Particle Simulation of Cyclotron Maser Cooling Experiment

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Abstract
Cyclotron Maser Cooling has been proposed as a process by which the width of the energy distribution of a beam of particles can be significantly reduced\(^1\). A numerical simulation of the experiment demonstrating this process has been performed using classical equations to propagate the electron beam across the resonant cavity described in [1]. Results show that for conditions described, the beam/wave interaction results in a spreading of the beam in energy. Only for either a modulated or correctly phased beam does a decrease in energy spread occur.

Simulation
The experimental setup is shown in Fig. 1. An axis-circling electron beam with \(E_{\|} \approx 50\)eV and \(8\)keV < \(E_\perp < 12\)keV is injected into a rectangular resonant cavity operating in the \(TE_{102}\) mode and fed with \(-100\)mW of rf power at \(f_{res} \approx 2.12\)GHz. Beam and cavity characteristics are shown in the figure. The beam is reported to emerge from the cavity with a significant decrease in the perpendicular energy spread accompanied by an overall increase in the perpendicular energy, i.e. \(E_\perp = 22\)keV and \(\Delta E_\perp \approx 0\)keV.

Figure 1 Experimental setup.

The above experiment has been simulated numerically by injecting a beam of particles whose energy is either fixed at a constant value or distributed as defined in the above figure. The simulation is run for a rf field defined in two ways: a) forward traveling wave - as defined in \([1]\) b) standing wave - a more realistic structure for a cavity field. The wave fields are defined for the \(TE_{102}\) mode as:

\[
E_y = E_0 \sin k_x x \ e^{-i \omega t} \times e^{i k_z z} \text{ traveling wave}
\]

\[
H_x = E_0 \frac{k_x}{\mu \omega} \sin k_x x \ e^{-i \omega t} \times -e^{i k_z z} \text{ traveling wave}
\]

\[
H_z = E_0 \frac{i k_x}{\mu \omega} \cos k_x x \ e^{-i \omega t} \times -e^{i k_z z} \text{ traveling wave}
\]

\[
E_x = E_z = H_y = 0.
\]

and \(k_x = \pi/a, k_z = 2\pi/L\). The above constant \(E_0\) is determined from an energy balance in the cavity using \(Q = \omega W_{stl}/P_{rf}\) where \(\omega = \text{excitation frequency}\) and

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\[ W_{st} = stored \ energy = \int \frac{L}{2} \varepsilon_0 E^2 \, dV + \int \frac{L}{2} \mu_0 H^2 \, dV \]
\[ = \varepsilon_0 E_0^2 \ abL \left( 1 + \frac{k^2_x + k^2_y}{\mu_0 \varepsilon_0 \omega^2} \right) \times \]
\[ 1/8 \ \text{traveling wave} \]
\[ 1/16 \ \text{standing wave} \]

Using Eqs. 1-2, \( E_0 \) is determined to be \( E_0 = 3.7 \times 10^3 \text{V/m} \) for the traveling wave case and \( E_0 = 7.4 \times 10^3 \text{V/m} \) for the standing wave case. At the input to the cavity the particles appear in the position and momentum planes as shown in Figs. 2. The phase of the injected beam relative to the rf field is random as inferred from the experiment. The classical equations of motion are used to propagate the beam through the rf field of the cavity - \( F = dp/dt = qE + \nu \times B, \) \( p = \gamma m \nu. \) The particle simulation is stopped when the last of the particles reaches the far end at \( z = L. \) It should be noted that effects of beam space charge are not included in the model. However, the experiment operates at such small currents (\( I_{beam} = 1 \mu \text{A} \)) that any space charge effects may be considered negligible (\( \alpha^2_p/\alpha^2_c < 1 \) [2]).

A run for a typical set of parameters is shown in Fig. 3. In this case all particles are injected with equal energy, \( E_{\perp} = 10 \text{keV}, \) \( E_{\parallel} = 50 \text{eV}. \) The relativistic gyro frequency is set slightly below the wave frequency, \( B = 0.0768T \) \( (\Delta = (2/\beta_z)(1-qB/\gamma m \omega) = 0.36). \) The figure shows the time evolution of the particles as they traverse the cavity. One can see that the initial phase quickly determines the path that the particles follow. The paths become straight lines when the particles reach the end of the cavity where they are viewed on the fluorescent screen. The passage through the cavity, the particles acquire a spread in longitudinal velocity due to the Lorentz force resulting in different transit times for particles of different initial phase. The run shown in Fig. 3(a) is typical of all runs performed for both the traveling and standing wave case. The mean energy of the distribution can either increase or decrease depending on the detuning between the wave frequency and the particles relativistic gyro frequency. There is though always a significant increase in the energy spread. For the case shown in Fig. 3(a), the spread becomes more than 20% of the final energy.

Figure 3 (b) and (c) shows the particles of Fig. 3(a) in the \( x-y \) and \( p_x-p_y \) plane at the end of the simulation. It is clear that, though the mean energy of the beam has increased, the spread in energy results in wide "rings" in the position and momentum planes of the beam. For this choice of parameters, "cooling" in the \( p_x-p_y \) momentum plane has not occurred.

Simulations similar to those of Figs. 3 are performed for the entire range of magnetic field, or detuning, values of interest. The mean energy change and energy spread is plotted in Figs. 4 as a function of magnetic field for both the forward traveling and standing wave fields. As is seen in both cases, the mean beam energy can be increased or decreased, given the correct magnetic field. However, the final energy spread is always greater than 10% for zero initial energy spread, and increases to as high as 40%, for any beam energy change of more than a few percent. The possibility therefore of injecting a random-phase beam with an initial energy spread and reducing this spread by the method described here seems unlikely.

Contrary to the experiment described in [1], if one could fix the phase of the injected beam with respect to the wave, the reduction in the energy spread of the beam may be possible. An example is shown in Figs. 5 where a distribution of particles with energies \( E_0 - 50 \text{eV} \) and \( 8 \text{keV} < E_{\perp} < 12 \text{keV} \) is injected into the cavity of Fig. 1. As seen by the time trajectories of the particles in Fig. 5(a), a reduction in the width of the energy distribution occurs as all particles, regardless of their initial energy state, reach the cavity end at a near constant energy. This though is only possible if the initial phase of all particles in the distribution is fixed at the same value. Figures 5(a) and (b) shows the required initial state of the particles to achieve this effect. At the cavity exit, the energy distribution is seen to be narrower, though the particles have now spread out over the entire \( 2\pi \) radians of phase.

**Conclusions**

A particle simulation of the cyclotron maser cooling effect has shown that the electron beam/wave interaction can result in either a decrease or increase in the mean energy of the injected beam. The energy spread induced however as a result of the interaction, is seen to be unacceptably large, varying between 10% and 40%, depending on the detuning between the electron gyro frequency and the wave frequency. These results are in contradistinction to those presented in [1] and cannot
explain the observed "cooling" of the electron beam in the $p_x$-$p_y$ momentum plane shown in the experiment. Contrarily, if the phase of the injected beam can be controlled, the simulation shows that a reduction in the energy spread of the injected electron beam can be attained. This requires a constant particle input phase and though the interaction results in a reduction of the energy spread it also results in a spreading of the beam to all phase values.

References
1. H. Ikegami, this conference, and references contained therein.

Figure 2. Initial phase space distribution of injected beam. Beam parameters are $E_{\parallel} = 50eV$ and $E_{\perp} = 10keV$
(a) $x$-$y$ plane (b) $p_x$-$p_y$ plane.

Figure 3. (a) time trajectories of beam during passage through resonant cavity. Simulation begins with 50 injected particles distributed randomly in phase. Beam parameters: $E_{\parallel} = 50eV$ and $E_{\perp} = 10keV$  (b) (c) phase space distribution of particles at exit of cavity.
Figure 4. Mean energy of distribution and energy spread of beam at exit of resonant cavity vs. magnetic field for (a) forward traveling rf wave field (b) standing rf wave field.

Figure 5. (a) (b) Initial phase space distribution of injected beam at constant phase (c) time trajectories of beam during passage through resonant cavity. Simulation begins with 30 injected particles. Beam parameters: traveling wave with $B = 0.0768T$, $E_{\parallel} = 50eV$ and $5keV < E_{\perp} < 12keV$ (d) (e) phase space distribution of particles at exit of cavity.