbit. Hence any beam dynamical effects cannot cause the accumulation of these field integrals for a ring of symmetric lattice. This means that a ring of symmetric lattice does not break the phase bunching mechanism under the condition of phase locking.

Phase debunching may occur due to transverse impulses received by the particles in the fringing fields of solenoidal and lattice magnets. However, the impulses received at the entry and the exit of a magnet are oppositely directed and hence the perturbation is of second order. Moreover, the duration of one pass is small compared with the time period of the stimulating rf field $2\pi/\omega_\mathrm{rf}$ and this fact will reduce the perturbation further.

The particles may exchange longitudinal and transverse energies during their passage through a rippled magnetic field of averaged value $B_\mathrm{av}$ defined by the relation $eB_\mathrm{av}/m_0 =$
$2\pi \beta \gamma c/L_p$, $L_p$ being the periodicity of the field [13]. This nonadiabatic process has a resonance feature like a depolarizing resonance in the acceleration of spin-polarized particles and hence, if necessary, a correction can be applied to the accelerator working point.

There are many other phase debunching mechanisms such as beam dynamical effects caused by interactions between the beam particles and lattice elements including vacuum chambers of ring, intrabeam scattering, frequency band widths of the stimulating and synchrotron accelerating rf fields, phase mismatching between multiple CMC rf cavities and so on. Some of them may be useful to adjust the value of coolable energy range, $\Delta \gamma \perp /\gamma \perp$ given in Eq. (14). Detailed investigations on these matters are left for further studies and developments.

6 EXPERIMENTAL TEST OF CMC

The most crucial problems to be tested experimentally are: does CMC work under the CMC conditions on the resonance, Eq. (13), can particles be cooled in the energy range according to Eq. (14) and can the extraordinarily rapid cooling be realized as predicted by Eq. (18). These problems have, in fact, so far never been investigated. On the other hand the maser mechanism has been utilized to extract radiation of high power from gyrating electrons. Since CMC is, in principle, applicable to any kind of charged particles with arbitrary energy, experimental tests using low energy electrons will provide indications of CMC, from which one may deduce features of CMC applied to high energy electron and ion beams.

In Fig. 3 is shown the experimental arrangement employed to test CMC. An electron beam of about 1 µA and 10 keV is accelerated by a thermionic electron gun with an active domain of about 0.5 mm in diameter. The gun is located outside a solenoidal magnetic field. The electrons pass through dual sets of electrostatic horizontal and vertical deflectors towards an orifice of 40 mm in diameter in a 15 mm thick iron end plate attached to the cylindrical iron yoke at the entrance of the solenoidal coil 500 mm long and 200 mm in inner diameter. The dual deflector system is arranged so as to adjust the position and direction of the electron beam at the fringing region of the magnetic field. This serves to transform adiabatically any desired fraction of the total kinetic energy into transverse kinetic energy with respect to the solenoid axis. Computer calculations of magnetic field distribution and electron trajectories also verified that this way of generating a helical motion worked with full efficiency up to the maximum beam energy but at the sacrifice in the very broad longitudinal energy, i.e. energy associated with the longitudinal momentum. After passing through the orifice, the gyrating electrons drift along the magnetic field and pass through an rf cavity with rectangular cross section located at the center of the solenoidal coil. Though three series of experiments are carried out through at different cavity modes, here described are typical results obtained using a cavity which is 100 mm wide, 50 mm high and 200 mm long and resonant at 2.09 GHz in the TE_{102} mode with the quality factor $Q = 8000$. The rf power fed to the cavity, adjustable between 0.1 µW and 400 mW, is monitored by means of a spectrum analyzer connected to the cavity. Over the cavity space the magnetic field is axially homogeneous within $\pm 10^{-4}$. The gyrating electrons are collected at a conductive ZnS screen of 25.3 mm in diameter, which is placed just behind an rf grid ring attached to the end window of the rf cavity. The ZnS screen can be electrostatically biased to measure the longitudinal drift energy of the electrons. Following the cavity, a video camera equipped with a read out system is set to measure the intensity of the ring shaped trace on the ZnS screen. Evidence as to the effectiveness with which the fringing region of the magnetic field and the dual deflector system function is also provided by comparing the value of accelerating voltage and the transverse kinetic energies of electrons obtained from the Larmor radius of the rings. The entire apparatus consisting of steel, copper and aluminum is evacuated by a turbo-molecular pump to about $6 \times 10^{-5}$ Pa without any treatment such as baking out for desorption of gasses.
Electrons pass through the rf cavity of axial length $L_0 = 0.20$ m only once, which mostly determines the maximum permissible phase debunching time $L_0/c\beta_\parallel$ and hence $a$ as, 

$$a < (2\omega_c L_0/c\beta_\parallel)(1 - \gamma_\perp^{-1}) \ .$$  

The phase debunching time was estimated for the collision of electrons with the residual gas and also for effects of the non-uniformities of the rf field and solenoidal magnetic field. It was found to be longer as much as two orders of magnitudes than the transit time of electrons in the cavity and thus neglected in Eq. (31). In the cavity, the rf field of the $TE_{102}$ mode consists of two travelling waves of $\beta_{rf} = \pm 0.72$ propagating along the axis. The interaction of electrons with the rf field will thus be characterized by two different values of $\omega_{rf}^*$ and hence $\omega_{rf}^* / \omega_0$ due to the Doppler effect. The resulting two response functions $F(x)$ will have to be taken into account for analyzing the present experimental data based on electrons with extremely low longitudinal drift energy less than 100 eV. This consideration is, of course, unnecessary in the practical cases of high energy particles, since they interact with only one of the forward- or backward-travelling waves. In the present experiment using a short rf cavity, it is essential to reduce the drift energy of the electrons to realize a sufficient phase debunching time and hence a sufficiently large value of coherency factor $a$ for fully developing the CMC effect. Further, the low drift energy excludes strictly the possibility of the transfer of the longitudinal energy to the transverse energy.

To find the resonance, the magnetic field was swept around the value $B_0 = 0.075$ T which corresponds to the non-relativistic cyclotron resonance, $\omega_{rf} = \omega_c$. A very strong rf power loss was found at $\omega_{rf} = \omega_c$ regardless electron sources such as the electron gun and the ionization vacuum gauge. Most likely the rf power loss is caused by stray electrons emitted from the walls of the apparatus generating an avalanche driven by the rf field, so called multifactoring effect. Any CMC effect close to the resonance at $\omega_{rf} = \omega_c$ is therefore difficult to observe. The difficulty of resolving the CMC effect due to the absorption of stray electrons is related to the particular condition of present small scale experiment with low energy electron beam and a small drift velocity resulting in two overlapping response functions mentioned above. For instance, the response function $F(x)$ for 10 keV electrons is partially located in the region where the rf power is absorbed by the stray electrons resulting in an insufficient rf power for the CMC effect to take place in the coherent emission mode, $x < 0$. Throughout the present experiments, only the
CMC condition of coherent absorption mode,

\[ \frac{\omega_c}{(\gamma_\perp)_{\text{max}}} = (1 - \beta_1 \beta_\text{rf}) \omega_{\text{rf}}, \]

was therefore carefully investigated, \((\gamma_\perp)_{\text{max}}\) denoting the maximum value of \(\gamma_\perp\). In fact, it was found that the coherent absorption mode was much more clearly observed than the coherent emission mode due to less interference with the stray electron resonances.

7 OBSERVATION OF CMC – COMPRESSION OF ENERGY SPREAD

In spite of the problems outlined in the previous section, we have obtained some first inspiring indications of cooling of the transverse energy-time phase space.

In the experiments, the transverse kinetic energy ranged from below 10 keV to beyond 20 keV while the longitudinal drift energy was set at 50 – 300 eV. It is thus possible to deduce the transverse kinetic energy distribution precisely by measuring the broadening of the Larmor radii of gyrating electrons depicted on the ZnS screen assuming that the centers of gyration are remained fixed. The latter point was verified experimentally as explained below.

As a demonstration of CMC, the Larmor radii arising from heating and absorptive cooling of electrons are compared in Fig. 4 at a fixed rf power of 100 mV for the cases of initial transverse kinetic energies of 8, 10 and 12 keV. The cyclotron frequency \(\omega_c/2\pi = 2.16 \text{ GHz} (B_0 = 0.772 \text{ T})\) was fixed so that electrons of \((\gamma_\perp)_{\text{max}} = 1.04\) were resonant with the rf frequency \(\omega_{\text{rf}}/2\pi = 2.09 \text{ GHz}\). The longitudinal drift energy was adjusted to be in the range 50 – 300 eV independently of the initial transverse kinetic energy values. Because of this large spread of longitudinal drift energy, the electrons were expected to consist of groups, such as those of \(a \sim 1\) and \(a \gg 1\) depending on their drift velocity \(\beta_\parallel c\) (see Eq. (31)). The main feature of the pictures with rf on in Fig. 4, in which circular patterns are found, is the formation of discrete circles enclosing the broad band of circles.

For the electrons of \(a \sim 1\), only incoherent transitions are expected from discussions in Section 2 (see also Appendix), which are verified by the inner broad band of circles as seen in the pictures of rf on in Fig. 4. The broad band is generated by the rf field and expands with the increase of initial transverse kinetic energy. The mean radius is almost same as the initial Larmor radius with rf off. It is thus a crucial problem whether the formation of the broad band of circles is an effect of the possible oscillation of the centers of gyration or a result from axially symmetric broadening of the Larmor radius due to heating. In order to settle the problem, an experiment was performed. A copper baffle was inserted perpendicular to the transverse plane outside the cavity exit aperture. The baffle served to cut out a part of the gyrating electrons.

The electrons were shifted by means of the deflector system, laterally towards the copper baffle. If the broad circles were formed by electrons gyrating about different centers, the successive cutting of the outer periphery would have resulted in visible effects also at the inner periphery. By cutting away so much of the broad circle that the remaining width was less than the width of the circle observed without the rf field applied, it was confirmed that the oscillations of centers caused by the rf field were negligible. The rf field thus accelerated or decelerated the electrons transversely without any marked change of the center of curvature. The feature should be associated with the broad band of circles, where the phases of gyration are homogeneously distributed with respect to the rf field.

The sharp outer circle in Fig. 4 corresponds to a transverse kinetic energy of 23 keV and its radius is independent of the initial transverse kinetic energy. This energy coincides with the transverse energy at the resonance, \(z = 0\), for a longitudinal drift energy around 50 eV. If the
Figure 4: Comparison of the Larmor radii arising from *heating* and *absorptive cooling* of electrons. The patterns depicted on the ZnS screen for rf on and rf off at $\omega_c/2\pi = 2.16$ GHz and $\omega_H/2\pi = 2.09$ GHz for the initial transverse kinetic energies: (a) 8 keV, (b) 10 keV, (c) 12 keV. The sharp outer circles correspond to the electron energy of 23 keV, which were observed without accompanied by the broad band of inner circles when the longitudinal drift energy spread was reduced.
pictures for the three cases in Fig. 4 are superimposed, under the assumption that electrons were strictly governed by the rf field only, it can be visualized how three initial sets of electrons, 8, 10 and 12 keV will reach a final energy 23 keV. Hence, an energy spread of 4 keV has been eliminated as far as these electrons concerned. The above assumption is reasonable for electrons undergoing coherent emission or absorption of radiation since any spontaneous or incoherent effects such as intrabeam scattering effect can be neglected. From the reduction of the energy spread, \( \Delta \gamma_L/(\Delta \gamma_L)_0 < 0.1 \), the cooling time \( \tau_L \) was estimated to be \( \tau_L < L_0/\beta_\parallel c \ln 10 \approx 20 \text{ ns} \) using the transit time of electrons in the cavity \( L_0/\beta_\parallel c = 50 \text{ ns} \). The value is consistent with the value \( \tau_L = 10 \text{ ns} \) predicted from Eq. (18) using the values \( B_0 = 0.077 \text{T} \), \( I(\omega_{rf}) = 2 \times 10^4 \text{ Wm}^{-2} \), \( \gamma_L = 1.04 \), \( \beta_\parallel = 0.014 \) and \( \delta_{rf} = -0.72 \). The value of \( I(\omega_{rf}) \) was obtained from the volume \( 10^{-3} \text{m}^3 \) and \( Q = 8000 \) of the cavity at a power of 100 mW.

The simultaneous appearance of broad inner band of circles and sharp outer circle is due to the extremely large fractional spread of longitudinal drift energy. In fact, the broad circle and the sharp circle were separately observed in the other experiments by reducing the spread of longitudinal energy. This indicates that the cooling and heating of electrons are controllable. It is however very curious that the electrons with a broad band of longitudinal energies split clearly into two groups, one of heating and one of cooling.

Once electrons with \( a > 1 \) gain their transverse kinetic energy through the coherent absorption of rf energy, the coherency factor \( a \) increases drastically with time together with the gain of the transverse kinetic energy. The increase of \( a \) is further intensified by the decrease of drift velocity \( \beta_\parallel c \) due to the recoil effect in the coherent absorption of backward travelling rf wave energy. The amount of the recoil energy, \( \Delta T_\parallel = \beta_\parallel \delta_{rf} \Delta T_L \) is of the same order as the longitudinal energy \( T_L \). For instance, \( \Delta T_\parallel = 70 \text{ eV} \) for a gain of transverse energy \( \Delta T_L = 5 \text{ keV} \). For these electrons the effective value of the coherency factor was estimated to be \( a > 15 \). Then the electrons reach the resonance point \( x = 0 \) instantly. On the contrary, for electrons with higher drift energy, \( T_\parallel > 100 \text{ eV} \) and hence \( a = 1 \), their transverse kinetic energy is almost unchanged due to their incoherent interaction with the rf field losing a chance of coherent absorption of stimulating rf energy.

### 8 OBSERVATION OF CMC – COMPRESSION OF PHASE ANGLE SPREAD

The CMC principle so far discussed has been based on the mechanism of bunching of phases of gyrating particles with respect to the applied rf field caused by the relativistic effects. However, in the experiments, the indication of phase bunching was obtained only indirectly through the drastic energy gain associated with the outer circles in Fig. 4. In fact, there was no way of explaining the energy gain and the sharpness of the outer circles other than assuming that the electrons are in resonance at a defined phase with the rf field. However, the arc or spot pattern was not observed on the ZnS screen. This was again caused by the very broad spectrum of longitudinal energies. Further, this rounding off effect on the patterns was enhanced by the recoil effect due to the radiation absorbed coherently by the electrons as described in the previous section.

The indication of phase bunching in CMC was provided by another experiment in which the sharp outer circle was not necessarily concentric with the broad band of inner circles. When the initial center of gyration was shifted away from the central axis of rf cavity, the center of the sharp circle shifted towards the central axis of cavity, where the strength of rf field was maximum, while the broad band of inner circle was always concentric with the initially shifted center of gyration. This effect can not be explained unless the electrons forming the sharp circle are assumed to have a certain degree of phase correlation with respect to the rf field,
RF OFF

RF ON

Figure 5: Indication of phase bunching for the cooled electrons of sharp circle. When the initial center of circle is shifted away from the central axis of the rf cavity, the sharp outer circle with rf on picture shifts back towards the central axis where the strength of the rf field is maximum. The broad band of inner circle is symmetric with the initial center of rf off circle. The shift tends to be larger for the further away from the central axis of the rf cavity. The electron beam is prepared with a high transverse to longitudinal kinetic energy ratio. The total energy 9 keV, cathode current 0.5 µA, rf field of 2.091 GHz and 200 mW, cyclotron frequency 2.165 GHz. (The pictures are displayed without inversion.)

since electrons with phase matched gyration of absorption mode are always pulled towards the maximum field area. In Fig. 5, the magnetic field is directed out of the paper and the electrons gyrate in the counter clockwise direction. The axis of the solenoid as well as the central axis of the cavity crosses the ZnS screen slightly to the left of the top of the rf off circle. The electric field vector of the rf field is in the vertical direction. When the rf field is on, the broad band of circles is always concentric with the initial rf off circle. The sharp circle is however laterally shifted as seen in Fig. 5. The shift tends to be larger for the further away from the central axis of cavity.

9 SUMMARY

A new principle for the cooling of charged particle beams has been presented. The formalism for this coherent microwave cooling (CMC) was derived in a quantum mechanical oscillator approach and in a classical dynamical approach (Appendix). Both approaches lead to the same results. It was shown that both the energy cooling and the phase bunching of particles are caused by the relativistic effects on the particle gyro-frequency resulting in the compression in the energy-time phase space of gyrating particles.
The new principle has been tested experimentally. The experimental results provided some inspiring indications of the cooling of the transverse energy-time phase space of electrons. Dramatically rapid cooling, < 20 ns as predicted, was observed for the transverse energy. Electrons initially in the range 8 - 12 keV, were all found to accumulate at one discrete energy, 23 keV. A phase bunching effect was observed indirectly in the gyration of the accumulated electrons of 23 keV energy.

The agreement between the prediction and the experimental results implies that in principle any kind of charged particle beams can be cooled in a single pass through the CMC sections regardless of their energy.

10 ACKNOWLEDGEMENT

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APPENDIX

MECHANISM OF CYCLOTRON MASER
- PARTICLE DYNAMICAL APPROACH

We start from the equation of motion and rewrite it in the instantaneous transverse energy transition rate for a particle as,

\[
\frac{d\gamma_\perp}{dt^*} = \frac{e\beta_\perp E_r^*}{m_0c} \cos \phi(t^*) ,
\quad \text{with} \quad \begin{cases} 
\phi(t^*) = \Omega^* t^* + \phi_0 , \\
\Omega^* = (\omega_c^* / \gamma_\perp) - \omega_r^* .
\end{cases}
\] (33)

\[\phi(t^*), \phi_0 \text{ and } \Omega^* \text{ being the phase, the initial phase and the mismatching frequency of particle gyration, respectively, with respect to the rf field.}\]

If the energy of the system of gyrating particles and the rf field were conserved, the speed \(\beta_\perp c\) and the frequency \(\omega_c^*/\gamma_\perp\) are constant on average. Eq. (33) gives only the fluctuation in the transition rate occurring during the course of a single period of gyration and thus the state of gyration is stable. That is to say, nothing happens in the energy-time phase space of gyrating particles yielding a typical Liouvillean system. However, according to the electromagnetic theory, gyrating charged particles generally emit radiation associated with their acceleration or deceleration resulting in the loss of energy of the system. Such radiation loss introduces kind of frictional forces, such as the speed dependent – and the gyro-frequency dependent – forces yielding a non-Liouvillean system of unharmonic gyration. These frictional forces cause the radiation loss of the system in proportion to \(d\beta_\perp\) and \(d(\omega_c^*/\gamma_\perp)\) as,

\[
\frac{d}{dt^*} \left( \frac{d\gamma_\perp}{dt^*} \right) = \frac{d}{dt^*} \left( \frac{d\gamma_\perp}{dt^*} \right)_1 + \frac{d}{dt^*} \left( \frac{d\gamma_\perp}{dt^*} \right)_2 ,
\] (34)
with
\[ d \left( \frac{d \gamma_{\perp}}{dt^*} \right)_1 = \frac{\partial}{\partial \beta_{\perp}} \left( \frac{d \gamma_{\perp}}{dt^*} \right) d \beta_{\perp} , \]  
(35)

\[ d \left( \frac{d \gamma_{\perp}}{dt^*} \right)_2 = \frac{\partial}{\partial (\omega_{\perp}^* / \gamma_{\perp})} \left( \frac{d \gamma_{\perp}}{dt^*} \right) d \left( \frac{\omega_{\perp}^*}{\gamma_{\perp}} \right) , \]  
(36)

where both \( d \beta_{\perp} \) and \( d (\omega_{\perp}^* / \gamma_{\perp}) \) are functions of \( t^* \) through Eq. (33) and hence,
\[ d \beta_{\perp} = \frac{\partial}{\partial \gamma_{\perp}} d \gamma_{\perp} = \frac{1}{\beta_{\perp} \gamma_{\perp}^3} \frac{d \gamma_{\perp}}{dt^*} dt^* = \frac{eE_{\perp}^*}{m_0 c} \frac{1}{\gamma_{\perp}^3} \frac{d \gamma_{\perp}}{dt^*} \cos(\Omega^* t^* + \phi_0) dt^* , \]  
(37)

\[ d \left( \frac{\omega_{\perp}^*}{\gamma_{\perp}} \right) = \frac{\partial}{\partial \gamma_{\perp}} \left( \frac{\omega_{\perp}^*}{\gamma_{\perp}} \right) d \gamma_{\perp} = - \frac{\omega_{\perp}^* \frac{d \gamma_{\perp}}{dt^*}}{\gamma_{\perp}^2} dt^* = - \frac{eE_{\perp}^* \beta_{\perp} \omega_{\perp}^*}{m_0 c} \frac{1}{\gamma_{\perp}^2} \cos(\Omega^* t^* + \phi_0) dt^* . \]  
(38)

Here, the factor \( d \gamma_{\perp} / dt^* \) in Eqs. (37) and (38) has been replaced by the right-hand side of Eq. (33). Substituting Eqs. (37) and (38) into Eqs. (35) and (36), respectively, we obtain
\[ d \left( \frac{d \gamma_{\perp}}{dt^*} \right)_1 = \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 \gamma_{\perp}^{-1} \cos(2\Omega^* t^*) dt^* , \]  
(39)

\[ d \left( \frac{d \gamma_{\perp}}{dt^*} \right)_2 = \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 (1 + \gamma_{\perp}^{-1}) \omega_{\perp}^* t^*(1 - \gamma_{\perp}^{-1}) \sin(2\Omega^* t^*) dt^* . \]  
(40)

The energy loss in Eqs. (39) and (40) is the sum of energy of radiation induced by the rf field. The induced radiation has the same phase with the rf field which means \( \phi(t^*) = 0 \) at the resonance \( \Omega^* = 0 \). The initial phase \( \phi_0 \) has therefore been dropped in Eqs. (39) and (40). Due to the presence of the frictional forces, the gyration of particles is not stable but damps with the mean life \( \tau^* \), so called phase debunching time. For the particles of damped gyration, the transition rate due to the radiation loss can be obtained by means of Laplace transformation.

\[ \frac{d \gamma_{\perp}}{dt^*} = \int_{t^*=0}^{\infty} e^{-t^*/\tau^*} d \left( \frac{d \gamma_{\perp}}{dt^*} \right)_1 + \left( \frac{d \gamma_{\perp}}{dt^*} \right)_2 , \]  
(41)

with
\[ \left( \frac{d \gamma_{\perp}}{dt^*} \right)_1 = \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 \gamma_{\perp}^{-1} \frac{\tau^*}{1 + x^2} \approx \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 \frac{\tau^*}{1 + x^2} , \]  
(42)

\[ \left( \frac{d \gamma_{\perp}}{dt^*} \right)_2 = \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 (1 + \gamma_{\perp}^{-1}) \frac{\tau^* a z}{(1 + x^2)^2} \approx \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 \frac{2\tau^* a z}{(1 + x^2)^2} , \]  
(43)

where
\[ a \equiv 2\omega_{\perp}^* (1 - \gamma_{\perp}^{-1}) \]  
(44)

\[ z \equiv 2\Omega^* \tau^* \]  
(45)

The rate formula, Eq. (41), is now represented as,
\[ \frac{d \gamma_{\perp}}{dt^*} = \frac{1}{2} \left( \frac{eE_{\perp}^*}{\gamma_{\perp} m_0 c} \right)^2 \tau^* F(x) = \frac{4\pi r_p}{\gamma_{\perp} m_0 c} \Omega^* \tau^* F(x) , \]  
(46)
with
\[ F(z) = \frac{1}{1 + z^2} + \frac{2ax}{(1 + z^2)^2}. \]

The physical implication behind the derivation of Eqs. (41-47) is now found to be the extraction of the non-Liouvillean components caused by the stimulated radiation through elimination of the Liouvillean components, i.e. the fluctuation part from Eq. (33).

Comparing Eqs. (7) and (46), it is seen that both the particle dynamical approach and the oscillator approach lead to the same result. Though both approaches have so far been contrasted in the last three decade discussions on the cyclotron maser, nevertheless they are essentially identical and further they need not actually any assumptions such as the very weakly relativistic. However, the physics implicated behind the two different approaches is of special importance to note. As seen in the particle dynamical approach, the transition rate of unharmonic quadrupole oscillator term, Eq. (43), is derived as the first order effect as well as that of damped dipole oscillator term, Eq. (42). Therefore, the quadrupole oscillator term is, of course, not a second order effect, which has so far been misinterpreted after Schneider's theory based on the very weakly relativistic approximation [11].

The second order effects due to the second order derivatives of \( \beta_\perp \) and \( \omega^*_\perp / \gamma_\perp \) are small compared with the first order effects as much as the factor \( (\omega^*_\perp / \gamma_\perp - \omega^*_\parallel) / \omega^*_\parallel \) being negligibly small at the resonance. Another ones yield from the coupled two first order derivatives of \( \beta_\perp \) and \( \omega^*_\parallel / \gamma_\parallel \) which are, however, smaller than the first order effects by the factor of \( (eE_\parallel^* t^*/2\gamma_\parallel m_0 c) \) and thus practically again negligible.

MECHANISM OF PHASE BUNCHING OF GYRATION
- PARTICLE DYNAMICAL APPROACH

We are ready to elucidate the mechanism of the phase bunching of particle gyration caused by relativistic effects. Due to the relativistic effects, the phase \( \phi(t^*) \) is not a linear function of \( t^* \). Recalling Eqs. (38) and (33), we obtain the derivative of \( \cos \phi(t^*) \) as,
\[ d [\cos \phi(t^*)] = -\sin \phi(t^*) \frac{\partial \phi(t^*)}{\partial t^*} \frac{d t^*}{d t^*} = \frac{e\beta_\perp E_\parallel^* \omega^*_\perp t^*}{2\gamma_\parallel m_0 c} \sin(2\Omega^* t^* + 2\phi_0) dt^*. \]

When particles with the random distribution of initial phase \( \phi_0 \) are subjected to the rf field during the phase debunching time \( t^* \), the average value of \( \cos \phi(t^*) \) is found through Laplace transformation. Recalling the discussion on the phase of induced radiation regarding Eqs. (39) and (40), we have,
\[ \overline{\cos \phi(t^*)} = \int_{t^*=0}^{\infty} e^{-t^*/ \tau^*} d [\cos \phi(t^*)] = \left( \frac{eE_\parallel^* \beta_\perp \epsilon \tau^*}{\gamma_\parallel m_0 c^2} \right) \left( \frac{\omega^*_\perp t^*}{\gamma_\parallel} \right) \frac{2\Omega^* t^*}{[1 + (2\Omega^* t^*)^2]^2}. \]

In the right-hand side of Eq. (49), the numerator in the first parenthesis denotes the energy transferred through the rf field to a particle during the phase debunching time \( \tau^* \) and the second parenthesis is the gyration angle of the particle. Along the discussion in Section 3, \( \tau^* \) is replaced by \( t^* \) for the case of single pass cooling and hence Eq. (49) may be replaced by
\[ \overline{\cos \phi(t^*)} \approx \left( \frac{eE_\parallel^* a t^*}{\beta_\perp \gamma_\parallel m_0 c} \right) \frac{2\Omega^* t^*}{[1 + (2\Omega^* t^*)^2]^2}. \]

In Fig. 6 are shown phase diagrams, \( |\cos \phi(t^*)| \) vs \( 2|\Omega^*| t^* \). Here, \( \Omega^* > 0 \) and \( \Omega^* < 0 \) correspond to the cases of absorption and emission, respectively. The curve a) corresponds to the case \( (eE_\parallel^* a t^* / \beta_\perp \gamma_\parallel m_0 c) \gg 1 \) at \( 2|\Omega^*| t^* = 3^{-1/2} \). Along the curve, a series of arrows indicates how the averaged value of \( \cos \phi(t^*) \) changes with \( \Omega^* t^* \). At \( t^* = 0 \), the phases of
Figure 6: Phase diagram: $|\cos \phi(t^*)|$ vs $2|\Omega^*|t^*$. Here the absorption and emission correspond to $\Omega^* > 0$ ($\cos \phi > 0$) and $\Omega^* < 0$ ($\cos \phi < 0$), respectively. The curve a) corresponds to the case $(eE_0^*at^*/\beta_1 \gamma_1 m_0 c) \gg 1$ at $2|\Omega^*|t^* = 3^{-1/2}$. At $t^* = 0$, the phases of particle gyration distribute at random and hence $\cos \phi(t^*) = 0$. However, the phase bunching caused by the relativistic effect grows up with time $t^*$ towards the phase matching point $M$ where $|\cos \phi(t^*)| \sim 1$. Then the mismatching frequency $|\Omega^*|$ falls off towards the resonance point $O$ where $\Omega^* = (\omega_c^*/\gamma_1) - \omega_m^* = 0$. At this $M \rightarrow O$ process, the bunched phase shifts such that $|\cos \phi(t^*)| \rightarrow 0$. If the value of $(eE_0^*at^*/\beta_1 \gamma_1 m_0 c)$ is not sufficiently large, the phase bunching cannot be fully developed missing both the matching point $M$ and the resonance $\Omega^* = 0$ as seen in the curve b). In this case, $\cos \phi(t^*)$ vanishes finally resulting in the random phase gyration.

particle gyration distribute at random and hence $\cos \phi(t^*) = 0$. However, the phase bunching caused by the relativistic effects grows up with flight time $t^*$ towards the phase matching point $M$ where $|\cos \phi(t^*)| \sim 1$. At the matching point, the cooling action is maximum resulting in the drastic change of $\Omega^*$ towards the resonance point $O$ where $\Omega^* = 0$. In the process $M \rightarrow O$, the bunched phase shifts such that $|\cos \phi(t^*)| = 1 \rightarrow 0$.

In Fig. 6, the curve b) is another phase diagram where the value of $(eE_0^*at^*/\beta_1 \gamma_1 m_0 c)$ is not sufficiently large at $2|\Omega^*|t^* = 3^{-1/2}$ so that the phase bunching can not be fully developed missing both the matching point $M$ and the resonance $\Omega^* = 0$. In this case, the value of $\cos \phi(t^*)$ vanishes finally going back to the random phase gyration.
### Table of Radiation of Gyrating Charged Particles

<table>
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<td>Cyclotron Radiation Incoherent emission (P \propto n) (G = 1)</td>
<td>Bunched Cyclotron Radiation Coherent emission (P \propto n^2) (G = n)</td>
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<tr>
<td>Stimulated radiation ((E_{rf} \neq 0))</td>
<td>Damped Harmonic Oscillator Emission or absorption in phase with rf field (P \propto nE_{rf}^2) (G = N_{\text{photon}})</td>
<td>Super-damped Oscillator Emission or absorption in phase with rf field (P \propto aN_{\text{photon}}E_{rf}^2) (G = aN_{\text{photon}})</td>
</tr>
</tbody>
</table>

Table 1: Radiation of gyrating charged particles. \(E_{rf}\): Electric field amplitude of stimulating rf field, \(P\): Power of radiation, \(G\): Enhancement of radiation rate, \(n\): Number of gyrating charged particles, \(N_{\text{photon}}\): Number density of photons of stimulating rf field, \(a \equiv 2\omega_c\tau(1 - \gamma^{-1})\): Coherency factor \((a > 10\) for CMC).
REFERENCES

  (1978) 277];


  Physics Course, Uppsala, Sweden, 18-29 september 1989.)


  (Proc. Second International Conference on Particle Production near Threshold, (Opening
  Talk), ed. C. Ekström).


  1952.

  J. Schneider, Phys. Rev. Lett. 2 (1959) 504;
  A.V. Gaponov, Izv. VUSOV. Radiofiz. 2 (1959) 450; 2 (1959) 837;
